Binomial and Poisson Distributions

1 The Binomial Distribution

1.1 Introduction to the binomial distribution

The binomial distribution is a **discrete** distribution.
Conditions for a binomial distribution:

- There are a fixed number of trials, **n**
- There are two outcomes (success and failure)
- Each trial is independent
- The probability of success is constant, **p**

Formula:
For $X\ B(n, p)$
$P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$
Example:
If 25 dice are thrown, find the probability three sixes are obtained.
$X\ B(25, \frac{1}{6})$
$P(X = 3) = \binom{25}{3} (\frac{1}{6})^3 (\frac{5}{6})^{22} = 0.1929$

1.2 Use of tables

Tables give "less than or equal to" probabilities.
Example:
$X\ B(5, 0.35)$
$P(x \leq 3) = 0.9640$ (in the table where n=5, p=0.35 and x=3)
$P(x < 4) = 0.9640$ (the data is discrete)
$P(x = 3) = P(x \leq 3) - P(x \leq 2)$
$P(x = 3) = 0.9640 - 0.7648$

$P(x \geq 3) = 1 - P(X \leq 2)$
$P(x \geq 3) = 1 - 0.7648$
$P(x \geq 3) = 0.2352$

1.3 Dealing with $P > \frac{1}{2}$

Example:
In the production of a car it is found that 85 % are without defects.
The cars are produces in batches of 50
Find the probability there are at least 40 defect-free cars in a batch
X=Number of cars without defects
$X\ B(50, 0.85)$
$P(X \geq 40)$
Y=Number of cars with defects
$Y\ B(50, 0.15)$
$P(Y \leq 10) = 0.8801 = P(x \geq 40)$

1.4 Mean and variance of the binomial distribution

$E(X) = np$
$Var(x) = np(1 - p)$
$\sigma(x) = \sqrt{Var(x)}$
Example 1:
$X\ B(80, 0.4)$ *Find* $E(x)$ *and* σ
$E(x) = 80 \times 0.4 = 32$
$\sigma = 80 \times 0.4(1 - 0.4) = 4.38$

Example 2:
$E(x) = 8$
$Var(x) = 6.4$
$Find\ n\ and\ p$
$np - np^2 = 6.4$
$8 - np^2 = 6.4$
$np^2 = 1.6$

$$p = \frac{np^2}{np} = \frac{1.6}{8} = 0.2$$
$$n = \frac{np}{p} = \frac{8}{0.2} = 40$$

2 The Poisson Distribution

2.1 Introduction to the Poisson Distribution

The conditions for a poisson distribution are:

- Events occur at random
- Events occur independently of each other
- The average rate of occurrences remains constant
- There is zero probability of simultaneous occurrences

$P(x = r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$
λ represents the mean number of occurrences in the time period.
Example
$x \sim P_o(4)$
$P(x = 2) = \dfrac{e^{-4} \times 4^2}{2!} = 0.1465$

2.2 Mean and variance of the poisson distribution

If $X\ P_o(\lambda)$ then $E(x) = \mu = \lambda$ and $Var(x) = \lambda$

2.3 Using the Poisson as an approximation of the binomial

You can use the Poisson distribution as an approximation if:

- P is small (< 0.1)
- n is large (> 50)

$\lambda = np$
$X \sim B(n,p) \approx Y \sim P_o(np)$
Example
$X \sim B(250, 0.01) \approx X \sim P_o(2.5)$