Statics of rigid bodies

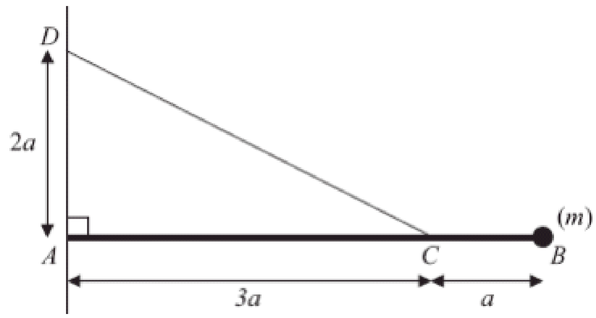
1 Equilibrium of rigid bodies

When calculating moments, only use the component of the force acting perpendicular to the rod.

A rigid body is in equilibrium if:

- The vector sum of the forces is zero
- The sum of the moments around any point is zero

Equilibrium of rigid bodies example



The diagram above shows a uniform rod AB of mass m and length 4a. The end A of the rod is freely hinged to a point on a vertical wall. A particle of mass m is attached to the rod at B. One end of a light inextensible string is attached to the rod at C, where AC = 3a. The other end of the string is attached to the wall at D, where AD = 2a and D is vertically above A. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is T.

Show that $T = mg\sqrt{13}$

Take moments about A

$$3a \times T\cos\theta = 2amg + 4amg$$

Calculate $\cos\theta$ from the lengths on the diagram

$$\cos\theta = \frac{2}{\sqrt{3^2+2^2}} = \frac{2}{\sqrt{13}}$$

Substitute in the value of $\cos\theta$ and divide through by a

$$\frac{6}{\sqrt{13}}T = 6mg$$

Multiply both sides by $\frac{\sqrt{13}}{6}$

$$T = mg\sqrt{13}$$

The particle of mass m at B is removed from the rod and replaced by a particle of mass M which is attached to the rod at B. The string breaks if the tension exceeds $2mg\sqrt{13}$*. Given that the string does not break, show that* $M \leqslant \frac{5}{2}m$

Rewrite the moments equation with the new information from the question

$$3a \times T\cos\theta = 2amg + 4aMg$$

Write the inequality given

$$T \leqslant 2mg\sqrt{13}$$

Substitute in the value for T

$$\frac{2mg+4Mg}{6}\sqrt{13} \leqslant 2mg\sqrt{13}$$

Simplify

$$mg + 2Mg \leqslant 6mg$$

$$M \leqslant \frac{5}{2}m$$