Allocation Problems

1 Cost Matrices

- Require the same number of tasks as workers
- Only relative costs are important

To find a reduced cost matrix:

1. Subtract the least value in each row from each element of that row
2. Using the new matrix, subtract the least value in each column from each element in that column

2 Hungarian algorithm

1. Find the reduced cost matrix
2. Determine the minimum number of horizontal or vertical straight lines which will cover all the zeros in the matrix
   - If lines $\geqslant$ n, where the matrix is n×n then STOP
   - If lines <n, the matrix can be improved and continue
3. Draw in the lines and look for the smallest uncovered element, call it e
4. Add e to all covered elements, add it twice if covered twice
5. Subtract e from **all** elements of the matrix (even those that just had e added)
6. Repeat steps 2-5 until an optimal solution is found

2.1 Example

	Dig	Weed	Cut
Boris	250	80	160
Percival	230	90	150
Spike	230	110	140

Find the reduced cost matrix - First process rows

	Dig	Weed	Cut
Boris	170	0	80
Percival	140	0	60
Spike	120	0	30

Find the reduced cost matrix - process columns

	Dig	Weed	Cut
Boris	50	0	50
Percival	20	0	30
Spike	0	0	0

Draw lines to cover zeros

	Dig	*Weed*	*Cut*
Boris	50	0	50
Percival	20	0	30
Spike	0	0	0

Compare to dimension of matrix

2 Lines$\leqslant$3×3 matrix

Find the smallest uncovered element

20 (Percival digging)

Add e to all covered elements, and add twice for double covering

	Dig	Weed	Cut
Boris	50	20	50
Percival	20	20	30
Spike	20	40	20

Subtract e from all elements

	Dig	Weed	Cut
Boris	30	0	30
Percival	0	0	10
Spike	0	20	0

Draw lines to cover zeros

	Dig	*Weed*	*Cut*
Boris	30	0	30
Percival	0	0	10
Spike	0	20	0

Compare to dimension of matrix

3 lines=3 $\times$ 3 matrix - Optimal solution

3 Dummy Locations

When the problem is not n$\times$n a dummy location is used, this has zeroes in all the elements.

3.1 Example

	Task A	Task B	Task C
Mark	12	23	15
Nicky	14	21	17
Nigel	13	22	20
Susie	14	24	13

Add another task with zero cost

	Task A	Task B	Task C	Task D
Mark	12	23	15	0
Nicky	14	21	17	0
Nigel	13	22	20	0
Susie	14	24	13	0

This is then processed as usual

4 Incomplete data

The algorithm can also work on incomplete data (where a person cannot to a task). To do this assign values to the tasks that cannot be done that are at least **twice as large** as the largest value.

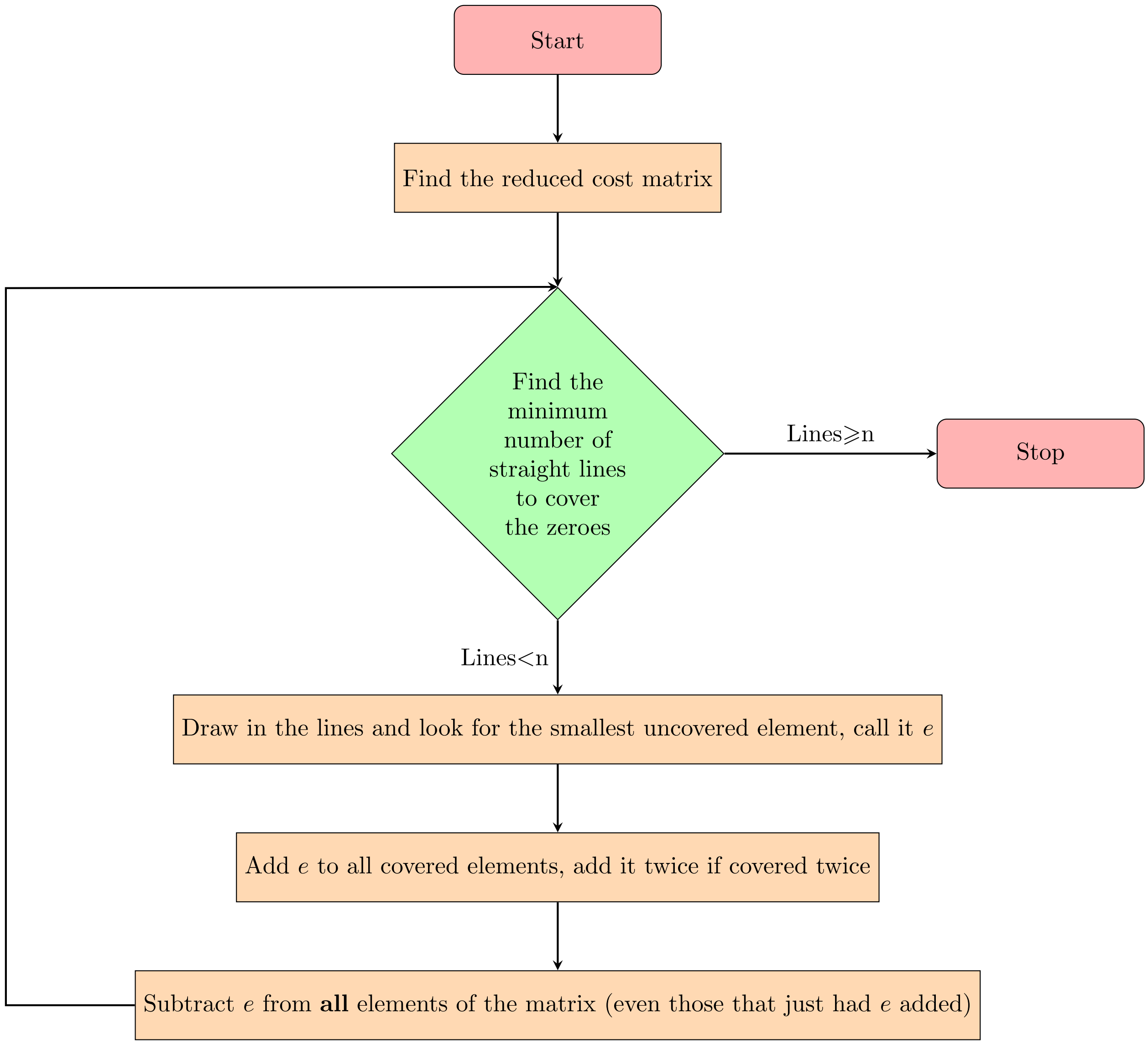
4.1 Example

	Chinese	French	Indian	Italian
Denis	-	27	15	40
Hilary	14	21	17	13
Robert	20	-	13	-
Trudy	14	24	10	30

The cells with dashes in are then replaced with 100

	Chinese	French	Indian	Italian
Denis	100	27	15	40
Hilary	14	21	17	13
Robert	20	100	13	100
Trudy	14	24	10	30

This can then be processed as normal


Start
Find the reduced cost matrix
Find the minimum number of straight lines to cover the zeroes
Lines⩾n
Stop
Lines<n
Draw in the lines and look for the smallest uncovered element, call it e
Add e to all covered elements, add it twice if covered twice
Subtract e from **all** elements of the matrix (even those that just had e added)