C1

1 Algebra and functions

1.1 Indices

$a^m \times a^n = a^{m+n}$
$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{mn}$
$a^{-m} = \frac{1}{a^m}$
$a^{\frac{1}{m}} = \sqrt[m]{a}$
$a^{\frac{n}{m}} = \sqrt[m]{a^n}$

2 Quadratic functions

2.1 Discriminant

- $b^2 > 4ac$, Two real solutions
- $b^2 = 4ac$, One repeated real solution
- $b^2 < 4ac$, No real solution

3 Sketching curves

3.1 Graph transformations

- f(x)+a, y coordinates increased by a
- af(x), y coordinates multiplied by a
- -f(x), reflection in the x axis
- f(x+a), x coordinates reduced by a
- f(ax), x coordinates divided by a
- f(-x), reflection in the y axis

4 Coordinate geometry

If two lines are perpendicular, the product of their gradients is -1.

5 Sequences and series

5.1 Deriving the formula for the sum of an arithmetic series

$$S_n = a + (a + d) + (a + 2d) + ... + (a + (n - 1)d)$$

Reverse the sun

$$S_n = (a + (n - 1)d) + (a + (n - 2)d) + a(a + (n - 3)d) + ... + (a + d) + a$$

Add the two sums

$$2S_n = [2a + (n - 1)d] + [2a + (n - 1)d] + [2a + (n - 1)d] + ... + [2a + (n - 1)d]$$

$$2S_n = n[2a + (n - 1)d]$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

5.2 Forming a recurrence relation

To create a recurrence relation formula from a formula given in n, substitute n+1 in place of n and rearrange.
Example

$$U_n = 2^n + 4n$$
$$U_{n+1} = 2^{n+1} + 4n + 1$$
$$U_{n+1} = 2U_n - 4n + 4$$

5.3 Finding a and d

Remember that the formula for the n$^{\text{th}}$ term may not be given in the form $a + (n-1)d$, and if it is not, it must be changed.
Example
$u_r = 4r - 7$
$u_1 = 4 - 7 = -3 = a$
$4r - 7 = -3 + (r-1)d \quad \Rightarrow 4r - 4 = (r-1)d \quad \Rightarrow d = 4$

5.4 Finding the nth term from a recurrence relation

To find the nth term, use a standard form, deducted from what is happening to the previous term.
Example
$U_{n+1} = 2U_n - 5$
The multiplier by 2 implies doubling, and so the nth term will contain $A \times 2^n$, the subtraction of 5 suggests there will also be a constant term, B.

$$U_n = A \times 2^n + B$$

Given $U_1 = 6$, A and B can be found to be $\frac{1}{2}$ and 5.