# C3

## 1 Algebraic fractions

Algebraic fractions can be simplified by equating the fraction to an equation made of constants

### 1.1 Example

$$\frac{x^3+2x^2-6x+1}{x-1} \equiv Ax^2+Bx+C+\frac{D}{x-1}$$

$$x^3+2x^2-6x+1 \equiv (x-1)(Ax^2+Bx+C)+D$$

$$x^3+2x^2-6x+1 \equiv Ax^3+(B-A)x^2+(C-B)x+(D-C)1$$

| Term | Calculation | Final Value |
|---|---|---|
| $x^3$ coefficient | $1=A$ | $\mathbf{A=1}$ |
| $x^2$ coefficient | $2=B-A$ | $\mathbf{B=3}$ |
| $x$ coefficient | $-6=C-B$ | $\mathbf{C=-3}$ |
| Constant term | $1=D-C$ | $\mathbf{D=-2}$ |

$$x^2+3x-3+\frac{-2}{x-2}$$

## 2 Functions

### 2.1 Definitions

**Domain** - The input to a function
**Range** - The output from a function

### 2.2 Function mapping

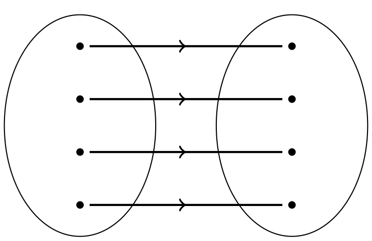


Figure 1: One-to-one function

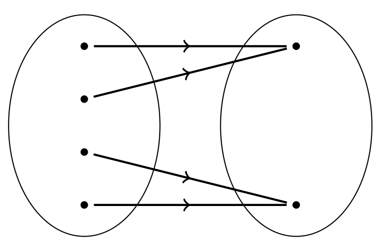


Figure 2: Many-to-one function

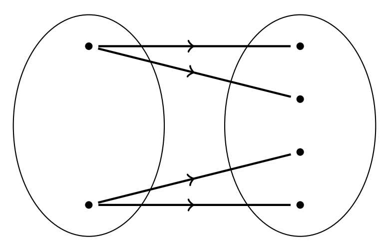
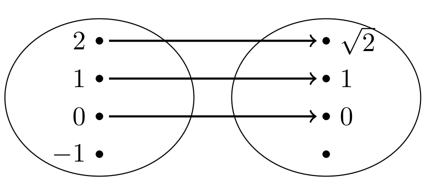


Figure 3: Not a function

A function is a mapping so that every element of the domain maps to exactly one element of the range

#### 2.2.1 Changing non functions to functions

Some non functions can be changed to functions by restricting the domain
For example for $f(x)=\sqrt{x}$ where $x \in \mathbb{R}$ all positive values get mapped, however negative numbers don't, see below:




This means that the domain must be restricted to $x \geqslant 0$

## 2.3 Inverse functions

The inverse of $f(x)$ is written as $f^{-1}(x)$
The domain and range of inverse function are the opposite of the normal function.

### 2.3.1 Finding the inverse function

To find the inverse function isolate $x$ then replace $x$ with $f^{-1}(x)$ and $y$ with $x$

$y = 2x^2 - 7$

$y = 7 = 2x^2$

$\dfrac{y+7}{2} = x^2$

$x = \sqrt{\dfrac{y+7}{2}}$

$f^{-1}(x) = \sqrt{\dfrac{x+7}{2}}$

**To show this graph it is the reflection of the normal function in the line y = x**

# 3 Exponential and log functions

## 3.1 Formulas for exponential growth or decay

Example: $P = 16000e^{-\dfrac{t}{10}}$ Where P is the Price in £s and t is the years from new

*What was the price when new?*
**Substitute t=0**

$P = 16000e^{-\dfrac{0}{10}}$
$P = 16000 \times 1$

*What is the value at 5 years old*
**Substitute t=5**
$P = 16000e^{-\frac{5}{10}}$
$P = £9704.49$

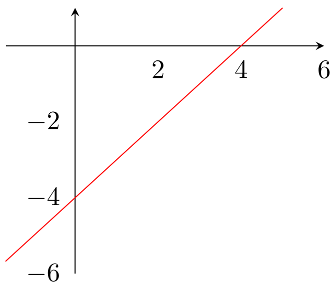
*What does the model say about the eventual value of the car*
As $t \to \infty$, $e^{-\frac{t}{10}} \to \infty$
Therefore $P \to 16000 \times 0 = 0$
The eventual value is zero.

## 3.2 The inverse of the exponential function

The function $f(x) = \ln x$ has domain $\{x \in \mathbb{R},\, x > 0\}$ and range $\{f(x) \in \mathbb{R}\}$

# 4 Numerical methods

## 4.1 Approximations for roots based on graphs




On this graph if the value of $f(x)$ was to be found at 2 it would be **negative**, whereas if it was to be found at 6 it would be **positive**, this implies that there is a root in-between 2 and 6

The exception to this rule is $f(x) = \frac{1}{x}$ as there is a discontinuity at $x = 0$, however there is no root

## 4.2 Iteration for finding approximations of roots

To solve an equation of the form $f(x) = 0$ by an iterative method, rearrange $f(x) = 0$ into a for $x = g(x)$ and use the iterative formula $x_{n+1} = g(x_n)$.

Example, find a root of the equation $x^2 - 4x + 1 = 0$
Re-write as $x = 4 - \dfrac{1}{x}$

Create the formula $x_{n+1} = 4 - \dfrac{1}{x_n}$
You get given a rough approximation, $x_0 = 3$
Substitute
$x_1 = 4 - \dfrac{1}{x_0}$
$x_1 = 4 - \dfrac{1}{3}$
$x_1 = \dfrac{11}{3}$

$x_2 = 4 - \dfrac{1}{\frac{11}{3}}$
$x_2 = \dfrac{41}{11}$
Continuing this increases the accuracy of the result.
This may not work and will not converge to a root.

# 5 Further Trigonometric identities and their applications

## 5.1 The R formula

For positive values of $a$ and $b$
$a \sin\theta \pm b \cos\theta$ can be expressed in the form $R \sin(\theta \pm \alpha)$, where $0 < \alpha < 90$
$a \cos\theta \pm b \sin\theta$ can be expressed in the form $R \cos(\theta \mp \alpha)$, where $0 < \alpha < 90$
$R = \sqrt{a^2 + b^2}$
$\alpha = \arctan(\frac{b}{a})$