

Series

1 The method of differences

This method is used when large parts of a series cancel, allowing the series to be expressed in a simple form in terms of n .

1.1 Example

Express the below summation in terms of partial fractions

$$\sum_{r=1}^n \frac{1}{r(r+1)}$$

Write using unknown numerators

$$\frac{1}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$$

Solve

$$A(r+1) + B(r) = 1$$

$$r = 0 \quad A = 1$$

$$r = -1 \quad B = -1$$

Substitute

$$\sum_{r=1}^n \frac{1}{r(r+1)} = \sum_{r=1}^n \frac{1}{r} - \frac{1}{r+1}$$

Find a formula for this without summation

Write down values for the start and end of the summation, crossing out cancelling entries:

$$\begin{array}{rcl} = & \frac{1}{1} & - \quad \frac{1}{2} \\ + & \frac{1}{2} & - \quad \frac{1}{3} \\ + & \frac{1}{3} & - \quad \frac{1}{4} \\ + & \dots & - \quad \dots \\ + & \cancel{\frac{1}{n-1}} & - \quad \frac{1}{n} \\ + & \frac{1}{n} & - \quad \frac{1}{n+1} \end{array}$$

Write down remaining values as the answer:

$$1 - \frac{1}{n+1}$$

2 Summations not starting at 1

Remember:

$$\sum_{r=21}^n = \sum_{r=1}^n - \sum_{r=1}^{20}$$