C3 Cheat Sheet

1 Algebraic fractions

Algebraic long division can help to simplify a fraction, remember fractions can be expressed as:

$$Q(x) + \frac{R(x)}{D(x)}$$

Where $Q(x)$ is the quotient, $R(x)$ is the remainder and $D(x)$ is the divisor

2 Functions

Domain - Inputs
Range - Outputs
Reasons for a restricted domain:

- The denominator of the fraction can't be zero
- You can't square root a negative number
- You can't log numbers $\leqslant 0$

Reasons for a restricted range:

- A restricted domain
- Asymptotes
- Minimum or maximum of a quadratic/trig graph

If the function doesn't have any obvious restrictions, still remember to put $x \in \mathbb{R}$, or $f(x) \in \mathbb{R}$

2.1 Finding the inverse of a function

Rearrange the function to make x the subject, then swap y for x and x for $f^{-1}(x)$

The inverse function is a reflection in the line $y = x$ of the original function.

If asked to find the values where the inverse function equals the original function, find where $f(x) = x$ as this will not have been transformed by the reflection

A function will not have an inverse if it is a many to one function

2.2 Finding a composite function

Substitute the inner function as x into the outer function

2.3 What to do when not given an explicit function

When not given an explicit function, remember that given a pair of coordinates $f(x) = f^{-1}(y)$, this can be used to find the function

3 Exponential and log functions

$$\log a + \log b = \log(ab)$$

$$\log a - \log b = \log(\frac{a}{b})$$

$$\log a^b = b \log a$$

Remember that you can only apply e to both sides when there is one term on each side, combine terms

3.1 Quadratic functions

Some questions with varying powers of e are usually best solved as a quadratic, multiplying through by e^{ax} then substituting $y = e^x$ and solving for y then finding the natural log of the answer.

3.2 Drawing graphs of e^x and $\ln(x)$

$\ln(x)$ is the inverse function of e^x, so just reflect in the line $y = x$, other than that, apply graph transformations as normal, remember to draw in asymptotes.

4 Numerical methods

To prove a root (turning point) is in a range, there will be a change of sign of the gradient.
Use the structure of the equation you are trying to rearrange to to help your method.

5 Transforming graphs of functions

5.1 Modulus graphs

$y = f(|x|)$ - Reflect in y axis (there can't be the correct values for negative x values)
$y = |f(x)|$ - Reflect in x axis (there can't be negative y values)
Remember that for curved graphs, the point of transformation will likely be sharp, rather than smooth

5.2 Solving modulus equations

To solve a modulus equation, use both the positive and negative versions, for example:

$$|3x - 2| = x + 4|$$

$$3x - 2 = x + 4 \quad \text{and} - 3x + 2 = x + 4$$

Though remember to substitute the values found back into the original equation to check they are valid

6 Trigonometry

$$\sin(x) = \sin(180 - x)$$

$$\cos(x) = \cos(360 - x)$$

$$\sin \text{and} \cos \text{repeat every } 360^\circ$$

$$\tan \text{repeats every } 180^\circ$$

$$\sin(x) = \cos(90 - x)$$

Remember to get the other identities, divide:

$$\sin^2 x + \cos^2 x = 1$$

By either $\sin^2 x$ or $\cos^2 x$

6.1 The R formula

$a \sin\theta \pm b \cos\theta$ can be expressed in the form $R \sin(\theta \pm \alpha)$
$a \cos\theta \pm b \sin\theta$ can be expressed in the form $R \sin(\theta \mp \alpha)$
Where:

$$R = \sqrt{a^2 + b^2}$$

$$\alpha = \arctan(\frac{b}{a})$$

7 Differentiation

Product rule:

$$y = f(x)g(x) \quad \text{then} \quad \frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$$

Quotient rule:

$$y = \frac{f(x)}{g(x)} \quad \text{then} \quad \frac{dy}{dx} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Chain rule:

$$y = [f(x)]^n \quad \text{then} \quad \frac{dy}{dx} = n[f(x)]^{n-1}f'(x)$$

$$y = f[g(x)] \quad \text{then} \quad \frac{dy}{dx} = f'[g(x)]g'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$