Centres of mass

1 Centre if mass of a discrete mass distribution

$\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i}$

$\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i}$

Example

Mass	2	3	2
x	2	3	-3
y	3	6	2

$\bar{x} = \frac{2 \times 2 + 3 \times 3 + 2 \times -3}{2 + 3 + 2} = 1$

$\bar{y} = \frac{2 \times 3 + 3 \times 6 + 2 \times 2}{2 + 3 + 2} = 4$

2 Uniform laminae

For a triangular lamina the centre of mass is $\frac{2}{3}$ along the line from the vertex to the middle of the line opposite.

For a sector of a circle, radius r, where the angle at the centre is 2α the centre of mass is $\frac{2r \sin \alpha}{3\alpha}$

3 Rods

In a circular arc, radius r, where the angle at the centre is 2α, the centre of mass is $\frac{r \sin \alpha}{\alpha}$ away from the centre.

4 Equilibrium

To avoid tipping, the lime of action of the weight must be within the side of the lamina in contact with the plane.
If a lamina is suspended from a fixed point, the centre of mass will be vertically below the point of suspension.
Assumptions made in equilibrium calculations:

- No friction at the point of suspension
- The mass of each area is uniform
- The mass is uniform at the join