Estimation, confidence intervals and tests

1 Estimators

A statistic that is used to estimate a population parameter is an **estimator**.
A particular individual value is an **estimate**.
Bias is how far the estimator is from the true value

If a statistic T is used as an estimator for a population parameter θ then the bias is:

$$E(T) - \theta$$

If $E(T) = \theta$ then the statistic is unbiased

1.1 Proving $\mathbf{E}(\overline{\mathbf{X}}) = \mu$

Prove that $\overline{X}$ is an unbiased estimator for μ when the population is normally distributed

A random sample $X_1, X_2, X_3 ... X_N$ is taken for a population with $X \sim N(\mu, \sigma^2)$

$$\overline{X} = \frac{1}{n} \sum X$$

$$E(\overline{X}) = E\Big(\frac{1}{n} \sum X\Big)$$

$$E(\overline{X}) = \frac{1}{n}\big(E(X_1) + E(X_2) + E(X_3)... + E(X_N)\big)$$

$$E(\overline{X}) = \frac{1}{n}(\mu + \mu + \mu... + \mu)$$

$$E(\overline{X}) = \frac{1}{n}(n\mu)$$

$$E(\overline{X}) = \mu$$

1.2 Variance

$$Var(\overline{X}) = \frac{\sigma^2}{\text{Sample size}}$$

We use S^2 as an estimator for σ^2

$$S^2 = \frac{1}{n-1}\Big(\Sigma X^2 - n\overline{X}^2\Big)$$

1.3 Uniform Distributions

Random variable X is continuously uniform $[0, \alpha]$. A sample $X_1, X_2..X_N$ is taken
Show that $\overline{X}$ is a biased estimate and state the bias

$$\overline{X} = \frac{0 + \alpha}{2} = \frac{\alpha}{2}$$

$$E(\overline{X}) = E(\frac{\alpha}{2}) = \frac{1}{2}E(\alpha) = \frac{\alpha}{2}$$

$$\text{Bias} = \frac{\alpha}{2} - \alpha = -\frac{\alpha}{2}$$

2 Standard error

$$\text{Standard error} = \frac{\sigma}{\sqrt{n}} \text{ or } \frac{s}{\sqrt{n}}$$

3 Central limit theorem

The central limit theorem (C.L.T.) states that if $X_1, X_2, X_3...$ is a random sample, from any distribution with mean μ and variance σ^2. Then the sample means, $\overline{X}$ are distributed with a normal distribution.

$$\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$$

For this, the sample size must be greater than 50