

Discrete Distributions Exam Questions

A bag contains a large number of coins. It contains only 1p and 2p coins in the ratio 1:3

1 Populations

1.1 Mean and variance

1.1.1 Mean

Find the mean of the population of coins

Use the formula on the data sheet:

$$\mu = \sum xP(X = x)$$

Create a table of probabilities and values of x

x	1p	2p
$P(X = x)$	$\frac{1}{4}$	$\frac{3}{4}$

Add together the columns multiplied together

$$\mu = 1 \times \frac{1}{4} + 2 \times \frac{3}{4} = \frac{7}{4}$$

1.1.2 Variance

Find the variance of the population of coins

Use the formula on the data sheet:

$$\mu = \sum x^2 P(X = x) - \mu^2$$

Create a table of probabilities and values of x

x^2	1	4
$P(X = x)$	$\frac{1}{4}$	$\frac{3}{4}$

Add together the columns multiplied together to find $E(X^2)$

$$E(X^2) = 1 \times \frac{1}{4} + 4 \times \frac{3}{4} = \frac{13}{4}$$

Subtract find $E(X^2) - E(X)^2$ to find the variance

$$\frac{13}{4} - \left(\frac{7}{4}\right)^2 = \frac{3}{16}$$

2 Samples

List all the possible samples that can be drawn

When doing this remember:

$$\text{Number of samples} = \text{Number of options}^{\text{Sample size}}$$

For the example:

Number of options=2

Sample size=3

Number of samples= $2^3=8$

List of samples:

(1,1,1)

(1,1,2) $\times 3$

(1,2,2) $\times 3$

(2,2,2)

Find the sampling distribution of the mean value of these samples.

List the possible means

$$1, \frac{4}{3}, \frac{5}{3}, 2$$

Calculate the probability of each, given the probability of picking a coin

$$1 : \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

$$\frac{4}{3} : 3 \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{9}{64}$$

$$\frac{5}{3} : 3 \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$$

$$2 : \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

Write in a table

\bar{x}	1	$\frac{4}{3}$	$\frac{5}{3}$	2
$P(\bar{X} = \bar{x})$	$\frac{1}{64}$	$\frac{9}{64}$	$\frac{27}{64}$	$\frac{27}{64}$