Centres of mass

1 Centre if mass of a discrete mass distribution

$\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i}$

$\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i}$

Example

Mass	2	3	2
x	2	3	-3
y	3	6	2

$\bar{x} = \frac{2 \times 2 + 3 \times 3 + 2 \times -3}{2+3+2} = 1$

$\bar{y} = \frac{2 \times 3 + 3 \times 6 + 2 \times 2}{2+3+2} = 4$

2 Uniform laminae

For a triangular lamina the centre of mass is $\frac{2}{3}$ along the line from the vertex to the middle of the line opposite.

For a sector of a circle, radius r, where the angle at the centre is 2α the centre of mass is $\frac{2r\sin\alpha}{3\alpha}$

3 Rods

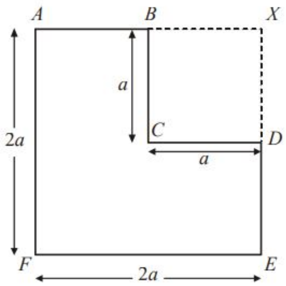
In a circular arc, radius r, where the angle at the centre is 2α, the centre of mass is $\frac{r\sin\alpha}{\alpha}$ away from the centre.

4 Equilibrium

To avoid tipping, the lime of action of the weight must be within the side of the lamina in contact with the plane.
If a lamina is suspended from a fixed point, the centre of mass will be vertically below the point of suspension.
Assumptions made in equilibrium calculations:

- No friction at the point of suspension
- The mass of each area is uniform
- The mass is uniform at the join

Centres of mass example - Equilibrium problems



(a) Find the distance of the centre of mass of the lamina from AF.

1. Find the masses and locations of all areas or rods and put in a table

Mass(m)	$4a^2$	$-a^2$
Distance From AF(x)	a	$\frac{3}{2}a$
mx	$4a^3$	$-\frac{3}{2}a^3$

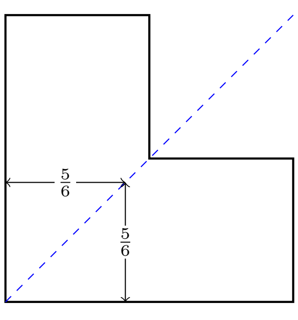
2. Use the formula to find the centre of mass

$$\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i}$$

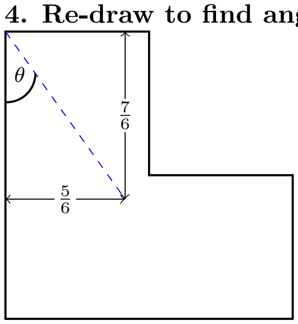
$$\bar{x} = \frac{\frac{5}{2}a^3}{3a^2} = \frac{5a}{6}$$

The lamina is freely suspended from A and hangs in equilibrium.
(b) Find, in degrees to one decimal place, the angle which AF makes with the vertical.

3. Use symmetry to find the y coordinate of the centre of mass



4. Re-draw to find angle



$$\tan(\theta) = \frac{5/6}{7/6}$$

$\theta = 35.5°$