# Continuous Distribution - Uniform - Exam Questions

## 1 Conditions

The conditions for a uniform distribution are:

- All values have the same probability

## 2 Finding probabilities

The probability of any result in a uniform distribution in a range from a to b is:

$$\frac{1}{b-a}$$

### 2.1 Example

*A rectangle has a perimeter of 20 cm. The length, x cm, of one side of this rectangle is uniformly distributed between 1 cm and 7 cm.*
*Find the probability that the length of the longer side of the rectangle is more than 6 cm long*
Write down the distribution

$$X \sim U[1, 7]$$

Find the probability wanted to be found

$$P(X < 4) + P(X > 6)$$

Find each probability

$$P(X > 6) = 1 \times \frac{1}{6} = \frac{1}{6}$$

$$P(X < 4) = 3 \times \frac{1}{6} = \frac{1}{2}$$

Add the probability to find the answer

$$\frac{1}{6} + \frac{1}{2} = \frac{2}{3}$$

## 3 Mean and Variance

*Jean regularly takes a break from work to go to the post office. The amount of time Jean waits in the queue to be served at the post office has a continuous uniform distribution between 0 and 10 minutes.*

### 3.1 Mean

*Find the mean time Jean spends in the queue*
Write down the distribution

$$X \sim U[0, 10]$$

Use the formula in the formula book

$$\mu = \frac{1}{2}(a + b)$$

$$\mu = \frac{1}{2}(0 + 10) = 5$$

### 3.2 Variance

*Find the variance of the time Jean spends in the queue*
Write down the distribution

$$X \sim U[0, 10]$$

Use the formula in the formula book

$$\text{Variance} = \frac{1}{12}(b - a)^2$$

$$\text{Variance} = \frac{1}{12}(10 - 0)^2 = \frac{25}{3}$$

### 3.2.1 Proof of variance formula

$$E(X) = \frac{1}{2}(a+b)$$

$$E(X^2) = \frac{1}{b-a}\int_a^b x^2 \ dx$$

$$E(X^2) = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$E(X^2) = \frac{b^3-a^3}{b-a}$$

$$E(X^2) = \frac{(b-a)(b^2+a^2+ab}{3(b-a)}$$

$$E(X^2) = \frac{b^2+a^2+ab}{3}$$

$$Var(X) = \frac{b^2+a^2+ab}{3} - \left(\frac{a+b}{2}\right)^2$$

$$Var(X) = \frac{b^2+a^2+ab}{3} - \frac{a^2+2ab+b^2}{4}$$

$$Var(X) = \frac{b^2-2ab+a^2}{12}$$

$$Var(X) = \frac{(b-a)^2}{12}$$