Hypothesis tests - Poisson - Exam Questions

1 One tailed test

An effect of a certain disease is that a small number of the red blood cells are deformed. Emily has this disease and the deformed blood cells occur randomly at a rate of 2.5 per ml of her blood. Following a course of treatment, a random sample of 2 ml of Emily's blood is found to contain only 1 deformed red blood cell.

Stating your hypotheses clearly and using a 5% level of significance, test whether or not there has been a decrease in the number of deformed red blood cells in Emily's blood.

Write the null and alternate hypothesis, using information from the text

$$H_0: \ \lambda = 5$$

$$H_1: \ \lambda < 5$$

Write down the distribution the hypothesis test will be done on

$$X \sim P_o(5)$$

Write down a probability to test based on the data, in this case 1 one deformed red blood cell

$$P(X \leqslant 1)$$

Find this value in the tables, or using an approximation if needed

$$P(X \leqslant 1) = 0.0404$$

Compare this to the significance level

$$0.0404 < 0.05$$

Write the conclusion, if in the critical region, reject H_0, if not accept it

As $0.0404 < 0.05$ the result is significant, so reject H_0
There is evidence in a **decrease** in the mean **rate** of **deformed blood cells**

2 Finding critical values

A test statistic has a Poisson distribution with parameter λ.
Given that:

$$H_0: \ \lambda = 9, \ H_1: \ \lambda \neq 9$$

Find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%.

Write down the distribution

$$X \sim P_o(9)$$

Find the lower value by looking on the tables at the column related to λ and look for the closest value to the probability wanted (in this case 2.5%)

$$P(X \leqslant 3) = 0.0212 \approx 0.025$$

To find the upper value look for the value closest to (1-wanted probability). Then add one to this value as:

$$P(X \geqslant c) = 1 - P(X \leqslant c - 1)$$

$$P(X \leqslant 15) = 0.9780 \approx 0.975$$

$$P(X \geqslant 16) = 1 - 0.9780 = 0.022 \approx 0.025$$

Critical values are 3 and 16

Write down the actual significance
Actual significance is the probability of a value being found in the critical region.

$$0.0212 + 0.022 = 0.0432$$

3 Two tailed test

Bacteria are randomly distributed in a river at a rate of 5 per litre of water. A new factory opens and a scientist claims it is polluting the river with bacteria. He takes a sample of 0.5 litres of water from the river near the factory and finds that it contains 7 bacteria. Stating your hypotheses clearly test, at the 5% level of significance, the claim of the scientist.

Write down the null and alternative hypothesis

$$H_0 : \ \lambda = 2.5$$

$$H_1 \ \lambda \neq 2.5$$

Write down the distribution to be tested

$$X \sim P_o(2.5)$$

Find a probability to test from the data

$$P(X \geqslant 7) = 1 - P(X \leqslant 6)$$

Look up the probability on the tables

$$1 - P(X \leqslant 6) = 1 - 0.9858 = 0.0142$$

Compare with the significance level, remembering to split it

$$0.0142 < 0.025$$

Write the conclusion

There is significant evidence that the factory is polluting the river with bacteria