Collisions

1 Impulse and momentum

Impulse=$mv - mu$=Ft
Total momentum before collision=total momentum after

2 Coefficient of restitution

This tells us how well something bounces, it is given the symbol **e**.
If $e = 1$ the ball returns to it's original height
If $e = 0$ the ball doesn't bounce

$e = \frac{\text{Speed of seperation}}{\text{Speed of approach}}$

2.1 Alternate form of coefficient of restitution formula

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

$e = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}}$

$e = \frac{\sqrt{h_2}}{\sqrt{h_1}}$

h_2 - the height the ball bounces back to

h_1 - the height the ball is dropped from

2.2 Calculations involving coefficient of restitution

When doing calculations involving the coefficient of restitution both the calculation for CoR and conservation of momentum will be needed.
Conservation of momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Coefficient of restitution

$e = \frac{v_1}{u_1}$

2.3 Successive Impacts

In some cases calculations will involve the impacts of multiple balls successively, like in newton's cradle.

2.3.1 Example

Three perfectly elastic particles A, B and C with masses 3kg, 2kg and 1kg respectively lie at rest in a straight line on a smooth horizontal table in alphabetical order. A is projected towards B with speed $5ms^{-1}$ and after A has collided with B, B collides with C

$5 \times 3 = 3V_A + 2V_B$

$1 = \frac{SoS}{SoA}$ therefore $SoS = SoA$ so $V_B - V_A = 5$ so $V_B = 5 + V_A$

$15 = 10 + 2V_A + 3V_A$

$15 = 2V_B + 3$ so after 1st collision $V_B = 6$

$15 = 10 + 5V_A$ so $\mathbf{V_A = 1}$

2nd Collision

$2 \times 6 = 2V_B + V_C$

$V_C - V_B = 6$

$12 = 3V_B + 6$

$3V_B = 6$

$\mathbf{V_B = 2}$

$V_C = 6 + 2 = 8$

Collisions example - Direct Impact

A small ball A of mass 3m is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$ The balls have the same radius and can be modelled as particles.

Find the speed of A immediately after the collision

1. Draw the diagram in its initial state

2. Apply the conservation of momentum formula (make sure to remember different directions)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$3mu - mu = 3mv + mw$

$\underline{2u = 3v + w}$

3. Apply Newton's law of restitution

$$e = \frac{\text{Speed of separation}}{\text{Speed of approach}}$$

$$\frac{1}{2} = \frac{w - v}{2u}$$

$u = w - v$

$\underline{u + v = w}$

4. Combine the results from momentum and restitution

$2u = 3v + u + v$

$u = 4v$

$v = \frac{1}{4}u$

$\underline{|v| = \frac{1}{4}u}$

Find the speed of B immediately after the collision

Combine the result from Newton's law of restitution and the speed of A

$u = w - \frac{1}{4}u$

$\underline{|w| = \frac{5}{4}u}$

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$

Find the speed of B immediately after hitting the wall

Apply Newton's law of restitution

$e = \dfrac{\text{Speed of separation}}{\text{Speed of approach}}$

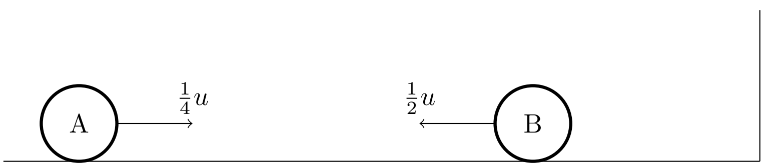
$\dfrac{2}{5} = \dfrac{V}{\frac{5}{4}u}$

$V = \frac{2}{5} \times \frac{5}{4}u = \underline{\frac{1}{2}u}$

The first collision between A and B occurred at a distance 4a from the wall. The balls collide again T seconds after the first collision.

Show that $T = \dfrac{112a}{15u}$

Re-draw diagram to show new information



Calculate the time it takes for B to approach the wall

$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

$\text{Time} = \dfrac{4a}{\frac{5}{4}u} = \dfrac{16a}{5u}$

Calculate the distance A moves towards the wall

$\text{Distance} = \dfrac{1}{4}u \times \dfrac{16a}{5u} = \dfrac{4}{5}a$

Calculate the speed A and B approach each other

$\dfrac{1}{4}u + \dfrac{1}{2}u = \dfrac{3}{4}u$

Calculate the distance between A and B immediately after B has collided with the wall

$4a - \dfrac{4}{5}a = \dfrac{16}{5}a$

Calculate time based on speed and distance

$t = \dfrac{\frac{16}{5}a}{\frac{3}{4}u} = \dfrac{64a}{15u}$

Add times to calculate total time

$\dfrac{16a}{5u} + \dfrac{64a}{15u} = \dfrac{112a}{15u}$