

C3

1 Algebraic fractions

Algebraic fractions can be simplified by equating the fraction to an equation made of constants

1.1 Example

$$\frac{x^3 + 2x^2 - 6x + 1}{x - 1} \equiv Ax^2 + Bx + C + \frac{D}{x - 1}$$

$$x^3 + 2x^2 - 6x + 1 \equiv (x - 1)(Ax^2 + Bx + C) + D$$

$$x^3 + 2x^2 - 6x + 1 \equiv Ax^3 + (B - A)x^2 + (C - B)x + (D - C)1$$

Term	Calculation	Final Value
x^3 coefficient	$1 = A$	A = 1
x^2 coefficient	$2 = B - A$	B = 3
x coefficient	$-6 = C - B$	C = -3
Constant term	$1 = D - C$	D = -2

$$x^2 + 3x - 3 + \frac{-2}{x - 2}$$

2 Functions

2.1 Definitions

Domain - The input to a function

Range - The output from a function

2.2 Function mapping

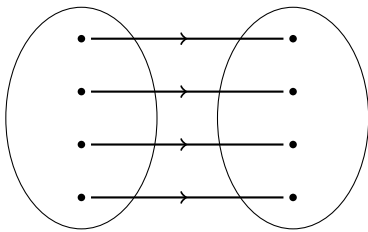


Figure 1: One-to-one function

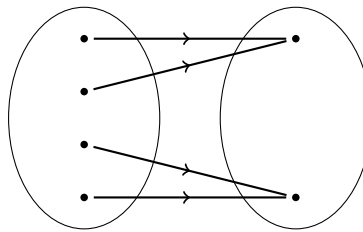


Figure 2: Many-to-one function

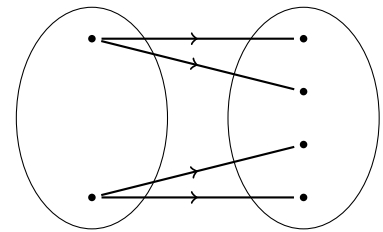


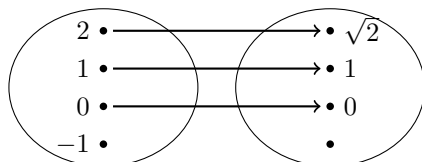
Figure 3: Not a function

A function is a mapping so that every element of the domain maps to exactly one element of the range

2.2.1 Changing non functions to functions

Some non functions can be changed to functions by restricting the domain

For example for $f(x) = \sqrt{x}$ where $x \in \mathbb{R}$ all positive values get mapped, however negative numbers don't, see below:



This means that the domain must be restricted to $x \geq 0$

2.3 Inverse functions

The inverse of $f(x)$ is written as $f^{-1}(x)$

The domain and range of inverse function are the opposite of the normal function.

2.3.1 Finding the inverse function

To find the inverse function isolate x then replace x with $f^{-1}(x)$ and y with x

$$y = 2x^2 - 7$$

$$y + 7 = 2x^2$$

$$\frac{y + 7}{2} = x^2$$

$$x = \sqrt{\frac{y + 7}{2}}$$

$$f^{-1}(x) = \sqrt{\frac{x + 7}{2}}$$

To show this graph it is the reflection of the normal function in the line $y = x$

3 Exponential and log functions

3.1 Formulas for exponential growth or decay

Example: $P = 16000e^{-\frac{t}{10}}$ Where P is the Price in £s and t is the years from new

What was the price when new?

Substitute $t=0$

$$P = 16000e^{-\frac{0}{10}}$$

$$P = 16000 \times 1$$

What is the value at 5 years old

Substitute $t=5$

$$P = 16000e^{-\frac{5}{10}}$$

$$P = £9704.49$$

What does the model say about the eventual value of the car

As $t \rightarrow \infty$, $e^{-\frac{t}{10}} \rightarrow 0$

Therefore $P \rightarrow 16000 \times 0 = 0$

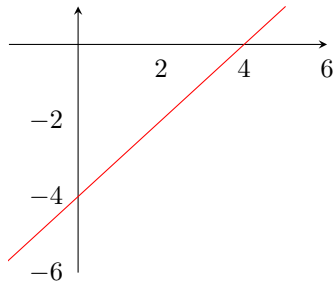
The eventual value is zero.

3.2 The inverse of the exponential function

The function $f(x) = \ln x$ has domain $\{x \in \mathbb{R}, x > 0\}$ and range $\{f(x) \in \mathbb{R}\}$

4 Numerical methods

4.1 Approximations for roots based on graphs



On this graph if the value of $f(x)$ was to be found at 2 it would be **negative**, whereas if it was to be found at 6 it would be **positive**, this implies that there is a root in-between 2 and 6

The exception to this rule is $f(x) = \frac{1}{x}$ as there is a discontinuity at $x = 0$, however there is no root

4.2 Iteration for finding approximations of roots

To solve an equation of the form $f(x) = 0$ by an iterative method, rearrange $f(x) = 0$ into a for $x = g(x)$ and use the iterative formula $x_{n+1} = g(x_n)$.

Example, find a root of the equation $x^2 - 4x + 1 = 0$

Re-write as $x = 4 - \frac{1}{x}$

Create the formula $x_{n+1} = 4 - \frac{1}{x_n}$

You get given a rough approximation, $x_0 = 3$

Substitute

$$x_1 = 4 - \frac{1}{x_0}$$

$$x_1 = 4 - \frac{1}{3}$$

$$x_1 = \frac{11}{3}$$

$$x_2 = 4 - \frac{1}{\frac{11}{3}}$$

$$x_2 = \frac{41}{11}$$

Continuing this increases the accuracy of the result.

This may not work and will not converge to a root.

5 Further Trigonometric identities and their applications

5.1 The R formula

For positive values of a and b

$a \sin \theta \pm b \cos \theta$ can be expressed in the form $R \sin(\theta \pm \alpha)$, where $0 < \alpha < 90$

$a \cos \theta \pm b \sin \theta$ can be expressed in the form $R \cos(\theta \mp \alpha)$, where $0 < \alpha < 90$

$$R = \sqrt{a^2 + b^2}$$

$$\alpha = \arctan\left(\frac{b}{a}\right)$$