Series

1 The method of differences

This method is used when large parts of a series cancel, allowing the series to be expressed in a simple form in terms of n.

1.1 Example

Express the below summation in terms of partial fractions

$$\sum_{r=1}^{n} \frac{1}{r(r+1)}$$

Write using unknown numerators

$$\frac{1}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$$

Solve

$$A(r+1) + B(r) = 1$$
$$r = 0 \ A = 1$$
$$r = -1 \ B = -1$$

Substitute

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \sum_{r=1}^{n} \frac{1}{r} - \frac{1}{r+1}$$

Find a formula for this without summation

Write down values for the start and end of the summation, crossing out cancelling entries:

$$= \quad \frac{1}{1} \quad - \quad \cancel{\frac{1}{2}}$$
$$+ \quad \cancel{\frac{1}{2}} \quad - \quad \cancel{\frac{1}{3}}$$
$$+ \quad \cancel{\frac{1}{3}} \quad - \quad \cancel{\frac{1}{4}}$$
$$+ \quad ... \quad - \quad ...$$
$$+ \quad \cancel{\frac{1}{n-1}} \quad - \quad \cancel{\frac{1}{n}}$$
$$+ \quad \cancel{\frac{1}{n}} \quad - \quad \frac{1}{n+1}$$

Write down remaining values as the answer:

$$1 - \frac{1}{n+1}$$

2 Summations not starting at 1

Remember:

$$\sum_{r=21}^{n} = \sum_{r=1}^{n} - \sum_{r=1}^{20}$$