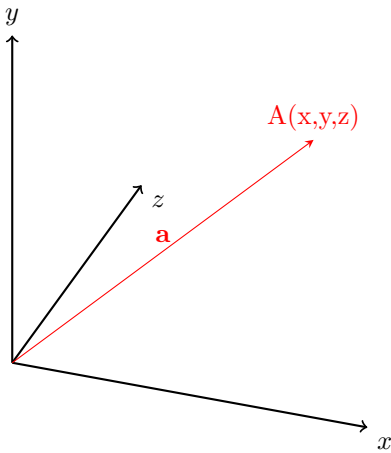


Vectors

1 3D Vectors



$$\overrightarrow{OA} = xi + yj + zk$$

$$\overrightarrow{OA} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Magnitude of \overrightarrow{OA}

$$|\overrightarrow{OA}| = \sqrt{x^2 + y^2 + z^2}$$

The vector between two vectors

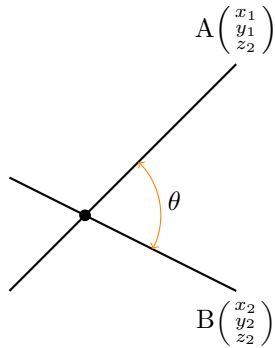
$$\overrightarrow{OA} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad \overrightarrow{OB} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2 Vector dot product

$$\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

$$\mathbf{a} \cdot \mathbf{b} = x_1x_2 + y_1y_2 + z_1z_2$$



$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

2.1 Perpendicular vectors

$$\cos 90 = 0$$

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

$$\mathbf{a} \cdot \mathbf{b} = 0$$

2.2 Parallel vectors

$$\theta = 1 \quad \cos \theta = 1$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|$$

3 Vector equation of a straight line

Types of situation:

1. Through one point parallel to a given vector

Find the equation of the line through \mathbf{a} which is parallel to \mathbf{b}

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}$$

2. A line through two points

Find the equation of a line through \mathbf{a} and \mathbf{b}

$$\mathbf{r} = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$$

4 Proofs involving vectors

4.1 Example of proving a point lies on a vector line

Show that $\begin{pmatrix} 9 \\ 9 \\ 3 \end{pmatrix}$ lies on the line $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix}$

Substitute the point equalling \mathbf{r}

$$\begin{pmatrix} 9 \\ 9 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix}$$

This is true when λ equals one so the point is on the line

4.2 Example of finding where two lines intersect

Where do the lines L_1 and L_2 intersect

$$L_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

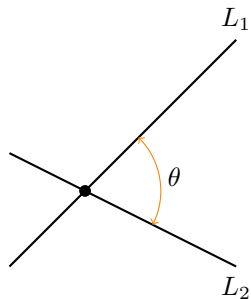
$$L_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Equal coordinates in a certain dimension to each other, preferably where constants can be eliminated

$$2 + 4\lambda = -2$$

$$\lambda = -1$$

5 Finding the angle between two straight lines



These vectors can be represented in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}_1$

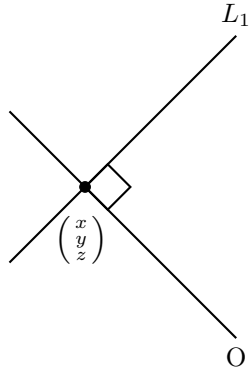
Example

$$\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 11 \\ 5 \\ 3 \end{pmatrix}$$

Here \mathbf{d}_1 is $\begin{pmatrix} 11 \\ 5 \\ 3 \end{pmatrix}$

$$\cos \theta = \left| \frac{\mathbf{d}_1 \cdot \mathbf{d}_2}{|\mathbf{d}_1| |\mathbf{d}_2|} \right|$$

6 Problems involving points on vector lines and perpendicular problems



$$L_1 : \mathbf{r} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$$

Using the vector dot product $\mathbf{a} \cdot \mathbf{b} = 0$ we can find the coordinates of X

$$\vec{OX} \cdot \mathbf{d} = 0$$

$$\begin{pmatrix} 9 - 3\lambda \\ -2 + 4\lambda \\ 1 + 5\lambda \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix} = 0$$

$$\lambda = \frac{3}{5}$$