Hypothesis tests - Binomial - Exam Questions

A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

1 Finding the critical region

Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025

Write down the distribution: $X \sim B(50, 0.25)$ For the lower value, look for the value closest to the probability of rejection

$$P(X \leqslant 6) = 0.0194$$

To find the upper value look for the value closest to (1-probability of rejection). Then add one to this value as:

$$P(X \geqslant c) = 1 - P(X \leqslant c - 1)$$

$$P(X \leqslant 18) = 0.9713$$

$$P(X \geqslant 19) = 0.0287$$

The critical values are 6 and 19

Find the actual significance
The actual significance is the true probability of rejection

$$0.0194 + 0.0287 = 0.0481$$

2 One tailed test

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty

Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly.

State the null and alternative hypothesis related to the $\frac{1}{4}$ probability mentioned earlier in the question

$$H_0 : \ p = 0.25$$

$$H_1 : \ p < 0.25$$

Write a probability based on the information given in the question

$$P(X \leqslant 5)$$

Look up the probability on the tables

$$P(X \leqslant 5) = 0.007$$

Compare this to the significance level

$$0.007 < 0.01$$

Write the conclusion

5 is in the critical region, reject H_0, the test is significant, there is evidence that the probability of faulty bolts has decreased.

3 Two tailed test

A teacher thinks that 20% of the pupils in a school read the Deano comic regularly.

He chooses 20 pupils at random and finds 9 of them read Deano.

Test, at the 5% level of significance, whether or not there is evidence that the percentage of pupils that read Deano is different from 20%. State your hypotheses clearly.

Write down the null and alternate hypothesis

$$H_0 : \ p = 0.2$$

$$H_1 : \ p \neq 0.2$$

Write down the distribution

$$X \sim B(20, 0.2)$$

Write down a probability based on information in the question

$$P(X \geqslant 9)$$

Find this probability using the tables

$$P(X \geqslant 9) = 1 - P(X \leqslant 8) = 1 - 0.99 = 0.01$$

Compare this value with the significance level, remembering to split it

$$0.01 < 0.025$$

Write conclusion

Probability is smaller than significance level, reject H_0 accept H_1, there is evidence that the percentage of pupils that read beano is not 20%.