Common integrals

$\int \sin f(x) \cos f(x)\ dx$	$\int \sin f(x) \cos g(x)\ dx$	$\int \sin^2 f(x) \cos^2 f(x)\ dx$
Example: $\int \sin 3x \cos 3x\ dx$ **Use addition formula to simplify** $\sin 2x = 2\cos x \sin x$ $\sin 3x \cos 3x = \frac{1}{2}\sin 6x$ $\frac{1}{2}\int \sin 6x\ dx$ **Find a value for u** $u = 6x$ **Find dx in terms of du** $\frac{du}{dx} = 6$ $dx = \frac{1}{6}du$ **Substitute two conversions from x to u** $\frac{1}{2} \times \frac{1}{6}\int \sin u\ du$ **Use integration of sin formula** $\frac{1}{12}\int \sin u\ du = -\frac{1}{12}\cos u + c$ **Substitute x for u** $-\frac{1}{12}\cos 6x + c$	**Example:** $\int \sin(3x)\cos(2x)\ dx$ **Add together sin addition formula and sin subtraction formula** $\sin(A+B) = \sin A\cos B + \cos A \sin B$ $\sin(A-B) = \sin A\cos B - \cos A \sin B$ $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$ $\sin 3x \cos 2x = \sin(3x+2x) + \sin(3x-2x)$ $\frac{1}{2}\int \sin 5x + \sin x\ dx = \frac{1}{2}\int \sin 5x\ dx + \frac{1}{2}\int \sin x\ dx$ **Integrate using sin standard result** $-\frac{1}{10}\cos 5x - \frac{1}{1}\cos x + c$	**Example:** $\int \sin^2(x)cos^2(x)\ dx$ **Give $\sin^2(x)\cos^2(x)$ in terms of $\sin^2(x)$** $2\sin(x)\cos(x) = \sin 2x$ $4\sin^2(x)\cos^2(x) = \sin^2 2x$ $\cos^2 x \sin^2 x = \frac{1}{4}\sin^2 2x$ **Sub simplified version** $\int \frac{1}{4}\sin^2 2x\ dx$ **Give $\sin^2 x$ in terms of cosx** $2\sin^2 2x = \sin^2 2x + (1 - \cos^2 2x)$ $2\sin^2 2x = 1 - (\cos^2 2x - \sin^2 2x)$ $2\sin^2 2x = 1 - \cos 4x$ - Use of cos subtraction formula $\sin^2 2x = \frac{1}{2} - \frac{1}{2}\cos 4x$ **Sub simplified version** $\frac{1}{4}\int \frac{1}{2} - \frac{1}{2}\cos 4x\ dx$ $\frac{1}{8}\int 1 - \cos 4x\ dx$ **Use cos integration standard result** $\frac{1}{8}(x - \frac{1}{4}\sin 4x) + c$ $\frac{1}{8}x - \frac{1}{32}\sin 4x + c$

$\int \cos^2 f(x)\ dx$	$\sin^2 f(x)\ dx$	$\int \ln f(x)\ dx$
$\int \cos x \times \cos x$ $\mathbf{u} = \cos x$ $\mathbf{v}' = \cos x$ **Use integration by parts** $\int \cos^2 x = \sin x \cos x - \int -\sin^2 x$ $\int \cos^2 x = \sin x \cos x + \int \sin^2 x$ $\int \cos^2 x = \sin x \cos x + \int 1 - \cos^2 x$ $\int \cos^2 x = \sin x \cos x + x - \int \cos^2 x$ $2\int \cos^2 x = \sin x \cos x + x$ $\int \cos^2 x = \frac{1}{2}\sin x \cos x + \frac{1}{2}x + c$	$\int \sin x \times \sin x$ $\mathbf{u} = \sin x$ $\mathbf{v}' = \sin x$ **Use integration by parts** $\int \sin^2 x = -\sin x \cos x - \int -\cos^2 x$ $\int \sin^2 x = -\sin x \cos x + \int \cos^2 x$ $\int \sin^2 x = -\sin x \cos x + \int 1 - \sin^2 x$ $\int \sin^2 x = -\sin x \cos x + x - \int \sin^2 x$ $2\int \sin^2 x = -\sin x \cos x + x$ $\int \sin^2 x = -\frac{1}{2}\sin x \cos x + \frac{1}{2}x + c$	$\int \ln x\ dx$ $\mathbf{u}$= $\ln x$ $\mathbf{v'}$= 1 **Use integration by parts** $\int \ln x = x \ln x - \int x \times \frac{1}{x}$ $\int \ln x = x \ln x - x$