Complex Numbers - Exam Questions

1 Example 1 - Finding roots of z^a

Find the four roots of the equation $z^4 = 8(\sqrt{3} + i)$ *in the form* $z = re^{i\theta}$

Convert into the form $z^4 = re^{i\theta}$

$$\theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$r = \sqrt{(8\sqrt{3})^2 + 8^2} = 16$$

$$z^4 = 16e^{i(\frac{\pi}{6} + 2k\pi)}$$

Take the fourth root of both sides

$$z = 2e^{i(\frac{\pi}{24} + \frac{1}{2}k\pi)}$$

Substitute in values of k to get values of θ between 0 and 2π

$$z = 2e^{i\frac{\pi}{24}}$$

$$z = 2e^{i\frac{13\pi}{24}}$$

$$z = 2e^{i\frac{25\pi}{24}}$$

$$z = 2e^{i\frac{37\pi}{24}}$$