

Coordinate Geometry

1 Parametric equations

In parametric equations, coordinates x and y are expressed as functions.

$x = f(t)$ and $y = g(t)$ where t is the independent variable (called a parameter)

1.1 Converting parametric equations into Cartesian form

Example 1

$$x = 2t$$

$$y = t^2$$

$$t = \frac{x}{2}$$

$$y = \frac{1}{4}x^2$$

Example 2

$$x = \sin(t) + 2$$

$$y = \cos(t) - 3$$

$$\sin(t) = x - 2$$

$$\cos(t) = y + 3$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$(x - 2)^2 + (y + 3)^2 = 1$$

2 Parametric differentiation

The chain rule can be used to differentiate parametric equations

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

This can be used to find the gradient of normals and tangents.

Example

$$x = t^3 + t$$

$$y = t^2 + 1$$

Differentiate the x and y terms

$$\frac{dy}{dt} = 2t$$

$$\frac{dx}{dt} = 3t^2 + 1$$

Invert the differential of x to fit the formula

$$\frac{dt}{dx} = \frac{1}{3t^2 + 1}$$

Multiply together to make one fraction

$$\frac{dy}{dx} = \frac{2t}{3t^2 + 1}$$