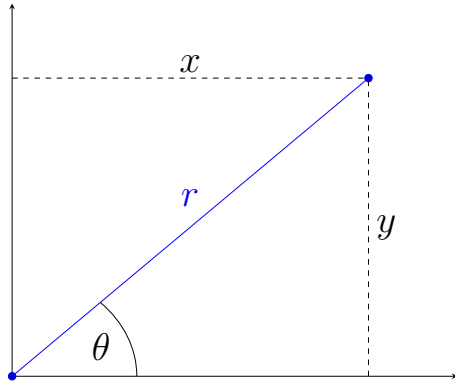


# Polar Coordinates



Usual conventions are either  $-\pi < \theta \leq \pi$  or  $0 \leq \theta < \pi$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

## 1 Converting between polar and Cartesian form

### 1.1 Example 1

Find the Cartesian equation of:  $r = 5$

$$\sqrt{x^2 + y^2} = 5$$

$$x^2 + y^2 = 25$$

### 1.2 Example 2

Find the Cartesian equation of:

$$r = 2 + \cos 2\theta$$

Replace  $r$  and convert  $\cos 2\theta$

$$\sqrt{x^2 + y^2} = 2 + \cos^2 \theta - \sin^2 \theta$$

Convert  $\sin^2 \theta$  and  $\cos^2 \theta$

$$\sqrt{x^2 + y^2} = 2 + \frac{x^2}{x^2 + y^2} - \frac{y^2}{x^2 + y^2}$$

Multiply all terms by  $x^2 + y^2$

$$(x^2 + y^2)^{\frac{3}{2}} = 3x^2 + y^2$$

## 2 Sketching polar curves

To plot less standard types we look for axes intercepts and max and min values

### 2.1 Standard types

#### 2.1.1 $r=a$

A circle, centre  $(0,0)$  with a radius of  $a$

#### 2.1.2 $\theta = \alpha$

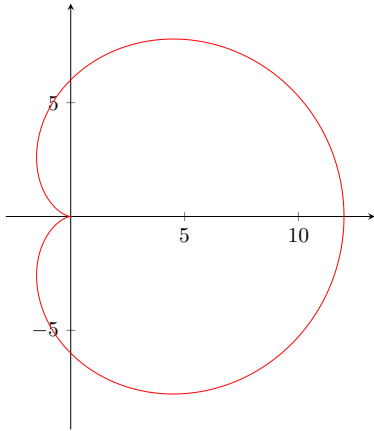
A half line starting from  $(0,0)$  making an angle of  $\alpha$  with the initial line (positive  $x$  axis)

#### 2.1.3 $r = a\theta$

A spiral starting at the origin

### 2.2 Cardioid type

#### 2.2.1 $r = a(1 + \cos \theta)$

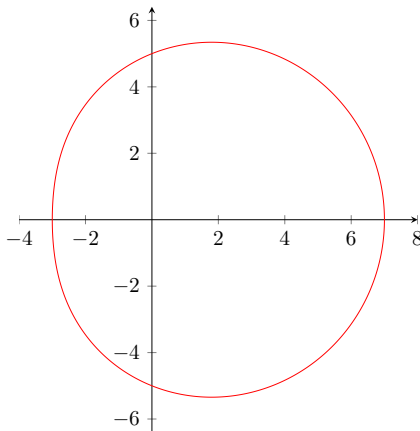


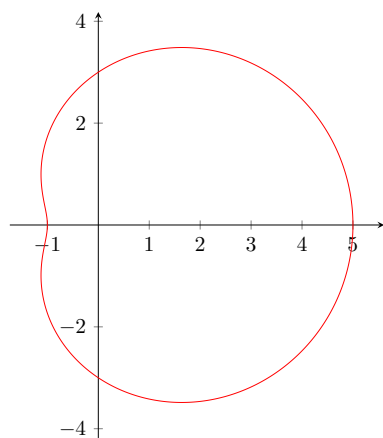
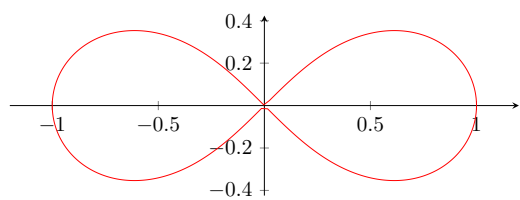
#### 2.2.2 $r = a(p + q \cos \theta)$

##### 2.2.2.1 $p=q$

Factor out the value of  $p$  and plot as normal

##### 2.2.2.2 $p \geq 2q$



**2.2.2.3**  $q \leq p < 2q$ **2.2.3**  $r^2 = a^2 \cos 2\theta$ 

The 4 asymptotes are half lines at  $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$