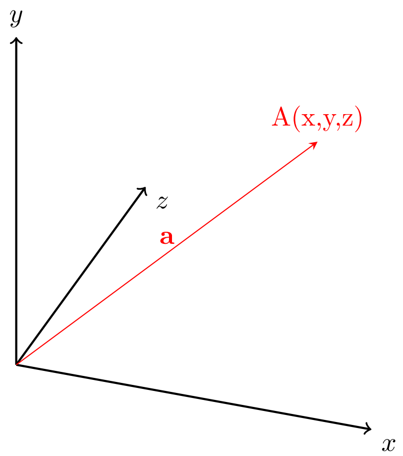
Differentiation

1 3D Vectors



$\overrightarrow{OA} = xi + yj + zk$

$\overrightarrow{OA} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Magnitude of $\overrightarrow{OA}$

$|\overrightarrow{OA}| = \sqrt{x^2 + y^2 + z^2}$

The vector between two vectors

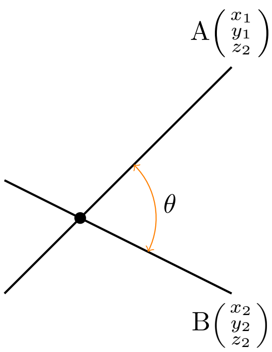
$\overrightarrow{OA} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad \overrightarrow{OB} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

2 Vector dot product

$\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$

$\mathbf{a}{\cdot}\mathbf{b} = x_1x_2 + y_1y_2 + z_1z_2$



$\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

2.1 Perpendicular vectors

$\cos 90 = 0$

$\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

$\mathbf{a}\cdot\mathbf{b} = 0$

2.2 Parallel vectors

$\theta = 1 \quad \cos\theta = 1$

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|$

3 Vector equation of a straight line

Types of situation:

1. Through one point parallel to a given vector

Find the equation of the line through ***a*** *which is parallel to* ***b***

r=a+t**b**

2. A line through two points

Find the equation of a line through a and b

$\mathbf{r} = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$

4 Proofs involving vectors

4.1 Example of proving a point lies on a vector line

Show that $\begin{pmatrix}9\\9\\3\end{pmatrix}$ lies on the line $\mathbf{r}=\begin{pmatrix}1\\4\\2\end{pmatrix}+\lambda\begin{pmatrix}8\\5\\1\end{pmatrix}$

Substitute the point equalling r

$$\begin{pmatrix}9\\9\\3\end{pmatrix}=\begin{pmatrix}1\\4\\2\end{pmatrix}+\lambda\begin{pmatrix}8\\5\\1\end{pmatrix}$$

This is true when λ equals one so the point is on the line

4.2 Example of finding where two lines intersect

Where do the lines L_1 and L_2 intersect

$$L_1:\mathbf{r}=\begin{pmatrix}3\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\4\end{pmatrix}$$

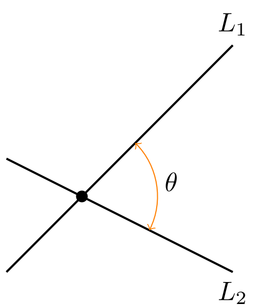
$$L_2:\mathbf{r}=\begin{pmatrix}0\\4\\-2\end{pmatrix}+\mu\begin{pmatrix}1\\1\\0\end{pmatrix}$$

Equal coordinates in a certain dimension to each other, preferably where constants can be eliminated

$2+4\lambda=-2$

$\lambda=-1$

5 Finding the angle between two straight lines



These vectors can be represented in the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d_1}$

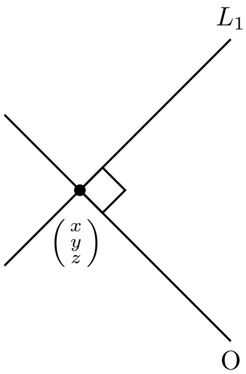
Example

$$\mathbf{r}=\begin{pmatrix}2\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}11\\5\\3\end{pmatrix}$$

Here $\mathbf{d_1}$ is $\begin{pmatrix}11\\5\\3\end{pmatrix}$

$$\cos\theta=\left|\frac{\mathbf{d_1}\cdot\mathbf{d_2}}{|\mathbf{d_1}||\mathbf{d_2}|}\right|$$

6 Problems involving points on vector lines and perpendicular problems



$L_1 : \mathbf{r} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$

Using the vector dot product $a \cdot b = 0$ we can find the coordinates of X

$\overrightarrow{OX} \cdot \mathbf{d} = 0$

$\begin{pmatrix} 9 - 3\lambda \\ -2 + 4\lambda \\ 1 + 5\lambda \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix} = 0$

$\lambda = \frac{3}{5}$