Binomial Expansion

1 Introduction to Binomial expansion

Expansion can be done using the $(1+x)^n$ expansion, including with $(1+ax)^n$

2 Negative powers

Example To expand $\frac{1}{1+x}$ turn it into $(1+x)^{-1}$ an use the formula from the book.
$1-x+x^2-x^3+x^4-x^5...$
As n is not a positive integer there will be no x coefficient equalling zero, meaning the expansion is infinite and convergent.
This gives valid values when $|x|<1$

3 Fractional powers

$\sqrt{1-3x}$
Simplify
$(1-3x)^{\frac{1}{2}}$

Find n and x
$n=\frac{1}{2}$
$x=-3x$

Substitute into the formula
$1+\frac{1}{2}\times -3x+\frac{\frac{1}{2}(\frac{1}{2}-1)}{1\times 2}\times(-3x)^2$

Simplify
$1-\frac{3}{2}x-\frac{9}{8}x^2$

Write conclusion
Convergent and infinite when: $|3x|<1$ $|x|<\frac{1}{3}$

4 Applying $(1+x)^n$ to $(a\pm bx)^n$

$(a\pm bx)^n$ can be rewritten as $a^n(1\pm\frac{b}{a}x)^n$

4.1 Example

Expand $\sqrt{4+x}$ *to the* x^3 *term*
Turn square root into power
$(4-x)^{\frac{1}{2}}$

Rewrite with a 1 in the bracket
$4^{\frac{1}{2}}(1+\frac{1}{4}x)^{\frac{1}{2}}$

Find n and x
n=$\frac{1}{2}$
x=$\frac{1}{4}x$

Substitute into the formula

$$2\left[1+\frac{1}{2}\times\frac{1}{4}x+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}\left(\frac{1}{4}x\right)^2+\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}\left(\frac{1}{4}x\right)^3\right]$$

Simplify

$2\left[1+\frac{x}{8}-\frac{x^2}{128}+\frac{x^3}{1024}\right]$

$2+\frac{x}{4}-\frac{x^2}{64}+\frac{x^3}{512}$

Write conclusion

Valid if $\left|\frac{x}{4}\right|<1$ so valid if $|x|<4$

5 Unknown coefficient type

$(a+bx)^{-2}$ can be approximated by
$a(1+\frac{b}{a}x)^{-2}$
$\frac{1}{a^2}(1-2\frac{b}{a}x)$

6 Fractional type

Expand up to x^3 $\frac{1+x}{2+x}$

Re-Write using powers
$(1+x)(2+x)^{-1}$

Ensure there is only a 1 in the bracket
$2(1+\frac{1}{2}x)^{-1}$

Find n and x
$n=-1$
$x=\frac{1}{2}x$

Substitute into the formula
$\frac{1}{2}\left(1+-1\times\frac{1}{2}x\right)+\frac{-1(-1-1)}{2!}\left(\frac{1}{2}(x)^2\right)^2+\frac{-1(-1-1)(-1-2}{3!}\left(\frac{1}{2}x\right)^3$

Simplify
$(1+x)\left(\frac{1}{2}-\frac{1}{4}x+\frac{1}{8}x^2-\frac{1}{16}x^3\right)$
$\frac{1}{2}+\frac{1}{4}x-\frac{1}{8}x^2+\frac{1}{16}x^3$

Write conclusion
Valid if $x\neq 2$

7 Approximating roots

Find the expansion of $\sqrt{1-2x}$ *up to* x^3

Re-Write using powers

$(1-2x)^{\frac{1}{2}}$

Find n and x

$n = \dfrac{1}{2}$

$x = -2x$

Substitute into the formula

$$1 + \left(\frac{1}{2} \times -2x\right) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} \times (-2x)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} \times (-2x)^3$$

Simplify

$$1 - x + \frac{1}{2}x^2 - \frac{1}{3}x^3$$

By substituting $x = 0.01$, *find a suitable approximation of* $\sqrt{2}$

Substitute values

$$\sqrt{1 - \frac{2}{100}} = 1 - \frac{1}{100} - \frac{(\frac{1}{100})^2}{2} - \frac{(\frac{1}{100})^3}{2}$$

Simplify

$$\sqrt{\frac{98}{100}} = \frac{\sqrt{98}}{10} = \frac{7\sqrt{2}}{10}$$

Rearrange

$$\sqrt{2} \approx \frac{10}{7}\left(1 - \frac{1}{100} - \frac{(\frac{1}{100})^2}{2} - \frac{(\frac{1}{100})^3}{2}\right)$$

$\sqrt{2} = 1.414213571$