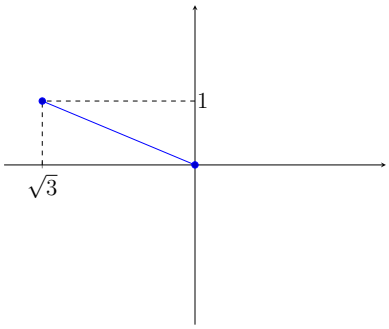
Further Complex Numbers

1 Expressions of complex numbers

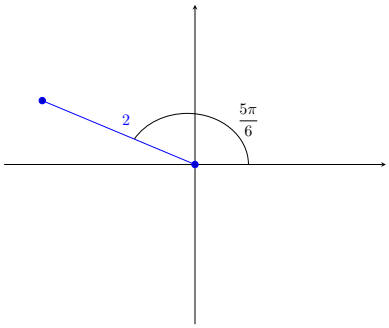
1.1 $\mathbf{x + iy}$

This expresses the coordinate of the point at the end of the vector on the argand diagram.



1.2 $\mathbf{r}(\cos\theta + \mathbf{i}\sin\theta)$

This expresses the length of the line and the angle anticlockwise from the positive x axis



$$2\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$$

1.3 $re^{i\theta}$

This uses the same parameters as $r(\cos\theta + i\sin\theta)$

2 Multiplying and dividing complex numbers

2.1 Multiplying

2.1.1 Trigonometric form

$$Z_1Z_2 = r_1(\cos\theta_1 + i\sin\theta_1) \times r_2(\cos\theta_2 + i\sin\theta_2)$$

$$= r_1r_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2)$$

Apply the cos addition formula to the first two terms

$$= r_1r_2(\cos(\theta_1 + \theta_2) + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2)$$

Apply the sin addition formula to the last two terms

$$= r_1r_2(\cos(\theta_1 + \theta_2) + \sin(\theta_1 + \theta_2))$$

2.1.2 Exponential form

$$Z_1Z_2 = r_1e^{i\theta_1} \times r_2e^{i\theta_2}$$

Apply laws of indices

$$Z_1Z_2 = r_1r_2e^{i(\theta_1+\theta_2)}$$

2.2 Dividing