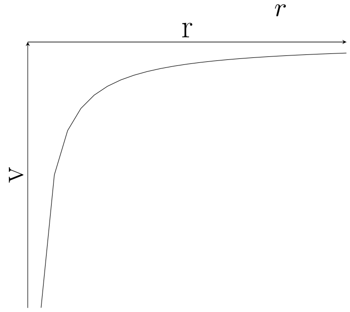
Gravitational Fields Notes

Gravitational field strength, g, is the force felt per unit mass on a unit mass placed at that point in a field.
$g = \frac{F}{m}$, unit=NKg^{-1}
The gravitational field is around all objects with mass and can be either **radial** or **uniform**.

1 Introduction to Gravitational potential

Definition: The potential at a point in a field is the work done in moving unit mass from infinity to that point.

$V = \frac{\text{Work done}}{\text{Unit mass}}$ Unit: JKg^{-1}
In a radial field $v = \frac{-GM}{r}$



Gravitational potential energy $= \Delta V \times m$
$V_o - V_s = \Delta V$ Unit:JKg^{-1}
$V_s =$ Potential on earth, $V_o =$ Potential in orbit.

2 Potential gradient

Potential gradient - Change in potential per unit change in distance

$$g = -\frac{\Delta V}{\Delta r}$$

3 Newton's law of gravitation

The force between two masses is attractive and is directly proportional to the product of the masses and is inversely proportional to the distance between them squared.

$$F = \frac{GM_1M_2}{r^2}$$

4 Escape Velocity

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$\frac{1}{2}v^2 = \frac{GM}{r}$$

$$v^2 = \frac{2GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

5 Satellite motion

The force between a planet of mass M and the satellite of mass m is described by Newton's law of gravitation.
This force provides the **centripetal force** acting towards the centre of the orbit.

$$\frac{GMm}{r^2}=\frac{mv^2}{r}$$
$$GM=v^2r$$
$$\sqrt{\frac{GM}{r}}=v$$
$$v\propto r^{-\frac{1}{2}}$$

6 Kepler's 3rd law

$$F=\frac{GMm}{r^2}=\frac{mv^2}{r}=m\omega^2r$$
$$\frac{GMm}{r^2}=m\omega^2r$$
$$\frac{GM}{r^3}=\left(\frac{2\pi}{T}\right)^2$$
$$\frac{GM}{r^3}=\frac{4\pi^2}{T^2}$$
$$\frac{r^3}{T^2}=\frac{GM}{4\pi^2}$$
$$\text{As } r\uparrow\ \ T\downarrow$$

7 Geostationary orbits

Sometimes called geosynchronous orbits
$T=24h=86400s$
$r=42Mm=35857km$ above the earth
Geostationary - Orbiting in plane of the equator
Geosynchronous - 24h orbit inclined at an angle to the equator

Geostationary orbits:

- Period of 24h
- 36,000 km above the earth's surface
- Circular
- Equatorial (in plane of the equator)
- Are in the same direction as the earth's rotation

8 The time period of satellites

$$\sqrt{\frac{GM}{r}}=\frac{2\pi r}{T}$$
$$\frac{GM}{r}=\frac{4\pi^2r^2}{T^2}$$
$$T^2=(\frac{4\pi^2}{GM})r^3$$
$$T^2\propto r^3$$
$$T=kr^{\frac{3}{2}}$$

$\log T=\frac{3}{2}\log r+\log k$ is in the form $y=mx+c$ This can also be used to fine the height that a satellite must be at to have any given orbit time.