

## Common integrals

| $\int \sin f(x) \cos f(x) \, dx$  | $\int \sin f(x) \cos g(x) \, dx$   | $\int \sin^2 f(x) \cos^2 f(x) \, dx$   |
|---|--|--|
| <p><b>Example:</b> <math>\int \sin 3x \cos 3x \, dx</math></p> <p>Use addition formula to simplify<br/> <math>\sin 2x = 2 \cos x \sin x</math><br/> <math>\sin 3x \cos 3x = \frac{1}{2} \sin 6x</math><br/> <math>\frac{1}{2} \int \sin 6x \, dx</math></p> <p>Find a value for u<br/> <math>u = 6x</math></p> <p>Find dx in terms of du<br/> <math>\frac{du}{dx} = 6</math><br/> <math>dx = \frac{1}{6} du</math></p> <p>Substitute two conversions from x to u<br/> <math>\frac{1}{2} \times \frac{1}{6} \int \sin u \, du</math></p> <p>Use integration of sin formula<br/> <math>\frac{1}{12} \int \sin u \, du = -\frac{1}{12} \cos u + c</math></p> <p>Substitute x for u<br/> <math>-\frac{1}{12} \cos 6x + c</math></p> | <p><b>Example:</b> <math>\int \sin(3x) \cos(2x) \, dx</math></p> <p>Add together sin addition formula and sin subtraction formula<br/> <math>\sin(A+B) = \sin A \cos B + \cos A \sin B</math><br/> <math>\sin(A-B) = \sin A \cos B - \cos A \sin B</math><br/> <math>\sin(A+B) + \sin(A-B) = 2 \sin A \cos B</math><br/> <math>\sin 3x \cos 2x = \sin(3x+2x) + \sin(3x-2x)</math><br/> <math>\frac{1}{2} \int \sin 5x + \sin x \, dx = \frac{1}{2} \int \sin 5x \, dx + \frac{1}{2} \int \sin x \, dx</math></p> <p>Integrate using sin standard result<br/> <math>-\frac{1}{10} \cos 5x - \frac{1}{1} \cos x + c</math></p> | <p><b>Example:</b> <math>\int \sin^2(x) \cos^2(x) \, dx</math></p> <p>Give <math>\sin^2(x) \cos^2(x)</math> in terms of <math>\sin^2(x)</math><br/> <math>2 \sin(x) \cos(x) = \sin 2x</math><br/> <math>4 \sin^2(x) \cos^2(x) = \sin^2 2x</math><br/> <math>\cos^2 x \sin^2 x = \frac{1}{4} \sin^2 2x</math><br/> <b>Sub simplified version</b><br/> <math>\int \frac{1}{4} \sin^2 2x \, dx</math></p> <p>Give <math>\sin^2 x</math> in terms of <math>\cos x</math><br/> <math>2 \sin^2 2x = \sin^2 2x + (1 - \cos^2 2x)</math><br/> <math>2 \sin^2 2x = 1 - (\cos^2 2x - \sin^2 2x)</math><br/> <math>2 \sin^2 2x = 1 - \cos 4x</math> - Use of cos subtraction formula<br/> <math>\sin^2 2x = \frac{1}{2} - \frac{1}{2} \cos 4x</math><br/> <b>Sub simplified version</b><br/> <math>\frac{1}{4} \int \frac{1}{2} - \frac{1}{2} \cos 4x \, dx</math><br/> <math>\frac{1}{8} \int 1 - \cos 4x \, dx</math></p> <p>Use cos integration standard result<br/> <math>\frac{1}{8} (x - \frac{1}{4} \sin 4x) + c</math></p> <p><math>\frac{1}{8} x - \frac{1}{32} \sin 4x + c</math></p> |

| $\int \cos^2 f(x) \, dx$   | $\int \sin^2 f(x) \, dx$   | $\int \ln f(x) \, dx$  |
|--|--|--|
| $\int \cos x \times \cos x$<br>$\mathbf{u} = \cos x$<br>$\mathbf{v}' = \cos x$<br><br><b>Use integration by parts</b><br>$\int \cos^2 x = \sin x \cos x - \int -\sin^2 x$<br>$\int \cos^2 x = \sin x \cos x + \int \sin^2 x$<br>$\int \cos^2 x = \sin x \cos x + \int 1 - \cos^2 x$<br>$\int \cos^2 x = \sin x \cos x + x - \int \cos^2 x$<br>$2 \int \cos^2 x = \sin x \cos x + x$<br>$\int \cos^2 x = \frac{1}{2} \sin x \cos x + \frac{1}{2} x + c$ | $\int \sin x \times \sin x$<br>$\mathbf{u} = \sin x$<br>$\mathbf{v}' = \sin x$<br><br><b>Use integration by parts</b><br>$\int \sin^2 x = -\sin x \cos x - \int -\cos^2 x$<br>$\int \sin^2 x = -\sin x \cos x + \int \cos^2 x$<br>$\int \sin^2 x = -\sin x \cos x + \int 1 - \sin^2 x$<br>$\int \sin^2 x = -\sin x \cos x + x - \int \sin^2 x$<br>$2 \int \sin^2 x = -\sin x \cos x + x$<br>$\int \sin^2 x = -\frac{1}{2} \sin x \cos x + \frac{1}{2} x + c$ | $\int \ln x \, dx$<br>$\mathbf{u} = \ln x$<br>$\mathbf{v}' = 1$<br><br><b>Use integration by parts</b><br>$\int \ln x = x \ln x - \int x \times \frac{1}{x}$<br>$\int \ln x = x \ln x - x$ |