FP2

1 First order differential equations

1.1 Solving first order DE using an integrating factor

Solving $\frac{dy}{dx}+P(x)y=Q(x)$

IF(Integrating factor) is found by finding $e^{\int p(x)\ dx}$ And multiplying the DE by the IF.

This will result in the DE being in the form:

$$f(x)\frac{dy}{dx}+f'(x)y$$

This form can then be shortened by integrating:

$$\int f'(x)g(x)+f(x)g'(x)dx=f(x)g(x)+c$$

Integrate both sides then simplify

2 Further complex numbers

2.1 Converting between forms

When converting from $x+iy$ to a form in r and θ, take the angle from the positive x axis

2.2 Multiplying and dividing complex numbers

It is easiest to use the exponential form, then convert if needed

2.2.1 Multiplying

$$Z_1Z_2=r_1e^{i\theta_1}\times r_2e^{i\theta_2}=r_1r_2e^{i(\theta_1+\theta_2)}$$

2.2.2 Dividing

$$\frac{Z_1}{Z_2}=r_1e^{i\theta_1}\div r_2e^{i\theta_2}=\frac{r+1}{r+2}e^{i(\theta_1-\theta_2)}$$

2.3 De Moivre's theorem

This is given on the data sheet

2.3.1 Z formulas

If $z=\cos\theta+i\sin\theta$

$$z+\frac{1}{z}=2\cos\theta$$

$$z-\frac{1}{z}=2i\sin\theta$$

$$z^n+\frac{1}{z^n}=2\cos n\theta$$

$$z^n-\frac{1}{z^n}=2i\sin n\theta$$

2.4 Loci on the complex plane

Equation	Description
$\lvert z - z_1 \rvert = r$	A circle centre (x_1, y_1) with a radius r
$\lvert z - z_1 \rvert = \lvert z - z_2 \rvert$	A perpendicular bisector of the line segment joining points z_1 and z_2
$\arg(z - z_1) = \theta$	The half line from a fixed point z_1, making an angle θ with the positive real axis
$\arg(\frac{z-z_1}{z-z_2}) = \theta$	An arc between the points z_1 and z_2 where the angle the lines from z_1 and z_2 to any point on the arc is θ

2.5 Translations

- $w = z + a + ib$ represents a translation with translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$
- $w = kz$ represents an enlargement with scale factor k centre $(0, 0)$
- $w = kz + a + ib$ represents an enlargement scale factor k centre $(0, 0)$ followed by a translation with translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$
- $w = z^2$ multiply a shape by itself, for example a circle of radius 4 would go to radius 16

3 Inequalities

We can build upon our previous algebraic skills in order to solve more complex inequalities
Remember:

- Don't multiply anything that could be negative - use "squared" things
- Find the critical values (f(x)=0)
- Sketch the graph to solve

4 Maclaurin and Taylor Series

Use the formulas on the data sheet

4.1 Solving differential equations using the Taylor expansion

From the differential equation, calculate the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ etc up to whatever is needed.
Then substitute those values into the Taylor series to solve the differential equation