

C4 Notes

1 Binomial Expansion

$(a \pm bx)^n$ can be rewritten as $a^n(1 \pm \frac{b}{a}x)^n$

2 Coordinate Geometry

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

3 Differentiation

3.1 Implicit differentiation

$$\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$$

Use the formula on the data sheet to differentiate $y = a^x$

$$\frac{d}{dx}(a^{f(x)}) = a^{f(x)} f'(x) \ln(a)$$

4 Integration

| $f(x)$ | $\int f(x) \, dx$ |
|---------------|---------------------------|
| x^n | $\frac{x^{n+1}}{n+1} + c$ |
| e^x | $e^x + c$ |
| $\frac{1}{x}$ | $\ln x + c$ |
| $\sin x$ | $-\cos x$ |
| $\cos x$ | $\sin x$ |
| a^x | $\frac{a^x}{\ln a } + c$ |

4.1 Parametric integration

$$\int y \frac{dx}{dt} \, dt$$

4.2 Integration by substitution - Fractional type

$$\int \frac{f'(x)}{f(x)} \, dx = \ln f(x) + c$$

4.3 Trapezium rule

$$\%error = \frac{\text{Approx-Actual}}{\text{Actual}} \times 100$$

4.4 Volumes of revolution

$$\text{Volume} = \pi \int_a^b y^2 \, dx$$

4.4.1 Parametric volumes of revolution

$$\text{Volume} = \pi \int_a^b y^2 \frac{dx}{dt} dt$$

5 Partial fractions

5.1 Partial fractions with same or higher denominator

When the denominator is the same or higher as the numerator, use long division to find the remainder and re-write it so that it can be converted into partial fractions.

6 Vectors

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

6.1 Vector dot product

$$\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

$$\mathbf{a} \cdot \mathbf{b} = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$