

Kinematics

1 Horizontal Projections

For a **constant speed** use $Speed = \frac{Distance}{Time}$

For a **constant acceleration** use **SUVAT**

For all projections:

- Assume air resistance to be zero
- Resolve horizontal and vertical motion
- Horizontal - Constant speed
- Vertical - Constant acceleration

2 Angular projections

The same as horizontal projections but the initial vertical velocity isn't zero.

2.1 Example

A particle is projected at a speed of 49ms^{-1} at an angle of 45° above the horizontal.

What is the time taken for the particle to reach its maximum height?

$$u = 49 \sin 45$$

$$v = 0$$

$$a = -g$$

$$t = ?$$

$$0 = 49 \sin 45 - gt$$

$$t = \frac{49 \sin 45}{g} = \frac{5\sqrt{2}}{2} \approx 3.54$$

What is the maximum height reached?

$$u = 49 \sin 45$$

$$v = 0$$

$$a = -g$$

$$s = ?$$

$$v^2 = u^2 + 2as$$

$$0 = (49 \sin 45)^2 - 2gs$$

$$S = \frac{(49 \sin 45)^2}{2g} = 61.3$$

What is the time of the flight?

$$u = 49 \sin 45$$

$$a = -g$$

$$S = 0$$

$$t = ?$$

$$S = ut + \frac{1}{2}at^2$$

$$0 = (49 \sin 45)t - \frac{1}{2}gt^2$$

$$49 \sin 45 = \frac{gt}{2}$$

$$t = \frac{2 \times 49 \sin 45}{g} = 7.07$$

What is the horizontal range of the particle?

$$t = 7.07$$

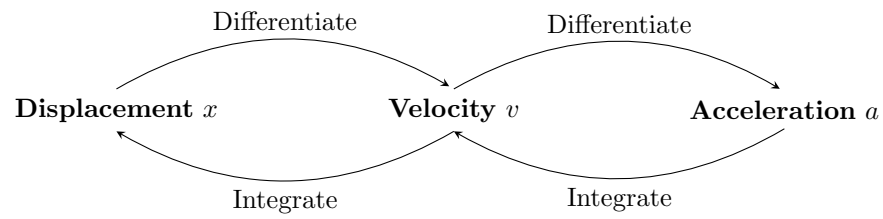
$$\text{Speed} = 49 \cos 45$$

$$S = 49 \cos 45 \times 7.07 = 245$$

3 Displacement, velocity and acceleration

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$



Kinematics Example - Problems with calculus

A particle P moves along the x -axis in a straight line so that, at time t seconds, the velocity of P is $v \text{ ms}^{-1}$, where

$$v = \begin{cases} 10t - 2t^2, & \text{for } 0 \leq t \leq 6 \\ -\frac{432}{t^2}, & t > 6 \end{cases}$$

At $t = 0$, P is at the origin O . Find the displacement of P from O when $t = 6$

Integrate velocity

$$x = \int v \, dt$$

$$x = \int 10t - 2t^2 \, dt = 5t^2 - \frac{2}{3}t^3 + c$$

Substitute in the value of t

$$t = 6$$

$$x = 5 \times 6^2 - \frac{2}{3} \times 6^3 = 36\text{m}$$

Find the displacement from O when $t = 10$

Integrate velocity for the second half of the journey

$$x = \int -\frac{432}{t^2} \, dt = \int -432t^{-2} \, dt = \frac{-432t^{-1}}{-1} + k = \frac{432}{t} + k$$

Find value of k by using known distance at $t=6$

$$t = 6 \quad x = 36$$

$$36 = \frac{432}{6} + k$$

$$k = 36 - 72 = -36$$

Find distance using value of k and t

$$t = 10$$

$$x = \frac{432}{10} - 36 = 7.2\text{m}$$

Kinematics Example - Finding direction of motion

At $t = 0$ a particle P is projected from a fixed point O with velocity $(7\mathbf{i} + 7\sqrt{3})\text{ms}^{-1}$. The particle moves freely under gravity. The position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})\text{m}$ relative to O .

Show that:

$$y = \sqrt{3}x - \frac{g}{98}x^2$$

x has constant velocity so write in terms of t

$$x = 7t \tag{1}$$

Write an equation for y using $s = ut + \frac{1}{2}at^2$

$$y = 7\sqrt{3}t - \frac{g}{2}t^2 \tag{2}$$

Substitute (1) into the (2)

$$y = \sqrt{3}x - \frac{g}{2} \times \left(\frac{x}{7}\right)^2$$

Simplify

$$y = \sqrt{3}x - \frac{g}{98}x^2 \tag{3}$$

Find the direction of motion of P when it passes through the point on the path where $x = 20$

Differentiate (3)

$$\frac{dy}{dx} = \sqrt{3} - \frac{2gx}{98}$$

Substitute in the value of x

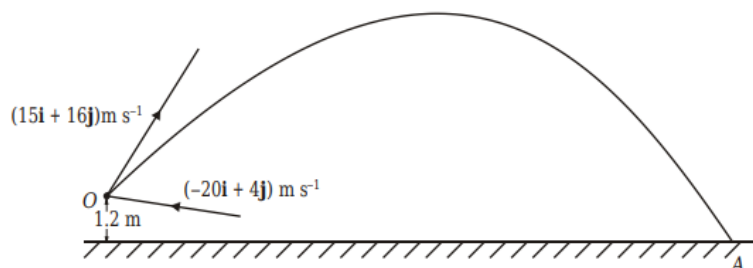
$$\frac{dy}{dx} = \sqrt{3} - \frac{40g}{98}$$

Arctan(gradient)=Angle to the positive horizontal as $\frac{dy}{dx}$ is the same as $\frac{O}{A}$

Perform arctan on the gradient to find the angle

$$\arctan\left(\sqrt{3} - \frac{40g}{98}\right) = -66.2$$

Problems with Vectors



A ball B of mass 0.4 kg is struck by a bat at a point O which is 1.2 m above horizontal ground. The unit vectors \mathbf{i} and \mathbf{j} are respectively horizontal and vertical. Immediately before being struck, B has velocity $(-20\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$. Immediately after being struck it has velocity $(15\mathbf{i} + 16\mathbf{j}) \text{ m s}^{-1}$.

After B has been struck, it moves freely under gravity and strikes the ground at the point A, as shown in the diagram above. The ball is modelled as a particle.

Calculate the magnitude of the impulse exerted by the bat on B.

$$I = m(v - u)$$

Calculate the impulse as a vector using information from the question

$$I = 0.4(15\mathbf{i} + 16\mathbf{j} - (-20\mathbf{i} + 4\mathbf{j})) = 14\mathbf{i} + 4.8\mathbf{j}$$

Calculate the magnitude of this vector

$$|I| = \sqrt{14^2 + 4.8^2} = 14.5 \text{ N s}$$

By using the principle of conservation of energy, or otherwise, find the speed of B when it reaches A. Initial energy:

$$\frac{1}{2} \times 0.4 \times \left(\sqrt{15^2 + 16^2} \right)^2 = 96.2 \text{ J}$$

Energy gained from loss in GPE

$$0.4 \times 9.8 \times 1.2 = 4.704 \text{ J}$$

Add energies and set equal to final kinetic energy

$$4.704 + 96.2 = \frac{1}{2} \times 0.4 \times v^2$$

Find v

$$v = \sqrt{\frac{4.704 + 96.2}{0.5 \times 0.4}} = 22.46$$

Calculate the angle which the velocity of B makes with the ground when B reaches A.

Using the knowledge that horizontal velocity is constant and the speed is known, use cos to find the angle

$$\arccos\left(\frac{15}{22.5}\right) = 48^\circ$$

State two additional physical factors which could be taken into account in a refinement of the model of the situation which would make it more realistic

- Air resistance
- Wind
- Rotation of ball (ball is not a particle)