C4 Cheat Sheet

1 Partial fractions

1.1 Split a fraction whose denominator is a product of linear expressions

Set the fraction equal to the sum of constants over each linear expression, then multiply by the denominator and solve by substitution.

1.2 Split a fraction where one or more of the factors in the denominator are squared

Each squared term gets two terms in the summation, one of the non squared term, and one of the squared term.

1.3 Deal with top heavy fractions

Use long division to simplify, for example simplifying $\frac{3x^2-3x-2}{(x-1)(x-2)}$

Long division to find remainder

$$\begin{array}{r|l} x^2-3x+2 & 3 \\ \hline & 3x^2-3x-2 \\ & -3x^2+9x-6 \\ \hline & 6x-8 \end{array}$$

Re-write with remainder

$$3+\frac{6x-8}{(x-1)(x-2)}$$

Partial fractions can then be applied normally

2 Parametric equations

$$\frac{dy}{dx}=\frac{(\frac{dy}{dt})}{(\frac{dx}{dt})}$$

$$\int y\ dx=\int y\frac{dx}{dt}dt$$

Remember to make use of $\sin^2 x+\cos^2 x=1$ when converting between parametric and Cartesian

3 Binomial expansion

$$(a+bx)^n=a^n(a+\frac{b}{a}x)^n$$

Remember that fractions can be simplified for binomial expansion using negative powers

4 Differentiation

On C3 data sheet:

$$a^x=e^{\ln(a^x)}=e^{x\ln(a)}$$

$$\frac{d}{dx}(a^x)=\ln a\times e^{x\ln a}=a^x\ln a$$

4.1 Implicit differentiation

$$\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$$

4.2 Setting up differential equations

Example:
Temperature falls at a rate proportional to its current temperature

$$\frac{dT}{dt} = -kT$$

4.3 Related rates of change

When given a rate and asked to find a different rate, use equations like

$$\frac{dA}{dx} = \frac{dA}{dt} \times \frac{dt}{dx}$$

Remember that finding relations between length, area and volume can be found using known formulas

5 Vectors

5.1 Point of intersection

Set the two vector lines equal to each other, find λ etc, substitute in to see if they agree, remember to write the final point

5.2 Angle between two lines

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

This can be used when lines are perpendicular as then

$$a \cdot b = |a||b|$$

5.3 Find a missing x/y/z value of a point on a line

Solve the equation for the two known points to find λ, then use that to find the other point

5.4 Find the length of a vector/distance between two points

Find the equation of the vector using the formula

$$\overrightarrow{AB} = \begin{pmatrix} x_b - x_a \\ y_b - y_a \\ z_b - z_a \end{pmatrix}$$

Then use

$$|AB| = \sqrt{x^2 + y^2 + z^2}$$

5.5 Find the nearest point on a line to a point not on the line

The shortest distance is always at a right angle, so you can use the fact that:

$$|a||b| = a \cdot b$$

Set up the equation of the line including λ, then solve the equation of the two of them being at right angles to find the value of λ and so find the point

5.6 Show that 3 points are collinear

Show that the vector between the first and second points is a multiple of the vector between the second and third points

5.7 Find the equation of a line from two points

Find the direction vector from $\overrightarrow{AB}$ then arrange it either the form:

$$A + \lambda\left(\overrightarrow{AB}\right) \quad \text{or} \quad B + \lambda\left(\overrightarrow{AB}\right)$$

5.8 Find the reflection of a point in a line

Find the shortest distance vector from the point to the line, then draw the vector coming out the other side of the line (negative of the vector)

5.9 Find the point after going a specific distance in the direction of a given vector

Example:

What is the position vector 10 units in the direction $\begin{pmatrix}1\\0\\3\end{pmatrix}$ *from the point* $\begin{pmatrix}2\\2\\2\end{pmatrix}$*?*

Convert the direction into a unit vector

$$\sqrt{1^2 + 0^2 + 3^2} = 2$$

This means that the vector is 2 units away from the origin, so to create the unit vector (1 unit away from the origin) multiply it by $\frac{1}{2}$

$$\frac{1}{2}\begin{pmatrix}2\\2\\2\end{pmatrix}$$

Then add 10 of these units to the initial vector point to get the answer

$$\begin{pmatrix}1\\2\\2\end{pmatrix} + 10 \times \frac{1}{2}\begin{pmatrix}2\\2\\2\end{pmatrix} = \begin{pmatrix}7\\2\\17\end{pmatrix}$$

6 Integration

6.1 Integral of Trig squares

$f(x)$	$\int f(x)\,dx$
$\sec^2 x$	On formula book
$\csc^2 x$	On differentiation side of formula book
$\cot^2 x$	Use the identity $1 + \cot^2 x = \csc^2 x$, then use known integral of $\csc^2 x$ to integrate
$\sin^2 x$	Use cos addition formula
$\cos^2 x$	Use cos addition formula
$\tan^2 x$	Use the identity $\tan^2 x + 1 = \sec^2 x$