

# Simple Harmonic Motion

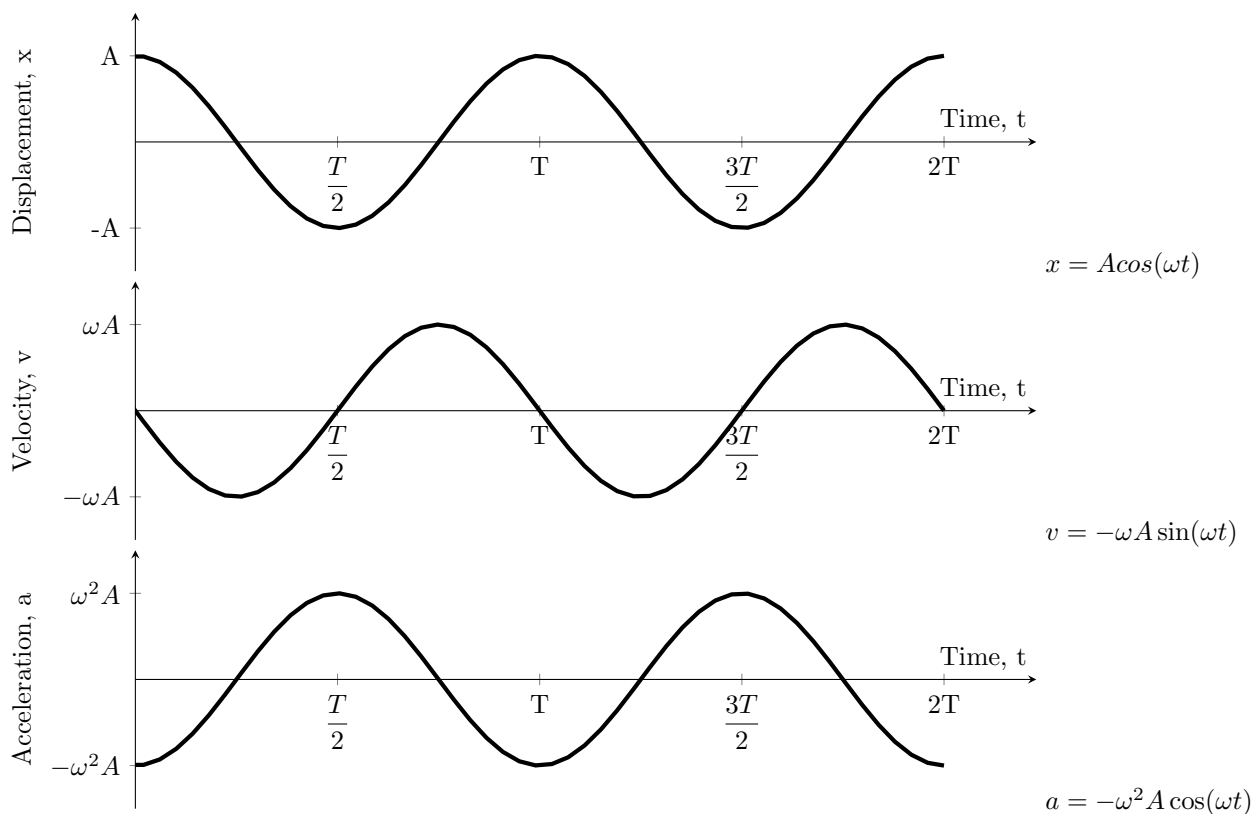
## 1 Introduction

An object is undergoing Simple harmonic motion if:

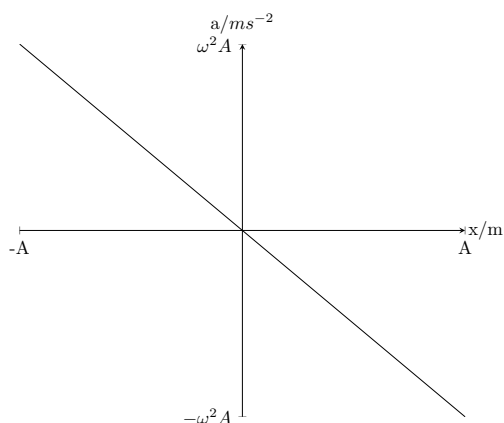
- Its acceleration is proportional to its displacement.
- Acceleration is directed to the equilibrium point (in the opposite direction to the displacement).

Rules of simple harmonic motion:

- The object can't have frictional forces (no damping)
- The oscillations must be small displacements



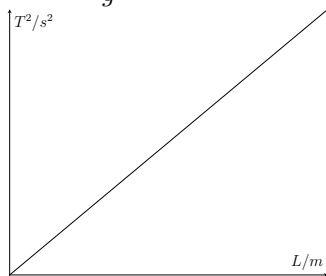
## 2 Acceleration vs Displacement SHM Graph



### 3 Period of a pendulum

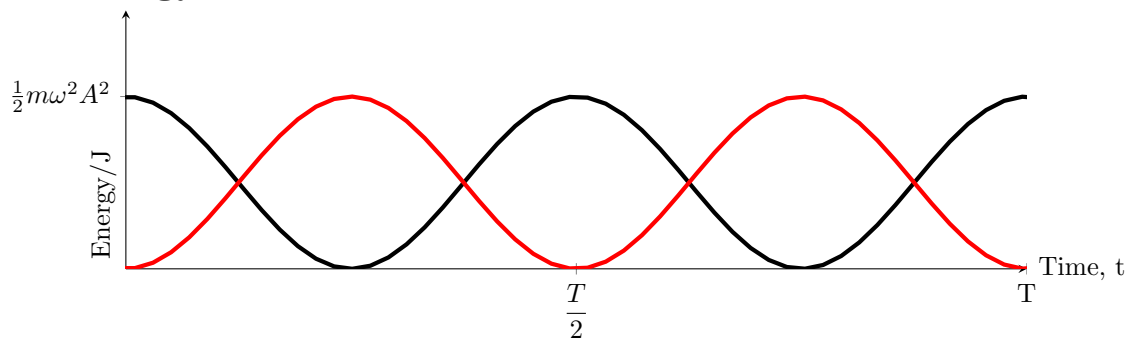
$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = \frac{4\pi^2 L}{g}$$



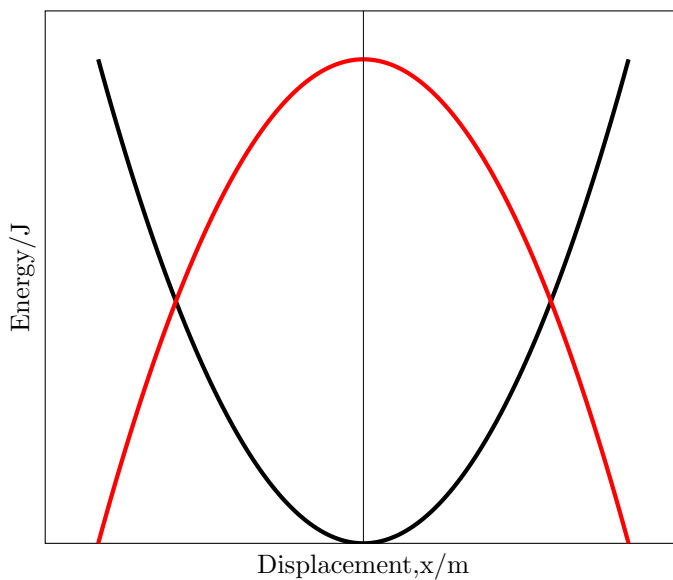
The gradient of the graph is  $\frac{4\pi^2}{g}$

### 4 Energy in SHM

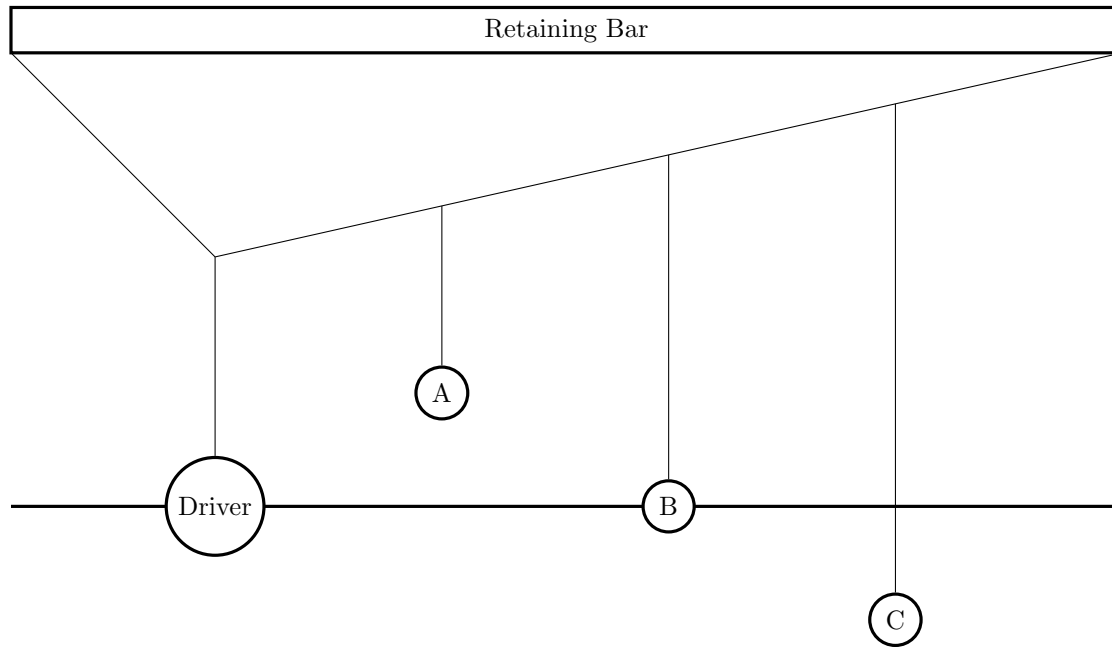


Red - Kinetic Energy

Black - Gravitational potential energy



## 5 Barton's Pendulum



All lengths are measured from the retaining bar to the centre of mass of the bob.

The **natural** frequency is the frequency of an object oscillating in the absence of any driving or damping force **Driver in diagram.**

The **driven** frequency is where an external force is provided, making the oscillator undergo forced vibrations. **A,B and C in diagram.**

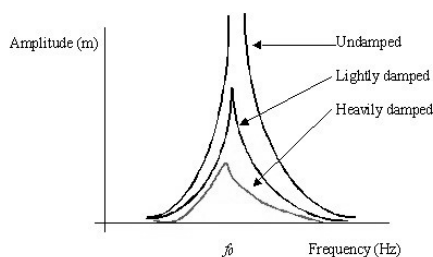
**Resonance** occurs when the frequency of the driving force is equal to the natural frequency and can result in large amplitude oscillations. **B in diagram.**

Pendulum	Natural frequency	Amplitude(wrt driver)	Phase(wrt driver)
A	Higher than driver	Small	In phase
B	Equal to driver (resonance)	Large	Out of phase( $90^\circ$ or $\frac{\pi}{2}$ )
C	Lower than driver	Smaller than resonant	Antiphase( $180^\circ$ or $\pi$ )

## 6 How do engineers prevent resonance

Ways to prevent resonance

- Use several materials - they all have different resonance properties
- Don't have symmetry - prevents standing waves
- Use very stiff materials - High frequency, small wavelengths



## 7 Damping

Damping is the reduction in amplitude due to energy losses (e.g. overcoming friction).

It will also cause the period to increase (it is essentially a driver).

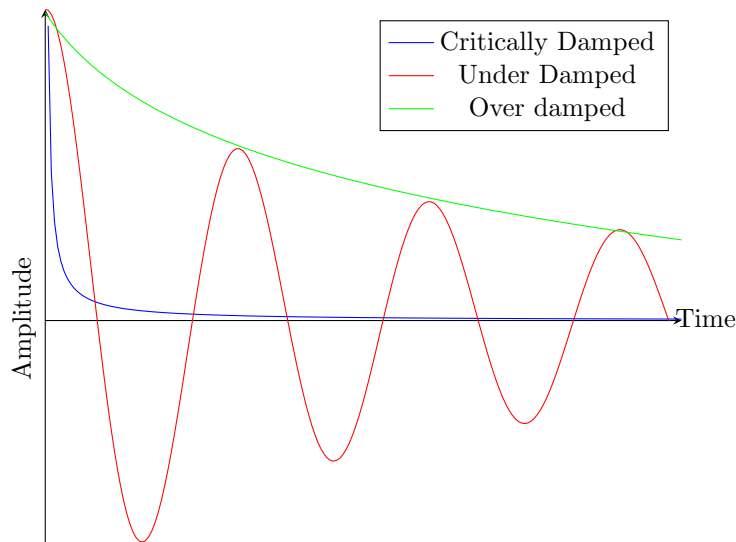
**Light(underdamping)** - Small amplitude change per oscillation

**Heavy(underdamping)** - Large amplitude change per oscillation

**Critical damping** - Reaches equilibrium in the shortest possible time **without** oscillating

**Over damping** - Reaches equilibrium after a long period of time

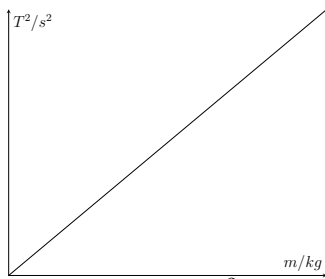
### 7.1 Damping graph



## 8 Mass spring system

### 8.1 Period of a mass-spring system

The period of a mass-spring system is  $T = 2\pi\sqrt{\frac{m}{k}}$  or  $T^2 = \frac{4\pi^2 m}{k}$



The gradient is  $\frac{4\pi^2}{k}$

The line passes through the origin

This is only true for small oscillations as large oscillations go past the limit of proportionality

### 8.2 Energy in a vertically oscillating spring

	KE	GPE	Elastic Energy
Top	0	$2mgA$	$\frac{1}{2}k(e - A)^2$
Middle	$\text{Max} = \frac{1}{2}m\omega^2 A^2$	$mgA$	$\frac{1}{2}ke^2$
Bottom	0	0	$\frac{1}{2}k(e + A)^2$