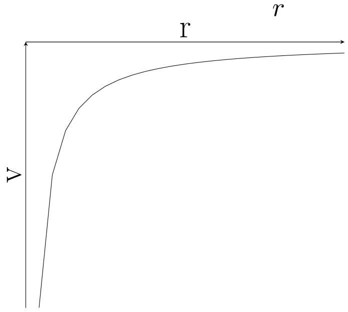
Gravitational Fields Notes

Gravitational field strength, g, is the force felt per unit mass on a unit mass placed at that point in a field.
$g = \frac{F}{m}$, unit=NKg^{-1}
The gravitational field is around all objects with mass and can be either **radial** or **uniform**.

1 Introduction to Gravitational potential

Definition: The potential at a point in a field is the work done in moving unit mass from infinity to that point.

$V = \frac{\text{Work done}}{\text{Unit mass}}$ Unit: JKg^{-1}
In a radial field $v = \frac{-GM}{r}$



Gravitational potential energy $= \Delta V \times m$
$V_o - V_s = \Delta V$ Unit:JKg^{-1}
$V_s =$ Potential on earth, $V_o =$ Potential in orbit.

2 Potential gradient

Potential gradient - Change in potential per unit change in distance
$g = -\frac{\Delta V}{\Delta r}$

3 Newton's law of gravitation

The force between two masses is attractive and is directly proportional to the product of the masses and is inversely proportional to the distance between them squared.
$F = \frac{GM_1 M_2}{r^2}$

4 Escape Velocity

$\frac{1}{2}mv^2 = \frac{GMm}{r}$
$\frac{1}{2}v^2 = \frac{GM}{r}$
$V^2 = \frac{2GM}{r}$
$V = \sqrt{\frac{2GM}{r}}$

5 Satellite motion

The force between a planet of mass M and the satellite of mass m is described by Newton's law of gravitation.
This force provides the **centripetal force** acting towards the centre of the orbit.
$\frac{GMm}{r^2} = \frac{mv^2}{r}$
$GM = v^2 r$
$\sqrt{\frac{GM}{r}} = V$
$V \propto r^{-\frac{1}{2}}$

6 Kepler's 3rd law

$F = \frac{GMm}{r^2} = \frac{mv^2}{r} = m\omega^2 r$
$\frac{GMm}{r^2} = m\omega^2 r$
$\frac{GM}{r^3} = (\frac{2\pi}{T})^2$
$\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$
$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ - Constant for a planet
As $r \uparrow T \downarrow$

7 Geostationary orbits

Sometimes called geosynchronous orbits
$T = 24h = 86400s$
$r = 42Mm = 35857kmabovetheearth$
Geostationary - Orbiting in plane of the equator
Geosynchronous - 24h orbit inclined at an angle to the equator

Geostationary orbits:

- Period of 24h
- 36,000 km above the earth's surface
- Circular
- Equatorial (in plane of the equator)
- Are in the same direction as the earth's rotation

8 The time period of satellites

$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$
$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$
$T^2 = (\frac{4\pi^2}{GM})r^3$
$T^2 \propto r^3$
$T = kr^{\frac{3}{2}}$
$\log T = \frac{3}{2}\log r + \log k$ is in the form $y = mx + c$