

# Coordinate Geometry

## 1 Parametric equations

In parametric equations, coordinates  $x$  and  $y$  are expressed as functions.

$x = f(t)$  and  $y = g(t)$  where  $t$  is the independent variable (called a parameter)

### 1.1 Converting parametric equations into Cartesian form

#### Example 1

$$x = 2t$$

$$y = t^2$$

$$t = \frac{x}{2}$$

$$y = \frac{1}{4}x^2$$

#### Example 2

$$x = \sin(t) + 2$$

$$y = \cos(t) - 3$$

$$\sin(t) = x - 2$$

$$\cos(t) = y + 3$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$(x - 2)^2 + (y + 3)^2 = 1$$

## 2 Parametric differentiation

The chain rule can be used to differentiate parametric equations

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

This can be used to find the gradient of normals and tangents.

#### Example

$$x = t^3 + t$$

$$y = t^2 + 1$$

**Differentiate the  $x$  and  $y$  terms**

$$\frac{dy}{dt} = 2t$$

$$\frac{dx}{dt} = 3t^2 + 1$$

**Invert the differential of  $x$  to fit the formula**

$$\frac{dt}{dx} = \frac{1}{3t^2 + 1}$$

**Multiply together to make one fraction**

$$\frac{dy}{dx} = \frac{2t}{3t^2 + 1}$$