Simple Harmonic Motion

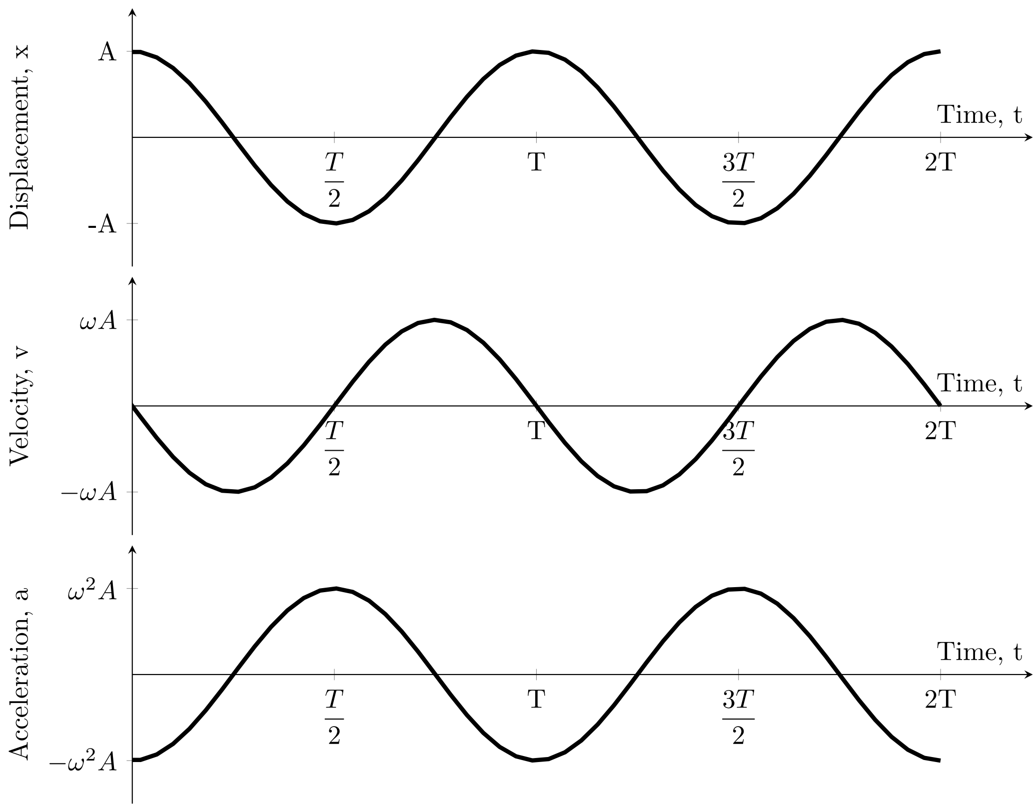
1 Introduction

An object is undergoing Simple harmonic motion if:

- Its acceleration is proportional to its displacement.
- Acceleration is directed to the equilibrium point (in the opposite direction to the displacement.

Rules of simple harmonic motion:

- The object can't have frictional forces (no damping)
- The oscillations must be small displacements

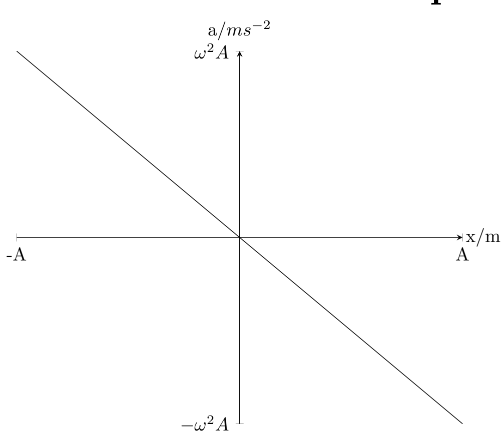


$x = Acos(\omega t)$

$v = -\omega A \sin(\omega t)$

$a = -\omega^2 A \cos(\omega t)$

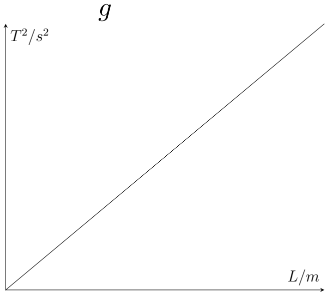
2 Acceleration vs Displacement SHM Graph



3 Period of a pendulum

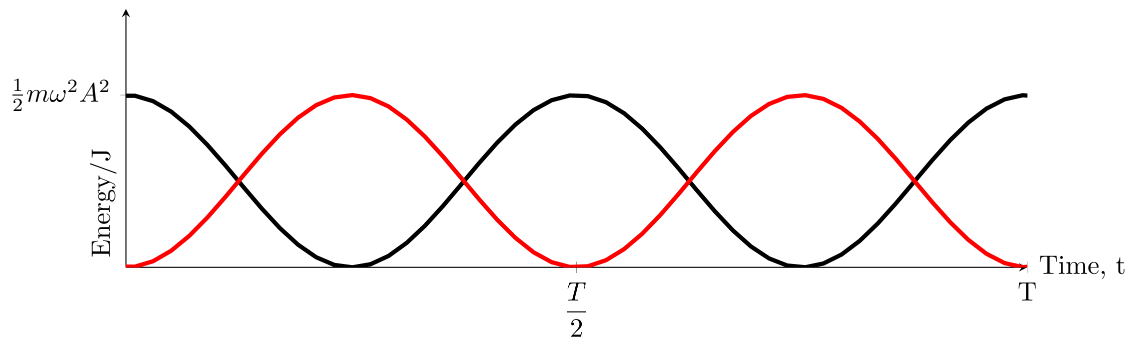
$T = 2\pi\sqrt{\frac{L}{g}}$

$T^2 = \frac{4\pi^2 L}{g}$

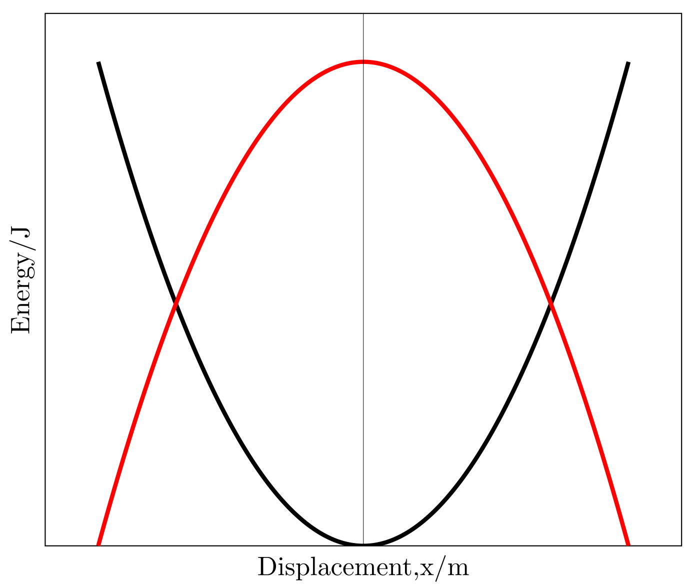


The gradient of the graph is $\frac{4\pi^2}{g}$

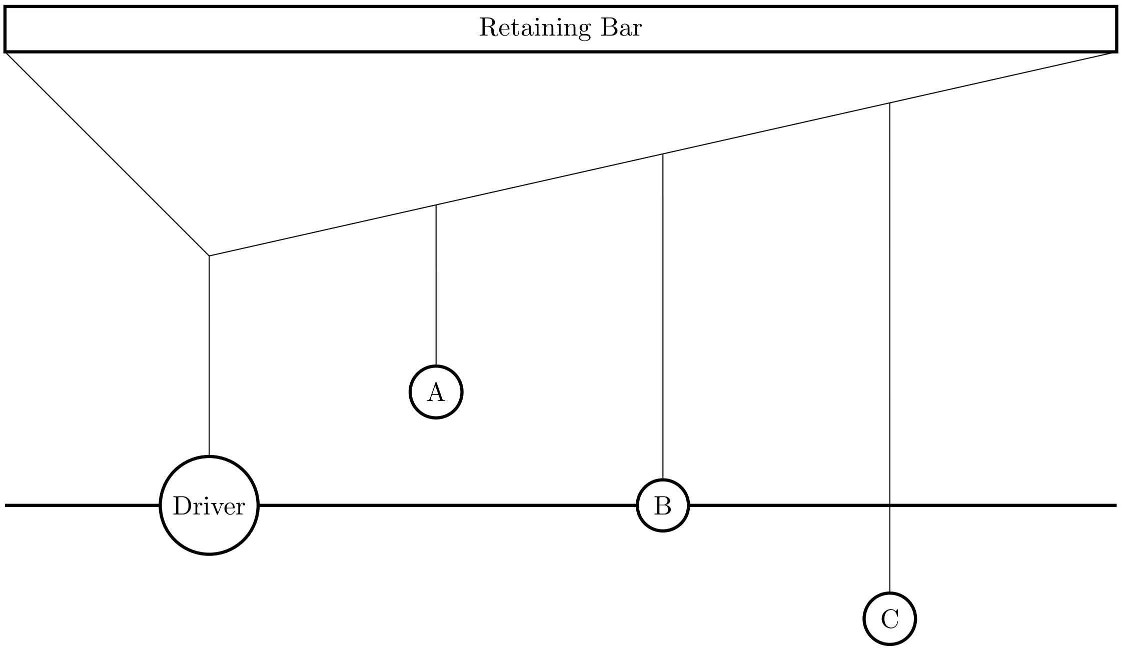
4 Energy in SHM



Red - Kinetic Energy
Black - Gravitational potential energy



5 Barton's Pendulum



All lengths are measured from the retaining bar to the centre of mass of the bob.

The **natural** frequency is the frequency of an object oscillating in the absence of any driving or damping force **Driver in diagram**.

The **driven** frequency is where an external force is provided, making the oscillator undergo forced vibrations. **A,B and C in diagram**.

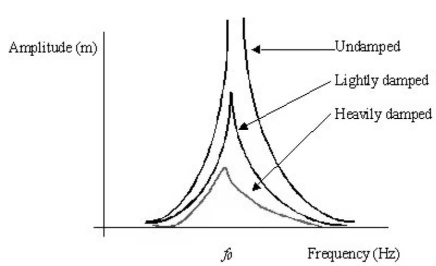
Resonance occurs when the frequency of the driving force is equal to the natural frequency and can result in large amplitude oscillations. **B in diagram**.

Pendulum	Natural frequency	Amplitude(wrt driver)	Phase(wrt driver)
A	Higher than driver	Small	In phase
B	Equal to driver (resonance)	Large	Out of phase($90^\circ or \frac{\pi}{2}$)
C	Lower than driver	Smaller than resonant	Antiphase($180^\circ or \pi$)

6 How do engineers prevent resonance

Ways to prevent resonance

- Use several materials - they all have different resonance properties
- Don't have symmetry - prevents standing waves
- Use very stiff materials - High frequency, small wavelengths



7 Damping

Damping is the reduction in amplitude due to energy losses (e.g. overcoming friction).

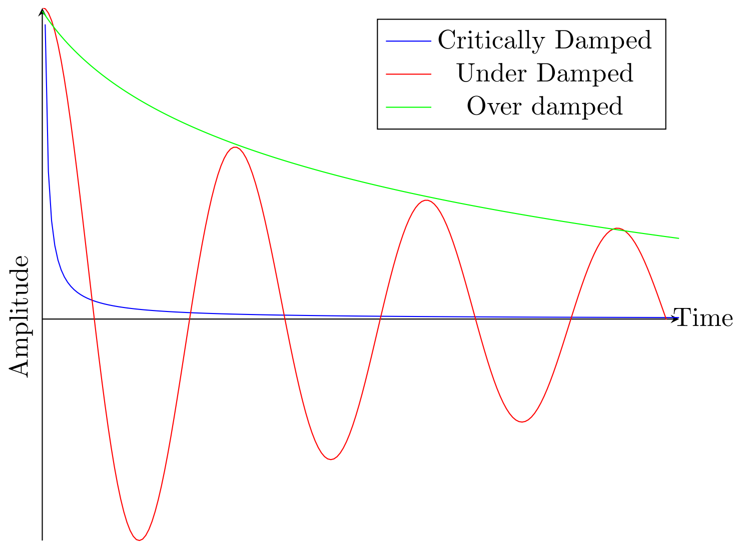
It will also cause the period to increase (it is essentially a driver).

Light(underdamping) - Small amplitude change per oscillation
Heavy(underdamping - Large amplitude change per oscillation

Critical damping - Reaches equilibrium in the shortest possible time **without** oscillating

Over damping - Reaches equilibrium after a long period of time

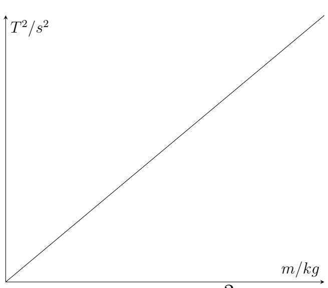
7.1 Damping graph



8 Mass spring system

8.1 Period of a mass-spring system

The period of a mass-spring system is $T = 2\pi\sqrt{\frac{m}{k}}$ or $T^2 = \frac{4\pi^2 m}{k}$



The gradient is $\frac{4\pi^2}{k}$
The line passes through the origin
This is only true for small oscillations as large oscillations go past the limit of proportionality

8.2 Energy in a vertically oscillating spring

	KE	GPE	Elastic Energy
Top	0	$2mgA$	$\frac{1}{2}k(e-A)^2$
Middle	Max=$\frac{1}{2}m\omega^2A^2$	mgA	$\frac{1}{2}ke^2$
Bottom	0	0	$\frac{1}{2}k(e+A)^2$