

# Centres of mass

## 1 Centre of mass of a discrete mass distribution

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i}$$

### 1.1 Example

Mass	2	3	2
$x$	2	3	-3
$y$	3	6	2

$$\bar{x} = \frac{2 \times 2 + 3 \times 3 + 2 \times -3}{2 + 3 + 2} = 1$$

$$\bar{y} = \frac{2 \times 3 + 3 \times 6 + 2 \times 2}{2 + 3 + 2} = 4$$

## 2 Uniform laminae

For a triangular lamina the centre of mass is  $\frac{2}{3}$  along the line from the vertex to the middle of the line opposite.

If given the coordinates of the vertices of a triangle, the centre of mass is the has coordinates of the mean of the x coordinates, and the mean of the y coordinates.

For a sector of a circle, radius  $r$ , where the angle at the centre is  $2\alpha$  the centre of mass is  $\frac{2r \sin \alpha}{3\alpha}$

## 3 Rods

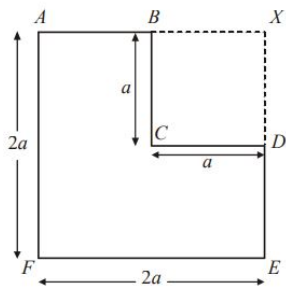
In a circular arc, radius  $r$ , where the angle at the centre is  $2\alpha$ , the centre of mass is  $\frac{r \sin \alpha}{\alpha}$  away from the centre.

## 4 Equilibrium

To avoid tipping, the line of action of the weight must be within the side of the lamina in contact with the plane. If a lamina is suspended from a fixed point, the centre of mass will be vertically below the point of suspension. Assumptions made in equilibrium calculations:

- No friction at the point of suspension
- The mass of each area is uniform
- The mass is uniform at the join

## Centres of mass example - Equilibrium problems



(a) Find the distance of the centre of mass of the lamina from AF.

**1. Find the masses and locations of all areas or rods and put in a table**

Mass(m)	$4a^2$	$-a^2$
Distance From AF(x)	$a$	$\frac{3}{2}a$
mx	$4a^3$	$-\frac{3}{2}a^3$

**2. Use the formula to find the centre of mass**

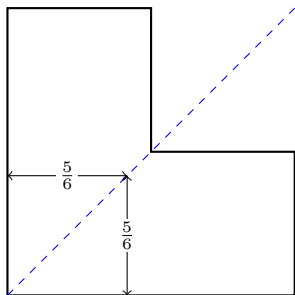
$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\bar{x} = \frac{\frac{5}{2}a^3}{3a^2} = \frac{5a}{6}$$

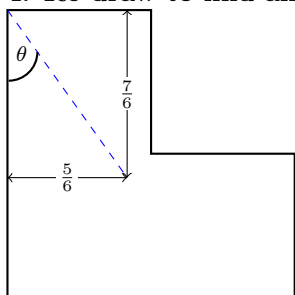
The lamina is freely suspended from A and hangs in equilibrium.

(b) Find, in degrees to one decimal place, the angle which AF makes with the vertical.

**3. Use symmetry to find the y coordinate of the centre of mass**



**4. Re-draw to find angle**



$$\tan(\theta) = \frac{5/6}{7/6}$$

$$\theta = 35.5^\circ$$