# Collisions

## 1 Impulse and momentum

Impulse=$mv - mu$=$Ft$
Total momentum before collision=total momentum after

## 2 Coefficient of restitution

This tells us how well something bounces, it is given the symbol **e**.
If $e = 1$ the ball returns to it's original height
If $e = 0$ the ball doesn't bounce

$$e = \frac{\text{Speed of seperation}}{\text{Speed of approach}}$$

### 2.1 Alternate form of coefficient of restitution formula

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

$$e = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}}$$

$$e = \frac{\sqrt{h_2}}{\sqrt{h_1}}$$

$h_2$ - the height the ball bounces back to

$h_1$ - the height the ball is dropped from

### 2.2 Calculations involving coefficient of restitution

When doing calculations involving the coefficient of restitution both the calculation for CoR and conservation of momentum will be needed.
**Conservation of momentum:** $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

#### Coefficient of restitution

$$e = \frac{v_1}{u_1}$$

### 2.3 Successive Impacts

In some cases calculations will involve the impacts of multiple balls successively, like in newton's cradle.

#### 2.3.1 Example

*Three perfectly elastic particles A, B and C with masses 3kg, 2kg and 1kg respectively lie at rest in a straight line on a smooth horizontal table in alphabetical order. A is projected towards B with speed $5ms^{-1}$ and after A has collided with B, B collides with C*

$5 \times 3 = 3V_A + 2V_B$

$1 = \frac{SoS}{SoA}$ therefore $SoS = SoA$ so $V_B - V_A = 5$ so $V_B = 5 + V_A$

$15 = 10 + 2V_A + 3V_A$

$15 = 2V_B + 3$ so after 1st collision $V_B = 6$

$15 = 10 + 5V_A$ so $\mathbf{V_A = 1}$

**2nd Collision**

$2 \times 6 = 2V_B + V_C$

$V_C - V_B = 6$

$12 = 3V_B + 6$

$3V_B = 6$

$\mathbf{V_B = 2}$

$V_C = 6 + 2 = 8$

# Collisions example - Direct Impact

*A small ball A of mass 3m is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$ The balls have the same radius and can be modelled as particles.*

*Find the speed of A immediately after the collision*

**1. Draw the diagram in its initial state**

**2. Apply the conservation of momentum formula (make sure to remember different directions)**

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$3mu - mu = 3mv + mw$

$\underline{2u = 3v + w}$

**3. Apply Newton's law of restitution**

$e = \dfrac{\text{Speed of separation}}{\text{Speed of approach}}$

$\dfrac{1}{2} = \dfrac{w - v}{2u}$

$u = w - v$

$\underline{u + v = w}$

**4. Combine the results from momentum and restitution**

$2u = 3v + u + v$

$u = 4v$

$v = \frac{1}{4}u$

$\underline{|v| = \frac{1}{4}u}$

*Find the speed of B immediately after the collision*

**Combine the result from Newton's law of restitution and the speed of A**

$u = w - \frac{1}{4}u$

$\underline{|w| = \frac{5}{4}u}$

*After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is* $\frac{2}{5}$

*Find the speed of B immediately after hitting the wall*

**Apply Newton's law of restitution**

$$e = \frac{\text{Speed of separation}}{\text{Speed of approach}}$$

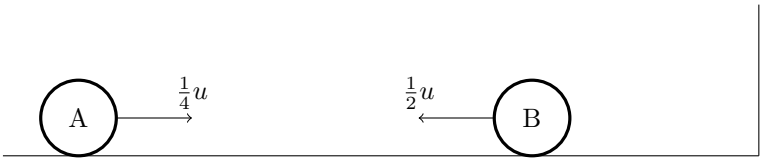
$$\frac{2}{5} = \frac{V}{\frac{5}{4}u}$$

$$V = \frac{2}{5} \times \frac{5}{4}u = \underline{\frac{1}{2}u}$$

*The first collision between A and B occurred at a distance 4a from the wall. The balls collide again T seconds after the first collision.*

*Show that* $T = \dfrac{112a}{15u}$

**Re-draw diagram to show new information**




**Calculate the time it takes for B to approach the wall**

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{4a}{\frac{5}{4}u} = \frac{16a}{5u}$$

**Calculate the distance A moves towards the wall**

$$\text{Distance} = \frac{1}{4}u \times \frac{16a}{5u} = \frac{4}{5}a$$

**Calculate the speed A and B approach each other**

$$\frac{1}{4}u + \frac{1}{2}u = \frac{3}{4}u$$

**Calculate the distance between A and B immediately after B has collided with the wall**

$$4a - \frac{4}{5}a = \frac{16}{5}a$$

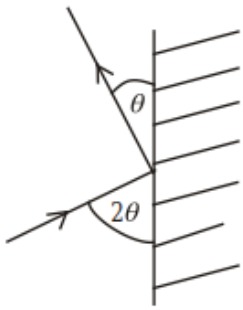
**Calculate time based on speed and distance**

$$t = \frac{\frac{16}{5}a}{\frac{3}{4}u} = \frac{64a}{15u}$$

**Add times to calculate total time**

$$\frac{16a}{5u} + \frac{64a}{15u} = \frac{112a}{15u}$$

# Collisions example - Oblique Impact




*A small smooth ball B, moving on a horizontal plane, collides with a fixed vertical wall.*
*Immediately before the collision the angle between the direction of motion of B and the wall is $2\theta$ where $0° < \theta < 45°$.*
*Immediately after the collision the angle between the direction of motion of B and the wall is $\theta$ as shown in the diagram above. Given that the coefficient of restitution between B and the wall is $\frac{3}{8}$ , find the value of $\tan\theta$.*

**Conservation of velocity**

$u_1 = v_1$

$u\cos 2\theta = v\cos\theta$

**Apply Newton's law of restitution**

$$e = \frac{\text{Speed of separation}}{\text{Speed of approach}}$$

$$\frac{3}{8} = \frac{v\sin\theta}{u\sin 2\theta}$$

$3u\sin 2\theta = 8v\sin\theta$

**Combine result from Conservation of velocity and Restitution by division**

$3\tan 2\theta = 8\tan\theta$

**Use tan addition formula**

$$3\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = 8\tan\theta$$

**Rearrange to solve**

$$\frac{6\tan\theta}{1-\tan^2\theta} = 8\tan\theta$$

$6\tan\theta = 8\tan\theta - 8\tan^3\theta$

$6 = 8 - 8\tan^2\theta$

$-2 = -8\tan^2\theta$

$$\frac{1}{4} = \tan^2\theta$$

$$\frac{1}{2} = \tan\theta$$