

Kinematics

1 Horizontal Projections

For a **constant speed** use $Speed = \frac{Distance}{Time}$

For a **constant acceleration** use **SUVAT**

For all projections:

- Assume air resistance to be zero
- Resolve horizontal and vertical motion
- Horizontal - Constant speed
- Vertical - Constant acceleration

2 Angular projections

The same as horizontal projections but the initial vertical velocity isn't zero.

Example:

A particle is projected at a speed of 49ms^{-1} at an angle of 45° above the horizontal.

What is the time taken for the particle to reach its maximum height?

- $u = 49 \sin 45$
- $v = 0$
- $a = -g$
- $t = ?$

$$0 = 49 \sin 45 - gt$$

$$t = \frac{49 \sin 45}{g} = \frac{5\sqrt{2}}{2} \approx 3.54$$

What is the maximum height reached?

- $u = 49 \sin 45$
- $v = 0$
- $a = -g$
- $s = ?$

$$v^2 = u^2 + 2as$$

$$0 = (49 \sin 45)^2 - 2gs$$

$$S = \frac{(49 \sin 45)^2}{2g} = 61.3$$

What is the time of the flight?

- $u = 49 \sin 45$
- $a = -g$
- $S = 0$
- $t = ?$

$$S = ut + \frac{1}{2}at^2$$

$$0 = (49 \sin 45)t - \frac{1}{2}gt^2$$

$$0 = t(49 \sin 45 - \frac{gt}{2})$$

$$t = 0$$

$$49 \sin 45 = \frac{gt}{2}$$

$$t = \frac{2 \times 49 \sin 45}{g} = 7.07$$

What is the horizontal range of the particle?

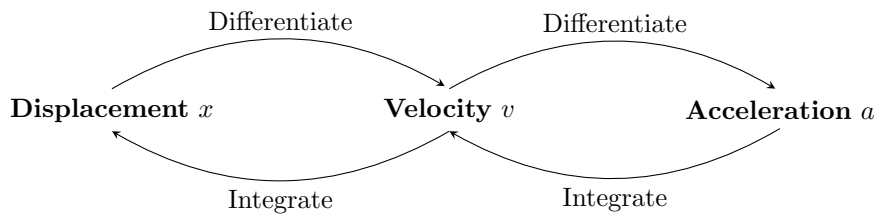
- $t=7.07$
- $\text{Speed}=49 \cos 45$

$$S = 49 \cos 45 \times 7.07 = 245$$

3 Displacement, velocity and acceleration

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$



Kinematics Example - Problems with calculus

A particle P moves along the x -axis in a straight line so that, at time t seconds, the velocity of P is $v \text{ ms}^{-1}$, where

$$v = \begin{cases} 10t - 2t^2, & \text{for } 0 \leq t \leq 6 \\ -\frac{432}{t^2}, & t > 6 \end{cases}$$

At $t = 0$, P is at the origin O . Find the displacement of P from O when

$$t = 6$$

Integrate velocity

$$x = \int v \, dt$$

$$x = \int 10t - 2t^2 \, dt = 5t^2 - \frac{2}{3}t^3 + c$$

Substitute in the value of t

$$t = 6$$

$$x = 5 \times 6^2 - \frac{2}{3} \times 6^3 = 36m$$

$$t = 10$$

Integrate velocity for the second half of the journey

$$x = \int -\frac{432}{t^2} \, dt = \int -432t^{-2} \, dt = \frac{-432t^{-1}}{-1} + k = \frac{432}{t} + k$$

Find value of k by using known distance at $t=6$

$$t = 6 \quad x = 36$$

$$36 = \frac{432}{6} + k$$

$$k = 36 - 72 = -36$$

Find distance using value of k and t

$$t = 10$$

$$x = \frac{432}{10} - 36 = 7.2m$$

Kinematics Example - Finding direction of motion

At $t = 0$ a particle P is projected from a fixed point O with velocity $(7\mathbf{i} + 7\sqrt{3})\text{ms}^{-1}$. The particle moves freely under gravity. The position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})\text{m}$ relative to O .

Show that:

$$y = \sqrt{3}x - \frac{g}{98}x^2$$

x has constant velocity so write in terms of t

$$x = 7t \quad [1]$$

Write an equation for y using $s = ut + \frac{1}{2}at^2$

$$y = 7\sqrt{3}t - \frac{g}{2}t^2 \quad [2]$$

Substitute [1] into the [2]

$$y = \sqrt{3}x - \frac{g}{2} \times \left(\frac{x}{7}\right)^2$$

Simplify

$$y = \sqrt{3}x - \frac{g}{98}x^2 \quad [3]$$

Find the direction of motion of P when it passes through the point on the path where $x = 20$

Differentiate [3]

$$\frac{dy}{dx} = \sqrt{3} - \frac{2gx}{98}$$

Substitute in the value of x

$$\frac{dy}{dx} = \sqrt{3} - \frac{40g}{98}$$

Arctan(gradient)=Angle to the positive horizontal as $\frac{dy}{dx}$ is the same as $\frac{O}{A}$

Perform arctan on the gradient to find the angle

$$\arctan\left(\sqrt{3} - \frac{40g}{98}\right) = -66.2$$