

Collisions

1 Impulse and momentum

$$\text{Impulse} = mv - mu = Ft$$

Total momentum before collision = total momentum after

2 Coefficient of restitution

This tells us how well something bounces, it is given the symbol e .

If $e = 1$ the ball returns to its original height

If $e = 0$ the ball doesn't bounce

$$e = \frac{\text{Speed of separation}}{\text{Speed of approach}}$$

2.1 Alternate form of coefficient of restitution formula

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$e = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}}$$

$$e = \frac{\sqrt{h_2}}{\sqrt{h_1}}$$

h_2 - the height the ball bounces back to

h_1 - the height the ball is dropped from

2.2 Calculations involving coefficient of restitution

When doing calculations involving the coefficient of restitution both the calculation for CoR and conservation of momentum will be needed.

Conservation of momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Coefficient of restitution

$$e = \frac{v_1}{u_1}$$

2.3 Successive Impacts

In some cases calculations will involve the impacts of multiple balls successively, like in newton's cradle.

2.3.1 Example

Three perfectly elastic particles A, B and C with masses 3kg, 2kg and 1kg respectively lie at rest in a straight line on a smooth horizontal table in alphabetical order. A is projected towards B with speed 5ms^{-1} and after A has collided with B, B collides with C

$$5 \times 3 = 3V_A + 2V_B$$

$$1 = \frac{SoS}{SoA} \text{ therefore } SoS = SoA \text{ so } V_B - V_A = 5 \text{ so } V_B = 5 + V_A$$

$$15 = 10 + 2V_A + 3V_A$$

$$15 = 2V_B + 3 \text{ so after 1st collision } V_B = 6$$

$$15 = 10 + 5V_A \text{ so } \mathbf{V_A = 1}$$

2nd Collision

$$2 \times 6 = 2V_B + V_C$$

$$V_C - V_B = 6$$

$$12 = 3V_B + 6$$

$$3V_B = 6$$

$$\mathbf{V_B = 2}$$

$$V_C = 6 + 2 = 8$$

Collisions example - Direct Impact

A small ball A of mass $3m$ is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

Find the speed of A immediately after the collision

