Transportation Problems

1 Terminology used in describing and modelling the transportation problem

In order to solve transportation problems you need to consider:

- The capacity of each of the supply points - The quantity of goods that can be produced at each factory or held at each warehouse. Called **supply** or **stock**
- The amount required at each of the demand points - The quantity of goods that are needed at each shop or by each customer. Called **demand** or **destination**
- The unit cost of transporting goods from the supply points to the demand points

1.1 Solving the transportation problem

1. Find an initial solution that uses all the stock and meets all the demands
2. Calculate the total cost of this solution and see if it can be reduced by using routes not currently in the solution (if not then optimal)
3. If the cost can be reduced by a different route, put as many units across it as possible
4. Check to see if optimal, if not, go to 3
5. When no further savings are possible, an optimal solution has been found

2 Finding an initial solution to the transportation problem

A method called the **north-west corner method** is used.
First, create a table with rows for source and columns for destination. Add demand at the bottom of each column and stock at the end of each row.

1. At the top left-hand corner of the table, allocate the maximum stock to meet demand
2. If stock is emptied, move down 1 and allocate
3. If demand is met, move 1 right and allocate
4. When all the stock is assigned and all the demands met, stop

2.1 Example

What would be given in the question:

	Depot W	Depot X	Depot Y	Depot Z	Stock
Supplier A	180	110	130	290	14
Supplier B	190	250	150	280	16
Supplier C	240	270	190	120	20
Demand	11	15	14	10	50

Set up the table with stock and demand

	W	X	Y	Z	Stock
A					14
B					16
C					20
Demand	11	15	14	10	50

Start by filling the top left corner
11 is put here as it is the demand for **W** but does not exceed the stock of **A**

	W	X	Y	Z	Stock
A	11				14
B					16
C					20
Demand	11	15	14	10	50

Move 1 right as the demand has been satisfied
3 is put here as it is the remainder of the stock of **A** and does not exceed the demand of **X**

	W	X	Y	Z	Stock
A	11	3			14
B					16
C					20
Demand	11	15	14	10	50

Move 1 down as the stock of A has been exhausted
12 is put here as it satisfies the demand for **X** but does not exceed the supply of **B**

	W	X	Y	Z	Stock
A	11	3			14
B		12			16
C					20
Demand	11	15	14	10	50

Move 1 right as the supply of B has not yet been exhausted
4 is put here as it is the maximum **B** is able to supply and does not exceed the demand of **Y**

	W	X	Y	Z	Stock
A	11	3			14
B		12	4		16
C					20
Demand	11	15	14	10	50

Move 1 down as the stock of B has been exhausted
10 is put this as it meets the demand of **Y** without exceeding the stock of **C**

	W	X	Y	Z	Stock
A	11	3			14
B		12	4		16
C			10		20
Demand	11	15	14	10	50

Move 1 right as the demand for Y has been met
10 is put here as it meets the demand of **Z** and exhausts the stock of **C**

	W	X	Y	Z	Stock
A	11	3			14
B		12	4		16
C			10	10	20
Demand	11	15	14	10	50

The total cost of transportation can then be found by multiplying the value in the square to be transported by the cost of that square.

	W	X	Y	Z	Stock
A	11×180	3×110			14
B		12×250	4×150		16
C			10×190	10×120	20
Demand	11	15	14	10	50

The sum of those multiplications is £9010, the cost of transportation.

3 Unbalanced transportation problems

When total supply>total demand, the problem is **unbalanced**
For an unbalanced transportation problem, add a dummy demand point with demand chosen to make demand and supply equal. All transportation costs of the dummy point is **zero**.

3.1 Example

	A	B	C	Supply
X	9	11	10	40
Y	10	8	12	60
Z	12	7	8	50
Demand	50	40	30	

Because 40+60+50¿50+40+30 the problem is unbalanced.

	A	B	C	D	Supply
X	9	11	10	0	40
Y	10	8	12	0	60
Z	12	7	8	0	50
Demand	50	40	30	30	

The problem can then be solved as normal

	A	B	C	D	Supply
X	40				40
Y	10	40	10		60
Z			20	30	50
Demand	50	40	30	30	

4 Degenerate solutions

If the number of cells filled in a solution is less than suppliers+demanders-1 then the solution is **degenerate**.

This happens when an entry, other than the last, satisfies both supply and demand at the same time, leading to the selector moving diagonally.

The algorithm requires for n+m-1 cells to be used in every solution, so a zero needs to be placed in a currently unused cell.

4.1 Example

	A	B	C	Supply
W	10	11	6	30
X	4	5	9	20
Y	3	8	7	35
Z	11	10	9	35
Demand	39	40	50	

This then gives the solution:

	A	B	C	Supply
W	30			30
X		20		20
Y		20	15	35
Z			35	35
Demand	39	40	50	

The problem with this is that it moves from AW to BX diagonally. This is solved by adding a zero in a place the selector could have moved after AW (BW or AX).

	A	B	C	Supply
W	30			30
X	0	20		20
Y		20	15	35
Z			35	35
Demand	39	40	50	

When doing this don't worry about where the zero goes, it can go in any empty space.

5 Shadow costs

Transportation costs are made up of two costs, one for source and one for destination, the cost of using that route is called the **shadow cost**.

5.1 Description

	W	X	Y	Z	Stock
A	180	110			14
B		250	150		16
C			190	120	20
Demand	11	15	14	10	50

Simultaneous equations can then be set up from this, for example:
S(A)+D(W)=180
S(A)+D(X)=110
S(B)+D(X)=250
As there are 5 equations and 6 unknowns relative costs need to be used.

5.2 Method

1. Set the source cost of the top left corner to zero
2. Move along the row and use **cost of destination =total cost-cost of supply**
3. When all the costs for that row have been found, move down one row and use the data already found to determine the supply cost
4. Repeat step 2 and step 3 until all costs have been found.

5.3 Example

Set up a table with space for shadow costs

Shadow costs						
		Depot W	Depot X	Depot Y	Depot Z	Stock
	Supply A	180	110			14
	Supply B		250	150		16
	Supply C			190	120	20
	Demand	11	15	14	10	50

Set the cost of source for the top left corner to zero and use to fill out the depot cost for the row

Shadow costs		180	110			
		Depot W	Depot X	Depot Y	Depot Z	Stock
0	Supply A	180	110			14
	Supply B		250	150		16
	Supply C			190	120	20
	Demand	11	15	14	10	50

Move 1 row down and use filled out values to determine all the values for that row

Shadow costs		180	110	10		
		Depot W	Depot X	Depot Y	Depot Z	Stock
0	Supply A	180	110			14
140	Supply B		250	150		16
	Supply C			190	120	20
	Demand	11	15	14	10	50

Repeat step above

Shadow costs		180	110	10	-60	
		Depot W	Depot X	Depot Y	Depot Z	Stock
0	Supply A	180	110			14
140	Supply B		250	150		16
180	Supply C			190	120	20
	Demand	11	15	14	10	50

Write all the costs individually

S(A)=0
S(B)=140
S(C)=180
D(W)=180
D(X)=110
D(Y)=10
D(Z)=-60

6 Improvement indices

Improvement indices look at each unused route and calculate the reduction in cost by using that route.
The formula for an improvement index for PX= $\mathbf{I_{PQ} = C(PQ) - S(P) - D(Q)}$
The route with the most negative improvement index will be introduced to the solution.
The route introduced to the solution is called the **entering cell** and the route it replaces is called the **exiting cell**.
If there are two equal entering or exiting cells choose either.
If there are no negative improvement indices the solution is **optimal**.

6.1 Example

Start with the shadow costs

Shadow costs		180	110	10	-60	
		Depot W	Depot X	Depot Y	Depot Z	Stock
0	Supply A	180	110			14
140	Supply B		250	150		16
180	Supply C			190	120	20
	Demand	11	15	14	10	50

Use the formula to calculate improvement indices

Shadow costs		180	110	10	-60	
		Depot W	Depot X	Depot Y	Depot Z	Stock
0	Supply A	0	0	120	350	14
140	Supply B	-130	0	0	200	16
180	Supply C	-120	20	0	0	20
	Demand	11	15	14	10	50

This shows that BW is the entering cell as it has the lowest improvement index.

7 Stepping stone method

The stepping stone method is a method to find a more optimal solution given improvement indices.
This looks for a cycle of adjustments, increasing the value in one cell and reducing in the next and so on.

7.1 Method

1. Create a cycle of adjustments with the rules:
   (a) Within any row or column there can only be one increasing and one decreasing cell
   (b) With the exception of the entering cell, adjustments can only be made to filled cells
2. Once the cycle of adjustments has been found, transfer the maximum number of units through this cycle. This will be equal to the smallest number in the decreasing cells to avoid negative units.
3. Adjust the solution to incorporate this solution

7.2 Example

Start with the initial solution

	W	X	Y	Z	Stock
A	11	3			14
B		12	4		16
C			10	10	20
Demand	11	15	14	10	50

Insert a theta in the value with the most negative improvement index

	W	X	Y	Z	Stock
A	11	3			14
B	θ	12	4		16
C			10	10	20
Demand	11	15	14	10	50

Subtract theta from AW to keep the demand at W correct. Then add theta to AX to keep the stock at AX correct, finally subtract theta from BX to keep the demand at X correct

	W	X	Y	Z	Stock
A	$11-\theta$	$3+\theta$			14
B	θ	$12-\theta$	4		16
C			10	10	20
Demand	11	15	14	10	50

Substitute 11 for theta as that is the smallest value in a decreasing cell

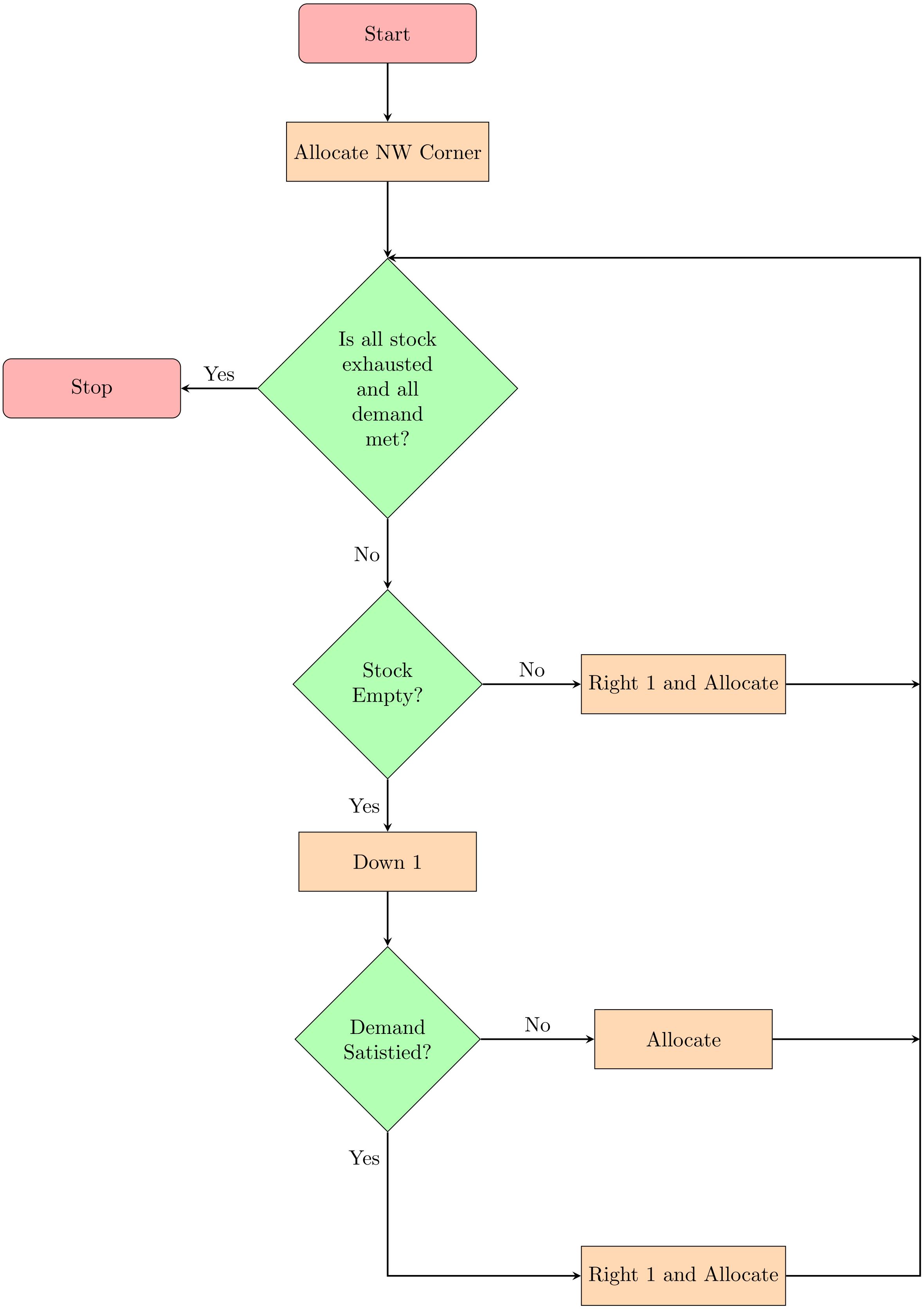
	W	X	Y	Z	Stock
A	$11-11$	$3+11$			14
B	11	$12-11$	4		16
C			10	10	20
Demand	11	15	14	10	50

Write the improved solution and find the cost

	W	X	Y	Z	Stock
A		14			14
B	11	1	4		16
C			10	10	20
Demand	11	15	14	10	50

The cost of this is £7580, a reduction from the initial solution.
AW has become empty, meaning it is the exiting cell
To find an optimal solution, new shadow costs and improvement indices would need to be found

8 Finding an initial solution to the transportation problem



9 Finding an improvement to the initial solution