

Gravitational Fields Notes

Gravitational field strength, g , is the force felt per unit mass on a unit mass placed at that point in a field.

$$g = \frac{F}{m}, \text{ unit} = \text{N Kg}^{-1}$$

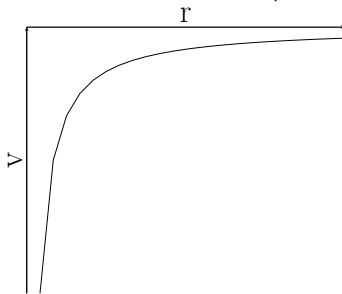
The gravitational field is around all objects with mass and can be either **radial** or **uniform**.

1 Introduction to Gravitational potential

Definition: The potential at a point in a field is the work done in moving unit mass from infinity to that point.

$$V = \frac{\text{Work done}}{\text{Unit mass}} \quad \text{Unit: } \text{JKg}^{-1}$$

$$\text{In a radial field } v = \frac{-GM}{r}$$



Gravitational potential energy = $\Delta V \times m$

$$V_o - V_s = \Delta V \quad \text{Unit: } \text{JKg}^{-1}$$

V_s = Potential on earth, V_o = Potential in orbit.

2 Potential gradient

Potential gradient - Change in potential per unit change in distance

$$g = -\frac{\Delta V}{\Delta r}$$

3 Newton's law of gravitation

The force between two masses is attractive and is directly proportional to the product of the masses and is inversely proportional to the distance between them squared.

$$F = \frac{GM_1M_2}{r^2}$$

4 Escape Velocity

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$\frac{1}{2}v^2 = \frac{GM}{r}$$

$$v^2 = \frac{2GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

5 Satellite motion

The force between a planet of mass M and the satellite of mass m is described by Newton's law of gravitation. This force provides the **centripetal force** acting towards the centre of the orbit.

$$\begin{aligned}\frac{GMm}{r^2} &= \frac{mv^2}{r} \\ GM &= v^2 r \\ \sqrt{\frac{GM}{r}} &= v \\ v &\propto r^{-\frac{1}{2}}\end{aligned}$$

6 Kepler's 3rd law

$$\begin{aligned}F &= \frac{GMm}{r^2} = \frac{mv^2}{r} = m\omega^2 r \\ \frac{GMm}{r^2} &= m\omega^2 r \\ \frac{GM}{r^3} &= \left(\frac{2\pi}{T}\right)^2 \\ \frac{GM}{r^3} &= \frac{4\pi^2}{T^2} \\ \frac{r^3}{T^2} &= \frac{GM}{4\pi^2} \\ \text{As } r \uparrow \quad T \downarrow\end{aligned}$$

7 Geostationary orbits

Sometimes called geosynchronous orbits

$$T = 24h = 86400s$$

$$r = 42Mm = 35857km \text{ above the earth}$$

Geostationary - Orbiting in plane of the equator

Geosynchronous - 24h orbit inclined at an angle to the equator

Geostationary orbits:

- Period of 24h
- 36,000 km above the earth's surface
- Circular
- Equatorial (in plane of the equator)
- Are in the same direction as the earth's rotation

8 The time period of satellites

$$\begin{aligned}\sqrt{\frac{GM}{r}} &= \frac{2\pi r}{T} \\ \frac{GM}{r} &= \frac{4\pi^2 r^2}{T^2} \\ T^2 &= \left(\frac{4\pi^2}{GM}\right)r^3 \\ T^2 &\propto r^3 \\ T &= kr^{\frac{3}{2}}\end{aligned}$$

$\log T = \frac{3}{2} \log r + \log k$ is in the form $y = mx + c$ This can also be used to find the height that a satellite must be at to have any given orbit time.