Linear Combinations of Normal Variables

1 Sums and Differences

$$\mu_{x+y} = \mu_x + \mu_y$$
$$\mu_{x-y} = \mu_x - \mu_y$$
$$\sigma^2_{x+y} = \sigma^2_x + \sigma^2_y$$
$$\sigma^2_{x-y} = \sigma^2_x + \sigma^2_y$$

2 Multiples

$$\mu_{ax+by} = a\mu_x + b\mu_y$$
$$\mu_{ax-by} = a\mu_x - b\mu_y$$
$$\sigma^2_{ax+by} = a^2\sigma^2_x + b^2\sigma^2_x$$
$$\sigma^2_{ax-by} = a^2\sigma^2_x + b^2\sigma^2_x$$

3 Addition vs Multiplication

$$E(X_1 + X_2) = E(2X_1)$$

However

$$Var(X_1 + X_2) \neq Var(2X_1)$$

$$Var(X_1 + X_2) = \sigma^2_1 + \sigma^2_1 = 2\sigma^2_2$$
$$Var(2X_1) = 4\sigma^2_1$$