

Hypothesis testing

1 Tests of hypotheses

Statistical hypothesis - An assertion or conjecture concerning a population.

To test the validity of a statement a random sample is taken from the population and that data can then be used to provide evidence that either supports or does not support the hypothesis.

Null hypothesis - H_0 - A hypothesis assumed to be true **Alternative hypothesis** - H_1 - The situation if H_0 is false.

If the data leads to rejection of the null hypothesis the alternative hypothesis will be accepted.

The sample data is used to evaluate the **test statistic**, probabilities related to it can be calculated using the null hypothesis.

If the test statistic is found in the **critical region** the null hypothesis will be rejected.

The **boundary values** of the critical region are called the critical values.

2 Method

1. Establish the null and alternative hypothesis (H_0 and H_1)
2. Define distribution under H_0
3. Decide on the significance level
4. Collect data, state the test statistic, $X=$
5. Calculate the probability of obtaining the test statistic or a more extreme result (same direction as H_1)
6. Compare this to the sig level as a decimal
 - If **greater** than the sig level, it is a **non significant** result, it is not in the critical region and we **do not** reject H_0
 - If **less** than sig level, it is a **significant result**, it is in the critical region and we **reject** H_0
7. Interpret the results in terms of the original claim

2.1 Example

Establish the null and alternative hypothesis

$$H_0 : p = 0.5$$

$$H_1 : p > 0.5$$

Define the distribution under H_0

$$\text{Under } H_0 \quad X \sim B(15, 0.5)$$

Decide on the significance level

$$5\%$$

Collect data, state the test statistic

$$X=12$$

Calculate the probability of obtaining the test statistic or a more extreme result

$$\begin{aligned} P(X \geq 12) &= 1 - P(X \leq 11) \\ &= 1 - 0.9824 \\ &= 0.0176 \end{aligned}$$

Compare this to the sig level as a decimal $0.0176 < 0.05$

Interpret the results in terms of the original claim

There is evidence to reject H_0 in favour of H_1 . The test is significant.

2.2 Finding critical values

We require a value c such that:

$$P(X \geq c) < 0.05$$

$$1 - P(X \leq c - 1) < 0.05$$

$$P(X \leq c - 1) > 0.95$$

Test against tables

$$P(X < 11) = 0.9824$$

$$c - 1 = 11$$

$$c = 12$$

2.3 Two tailed tests

When doing a two tailed test the significance level must be split in two. For example if the significance level is 5% then it must be split into 2.5% for each tail.

Two tailed test also concern equal or not equal, rather than inequalities, for example:

$$H_0 : p = 0.15$$

$$H_1 : p \neq 0.15$$