

# Continuous Distribution - Normal- Exam Questions

## 1 Approximations

### 1.1 Poisson

#### 1.1.1 Cases where normal can be used to approximate poisson

- $\lambda > 10$

#### 1.1.2 Conversion

$$X \sim P_o(\lambda)$$

$$X \sim N(\lambda, \lambda)$$

#### 1.1.3 Continuity correction

As the poisson distribution is discrete and the normal distribution is continuous continuity correction must be used.

| Probability   | Corrected Probability      |
|---------------|----------------------------|
| $P(X = n)$    | $P(n - 0.5 < X < n + 0.5)$ |
| $P(X > n)$    | $P(X > n + 0.5)$           |
| $P(X \leq n)$ | $P(X < n + 0.5)$           |
| $P(X < n)$    | $P(X < n - 0.5)$           |
| $P(X \geq n)$ | $P(X > n - 0.5)$           |

#### 1.1.4 Example question

*During the summer the mean number of yachts hired per week is 25.*

*The company has only 30 yachts for hire.*

*Using a suitable approximation find the probability that the demand for yachts cannot be met on a particular Saturday in the summer.*

Let X Represent the number of yachts hired

$$X \sim P_o(25)$$

$$X \sim N(25, 25)$$

Write inequality

$$P(X > 30)$$

Apply continuity correction

$$P(X > 30.5)$$

Do Z transformation

$$Z = \frac{30.5 - 25}{5} = 1.1$$

Lookup value in tables

$$0.8643$$

Subtract from one as probability is greater than

$$1 - 0.8643 = 0.1357$$

## 1.2 Binomial

### 1.2.1 Conditions for normal to approximate binomial

- $n > 10$
- $p < 0.5$

### 1.2.2 Conversion

$$X \sim B(n, p)$$

$$X \sim N(np, np(1 - p))$$

### 1.2.3 Continuity correction

Binomial will also require continuity correction when approximated to normal as it is discrete and normal is continuous.

| Probability   | Corrected Probability      |
|---------------|----------------------------|
| $P(X = n)$    | $P(n - 0.5 < X < n + 0.5)$ |
| $P(X > n)$    | $P(X > n + 0.5)$           |
| $P(X \leq n)$ | $P(X < n + 0.5)$           |
| $P(X < n)$    | $P(X < n - 0.5)$           |
| $P(X \geq n)$ | $P(X > n - 0.5)$           |

### 1.2.4 Example Question

*In a large college 58% of students are female and 42% are male.*

*A random sample of 100 students is chosen from the college.*

*Using a suitable approximation find the probability that more than half the sample are female.* Write the question in the binomial for females

$$X \sim B(100, 0.58)$$

Use the conversion formula to convert to normal

$$X \sim N(58, 24.36)$$

Write the inequality then apply continuity correction

$$P(X > 50) \rightarrow P(X > 50.5)$$

Find the Z value

$$Z = \frac{50.5 - 58}{\sqrt{24.36}} \approx -1.52$$

Convert the Z value to a probability using the tables

$$p = 0.9357$$