Hypothesis testing

1 Tests of hypotheses

Statistical hypothesis - An assertion or conjecture concerning a population.

To test the validity of a statement a random sample is taken from the population and that data can them be used to provide evidence that either supports or does not support the hypothesis.

Null hypothesis - H_0 - A hypothesis assumed to be true **Alternative hypothesis** - H_1 - The situation if H_0 is false.
If the data leads to rejection of the null hypothesis the alternative hypothesis will be accepted.
The sample data is used to evaluate the **test statistic**, probabilities related to it can be calculated using the null hypothesis.
If the test statistic is found in the **critical region** the null hypothesis will be rejected.
The **boundary values** of the critical region are called the critical values.

2 Method

1. Establish the null and alternative hypothesis (H_0 and H_1)
2. Define distribution under H_0
3. Decide on the significance level
4. Collect data, state the test statistic, X=
5. Calculate the probability of obtaining the test statistic or a more extreme result (same direction as H_1)
6. Compare this to the sig level as a decimal
   - If **greater** than the sig level, it is a **non significant** result, it is not in the critical region and we **do not** reject H_0
   - If **less** than sig level, it is a **significant result**, it is in the critical region and we **reject** H_0
7. Interpret the results in terms of the original claim

2.1 Example

Establish the null and alternative hypothesis
$H_0 : p = 0.5$
$H_1 : p > 0.5$

Define the distribution under $\mathbf{H_0}$
Under H_0 $X \sim B(15, 0.5)$

Decide on the significance level
5%

Collect data, state the test statistic
X=12

Calculate the probability of obtaining the test statistic or a more extreme result

$$\begin{aligned} P(X \geqslant 12) &= 1 - P(X \leqslant 11) \\ &= 1 - 0.9824 \\ &= 0.0176 \end{aligned}$$

Compare this to the sig level as a decimal $0.0176 < 0.05$

Interpret the results in terms of the original claim
There is evidence to reject H_0 in favour of H_1. The test is significant.

2.2 Finding critical values

We require a value c such that:
$P(X \geqslant c) < 0.05$
$1 - P(X \leqslant c - 1) < 0.05$
$P(X \leqslant c - 1) > 0.95$

Test against tables
$P(X < 11) = 0.9824$

$c - 1 = 11$
$c = 12$

2.3 Two tailed tests

When doing a two tailed test the significance level must be split in two. For example if the significance level is 5% then it must be split into 2.5% for each tail.

Two tailed test also concern equal or not equal, rather than inequalities, for example:
$H_0 : p = 0.15$
$H_1 : p \neq 0.15$