Coordinate Geometry

1 Parametric equations

In parametric equations, coordinates x and y are expressed as functions.
$x = f(t)$ and $y = g(t)$ where t is the independent variable (called a parameter)

1.1 Converting parametric equations into Cartesian form

Example 1
$x = 2t$
$y = t^2$

$t = \frac{x}{2}$

$y = \frac{1}{4}x^2$

Example 2
$x = \sin(t) + 2$
$y = \cos(t) - 3$

$\sin(t) = x - 2$
$\cos(t) = y + 3$

$\sin^2\theta + \cos^2\theta = 1$
$(x-2)^2 + (y+3)^2 = 1$

2 Parametric differentiation

The chain rule can be used to differentiate parametric equations
$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$
This can be used to find the gradient of normals and tangents.

Example
$x = t^3 + t$
$y = t^2 + 1$

Differentiate the x and y terms
$\frac{dy}{dt} = 2t$
$\frac{dx}{dt} = 3t^2 + 1$

Invert the differential of x to fit the formula
$\frac{dt}{dx} = \frac{1}{3t^2+1}$

Multiply together to make one fraction
$\frac{dy}{dx} = \frac{2t}{3t^2+1}$