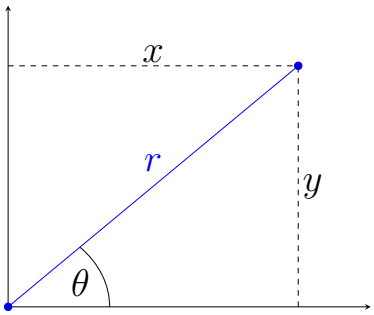
Polar Coordinates



Usual conventions are either $-\pi < \theta \leqslant \pi$ or $0 \leqslant \theta < \pi$

$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2 + y^2$
$\theta = \arctan\left(\frac{y}{x}\right)$

1 Converting between polar and Cartesian form

1.1 Example 1

Find the Cartesian equation of: $r = 5$

$$\sqrt{x^2 + y^2} = 5$$
$$x^2 + y^2 = 25$$

1.2 Example 2

Find the Cartesian equation of:

$$r = 2 + \cos 2\theta$$

Replace r and convert $\cos 2\theta$

$$\sqrt{x^2 + y^2} = 2 + \cos^2\theta - \sin^2\theta$$

Convert $\sin^2\theta$ and $\cos^2\theta$

$$\sqrt{x^2 + y^2} = 2 + \frac{x^2}{x^2 + y^2} - \frac{y^2}{x^2 + y^2}$$

Multiply all terms by $x^2 + y^2$

$$(x^2 + y^2)^{\frac{3}{2}} = 3x^2 + y^2$$

2 Sketching polar curves

To plot less standard types we look for axes intercepts and max and min values

2.1 Standard types

2.1.1 r=a

A circle, centre (0,0) with a radius of a

2.1.2 $\theta = \alpha$

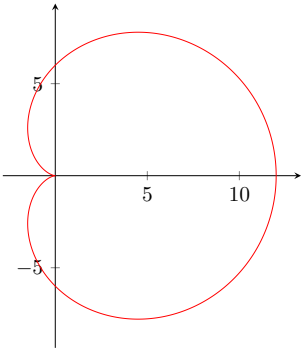
A half line starting from (0,0) making an angle of α with the initial line (positive x axis)

2.1.3 $r = a\theta$

A spiral starting at the origin

2.2 Cardioid type

2.2.1 $r = a(1 + \cos\theta)$

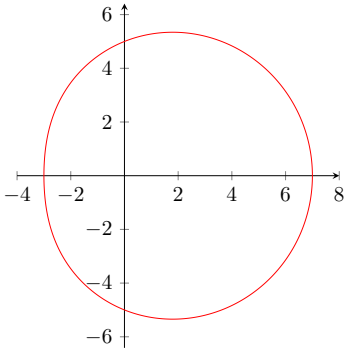


2.2.2 $r = a(p + q\cos\theta)$

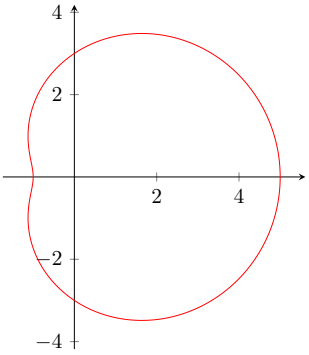
2.2.2.1 p=q

Factor out the value of p and plot as normal

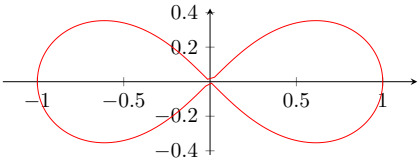
2.2.2.2 $p \geqslant 2q$



2.2.2.3 $q \leqslant p < 2q$

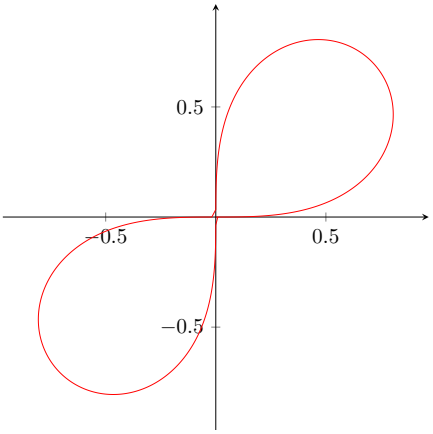


2.2.3 $r^2 = a^2 \cos 2\theta$



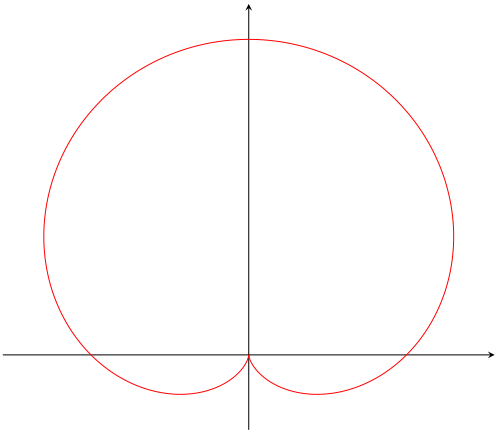
The 4 asymptotes are half lines at $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

2.3 $r^2 = a^2 \sin 2\theta$



This is an anticlockwise rotation of $r^2 = a^2 \cos 2\theta$ by $\frac{\pi}{4}$, it is this not $\frac{\pi}{2}$ as the phase difference between $\cos 2\theta$ and $\sin 2\theta$ is $\frac{\pi}{4}$ compared to $\frac{\pi}{2}$ with one θ.

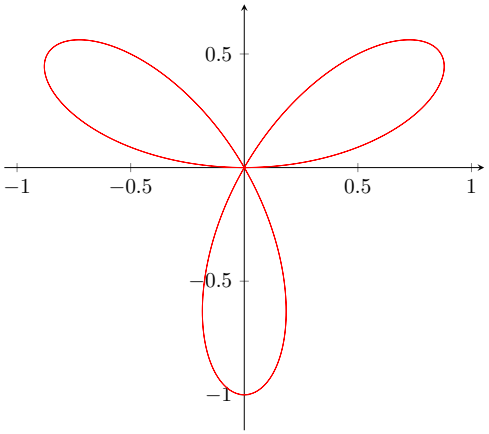
2.3.1 $r = a(p + q \sin \theta)$



This is a rotation of $r = a(p + q \cos \theta)$ anticlockwise by $\frac{\pi}{2}$

2.4 Other types

2.4.1 $r = a\sin 3\theta$



Asymptotes occur when $r = 0$

2.4.2 $r = 2\sec(\theta - \frac{\pi}{3})$