Kinematics

1 Horizontal Projections

For a **constant speed** use $Speed = \frac{Distance}{Time}$
For a **constant acceleration** use **SUVAT**
For all projections:

- Assume air resistance to be zero
- Resolve horizontal and vertical motion
- Horizontal - Constant speed
- Vertical - Constant acceleration

2 Angular projections

The same as horizontal projections but the initial vertical velocity isn't zero.
Example:
A particle is projected at a speed of $49ms^{-1}$ at an angle of 45° above the horizontal.
What is the time taken for the particle to reach its maximum height?

- u=$49\sin 45$
- v=0
- a=-g
- t=?

$0 = 49\sin 45 - gt$
$t = \frac{49\sin 45}{g} = \frac{5\sqrt{2}}{2} \approx 3.54$
What is the maximum height reached?

- u=$49\sin 45$
- v=0
- a=-g
- s=?

$v^2 = u^2 + 2as$
0=$(49\sin 45)^2 - 2gs$
$S = \frac{(49\sin 45)^2}{2g} = 61.3$
What is the time of the flight?

- u=$49\sin 45$
- a=-g
- S=0
- t=?

$S = ut + \frac{1}{2}at^2$

$0 = (49\sin 45)t - \frac{1}{2}gt^2$

0=$t(49\sin 45 - \frac{gt}{2}$
$t = 0$

$49 \sin 45 = \frac{gt}{2}$

$t = \frac{2 \times 49 \sin 45}{g} = 7.07$

What is the horizontal range of the particle?

- t=7.07
- Speed=$49 \cos 45$

$S = 45 \cos 45 \times 7.07 = 245$

3 Displacement, velocity and acceleration

$v = \frac{dx}{dt}$

$a = \frac{dv}{dt}$

$$\begin{tikzcd}
\textbf{Displacement } x \arrow[r, bend left, "\text{Differentiate}"] & \textbf{Velocity } v \arrow[l, bend left, "\text{Integrate}"] \arrow[r, bend left, "\text{Differentiate}"] & \textbf{Acceleration } a \arrow[l, bend left, "\text{Integrate}"]
\end{tikzcd}$$

Kinematics Example - Problems with calculus

A particle P moves along the x-axis in a straight line so that, at time t seconds, the velocity of P is v ms^{-1}, where

$$v = \begin{cases} 10t - 2t^2, & \text{for } 0 \leqslant t \leqslant 6 \\ -\dfrac{432}{t^2}, & t > 6 \end{cases}$$

At $t = 0$, P is at the origin O. Find the displacement of P from O when

$t = 6$

Integrate velocity

$x = \int v \, dt$

$x = \int 10t - 2t^2 \; dt = 5t^2 - \frac{2}{3}t^3 + c$

Substitute in the value of t

$t = 6$

$x = 5 \times 6^2 - \frac{2}{3} \times 6^3 = 36m$

$t = 10$

Integrate velocity for the second half of the journey

$$x = \int -\frac{432}{t^2} \; dt = \int -432t^{-2} \; dt = \frac{-432t^{-1}}{-1} + k = \frac{432}{t} + k$$

Find value of k by using known distance at t=6

$t = 6 \quad x = 36$

$36 = \dfrac{432}{6} + k$

$k = 36 - 72 = -36$

Find distance using value of k and t

$t = 10$

$x = \dfrac{432}{10} - 36 = 7.2\text{m}$

Kinematics Example - Finding direction of motion

At $t = 0$ a particle P is projected from a fixed point O with velocity $(7i + 7\sqrt{3})ms^{-1}$. The particle moves freely under gravity. The position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})m$ relative to O.
Show that:

$$y = \sqrt{3}x - \frac{g}{98}x^2$$

x has constant velocity so write in terms of t

$$x = 7t \tag{1}$$

Write an equation for y using $s = ut + \frac{1}{2}at^2$

$$y = 7\sqrt{3}t - \frac{g}{2}t^2 \tag{2}$$

Substitute (1) into the (2)

$$y = \sqrt{3}x - \frac{g}{2} \times \left(\frac{x}{7}\right)^2$$

Simplify

$$y = \sqrt{3}x - \frac{g}{98}x^2 \tag{3}$$

Find the direction of motion of P when it passes through the point on the path where $x = 20$

Differentiate (3)

$$\frac{dy}{dx} = \sqrt{3} - \frac{2gx}{98}$$

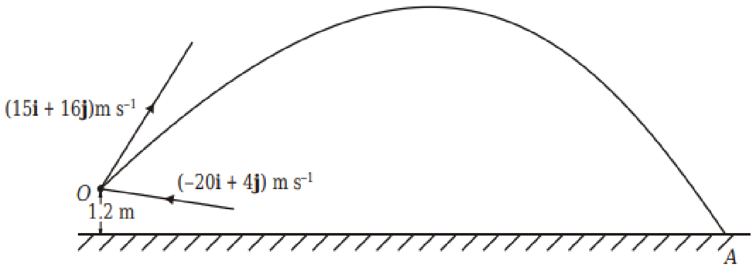
Substitute in the value of x

$$\frac{dy}{dx} = \sqrt{3} - \frac{40g}{98}$$

Arctan(gradient)=Angle to the positive horizontal as $\frac{dy}{dx}$ is the same as $\frac{O}{A}$
Perform arctan on the gradient to find the angle

$$\arctan\left(\sqrt{3} - \frac{40g}{98}\right) = -66.2$$

Problems with Vectors



A ball B of mass 0.4 kg is struck by a bat at a point O which is 1.2 m above horizontal ground. The unit vectors i and j are respectively horizontal and vertical. Immediately before being struck, B has velocity $(-20i+4j)$ ms^{-1}. Immediately after being struck it has velocity $(15i+16j)$ ms^{-1}.
After B has been struck, it moves freely under gravity and strikes the ground at the point A, as shown in the diagram above. The ball is modelled as a particle.

Calculate the magnitude of the impulse exerted by the bat on B.

$$I = m(v-u)$$

Calculate the impulse as a vector using information from the question

$$I = 0.4(15i + 16j - (-20i + 4j) = 14i + 4.8j$$

Calculate the magnitude of this vector

$$|I| = \sqrt{14^2 + 4.8^2} = 14.5Ns$$

By using the principle of conservation of energy, or otherwise, find the speed of B when it reaches A. Initial energy:

$$\frac{1}{2} \times 0.4 \times \left(\sqrt{15^2 + 16^2}\right)^2 = 96.2J$$

Energy gained from loss in GPE

$$0.4 \times 9.8 \times 1.2 = 4.704J$$

Add energies and set equal to final kinetic energy

$$4.704 + 96.2 = \frac{1}{2} \times 0.4 \times v^2$$

Find v

$$v = \sqrt{\frac{4.704 + 96.2}{0.5 \times 0.4}} = 22.46$$

Calculate the angle which the velocity of B makes with the ground when B reaches A.
Using the knowledge that horizontal velocity is constant and the speed is known, use cos to find the angle

$$\arccos\left(\frac{15}{22.5}\right) = 48°$$

State two additional physical factors which could be taken into account in a refinement of the model of the situation which would make it more realistic

- Air resistance
- Wind
- Rotation of ball (ball is not a particle)