Exam Questions - Estimation

Given that $E(S^2) = \sigma^2$, where S^2 is an unbiased estimator of σ^2 and the statistic

$$Y = \frac{1}{8}\left(\sum_{i=1}^{8} X_i^2 - 8\overline{X}^2\right)$$

Find $E(Y)$ in terms of σ^2

$$S^2 = \frac{1}{7}\left(\sum X^2 - \frac{(\sum X)^2}{8}\right) = \frac{1}{7}\left(\sum X^2 - 8\overline{X}^2\right)$$

$$7S^2 = \sum X^2 - 8\overline{X}^2$$

$$\frac{7}{8}S^2 = \frac{1}{8}(\sum X^2 - 8\overline{X}^2) = Y$$

$$E(Y) = E\left(\frac{7}{8}S^2\right) = \frac{7}{8}\sigma^2$$