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SUPERSYMMETRY, SUPERGRAVITY AND PARTICLE PHYSICS

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Abstract:

We give a short introduction to $N = 1$ supersymmetry and supergravity and review the attempts to construct models in which the breakdown scale of the weak interactions is related to supersymmetry breaking.

1. Introduction

Since its discovery some ten years ago, supersymmetry [288, 544, 561] has fascinated many physicists. This has happened despite the absence of even the slightest phenomenological indication that it might be relevant for a description of nature. The only fact we know for sure is that supersymmetry has to be broken badly in the energy range explored by the presently available accelerators.

The motivation for supersymmetry is thus purely theoretical, for good reasons. First of all – it is a new symmetry, and symmetry considerations have in the past often led to progress in particle physics. It is, moreover, a symmetry with properties not shared by any symmetry discussed so far in the context of high energy physics – it transforms bosons into fermions. This symmetry between bosons and fermions gives rise to special (sometimes called miraculous) ultraviolet behavior of supersymmetric field theories. The simplest theory, $N = 1$ supersymmetry (where N counts the number of supersymmetry generators), is essentially free of quadratic divergences. Theories of higher N for example $N = 4$ are shown to be finite [410, 242, 515a]. The fact that $N = 4$ supersymmetric Yang–Mills theory is finite has led some famous physicists to make statements which I cannot resist the temptation to repeat here. "One is tempted to argue that a theory with such unique properties must have some relevance somewhere for nature" is one of them, and "Well, now at least we know that $N = 4$ supersymmetric Yang–Mills theory has nothing to do with nature" is the second. Controversial as these statements might seem, there is the hope that in the local version of supersymmetry (supergravity) [248, 113] it might be possible to arrive at an acceptable quantum theory that includes gravity. If this would work out one would have a motivation to believe that supersymmetry might be relevant at energies comparable to the Planck mass $M_{\rm P} \approx 1.2 \times 10^{19}$ GeV but could still be broken just below this scale, such that it might be irrelevant for the mass scales of present day particle physics. In this paper we will study the question, whether supersymmetry could have implications at scales of the order of 100 GeV. For reasons which will become clear later we will only consider $N = 1$ supersymmetry. The main motivations to consider supersymmetry are the improved convergence properties of supersymmetric field theories. To make this more specific let us look at the status of present day particle physics.

With the recent discovery of the intermediate vector bosons at the CERN $p\bar{p}$ collider [18, 19, 26, 31] one of the central predictions of the standard model [59, 280, 500, 547, 495, 281, 368] has been confirmed. This standard model is based on the gauge group $SU(3) \times SU(2) \times U(1)$ for the strong electromagnetic and weak interactions of three families of quarks and leptons. Apart from the gauge coupling constants this model needs of the order of twenty parameters used to adjust masses, mixing angles and Yukawa couplings. The model is believed to describe physics correctly at least up to the energy region of (let us say) 100 GeV. This belief is based on the fact that all experimental tests of the predictions of this model have turned out positive. Of course the model is not completely tested; there are indeed many predictions of the model which are not checked at all. These include for example the existence of the gauge boson trilinear and quadrilinear couplings as well as the existence of the Higgs sector. Thus there are still many places where physics might hide surprises, which are not contained in the standard model. Nonetheless, up to now this model is in accord with all experimental findings.

Theoretically the model is also very appealing. It is not just based on an effective Lagrangian, like e.g. the Fermi theory of weak interactions, but it is a renormalizable field theory. Actually we have to be more careful here. For the model to be an acceptable field theory, one condition has to be fulfilled by the spectrum, which can in turn be regarded as a prediction of the theory: at least one not yet detected fermion has to exist – the so-called top quark. According to the model it should be found in an energy region between 20 and 200 GeV [538]. The fact that the standard model is renormalizable could have drastic consequences on the range of validity of the model; would it be nonrenormalizable it necessarily would only be defined with a cutoff Λ (of dimension of a mass) and one would expect that its region of validity is bounded from above by Λ. Above Λ one expects new things to happen which are not described by the model. Ultimately one would think that such a model would be an approximation to a more complete theory which is also valid above Λ. Since the standard model is renormalizable it could however be valid in a much larger energy range, i.e., scales which are much larger than 100 GeV, the scale of the intermediate vector boson masses. This, however, does not prove that nothing new happens above 100 GeV. Experiments will check this region in the near future both by direct experiments above 100 GeV as well as by precise measurements at lower energies. It might also be that new particles are found below 100 GeV (e.g. the top quark); still many possibilities are open. What do we actually know about the region above 100 GeV? Well, there is gravity whose dimensionful coupling constant can be traded for a mass scale just as the weak interaction scale came from the Fermi coupling G_F. But this mass scale is sufficiently large and probably not of immediate interest to the 100 GeV region. The very precise measurements of the K_L–K_S mass difference and CP violation in the kaon system could indicate something new at the TeV scale, but they can also be described by the parameters of the standard model without invoking new physics at higher mass scales. The recent efforts to detect proton decay, if positive, would provide us with a new mass scale of 10^{15} GeV as predicted by grand unified models [271, 473]. In such a theory the $SU(3)\times SU(2)\times U(1)$ gauge group is unified in a larger group, e.g., SU(5) at this mass scale. These theories are able to predict one of the parameters, the weak mixing angle correctly. If these grand unified models are the correct description of nature they would probably imply the validity of the standard model up to 10^{14}–10^{15} GeV (the "great desert") which in term is only possible since the standard model is a renormalizable field theory. Recent experimental limits on proton decay [56], however, have not confirmed this picture up to now, and have reached values that might be already inconsistent with the simplest grand unified models.

Let us therefore return to the 100 GeV region and examine the standard model more closely. The standard model is probably not the final theory of the world since it contains some twenty free parameters. There is especially one sector in the theory, the Higgs sector [317], which remains rather mysterious and which is not checked at all: not even the predicted particles have been found. This sector, however, is very important for the model since the mass scale of the W and Z is given by the parameters in this sector. The Higgs particles are scalar (spin zero) particles and if found they would be the first *fundamental* scalar particles shown to exist. Scalar particles have several nice properties. They are the only particles that can have nonvanishing vacuum expectation values without breaking Lorentz invariance. They can induce a spontaneous breakdown of gauge symmetries and also allow Yukawa couplings to the fermions which lead to a mass generation after the breakdown of the gauge symmetries. Scalar particles have another property which is not considered as nice: their masses are subject to quadratic divergences in perturbation theory (fig. 1.1). As long as these masses are free parameters there is of course

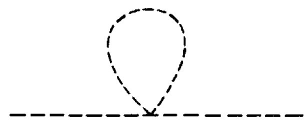

Fig. 1.1. Quadratically divergent contribution to the mass of a scalar particle.

nothing wrong with this, the input parameters being just the renormalized masses. Suppose, however, that one would in the future try to understand the magnitude of these masses. Such an understanding could only survive perturbation theory if the quadratic divergence would be physically cut off at a certain mass scale which would correspond to the masses of the scalar particles. The standard model now requires the mass of the scalar particles to be not much larger than a TeV, since otherwise the couplings in the Higgs sector have to be chosen so large that the model becomes at least ill-defined perturbatively [538]. Thus a model in which the scale of the breakdown of the weak interactions is understood in terms of more fundamental parameters there is hardly any other possibility than to expect new physics in the TeV region, which cuts off the quadratic divergence of the scalar particles. In this sense the Higgs sector in the standard model is the sector that is most sensitive to the region above 100 GeV. It is, at the same time the sector that is the least tested experimentally.

This "problem" [567, 523] of the quadratic divergencies of the scalar particles has led people to explore models that do not contain fundamental scalar particles: the so-called technicolor or composite models [523, 551, 475]. There the Higgs sector of the standard model is replaced by a strongly interacting gauge system. Fermion–antifermion bound states (in analogy to quantum chromodynamics) are assumed to condense and break the weak interaction symmetry at a scale of a few hundred GeV and lead to massive intermediate vector bosons at 100 GeV. A variety of new composite particles, like e.g. technipions, is predicted in the TeV range, as expected in any model in which the scale of the weak interactions is understood in terms of more fundamental parameters. Much effort has been devoted to the study of this class of models, especially whether it is possible to construct a "realistic" model, which does not contradict known experimental results [191]. These models contain a gauge sector strongly interacting at 1 TeV, and our knowledge about such theories is strongly restricted. One has to make simple assumptions about the dynamics of such systems mostly assuming the technicolor forces to be a scaled up version of QCD. With these assumptions severe problems arise. The models predict a variety of composite pseudo-Goldstone bosons some of which might be below 10 GeV in mass, and which are certainly not yet experimentally detected. A second problem is the absence of a satisfactory mechanism to generate the quark and lepton masses after the breakdown of the weak interactions. Extended technicolor [128] models have been invented to tackle these questions. A mass mechanism had been found but it led to the prediction of very large flavor changing neutral currents, inconsistent with experimental findings [121]. This is the central problem of all these models, and it is not resolved. Maybe we do not know enough about these strongly interacting gauge systems which are dealt with in these models, and this question can only be answered at a time where we have learnt more about these systems. The mass generation in the standard model is done by the Higgs–Yukawa couplings. As a result the masses are pure input parameters. In an extended technicolor model these parameters are determined by the dynamics of the system. With our limited knowledge and simplified assumptions they usually turn out to be wrong. We do not know at this time whether these models might give a satisfactory phenomenological prescription or not. However, it might very well be that these models without fundamental scalars do not work because of the absence of the fundamental scalars with which the masses of quarks and leptons can be parametrized so effectively.

One might thus be led to the conclusion to reconsider fundamental scalars. Let us suppose that the standard model is valid up to a grand unification scale of 10^{15} GeV or even to the Planck scale 10^{19} GeV. The weak interaction scale of 100 GeV is very tiny compared to these two scales. If these three scales were input parameters in the theory the $(\text{mass})^2$ of the scalar particles in the Higgs sector have to be chosen with an accuracy of 10^{-34} compared to the Planck mass. Theories where such adjustments of incredible accuracy have to be made are sometimes called "unnatural", since there is no way to understand the smallness of the scale [531]. The only way to render such a theory "natural" would be a symmetry that implies the small parameters of the theory to be exactly zero. The actual values of the small parameters would then be related to the breakdown of this symmetry. With this definition of naturalness, we conclude that the standard model is not natural. In the limit of vanishing mass parameters in the Higgs sector the symmetry of the model is not enlarged. This manifests itself in the fact that radiative corrections induce masses in the scalar sector. This aspect is sometimes called the gauge hierarchy problem [274, 275] in grand unified theories: it is in general not enough to fine tune the small parameters at the tree level, but in general several orders in perturbation theory have to be computed to consistently fine tune the small parameters of the theory. We will call this the technical aspect of the naturalness problem. One should however keep in mind that the reason for this behaviour is the existence of a tiny understood parameter in the theory, and that with an "understanding" of this parameter, all these problems will necessarily have to be solved.

Now the question arises, how one could render the standard model natural? Since the scale of the breakdown of the weak interactions is entirely given by the vacuum expectation value of scalar particles one needs a symmetry that could imply vanishing masses of scalar particles. There is only one known of such symmetries [542, 409, 569]: *supersymmetry*. Usually, e.g. chiral, symmetries cannot enforce the masslessness of a scalar φ, since always $\varphi\varphi^*$ is a possible mass term that respects the symmetry. If one, however, has a chiral symmetry that forbids certain fermion masses the corresponding scalar partners are also massless in a supersymmetric model. The motivation to apply supersymmetry to particle physics comes from this fact that supersymmetry might render the standard model natural. Notice that this kind of argumentation is in analogy to the situation of spin 1 particles. To have massless fundamental spin 1 particles in a theory they are usually introduced as gauge particles, i.e., a symmetry [577] is invoked to keep these particles massless. Nonzero masses arise through a spontaneous breakdown of the corresponding symmetries.

To render the standard model supersymmetric a price has to be paid. For every boson (fermion) in the standard model, a supersymmetric partner fermion (boson) has to be introduced, and to construct phenomenologically acceptable models an additional Higgs supermultiplet is needed. Compared to the alternatives, however, the introduction of supersymmetry is the most conservative way to render the standard model natural.

In the supersymmetric standard model the $(\text{mass})^2$ of the scalars are no longer quadratically divergent, the quadratic divergence is cancelled by the partners (fig. 1.2). The masses of these scalars are actually not renormalized at all as a result of certain nonrenormalization theorems [562, 336, 231, 296, 240] in supersymmetric field theories. These nonrenormalization theorems are actually necessary for supersymmetry to survive perturbation theory in the sense that radiative corrections do not induce a spontaneous supersymmetry breakdown.

We are, however, not interested in the exact supersymmetric limit of the standard model. Charged scalar particles with mass and quantum numbers of, e.g., the electron are not found and thus supersymmetry has to be broken in nature. Also, vanishing mass parameters in the Higgs sector would not induce the breakdown of the weak interactions. The model has to contain a mechanism for the

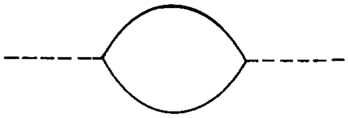

Fig. 1.2. Quadratically divergent contribution to the mass of a scalar particle (dashed line) that cancels the contribution of fig. 1.1 in a supersymmetric model. The solid line represents the fermionic partner of the scalar particle.

breakdown of supersymmetry that splits the masses of the different members of the supermultiplets and in addition induces the scale of the breakdown of the weak interactions of 0 (200) GeV. This breaking, however, should not be arbitrary since we still want the scalar masses not to be quadratically divergent. Such breakings have a name: they are called soft [525, 277]. A particular soft breaking is a spontaneous breakdown of supersymmetry. With broken supersymmetry the cancellation between the graphs in figs. 1.1 and 1.2 is no longer exact since the different partners in the supermultiplets are no longer degenerate. If the breaking of supersymmetry, however is spontaneous, this cancellation still takes place partially and leads to a finite result, the result being related to the breakdown scale M_S of supersymmetry. The supersymmetric partners of the known particles cut off the divergent parts in the integrals; they represent that additional sector needed in any theory that explains masses of scalar particles in a natural way.

One can now imagine a model in which $SU(2)\times U(1)$ is unbroken in the supersymmetric limit and where the breakdown of the weak interaction is induced by the breakdown of supersymmetry. In such a model we would then understand the breakdown scale M_W of $SU(2)\times U(1)$ in terms of M_S. In fact one has replaced one misunderstood scale by another one and one might ask the question whether one has gained something. One has. M_W and M_S are now related in a natural framework: this has as a consequence that these scales are stable in perturbation theory. This for example solves the technical aspect of the gauge hierarchy problem in grand unified theories [569, 122, 490, 449]. Parameters that are small at the tree level remain small to all orders in perturbation theory, their finite radiative corrections being governed by supersymmetry and its breakdown scale. Due to this extreme stability one might even be able to understand M_S in terms of the large scales like the Planck scale and the grand unification scale [444]. This comes from the fact that the scale of M_S at this point is not uniquely fixed, still various M_S can lead to the same M_W. To see this we have to look more closely to this relationship. Naively one might think that M_S should be of the order of M_W, i.e. a few hundred GeV. This is in fact true in models where supersymmetry is broken explicitly: the splitting in the supermultiplets is denoted by M_S and should be of order of M_W. Such models of explicitly broken supersymmetry, however, are arbitrary and lack predictive power. One would therefore prefer models in which supersymmetry is broken spontaneously. In these models the splittings of the supermultiplets are proportional to gM_S where g is a coupling constant (or a product of coupling constants). This fact has far reaching consequences. One can construct models where M_S is as large as 10^{11} GeV, but still sufficiently small to protect certain mass splittings and the scale M_W to remain in the 100 GeV region. In fact it turned out that models with such a large scale of supersymmetry breaking are actually preferred because of phenomenological reasons. The scale $M_S = 10^{11}$ GeV is actually so large that gravity cannot be neglected any longer. We can see this as follows. Since all particles participate in the gravitational interactions a supersymmetry breaking at scale M_S will reflect itself in mass splittings of the supermultiplets of order M_S^2/M_P where M_P^{-1} denotes the gravitational coupling constant. With $M_S \sim 10^{11}$ GeV one arrives at $M_S^2/M_P \sim 10^3$ GeV comparable to the breakdown scale of the weak interactions [42, 444]. The appropriate framework for this type of models is thus local supersymmetry. Local supersymmetry necessarily includes gravity and is therefore called supergravity. In these models the partner of the spin 2 graviton is a spin 3/2 particle: the gravitino. If supergravity is broken at a scale M_S, the gravitino mass will be approximately $m_{3/2} = M_S^2/M_P$ [114, 100, 101]. The breakdown of supergravity can induce the breakdown of the weak interaction and M_W is typically [97, 98, 77] of the order of the gravitino mass $m_{3/2}$. These models based on spontaneously broken supergravity actually turned out to be most promising on the way to construct phenomenologically acceptable models, the weak interaction scale governed by the gravitino mass.

One might be worried because $N = 1$ supergravity is not renormalizable. The supergravity models we discuss will not make sense unless one introduces a cutoff. Such a cutoff will be the Planck scale, and in this case the predictions of the discussed models will remain stable [37]. In this sense we are in the same situation as in all other cases. Whenever we have a model, even if it is renormalizable, that does not include gravity, we know that its range of validity will terminate at latest at the Planck scale, whenever it includes gravity it is (up to now) nonrenormalizable. In the models under consideration, the low energy sector, however, can be described as a renormalizable field theory. This sector has an explicitly softly broken supersymmetry with certain definite breaking terms.

Within this framework several classes of models have been constructed that result in a low energy (100 GeV) sector that still deserve the label "phenomenologically acceptable". These several classes of models exist because there are still some free parameters which reflect the ambiguity of the supergravity breakdown. These parameter include, for example, the gravitino mass, the gaugino masses and a supersymmetry breaking parameter A of the trilinear scalar couplings. The fact, however, that acceptable models can be constructed is nontrivial. Since there are several unconstrained input parameters, the phenomenological prediction of these models are not unique. This, however, had to be expected, since the models arise essentially from an extension of the Higgs sector. This Higgs sector in the standard model does not make very definite predictions: a neutral scalar particle should exist with mass somewhere between 10 and 1000 GeV. Compared to this, supergravity models have more predictive power. One can hardly imagine a model in which the weak interaction breakdown is induced by the breakdown of supergravity that does not predict at least one new particle with mass below 100 GeV. The different classes of models just differ in their predictions whether these "light" particles are photinos, gluinos or perhaps scalar partners of quarks of leptons. Supersymmetric models might also make some predictions for proton decay but these predictions are very model dependent. The low energy predictions of the supersymmetric models are specific enough to be tested in the foreseeable future. Experiments within the next five to ten years will enable us to decide whether supersymmetry as a solution of the naturalness problem of the weak interactions scale is a myth or reality.

This far for the 100 GeV region. The extension of the models to higher energies is of course only mildly constrained. Supersymmetric grand unified models are flexible enough to escape experimental falsification in the near future. A detection of proton decay $p \to K\nu$ could, however, support the idea of supersymmetric grand unification.

Also the origin of the mechanism of supersymmetry breaking remains rather mysterious. Whether M_S could be understood through a dynamical reason or through a conspiracy of the large mass scales in the theory must remain speculation for a while. Theoretical information might come from a better understanding of strongly interacting gauge theories and/or extended supergravity.

Needless to say that there are many problems of particle physics which are even not addressed in this framework. One of them is the question why there are three generations of quarks and leptons and to understand the origin of their masses. Supersymmetric (grand unified) models do not improve our knowledge with respect to these questions. One could argue that these questions are solved in the context of models based on extended ($N > 1$) supersymmetry [533] since there gauge and Yukawa couplings become unified, but at the moment this remains pure speculation.

Another unsolved question remains the naturalness problem of the cosmological constant [402, 536], although one would have thought that supersymmetry might help in this respect. This hope is based on the fact that for unbroken global supersymmetry the vacuum energy is well defined and can be shown to vanish. This remarkable property, however, ceases to hold if the supersymmetry is broken and/or gravity is coupled. In the models under consideration the only improvement (compared to the usual theories) is the vanishing quartic divergence of the cosmological constant. Again one might hope this problem to be solved in the framework of extended supergravities. In this report we will, however, restrict ourselves to models based on $N = 1$ supersymmetry, since the known low energy spectrum is not compatible with extended supersymmetry, which only allows real particle representations with respect to the gauge groups under consideration.

Chapters 2 and 3 will serve as an introduction to global and local supersymmetry, respectively. They are not intended to fully review this subject and we will also restrict ourselves to $N = 1$ supersymmetry. The content, however, will be sufficient to follow the remainder of this report. For a more exhaustive treatment we refer to the existing excellent reviews [217, 262, 510, 533, 560].

In Chapters 4 and 5 we will review the application of global supersymmetry to models for particle physics. Chapter 4 discusses the basic ingredients of these models as well as model independent results, whereas in Chapter 5 specific representative models will be presented. Chapters 6 and 7 discuss $N = 1$ supergravity models in this context. Again we will first give the recipe to construct such models and will then discuss specific models. Chapter 8 is devoted to cosmological considerations and the resulting constraints on model building. In Chapter 9 we will try to list the most important phenomenological predictions of the considered classes of models with special emphasis on the experimental possibilities in the near future. Chapter 10 will finally give a summary and an outlook.

2. Global supersymmetry

2.1. The supersymmetry algebra and its immediate implications

Supersymmetry is a symmetry that transforms bosons into fermions and vice versa. The generator Q of these transformations should therefore have fermionic character. In fact Q_α ($\alpha = 1, 2$) can be chosen to have the transformation properties of a left handed Weyl spinor, a $(1/2, 0)$ representation under Lorentz transformations. (For our notations and conventions see Appendix A.) Its Hermitean adjoint is denoted by $\bar{Q}_{\dot\beta}$ a right handed Weyl spinor. Since the supersymmetry generators carry one half unit of spin they obey anticommutation relations. The anticommutator of Q and $\bar{Q}$ does not vanish as the anticommutator of any operator with its adjoint is nonzero. $\{Q_\alpha, \bar{Q}_{\dot\beta}\}$ transforms as $(1/2, 1/2)$ under Lorentz transformations. The Coleman Mandula theorem [92] permits the conservation of one operator which transforms as $(1/2, 1/2)$: the energy momentum operator P_μ. The algebra that defines supersymmetry is now given by

$$[Q_\alpha, P_\mu] = 0\,, \tag{2.1}$$

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu\,, \tag{2.2}$$

where σ denotes the Pauli matrices. Haag, Lopuszanski and Sohnius [302] have shown that the supersymmetry algebras, with the possible extensions to include central charges, are the only graded Lie algebras of symmetries of the S-matrix that are consistent with relativistic quantum field theory.

Equation (2.1) for $\mu = 0$ shows that Q commutes with the Hamiltonian $H = P^0$, which leads to the observation that the states of nonzero energy are paired by the action of Q. Due to the fermionic character of Q they contain an equal number of bosonic and fermionic states.

Equation (2.2) has immediate consequences since it relates the supercharges to the Hamiltonian $H = P^0$. Using $\sigma^{\mu}_{\alpha\dot\beta}\sigma^{\dot\alpha\beta}_{\nu} = 2g^{\mu}_{\nu}$ one derives

$$H = P^0 = \tfrac{1}{4}(\bar{Q}_1 Q_1 + Q_1\bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2\bar{Q}_2)\,, \tag{2.3}$$

which implies that the spectrum of H is semipositive definite, $H \geq 0$ remember that $\bar{Q}$ is the Hermitean adjoint of Q). This is a very strong result. It implies that in supersymmetric theories, e.g. the vacuum energy, is well defined. Let us define $|0\rangle$ to be the vacuum state and suppose that the vacuum state is supersymmetric (this means that supersymmetry is not spontaneously broken). This implies that the supercharges annihilate the symmetric vacuum state

$$Q_\alpha|0\rangle = 0\,. \tag{2.4}$$

Since H is given by the anticommutator of the charges this leads to [584]

$$E_{\text{vac}} \equiv \langle 0|H|0\rangle = 0\,. \tag{2.5}$$

In a supersymmetric theory the vacuum energy thus is not only well defined, but is also bound to vanish. Since $H \geq 0$ this implies that if a supersymmetric vacuum state exists it is always at the absolute minimum of the potential. This is quite different from the situation with ordinary symmetries where the existence of a symmetric state does not necessarily imply that it is a ground state, as is illustrated [569] in fig. 2.1.

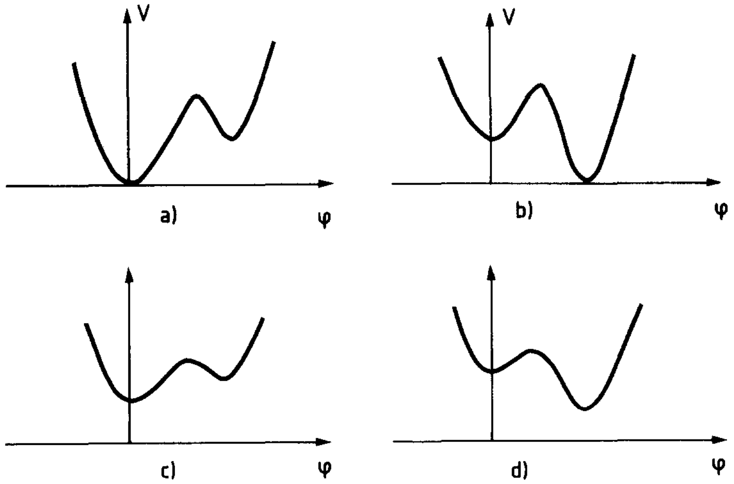

Fig. 2.1. Potentials $V(\varphi)$ with unbroken supersymmetry (a and b) and spontaneously broken supersymmetry (c and d). If φ transforms nontrivially under a gauge group the gauge symmetry will be spontaneously broken in cases b and d.

Supersymmetric ground states are always at $E_{\mathrm{vac}} = 0$. They, however, might still be degenerate with other states that have $E_{\mathrm{vac}} = 0$ and the question remains whether these states are supersymmetric or not? To answer this question let us consider the case of spontaneously broken supersymmetry, where the vacuum state is not annihilated by the supercharges

$$Q_\alpha|0\rangle \neq 0\,, \tag{2.6}$$

which implies $E_{\mathrm{vac}} = \langle 0|H|0\rangle \neq 0$ through eq. (2.3). Thus supersymmetry is unbroken if and only if $E_{\mathrm{vac}} = 0$. It can be broken spontaneously only if the potential is strictly positive.

The spontaneously broken case deserves further discussion. The right hand side of eq. (2.6) is necessarily a fermionic state which we denote by $|\psi_\alpha\rangle$. We can then write [571]

$$\langle\psi_{\dot\beta}|J_{\mu\alpha}|0\rangle = f\sigma_{\mu\dot\beta\alpha}\,, \tag{2.7}$$

where $J_{\mu\alpha}$ is the (conserved) supercurrent with

$$Q_\alpha = \int \mathrm{d}^3x J_{0\alpha}\,. \tag{2.8}$$

The supercurrent thus creates a fermion out of the vacuum with coupling f, which is a measure for the breakdown of supersymmetry. This situation is analogous to the case of ordinary spontaneously broken symmetries where Goldstone bosons can be created out of the vacuum. In fact an analogue of the Goldstone theorem can be proven for supersymmetry. It proves the existence of a massless fermion in theories with spontaneously broken supersymmetry. This particle is called Goldstone fermion or goldstino. f is the "goldstino decay constant". Since the energy momentum tensor in a supersymmetric theory can be written as $T_{\mu\nu} = \sigma^{\dot\beta}_{\mu\alpha}\{Q_{\dot\beta}, J^{\alpha}_{\nu}\}$ it can be related to the vacuum energy $E_{\mathrm{vac}} = f^2$, and E_{vac} measures the strength of supersymmetry breakdown.

2.2. Superfields

In order to construct supersymmetric models one would like to have a formalism in which supersymmetry is manifest. Such a formalism exists: it is the superfield formalism introduced by Salam and Strathdee [496, 236] which we will discuss in this section.

Let us first try to construct representations of the algebra given in (2.1) and (2.2). To take into account the anticommutator in (2.2) we introduce anticommuting parameters θ^α $(\alpha = 1, 2)$ and $\bar\theta^{\dot\beta}$ $(\dot\beta = 1, 2)$ which are elements of a Grassmann algebra

$$\{\theta^\alpha, \theta^\beta\} = \{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\} = \{\theta^\alpha, \bar\theta^{\dot\beta}\} = 0\,. \tag{2.9}$$

The supersymmetry algebra can then be rewritten as

$$[\theta Q, \bar Q\bar\theta] = 2\theta\sigma_\mu\bar\theta P^\mu\,, \tag{2.10}$$

$$[\theta Q, \theta Q] = [\bar Q\bar\theta, \bar Q\bar\theta] = 0\,, \tag{2.11}$$

where we have dropped the indices ($\theta Q = \theta^\alpha Q_\alpha = \theta^\alpha Q^\beta \varepsilon_{\alpha\beta}$ where $\varepsilon_{\alpha\beta}$ is antisymmetric and $\varepsilon_{12} = +1$). A "finite" supersymmetry transformation depends now on the parameters x_μ, θ and $\bar\theta$ and can be defined as follows

$$S(x, \theta, \bar\theta) = \exp \mathrm{i}(\theta Q + \bar Q \bar\theta - x_\mu P^\mu)\,. \tag{2.12}$$

The "multiplication" of two successive transformations can be computed with the help of Hausdorf's formula

$$S(x, \theta, \bar\theta) S(y, \alpha, \bar\alpha) = S(x + y - \mathrm{i}\alpha\sigma_\mu\bar\theta + \mathrm{i}\theta\sigma_\mu\bar\alpha, \theta + \alpha, \bar\theta + \bar\alpha)\,. \tag{2.13}$$

A superfield $\phi(x, \theta, \bar\theta)$ is now defined as a function of the parameters θ, $\bar\theta$ in addition to x_μ such that it transforms as follows under a supersymmetry transformation:

$$S(y, \alpha, \bar\alpha)[\phi(x, \theta, \bar\theta)] = \phi(x + y - \mathrm{i}\alpha\sigma_\mu\bar\theta + \mathrm{i}\theta\sigma_\mu\bar\alpha, \theta + \alpha, \bar\theta + \bar\alpha)\,. \tag{2.14}$$

To find a representation of the supercharges it is enough to consider infinitesimal transformations on the superfield ϕ

$$\delta_S \phi = \left[\alpha \frac{\partial}{\partial\theta} + \bar\alpha \frac{\partial}{\partial\bar\theta} - \mathrm{i}(\alpha\sigma_\mu\bar\theta - \theta\sigma_\mu\bar\alpha)\partial^\mu\right]\phi\,, \tag{2.15}$$

which lead to

$$P_\mu = \mathrm{i}\frac{\partial}{\partial x^\mu} \equiv \mathrm{i}\partial_\mu\,, \tag{2.16}$$

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu\,, \tag{2.17}$$

$$\bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu\,, \tag{2.18}$$

a representation of the superalgebra in terms of differential operators. One can now define covariant derivatives. (For an early review see ref. [557].) These are derivatives that anticommute with the variation

$$D_\alpha(\delta_S\phi) = -\delta_S(D_\alpha\phi)\,. \tag{2.19}$$

This gives

$$D_\alpha\phi = (\partial_\alpha + \mathrm{i}\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu)\phi\,, \tag{2.20}$$

$$\bar D_{\dot\alpha}\phi = (-\bar\partial_{\dot\alpha} - \mathrm{i}\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu)\phi\,, \tag{2.21}$$

where we have denoted $\partial/\partial\theta^{\alpha}(\partial/\partial\bar{\theta}^{\dot{\alpha}})$ by $\partial_{\alpha}(\bar{\partial}_{\dot{\alpha}})$. Before we proceed to discuss specific superfields let us introduce two different representations of the superalgebra because they will simplify certain formulae.

Instead of (2.12) one might also define

$$S_{\mathrm{L}} = \exp[\mathrm{i}(\theta Q - xP]\exp[\mathrm{i}\bar{Q}\bar{\theta}]\,, \tag{2.22}$$

or

$$S_{\mathrm{R}} = \exp[\mathrm{i}(\bar{Q}\bar{\theta} - xP)]\exp[\mathrm{i}\theta Q]\,. \tag{2.23}$$

Computing the supersymmetry transformation $S(y, \alpha, \bar{\alpha})\cdot S_{L(R)}(x, \theta, \bar{\theta})$ gives

$$S_{\mathrm{L}}(x + y + 2\mathrm{i}\theta\sigma\bar{\alpha} + \mathrm{i}\alpha\sigma_{\mu}\bar{\alpha}, \theta + \alpha, \bar{\theta} + \bar{\alpha})\,, \tag{2.24}$$

$$S_{\mathrm{R}}(x + y - 2\mathrm{i}\alpha\sigma_{\mu}\bar{\theta} - \mathrm{i}\alpha\sigma_{\mu}\bar{\alpha}, \theta + \alpha, \bar{\theta} + \bar{\alpha})\,, \tag{2.25}$$

respectively. ϕ_{L} and ϕ_{R} superfields can now be introduced in analogy to (2.14). They transform under infinitesimal supersymmetry transformations as

$$\delta\phi_{\mathrm{L}} = (\alpha\partial_{\theta} + \bar{\alpha}\partial_{\bar{\theta}} + 2\mathrm{i}\theta\sigma^{\mu}\bar{\alpha}\partial_{\mu})\phi_{\mathrm{L}}\,, \tag{2.26}$$

$$\delta\phi_{\mathrm{R}} = (\alpha\partial_{\theta} + \bar{\alpha}\partial_{\bar{\theta}} - 2\mathrm{i}\alpha\sigma^{\mu}\bar{\theta}\partial_{\mu})\phi_{\mathrm{R}}\,, \tag{2.27}$$

and lead to the additional L(R)-representations of the superalgebra

$$Q_{\mathrm{L}} = \partial_{\theta}\,; \qquad \bar{Q}_{\mathrm{L}} = -\partial_{\bar{\theta}} + 2\mathrm{i}\theta\sigma_{\mu}\partial^{\mu}\,, \tag{2.28}$$

$$Q_{\mathrm{R}} = \partial_{\theta} - 2\mathrm{i}\sigma^{\mu}\bar{\theta}\partial_{\mu}\,; \qquad \bar{Q}_{\mathrm{R}} = -\partial_{\bar{\theta}}\,, \tag{2.29}$$

where ∂_{θ} denotes $\partial/\partial\theta$. The corresponding covariant derivatives are given by

$$D_{\mathrm{L}} = \partial_{\theta} + 2\mathrm{i}\sigma_{\mu}\bar{\theta}\partial^{\mu}\,; \qquad \bar{D}_{\mathrm{L}} = -\partial_{\bar{\theta}}\,, \tag{2.30}$$

$$D_{\mathrm{R}} = \partial_{\theta}\,; \qquad \bar{D}_{\mathrm{R}} = -\partial_{\bar{\theta}} - 2\mathrm{i}\theta\sigma_{\mu}\partial^{\mu}\,. \tag{2.31}$$

There is a simple relation between the three representations we have introduced:

$$\begin{aligned}\phi(x_{\mu}, \theta, \bar{\theta}) &= \phi_{\mathrm{L}}(x_{\mu} + \mathrm{i}\theta\sigma_{\mu}\bar{\theta}, \theta, \bar{\theta}) \\ &= \phi_{\mathrm{R}}(x_{\mu} - \mathrm{i}\theta\sigma_{\mu}\bar{\theta}, \theta, \bar{\theta})\end{aligned} \tag{2.32}$$

transform identically under the supersymmetry transformation.

We have now everything set up to discuss special examples of superfields. Superfields that satisfy the condition $\bar{D}\phi = 0$ or $D\phi = 0$ are called scalar superfields. $\phi(x, \theta, \bar{\theta})$ with $\bar{D}\phi = 0$ ($D\phi = 0$) is sometimes also called a left- (right-) handed chiral superfield. Let us now consider such a left-handed chiral superfield with $\bar{D}\phi = 0$. It is in general a function of x, θ and $\bar{\theta}$. It is most conveniently discussed in the

L-representation in which $\bar{D}$ has a simple form $\bar{D} = -\partial/\partial\bar{\theta}$. $\bar{D}\phi = 0$ then implies that ϕ_L is independent of $\bar{\theta}$. Let us now expand $\phi_L(x, \theta)$ as a power series in θ. Since θ is anticommuting and there exist only two independent variables θ^α ($\alpha = 1, 2$) this series terminates after the third term and we can write

$$\phi_L(x, \theta) = \varphi(x) + \theta^\alpha \psi_\alpha(x) + \theta^\alpha \theta^\beta \varepsilon_{\alpha\beta} F(x), \tag{2.33}$$

where φ and F are complex scalar fields and ψ is a left-handed Weyl spinor. The effect of a supersymmetry transformation on these "component" fields can be computed from (2.26). With

$$\delta\phi_L(x, \theta) = \delta\varphi + \theta\delta\psi + \theta\theta\delta F \tag{2.34}$$

one obtains

$$\delta\varphi = \sqrt{2}\alpha\psi, \tag{2.35}$$

$$\delta\psi = \sqrt{2}\alpha F + \mathrm{i}\sqrt{2}\sigma_\mu\bar{\alpha}\partial^\mu\varphi, \tag{2.36}$$

$$\delta F = -\mathrm{i}\sqrt{2}\partial^\mu\psi\sigma_\mu\bar{\alpha}. \tag{2.37}$$

Observe that the supersymmetry variation of the highest (F) component of the superfield is a total derivative, a fact which will become important when we construct Lagrangians in the next section. Equations (2.35)–(2.37) show that supersymmetry transforms fermions into bosons. The terms without derivatives transform lower components into higher ones $\varphi \to \psi$ and $\psi \to F$. Let us now give some useful formulae with respect to chiral superfields. In the L(R)-representation one can easily show that $D^3 = \bar{D}^3 = 0$. This implies that for a left-handed chiral superfield ϕ (with $\bar{D}\phi = 0$) the superfield $D^2\phi \equiv D^\alpha D_\alpha \phi$ is a right-handed chiral superfield $D(D^2\phi) = 0$. Its lowest component contains the highest component of the original superfield ϕ.

A scalar superfield contains spin 0 bosons and spin 1/2 fermions. One would also like to introduce superfields that contain spin 1 bosons. This leads us to the introduction of a vector superfield $V(x, \theta, \bar{\theta})$ [239, 498], a superfield that satisfies the (reality) condition $V^+ = V$. In components it can be written as [217]

$$\begin{aligned} V(x, \theta, \bar{\theta}) = {} & (1 + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box)C + (\mathrm{i}\theta + \tfrac{1}{2}\theta\theta\sigma^\mu\bar{\theta}\partial_\mu)\chi + \tfrac{1}{2}\mathrm{i}\theta\theta(M + \mathrm{i}N) \\ & + (-\mathrm{i}\bar{\theta} + \tfrac{1}{2}\bar{\theta}\bar{\theta}\theta\sigma_\mu\partial^\mu)\bar{\chi} - \tfrac{1}{2}\mathrm{i}\bar{\theta}\bar{\theta}(M - \mathrm{i}N) - \theta\sigma_\mu\bar{\theta}V^\mu + \mathrm{i}\theta\theta\bar{\theta}\bar{\lambda} - \mathrm{i}\bar{\theta}\bar{\theta}\theta\lambda + \tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D, \end{aligned} \tag{2.38}$$

where $\Box = \partial_\mu\partial^\mu$. C, M, N, D are real spin 0 fields, χ, λ are Weyl spinors and V^μ is a spin 1 field. The special way one does the component decomposition in (2.38) is hard to understand at this point. It will become more transparent when we explicitly perform supersymmetry transformation on the component fields. With (2.15)

$$\delta V = [\alpha\partial_\theta + \bar{\alpha}\partial_{\bar{\theta}} - \mathrm{i}(\alpha\sigma_\mu\bar{\theta} - \theta\sigma_\mu\bar{\alpha})\partial^\mu]V$$

we obtain

$$\begin{aligned}
&\delta C = \mathrm{i}\alpha\chi - \mathrm{i}\bar\alpha\bar\chi\,, \qquad \delta\chi = \alpha(M + \mathrm{i}N) + \sigma^\mu\bar\alpha(\partial_\mu C + \mathrm{i}\,V_\mu)\,,\\
&\delta\bar\chi = \bar\alpha(M - \mathrm{i}N) + \alpha\sigma^\mu(\partial_\mu C - \mathrm{i}\,V_\mu)\,, \qquad \delta N = \alpha(\mathrm{i}\lambda - \sigma_\mu\partial^\mu\bar\chi) + \bar\alpha(-\mathrm{i}\bar\lambda + \bar\sigma_\mu\partial^\mu\chi)\,,\\
&\delta M = \alpha(\lambda + \mathrm{i}\sigma_\mu\partial^\mu\bar\chi) + \bar\alpha(\bar\lambda + \mathrm{i}\bar\sigma_\mu\partial^\mu\chi)\,, \qquad \delta\, V_\mu = \alpha\partial_\mu\chi + \bar\alpha\partial_\mu\bar\chi + \mathrm{i}\alpha\sigma_\mu\bar\lambda + \mathrm{i}\bar\alpha\bar\sigma_\mu\lambda\,,\\
&\delta\lambda = \alpha\sigma^{\mu\nu}V_{\mu\nu} + \alpha D\,, \qquad \delta\bar\lambda = \bar\alpha\bar\sigma^{\mu\nu}V_{\mu\nu} + \bar\alpha D\,, \qquad \delta D = -\alpha\sigma^\mu\partial_\mu\bar\lambda + \bar\alpha\sigma^\mu\partial_\mu\lambda\,,
\end{aligned} \tag{2.39}$$

where $\bar\sigma^\mu = (1, -\boldsymbol{\sigma})$ if $\sigma^\mu = (1, +\boldsymbol{\sigma})$ and $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)$, $\bar\sigma^{\mu\nu} = \frac{1}{4}(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu)$. We have also introduced the field $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. Observe again that the highest component of the superfield D transforms into a total derivative. One can also see from (2.39) that the general vector superfield is reducible. The fields $V_{\mu\nu}$, λ, $\bar\lambda$ and D form a representation by themselves. As we will see later these correspond to the components of a "massless" vector superfield. It will become important when we discuss supersymmetric Yang–Mills theories. The special choice of the expansion of V in (2.38) was used in order to make this reducibility transparent.

2.3. *Lagrangians* [561, 562]

We have already seen in (2.37) and (2.39) that the highest components of the superfield (the F-term in the scalar superfield and the D-term in the vector superfield) transform under supersymmetry transformations into total derivatives. A space time integral $\int \mathrm{d}^4x$ of such a quantity is thus invariant under supersymmetry transformations (provided, of course, that the fields fall off at infinity fast enough, which we will always assume). This is the basic observation for the construction of supersymmetric field theories. The Lagrange density is a sum of superfields, which are in general products of the elementary superfields that we have introduced in the last section (see below for a discussion of the multiplication of superfields). There are two types of superfields: scalar and vector. Accordingly we can write the Lagrange density

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_D\,, \tag{2.40}$$

where $\mathcal{L}_F$ is a sum of scalar superfields and $\mathcal{L}_D$ is a sum of vector superfields. The highest components $\theta\theta F$ and $\theta\theta\bar\theta\bar\theta D$ can be projected with integration over θ and $\bar\theta$. Integration is defined by $\int \mathrm{d}\theta = 0$ and $\int \mathrm{d}\theta \cdot \theta = 1$. An invariant Lagrangian is then obtained through

$$L = \int \mathrm{d}^4x \Big[\int \mathrm{d}^2\theta\, \mathrm{d}^2\bar\theta \mathcal{L}_D + \int \mathrm{d}^2\theta \mathcal{L}_F + \text{h.c.} \Big]\,. \tag{2.41}$$

Observe that it contains two terms of different nature F- and D-terms, a fact which will become important when we discuss perturbation theory.

The terms in $\mathcal{L}_D$ and $\mathcal{L}_F$ arise as products of "single-particle" superfields since $\mathcal{L}$ has to contain terms at least bilinear in these elementary fields and we have to discuss the multiplication of superfields. Consider a scalar superfield ϕ with $\bar D\phi = 0$. The product ϕ^2 (or ϕ^n) of such a superfield is also a scalar left-handed superfield since it automatically satisfies $\bar D(\phi^2) = 0$. ϕ is a complex field: ϕ^+ the conjugate is a right-handed field. The product $(\phi^+)^n$ is then also a right-handed chiral superfield. The product of left- (right-) handed superfield is thus always a left- (right-) handed superfield and as such a candidate for a term in the Lagrange density $\mathscr{L}_F$ as defined in (2.40). To give an example let us compute the square of the superfield given in (2.33) $\phi_L(x, \theta) = \varphi(x) + \theta^\alpha \psi_\alpha + \theta\theta F$:

$$\phi_L^2(x, \theta) = \varphi^2 + 2\varphi\theta^\alpha\psi_\alpha + (\theta\theta)[2\varphi F - \tfrac{1}{2}\psi^\alpha\psi_\alpha]\,, \tag{2.42}$$

$$\int \mathrm{d}^2\theta\phi^2(x, \theta) = \tfrac{1}{4}[2\varphi F - \tfrac{1}{2}\psi^\alpha\psi_\alpha]\,, \tag{2.43}$$

an example for an F-term in the Lagrange density. Actually all F-terms in the Lagrange density will arise as products of scalar superfields with the same handedness. $\mathscr{L}_F$ is often called the superpotential.

Terms in $\mathscr{L}_D$ can, e.g., arise if we multiply left- with right-handed superfields. As an example let us consider the multiplication of ϕ with its complex conjugate ϕ^+. The superfield $\phi_L(x, \theta) = \varphi(x) + \theta\psi + \theta^2 F$ is given in the L-representation (where it is independent of $\bar\theta$). Consequently its conjugate ϕ^+ is given in the R-representation if we just naively take the complex conjugate

$$\phi_R^+(x, \bar\theta) = \varphi^* + (\bar\theta\bar\psi) + (\bar\theta\bar\theta)F^*\,. \tag{2.44}$$

It is independent of θ and satisfies $D_R\phi_R^+ = 0$. Before we can multiply ϕ with ϕ^+ we have first to bring them in the same representation. This can be done through the shift given in eq. (2.32). Let us bring $\phi_R^+(x, \bar\theta)$ back in the L-representation

$$\phi_L^+(x, \theta, \bar\theta) = \phi_R^+(x_\mu - 2\mathrm{i}(\theta\sigma_\mu\bar\theta), \theta, \bar\theta)\,. \tag{2.45}$$

We now expand ϕ_R^+ and obtain in components

$$\varphi^* - 2\mathrm{i}(\theta\sigma_\mu\bar\theta)\partial^\mu\varphi^* - 2(\theta\sigma_\mu\bar\theta)(\theta\sigma_\nu\bar\theta)\partial^\mu\partial_\nu\varphi^* + (\bar\theta\bar\psi) - 2\mathrm{i}(\theta\sigma_\mu\bar\theta)\partial^\mu(\bar\theta\bar\psi) + (\bar\theta\bar\theta)F^*\,. \tag{2.46}$$

In this form the components of ϕ and ϕ^+ can be multiplied naively,

$$\phi\phi^+(x, \theta, \bar\theta) = \phi(x, \theta)\phi^+(x - 2\mathrm{i}\theta\sigma\bar\theta, \bar\theta) = \phi(x, \theta)\exp[-2\mathrm{i}\theta\sigma^\mu\bar\theta\partial_\mu]\phi^+(x, \bar\theta)\,, \tag{2.47}$$

which can be shown to be a general vector superfield. With an explicit calculation (using formulas of the appendix) one obtains for its "highest" (D)-component

$$(\phi\phi^+)_D = FF^* - \varphi\Box\varphi^* + \tfrac{1}{2}\psi^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu\bar\psi^{\dot\alpha}\,, \tag{2.48}$$

where $\Box = \partial_\mu\partial^\mu$. According to (2.39) the quantity $\int \mathrm{d}^4x(\phi\phi^+)_D$ is invariant under supersymmetry transformations.

We are now in a situation to construct supersymmetric actions for chiral superfields. The expression (2.48) is obviously appropriate to describe the kinetic terms for a complex scalar φ and a Weyl spinor ψ. Mass terms and interactions can be added through F-terms (the superpotential), e.g.,

$$\mathscr{L}_F = m\phi^2 + \lambda\phi^3\,. \tag{2.49}$$

The Lagrangian

$$L = (\phi\phi^+)_D + m(\phi^2)_F + \lambda(\phi^3)_F + \text{h.c.} \tag{2.50}$$

reads in components

$$L = (\partial_\mu\varphi)(\partial^\mu\varphi^*) + \tfrac{1}{2}\mathrm{i}\psi\sigma_\mu\partial^\mu\bar{\psi} + FF^* + m(2\varphi F - \tfrac{1}{2}(\psi\psi) + \text{h.c.}) + \lambda(3\varphi^2 F - \tfrac{3}{2}\varphi(\psi\psi) + \text{h.c.})\,, \tag{2.51}$$

up to total divergences. It leads to the field equations

$$\begin{aligned} &\partial^\mu\partial_\mu\varphi = 2mF^* - \tfrac{3}{2}\lambda(\bar{\psi}\bar{\psi}) + 6\lambda\varphi^* F^*\,, \\ &\mathrm{i}\sigma^\mu\partial_\mu\bar{\psi} = 2m\psi + 3\lambda\varphi\psi\,, \qquad F^* + 2m\varphi + 3\lambda\varphi^2 = 0\,, \end{aligned} \tag{2.52}$$

(together with the complex conjugate equations). The field F can be eliminated via the equations of motion; it is an auxiliary field: $F = -2m\varphi^* - 3\lambda\varphi^{*2}$. Substituting this back in (2.51) one obtains

$$\begin{aligned} L = {} & |\partial_\mu\varphi|^2 + \tfrac{1}{2}\mathrm{i}\psi\sigma_\mu\partial^\mu\bar{\psi} + |2m\varphi + 3\lambda\varphi^2|^2 - m[2\varphi(2m\varphi^* + 3\lambda\varphi^{*2}) + \tfrac{1}{2}(\psi\psi) + \text{h.c.}] \\ & - \lambda[3\varphi^2(2m\varphi^* + 3\lambda\varphi^{*2}) + \tfrac{3}{2}\varphi(\psi\psi) + \text{h.c.}]\,, \end{aligned} \tag{2.53}$$

which describes an interacting complex scalar with a Weyl spinor both of mass $2m$. Notice first that the model contains an equal number of bosonic and fermionic degrees of freedom (namely two) as expected from our discussion of the algebra in section 2.1. The interactions include a trilinear $\varphi\psi\psi$ coupling as well as trilinear and quadrilinear scalar couplings. They are however related by supersymmetry and given by λ and m. The term ϕ^3 in the superpotential (2.49) gives rise to quadrilinear scalar selfcouplings. It can be shown that powers of higher than three in the fields contained in the superpotential leads to nonrenormalizable models since they imply scalar selfcouplings higher than quadrilinear. This can be seen easily by a trick which is also useful to eliminate the auxiliary fields from (2.51). It is the observation that the scalar potential (i.e. the part of the Lagrangian that does not contain derivatives or fermions) can be obtained easily from the superpotential [569]

$$V = |\partial W(\varphi)/\partial\varphi|^2\,, \tag{2.54}$$

where $W(\varphi)$ denotes (2.49) with the superfields replaced by its scalar component. (Observe that $F^* = -\partial W(\varphi)/\partial\varphi$.) In our case $V = |2m\varphi + 3\lambda\varphi^2|^2$ and this leads to

$$L = |\partial_\mu\varphi|^2 + \tfrac{1}{2}\mathrm{i}\psi\sigma_\mu\partial^\mu\bar{\psi} - \tfrac{1}{2}m(\psi\psi + \bar{\psi}\bar{\psi}) - \tfrac{3}{2}\lambda(\varphi\psi\psi + \varphi^*\bar{\psi}\bar{\psi}) - |2m\varphi + 3\lambda\varphi^2|^2\,, \tag{2.55}$$

which is equivalent to (2.53).

Up to now we have only discussed Lagrangians which describe spin 0 and spin 1/2 particles. One would of course also like to consider models with spin 1 particles. Since all spin 1 particles we will discuss are "gauge particles" of Yang–Mills gauge theories one has for this purpose to construct supersymmetric Yang–Mills theories. We have not the time and space here to do this in full detail. We will here restrict ourselves to the Abelian case [563, 498] and give the relevant formulas for the non-Abelian case at the end of this section. For a more detailed treatment we refer to the existing reviews [217, 560, 510].

The central ingredient is the vector superfield as given in (2.38). It contains a spin 1 field V^μ. In an

Abelian gauge theory (e.g. QED) the gauge field V^μ (e.g. the photon) transforms under gauge transformations as

$$V_\mu \to V_\mu + \partial_\mu \varphi\,, \tag{2.56}$$

where φ is a real scalar field.

The field V^μ is now a member of the supermultiplet $V(x, \theta, \bar\theta)$ and the gauge transformation (2.56) has to be extended in a way consistent with supersymmetry. This needs the extension of φ to a supermultiplet. According to Wess and Zumino [563] the transformation of the vector superfield

$$V \to V + \mathrm{i}(\Lambda - \Lambda^+) \tag{2.57}$$

is the correct generalization. Λ denotes a chiral superfield containing a complex scalar field η as well as a Weyl fermion. An inspection of (2.57) in component form shows that the fields C, χ, M and N are gauge artifacts while λ and D are gauge invariant and V_μ transforms as in (2.56), where $\varphi = (\eta + \eta^*)/2$. The fields C, χ, M and N can be gauged away by a particular choice of the gauge, the Wess–Zumino gauge. In this gauge the vector superfield takes the form

$$V_{WZ}(x, \theta, \bar\theta) = -\theta\sigma_\mu\bar\theta V^\mu + \mathrm{i}\theta\theta\bar\theta\bar\lambda - \mathrm{i}\bar\theta\bar\theta\theta\lambda + \tfrac{1}{2}\theta\theta\bar\theta\bar\theta D\,. \tag{2.58}$$

It only contains V^μ, λ and D. The Wess–Zumino gauge does not fix the gauge freedom completely. We still have $\delta V_\mu = \partial_\mu\varphi$ (λ and D were gauge invariant from the beginning). The Wess–Zumino gauge is a kind of unitary gauge for supersymmetric Yang–Mills theories and it is very convenient to construct Lagrangians. It is in general less suitable if one actually wants to do calculations in perturbation theory.

To obtain the supersymmetric generalization of the electromagnetic field strength we define a spinor chiral superfield W_α by

$$W_\alpha = (\bar D\bar D)D_\alpha V \tag{2.59}$$

where D_α and $\bar D_{\dot\alpha}$ denote the covariant derivatives as given in (2.19)–(2.21). In components it is given by

$$W_\alpha(x, \theta) = 4\mathrm{i}\lambda_\alpha - 4\theta_\alpha D + 4\mathrm{i}\theta^\beta\sigma_{\nu\alpha\dot\beta}\sigma^{\dot\beta}_{\mu\beta}(\partial^\mu V^\nu - \partial^\nu V^\mu) - 4(\theta\theta)\sigma_{\mu\alpha\dot\beta}\partial^\mu\bar\lambda^{\dot\beta}\,, \tag{2.60}$$

where we have performed the calculation in the L-basis. W_α contains D, λ and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu (= F_{\mu\nu})$ as component fields and is gauge invariant. The F-component of the chiral superfield $W^\alpha W_\alpha$ is thus invariant under supersymmetry transformations and gauge transformations. In components it reads

$$\tfrac{1}{32}(W^\alpha W_\alpha)_F = -\tfrac{1}{4}V_{\mu\nu}V^{\mu\nu} - \tfrac{1}{2}\mathrm{i}\lambda^\alpha\sigma_{\mu\alpha\dot\gamma}\partial^\mu\bar\lambda^{\dot\gamma} - \tfrac{1}{2}\mathrm{i}\sigma^\alpha_{\mu\dot\beta}(\partial^\mu\bar\lambda^{\dot\beta})\lambda_\alpha + \tfrac{1}{2}D^2\,. \tag{2.61}$$

It is the Lagrange density for pure supersymmetric Abelian Yang–Mills theory. The field D is an auxiliary field. It can be eliminated via the equations of motion. Supersymmetry requires a fermionic partner λ of the gauge boson V^μ which is called gauge fermion or gaugino. The minimal coupling of matter fields with charge g to the supersymmetric Yang–Mills theory is given by replacing the usual kinetic terms $(\phi^+\phi)_D$ of a chiral superfield $\phi = (\varphi, \psi, F)$ by

$$(\phi^+ \exp(2gV)\phi)_D = |D_\mu\varphi|^2 - \tfrac{1}{2}\mathrm{i}[\bar{\psi}^{\dot\beta}\sigma_{\mu\alpha\dot\beta}D^\mu\psi^\alpha] + g\varphi^* D\varphi + \mathrm{i}g[\varphi^*(\lambda\psi) - (\bar\lambda\bar\psi)\varphi] + |F|^2 \tag{2.62}$$

(in the Wess–Zumino gauge) where $D_\mu = \partial_\mu + \mathrm{i}gV_\mu$ is the (gauge) covariant derivative.

The generalization to the non-Abelian case has been given by Ferrara and Zumino [237], Salam and Strathdee [497]. Using $V_\mu = V_\mu^a T^a$ and $\Lambda^a = \Lambda T^a$ where T^a are the generators of the non-Abelian gauge group under consideration one constructs the spinor chiral superfield

$$W_\alpha = \bar{D}\bar{D}[\exp(-gV)D_\alpha \exp(gV)]\,, \tag{2.63}$$

where g is the gauge coupling. The gauge transformations $\exp(gV) \to \exp(-\mathrm{i}g\Lambda^+)\exp(gV)\cdot\exp(\mathrm{i}g\Lambda)$ transform W_α to $\exp(-\mathrm{i}g\Lambda)W_\alpha\exp(\mathrm{i}g\Lambda)$. The quantity $W_\alpha W^\alpha$ is gauge invariant and its F-component provides us again with a Lagrange density for pure supersymmetric Yang–Mills theory

$$\begin{aligned}\mathscr{L} = (1/32g^2)W^\alpha W_\alpha = -\tfrac{1}{4}G^{\mu\nu}G_{\mu\nu} + \tfrac{1}{2}D^2 - \tfrac{1}{2}\mathrm{i}[&\lambda^\alpha\sigma_{\mu\alpha\dot\beta}(\partial^\mu\bar\lambda^{\dot\beta} + \mathrm{i}g[V^\mu, \bar\lambda^{\dot\beta}]) \\ &- (\partial^\mu\bar\lambda^{\dot\beta} + \mathrm{i}g[V^\mu, \bar\lambda^{\dot\beta}])\sigma_{\mu\alpha\dot\beta}\lambda^\alpha]\,,\end{aligned} \tag{2.64}$$

where $G_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + \mathrm{i}g[V_\mu, V_\nu]$ and $\lambda = \lambda^a T_a$ is in the adjoint representation of the gauge group. The minimal coupling to matter is again given by $\phi^+\exp(2gV)\phi$.

2.4. *Spontaneous breaking of supersymmetry*

From the discussion in section 2.1 we know that a nonvanishing vacuum expectation value of the anticommutator $\{Q_\alpha, \psi\}$ signals a spontaneous breakdown of supersymmetry in connection with the existence of a Goldstone fermion ψ (Q_α denotes the supersymmetry charge). The fermions under consideration are either "chiral" fermions (i.e. members of a chiral supermultiplet) or gauge fermions (partners of the gauge bosons). Recalling the supersymmetry transformations

$$\delta\psi = \sqrt{2}\alpha F + \mathrm{i}\sqrt{2}\sigma_\mu\bar\alpha\partial_\mu\varphi\,, \tag{2.36}$$

$$\delta\lambda = \alpha\sigma^{\mu\nu}V_{\mu\nu} + \alpha D\,, \tag{2.39}$$

we see that the spontaneous breakdown of supersymmetry can be conveniently characterized by the auxiliary fields under consideration. We have

$$\langle 0|\{Q, \psi\}|0\rangle \sim \langle 0|F|0\rangle\,, \qquad \langle 0|\{Q, \lambda\}|0\rangle \sim \langle 0|D|0\rangle\,, \tag{2.65}$$

since the additional terms in (2.36) $\partial_\mu\varphi$ and (2.39) $V_{\mu\nu}$ contain derivatives of fields which are not supposed to receive vacuum expectation values (v.e.v.). *Thus supersymmetry is spontaneously broken if and only if auxiliary fields have nonvanishing vacuum expectation values.* The fermionic partner of the auxiliary field that receives a v.e.v. is the massless Goldstone fermion of broken supersymmetry. If there is more than one auxiliary field with nonvanishing v.e.v. the goldstino is a suitable linear combination of their fermionic partners.

In section 2.1 we had also realized that the vacuum energy is an order parameter for supersymmetry. This can be verified here as well. From (2.54) and (2.61) we know that the scalar potential can be written as (D is real)

$$V = FF^* + \tfrac{1}{2}D^2 \,, \tag{2.66}$$

and a nonvanishing v.e.v. for an auxiliary field implies $E_{\text{vac}} \neq 0$.

In the following we will give some simple examples of spontaneously broken supersymmetry. The first is a so-called O'Raifeartaigh model [461, 199] in which only auxiliary fields of chiral supermultiplets receive a v.e.v. We consider three scalar superfields $X = (x, \psi_x, F_x)$, $Y = (y, \psi_y, F_y)$ and $Z = (z, \psi_z, F_z)$ with

$$\mathscr{L} = (XX^*)_D + (YY^*)_D + (ZZ^*)_D + [\lambda X(Z^2 - M^2) + gYZ]_F + \text{h.c.} \tag{2.67}$$

The auxiliary fields are given through the derivatives of the superpotential ($F^* = -\partial W(\varphi)/\partial\varphi$)

$$F_x^* = -\lambda(z^2 - M^2)\,, \qquad F_y^* = -gz\,, \qquad F_z^* = -gy - 2\lambda xz\,. \tag{2.68}$$

Supersymmetry would be unbroken if (2.68) had a solution with $F_x = F_y = F_z = 0$. (Since the potential $V = F_xF_x^* + F_yF_y^* + F_zF_z^*$ is (semi)-positive definite $V = 0$ is the absolute minimum.) However, the model was chosen in such a way that this solution does not exist. $F_x = 0$ implies $z^2 = M^2$ which is inconsistent with $F_y = 0$. Thus, supersymmetry is broken. To obtain the ground state we have to minimize

$$V = \lambda^2|z^2 - M^2|^2 + g^2|z|^2 + |gy + 2\lambda xz|^2\,. \tag{2.69}$$

Suppose that $M^2 < g^2/2\lambda^2$. The absolute minimum is at $z = y = 0$, with $F_x = \lambda M^2$ and consequently $E_{\text{vac}} = \lambda^2 M^4$. The vacuum expectation value of x is undetermined which implies a vacuum degeneracy, there is a flat direction in the potential, a point to which we will come back later.

According to our general arguments we know that supersymmetry is broken and that ψ_x (the partner of F_x) is a Goldstone fermion. Let us check this explicitly in our example. From (2.67) and $y = z = 0$ it is obvious that ψ_x is massless. There are two massive fermions of mass g which are linear combinations of ψ_y and ψ_z. The scalar x is massless and y has mass g. This looks still completely supersymmetric. Splittings, however, occur in the z-scalars. Denoting $z = a + \mathrm{i}b$ where a and b are real fields, we obtain $m_a^2 = g^2 - 2\lambda^2M^2$ and $m_b^2 = g^2 + 2\lambda^2M^2$ and we see that the boson–fermion degeneracy is destroyed.

Several things can already be learned here. At the tree graph level, only the boson mass spectrum is affected by the fact that supersymmetry is broken. In our case it is actually only the z-scalar. The reason for this is that from all the scalars only z couples to the goldstino ψ_x (through the term λXZ^2 in the superpotential). The splittings are thus governed by the coupling to the Goldstone fermion: $\Delta m^2 = 2\lambda F_x = 2\lambda^2 M^2$. F_x is usually identified with the square of the supersymmetry breakdown scale $F_x = M_{\mathrm{S}}^2$. We see, however, that the splittings of the supermultiplets can be much smaller than M_{S}, provided that the coupling to the goldstino is weak enough, a point which will become important later, when we discuss the application of supersymmetry to particle physics.

An interesting property of the mass spectrum at the *tree graph level* is the fact that $m_a^2 + m_b^2 = 2g^2$ independent of the supersymmetry breaking, on average the masses remain the same as in the case of unbroken supersymmetry. At the three level the fermion masses remain unchanged whereas the scalars are shifted by the *same* amount in opposite directions. This result holds in general (with one exception which we will discuss in our next example) as shown by Ferrara, Girardello and Palumbo [227]. The result is that the supertrace of the squared mass matrix

$$\mathrm{S\,Tr}\,\mathcal{M}^2 = \sum_J (-1)^{2J}(2J+1)\mathcal{M}_J^2 = 0 \tag{2.70}$$

remains zero in the presence of spontaneous supersymmetry breaking. J denotes the spin of the particles in (2.70). We stress again that (2.70) only holds at the tree graph level, it is generally violated by radiative corrections [279]. Expression (2.70) implies that the vacuum energy is not quadratically divergent at the one loop level even in the presence of spontaneous supersymmetry breaking.

So far we have discussed a model where F-fields received vacuum expectation values. Our next example shows supersymmetry breaking through a D-term [218] and is due to Fayet [200]. It consists of a chiral multiplet $U = (\varphi, \psi, F)$ with charge e coupled to Abelian supersymmetric Yang–Mills theory with the addition of a Fayet–Iliopoulos [218] term $\int \mathrm{d}^4\theta V \sim (V)_D \sim D$ where V is the Abelian vector multiplet.

The Lagrangian density reads

$$\mathcal{L} = \tfrac{1}{32}[W^\alpha W_\alpha]_F + [U^* \exp(2eV)U + 2\xi V]_D\,. \tag{2.71}$$

This leads to the following field equations for the auxiliary fields (compare (2.62)):

$$F = 0\,, \qquad D + \xi + e\varphi^*\varphi = 0\,. \tag{2.72}$$

Eliminating the auxiliary fields leads to the expression

$$\mathcal{L} = -\tfrac{1}{4}V^{\mu\nu}V_{\mu\nu} - \tfrac{1}{2}\mathrm{i}\bar{\lambda}\not{\partial}\lambda - \tfrac{1}{2}\mathrm{i}\bar{\psi}\not{D}\psi + (D^\mu\varphi)(D_\mu\varphi^*) + \mathrm{i}e(\varphi^*\lambda\psi - \bar{\lambda}\bar{\psi}\varphi) - \tfrac{1}{2}|\xi + e\varphi^*\varphi|^2\,, \tag{2.73}$$

where $D_\mu = \partial_\mu + \mathrm{i}eV_\mu$. The scalar potential is $V = \frac{1}{2}D^2 = \frac{1}{2}|\xi + e\varphi^*\varphi|^2$. Several things can happen in this model, depending on the parameters e and ξ. If ξ and e have opposite sign $e\xi < 0$ the vacuum expectation value of D is zero and $\langle\varphi^*\varphi\rangle = -\xi/e$: supersymmetry is unbroken but gauge symmetry is spontaneously broken (fig. 2.2). If, however, $e\xi > 0$ the equation $D = 0$ has no solution and supersymmetry is broken (fig. 2.3). Let us briefly discuss the first case, because it gives us the supersymmetric generalization of the Higgs mechanism. We have $\langle\varphi^*\varphi\rangle = -\xi/e > 0$ at the minimum. The formerly

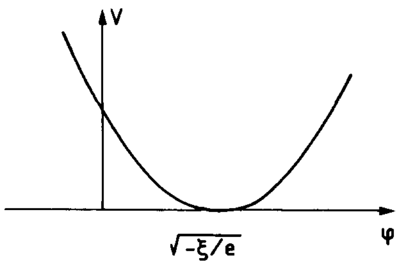

Fig. 2.2. The scalar potential for the case $e\xi < 0$. The U(1)-gauge symmetry is spontaneously broken but supersymmetry remains unbroken.

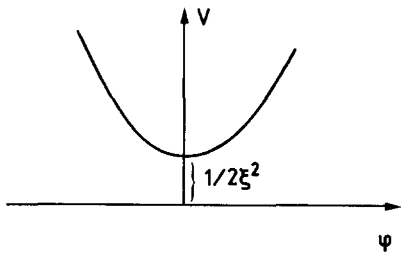

Fig. 2.3. The case $e\xi > 0$. Supersymmetry is spontaneously broken.

massless boson V^μ receives a mass by absorbing one real scalar. The second real scalar has the same mass $\sqrt{-e\xi}$ as the massive gauge boson. This is just the usual Higgs mechanism apart from the fact that here both masses are equal as a result of supersymmetry. Since supersymmetry is unbroken these bosons must have degenerate fermionic partners. This happens through the term $e\varphi^*(\lambda\psi)$ in (2.73); with the v.e.v. $\varphi^* = \sqrt{-\xi/e}$ ($e\xi < 0$) this gives rise to a mass $\sqrt{-\xi e}$. The two Weyl fermions ψ and λ combine to a massive Dirac fermion. This is the supersymmetric generalization of the Higgs effect. A chiral supermultiplet combines with a massless vector multiplet to a massive vector supermultiplet.

Let us now consider $e\xi > 0$. At the minimum of the potential we have $\langle\varphi\rangle = 0$ and $D = -\xi$ corresponding to $E_{\text{vac}} = \frac{1}{2}\xi^2$. Supersymmetry is broken through the v.e.v. of D. The partner of D, the gauge fermion λ, is the goldstino. Gauge symmetry is unbroken and V_μ as well as λ remain massless. From the scalar multiplet ψ remains massless and the scalars φ receive a mass as a result of its coupling e to the goldstino

$$m_\varphi^2 = e\xi \,. \tag{2.74}$$

Notice that this is completely different as in the example previously discussed. The two real scalars receive a mass shift in the same direction and the mass formula (2.70) is violated even at the tree graph level. The shift in (2.74) is proportional to the charge of the superfield. In the presence of an unbroken U(1) symmetry in which supersymmetry is broken by the D-term (2.70) has to be modified

$$\mathrm{S\,Tr}\,\mathcal{M}^2 = 2\,\mathrm{Tr}\,Q\langle D\rangle \,. \tag{2.75}$$

This is the final formula. The only possible term on the right hand side is the one given in (2.75) where Q is the charge matrix of the chiral superfields under consideration, and $\langle D\rangle$ is the vacuum expectation value of the auxiliary gauge field. If we had, e.g., added a chiral superfield with charge $-e$ to the model defined in (2.71), the right hand side in (2.75) would vanish since the scalars of this multiplet get shifted by $\Delta m^2 = -e\xi$.

So far we have discussed two examples where fundamental auxiliary fields F or D have received vacuum expectation values that lead to a breakdown of supersymmetry. In general, a combination of these two mechanisms might occur. The corresponding Goldstone fermion would then be a combination of the respective gauge and chiral fermions. One might also consider a situation where the Goldstone fermion is not a fundamental particle of the theory but a composite particle bound, e.g., by strong gauge forces in analogy to the pion as a composite Goldstone boson of broken chiral symmetry in quantum chromodynamics (QCD). Such a mechanism has been considered in "supercolor" models [123, 569, 137], a supersymmetric version of technicolor models [191]. Let us consider a supersymmetric SU(N) Yang–Mills theory coupled to chiral superfields ϕ_i and $\bar\phi^i$ in the N and $\bar N$ representation of SU(N). Suppose that these interactions lead to confinement in the infrared region. The spectrum below a certain scale Λ will thus consist of SU(N) singlet boundstates. They can be described supposedly in a manifest supersymmetric way. An example is the composite superfield $T = \phi\bar\phi$. With ϕ and $\bar\phi$ being left-handed chiral superfields the product will have the same property; it can be written in the L-basis as

$$T = \varphi_i\bar\varphi^i + \theta(\varphi_i\bar\psi^i + \bar\varphi^i\psi_i) + \theta\theta(\varphi_i\bar F^i + \bar\varphi^i F_i - \tfrac{1}{2}\psi_i\bar\psi^i) \,. \tag{2.76}$$

Now assume that like in ordinary nonsupersymmetric QCD the quantity $\psi\bar\psi$ receives a vacuum expectation value. Since we believe to be allowed to eliminate $F_i = 0$ via the equations of motions this would lead to a breakdown of supersymmetry because the highest component of a chiral superfield has a nonvanishing v.e.v. We have

$$\{Q_\alpha, (\varphi\bar{\psi} + \bar{\varphi}\psi)\} \sim \psi\bar{\psi}\,, \tag{2.77}$$

and the fermionic component of T would be a Goldstone fermion. Notice that this argument is restricted to this chiral superfield, the assumptions of a supersymmetry breakdown through gauge fermion condensates $(\lambda\lambda)$ are wrong since $\lambda\lambda$ is the lowest-component of the composite superfield $W_\alpha W^\alpha$ and not an auxiliary field [443]. Equation (2.77) looks quite convincing but the real question is whether $\psi\bar{\psi}$ receives a vacuum expectation value or not. After all we discuss a supersymmetric theory and we know that supersymmetric vacuum states have vanishing energy and are the lowest states (if they exist). A state with $\langle\psi\bar{\psi}\rangle \neq 0$ is nonsupersymmetric and has as such positive energy. One might argue that in a model with several chiral multiplets $\psi\bar{\psi}$ condensation is necessary to break certain chiral symmetries, but this role can as well be played by $\varphi\bar{\varphi}$ condensates and unbroken supersymmetry. There are in fact strong arguments by Witten [573, 75] that under some circumstances such a dynamical breakdown of supersymmetry is not possible. The situation is, however, not clear. Investigations of these models with the help of effective Lagrangians [543, 530] indicate that such a breakdown might happen [475a, 448] circumventing the no-go theorem of Witten. At the moment the question of this dynamical supersymmetry breakdown is still open and efforts should be made to reach a conclusion, a question which is also relevant for the study of supersymmetric composite models of quarks and leptons.

2.5. *Nonrenormalization theorems, absence of quadratic divergencies and soft breaking*

Supersymmetric theories have the property of vanishing vacuum energy as dictated by the algebra and as we have verified for the examples discussed up to now at the three graph level. Let us now consider the effects of perturbation theory. One might be worried that radiative corrections give contributions to the vacuum energy and as such supersymmetry would not survive in the perturbative expansion. To be more specific consider the model defined in (2.67) and let us suppose that $M^2 = 0$. This model then has the superpotential $\lambda XZ^2 + gYZ$ and one might ask the question whether a term $\lambda' M^2 X$ could be generated in perturbation theory. If it were, independent of how small the coefficient λ' might be, this would induce a breakdown of supersymmetry. For supersymmetry to remain unbroken such a term has to remain absent in all orders of perturbation theory. Thus supersymmetry requires nonrenormalization of certain parameters [336] (to remain well defined in perturbation theory). A second example of this potential instability of supersymmetry is the Fayet–Iliopoulos term in (2.71). If we would drop it in (2.71) supersymmetry would be unbroken in this model, but it could be generated in perturbation theory and supersymmetry would be broken. For this type of breakdown, however, one additional property of this model must be satisfied, there has to exist a massless fermion, to become a Goldstone fermion in the broken case [569].

Let us now discuss at the one loop level whether such things can happen. Consider for example the M^2 term in our first example. It corresponds to a term $M^2 \int \mathrm{d}^2\theta\phi = F_\phi$ in the superpotential where ϕ is a chiral superfield. At the one loop level there is a contribution to F by the graph in fig. 2.4, through the coupling $3\lambda(F\varphi^2 + F^*\varphi^{*2})$ from (2.51). F and φ are complex fields $F = f + \mathrm{i}g$, $\varphi = a + \mathrm{i}b$ and we have $6\lambda[f(a^2 - b^2) - 2gab]$ for the couplings of the real fields. The mass insertion in the model defined in (2.51) corresponds to $4m^2\varphi\varphi^* = 4m^2(a^2 + b^2)$ as can be read off from (2.55). The graph of fig. 2.4 for the

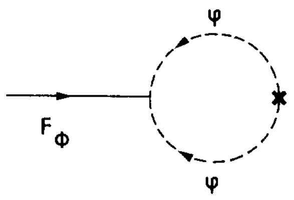

Fig. 2.4. A potential contribution to the auxiliary field F_ϕ at the one loop level.

external field g does not exist, there is no ab mixing term, whereas for the external auxiliary field f it splits into two contributions with a or b running around the loop. The couplings fa^2 and fb^2, however, have opposite signs. Thus these two contributions cancel and as a result a term $\int d^2\theta\phi$ is not generated at the one loop level. This is one of the so-called "miraculous" cancellations in supersymmetric theories. As we have seen a whole class of these cancellations have to exist to give supersymmetry a well-defined meaning in perturbation theory.

A convenient way to discuss these questions in general is the supergraph [231, 255, 107] formalism which we will briefly introduce here. For a more detailed treatment we refer to ref. [296], whose notation we shall follow. Consider

$$L = \int d^4x\, d^4\theta\bar{\phi}\, e^{gV}\phi + \frac{1}{64g^2}\int d^4x\, d^2\theta W^\alpha W_\alpha - \int d^4x\, d^2\theta\left(\tfrac{1}{2}m\phi^2 + \frac{1}{3!}\lambda\phi^3\right) + \text{h.c.} \tag{2.78}$$

We will choose the Fermi–Feynman gauge, which corresponds to adding $-\frac{1}{16}\int d^4x\, d^4\theta(D^2V)(\bar{D}^2V)$ to (2.78). Appropriate ghost terms should be added as well. This leads to the following Feynman rules for the effective action in superspace. The propagators are

$$\begin{aligned} \langle VV\rangle &= -\frac{1}{p^2}\delta^4(\theta-\theta'), \qquad \langle\bar{\phi}\phi\rangle = \frac{1}{p^2+m^2}\delta^4(\theta-\theta'), \\ \langle\phi\phi\rangle &= \frac{1}{4}\frac{mD^2(p,\theta)}{p^2(p^2+m^2)}\delta^4(\theta-\theta'), \end{aligned} \tag{2.79}$$

where D is the covariant derivative and $\delta^4(\theta) = \theta^2\bar{\theta}^2$. The vertices can be directly taken from L with additional factors $-\frac{1}{4}\bar{D}^2$ $(-\frac{1}{4}D^2)$ acting on the propagators for each ϕ $(\bar{\phi})$ line leaving the vertex. One such factor has to be omitted if the vertex under consideration contains only a $\int d^2\theta$ $(\int d^2\bar{\theta})$ integration rather than a $\int d^4\theta = \int d^2\theta\, d^2\bar{\theta}$ integration. The resulting amputated one-particle irreducible graphs should have each amputated external line multiplied by the corresponding superfield with $-\frac{1}{4}\bar{D}^2$ $(-\frac{1}{4}D^2)$ omitted at a vertex for each external $\phi(\bar{\phi})$ superfield. There are the usual combinatorial factors and integrations. In addition there is an integral $\int d^4\theta$ for each vertex. Sometimes it might be convenient to let these factors D^2 and $\bar{D}^2$ act directly on the propagators which then leads to the following propagators:

$$\langle\bar{\phi}\phi\rangle = \frac{1}{p^2+m^2}\exp[-\not{p}(\theta\bar{\theta} + \theta'\bar{\theta}' - 2\theta\bar{\theta}')], \qquad \langle\phi\phi\rangle = \frac{m}{p^2+m^2}(-\tfrac{1}{4}\bar{D}^2)\delta^4(\theta-\theta'), \tag{2.80}$$

with corresponding $\int d^2\theta$ $(\int d^2\bar{\theta})$ integrations at the chiral vertices.

Grisaru, Rocek and Siegel have now observed [296] that the θ structure of these supergraphs is very simple. All but one of the $\int d^4\theta$ integrations can be performed explicitly and the resulting terms in the effective action can be written as a single integral $\int d^4\theta$. What is most remarkable about this result is the fact that only the full integration $\int d^4\theta$ occurs, but no terms with $\int d^2\theta$ or $\int d^2\bar{\theta}$ alone. As a result the parameters m and λ in the superpotential are not renormalized in any order of perturbation theory, and as stated this holds for finite as well as infinite contributions. In components this nonrenormalization behaviour manifests itself through cancellations among a set of graphs. A simple example of this behaviour is the one which we discussed earlier where the contributions from the a and b particles cancel. We will later see more examples of these cancellations which, of course, are most conveniently discussed in terms of supergraphs.

Let us now discuss the ultraviolet behavior of supersymmetric theories and enumerate the possible counterterms. We will confine ourselves to renormalizable theories. What makes the Lagrangian in (2.78) renormalizable is the fact that *at most* four factors of D and $\bar{D}$ (two D and two $\bar{D}$) are present at each vertex and this can be used as a general criterion.

We have seen already that there are no counter terms for the superpotential. For possible other divergencies we have the following power counting rules. The degree of divergence of any graph is given by [231, 510] (except for contributions to the superpotential)

$$d = 2 - E_c - I_c\,, \tag{2.81}$$

where E_c is the number of external chiral lines and I_c is the number of massive internal chiral propagators. From this one then deduces the possible divergent contributions to the effective action. They are

$$\int d^4\theta\bar{\phi}\phi\,, \qquad \int d^4\theta\bar{\phi}V\phi\,, \qquad \int d^4\theta V\,, \qquad \int d^4\theta VV\,, \qquad \int d^4\theta VVV\,, \tag{2.82}$$

where in the *last two terms* two D and two $\bar{D}$ factors should be included at arbitrary positions. We have assumed that V appears as a *gauge* vector multiplet. The factors ϕ have dimension 1, D_α dimension 1/2, V is dimensionless and $d^4\theta$ has dimension two. As a result all contributions in (2.82) except for $\int d^4\theta V$ are only logarithmically divergent (remember that the last two terms contain $D^2\bar{D}^2$). They correspond to the usual wave function and gauge coupling constant renormalizations. The contribution $\int d^4\theta V$ to the Fayet–Iliopoulos term is quadratically divergent. It is gauge invariant only if the gauge group under consideration is Abelian (or contains a U(1) factor). We will come back to it later and assume for the moment that it is absent. We thus have only logarithmic divergencies. In particular the usual quadratic divergence of the masses of scalar particles is absent in a supersymmetric theory. They usually come from graphs like fig. 2.5 where the coupling constant is given by $9\lambda^2$ $(6\lambda m)$ for the model given in (2.55). In a supersymmetric theory there are now additional contributions to the $\varphi\varphi^*$ mass term as for example the one given in fig. 2.6. They cancel the quadratic divergent contribution of the first two graphs, confirming the general result given above.

Let us now come back to the Fayet–Iliopoulos term which can only appear in the presence of an Abelian gauge symmetry. It receives a quadratic divergent contribution in perturbation theory by the graph shown in fig. 2.7, and there is no other contribution to this order in the gauge coupling constant that could cancel this contribution. The graph, however, implies a sum of the contributions of all scalar particles in the theory weighted by the charges which define the coupling to the D-field. If one

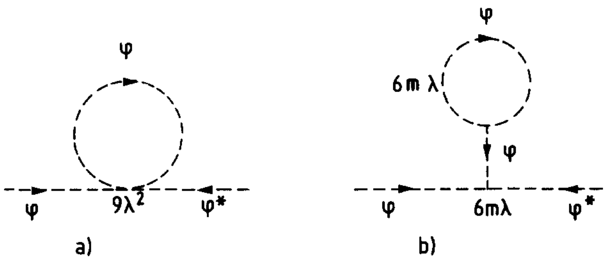

Fig. 2.5. Quadratically divergent contributions to $m^2\varphi\varphi^*$ due to the self-interactions of the scalar particles.

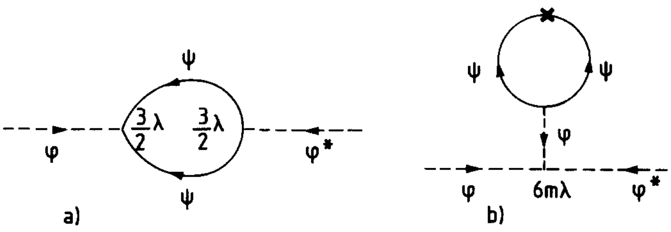

Fig. 2.6. The fermionic contributions to $m^2\varphi\varphi^*$ that cancel the contributions in fig. 2.5.

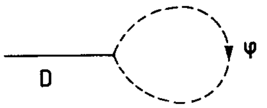

Fig. 2.7. Quadratically divergent contribution to the Fayet–Iliopoulos term at the one-loop level.

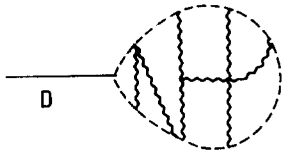

Fig. 2.8. A contribution to the D-term at the seven-loop level.

would now, for example, impose a parity symmetry these contributions would add up to zero (D is a pseudoscalar). More general the contribution at the one loop level is absent if the trace of the charges Tr Q of the scalar particles vanishes. But this takes only care of the first order of perturbation theory. Contributions in higher order, e.g. fig. 2.8, could still lead to quadratic divergencies. This could for example lead to a breakdown of supersymmetry as we have discussed earlier and it would spoil the naturalness of the supersymmetric models as discussed in the introduction. It actually turns out that this does not happen. The imposition of the constraint $\mathrm{Tr}\, Q = 0$ is sufficient to forbid the appearance of $\int d^4\theta V$ in perturbation theory [569, 240]. Independent of Tr Q no contributions appear in second and all higher orders of perturbation theory [240]. This result can be best seen by the use of supergraphs (fig. 2.9). The two contributions cancel. The blob denotes an arbitrary supergraph with external ϕ, $\bar{\phi}$ and V external lines and it is of course understood that the blobs in the two contributions are identical for the cancellation to hold.

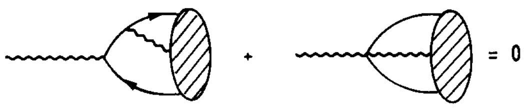

Fig. 2.9. The cancellation of the contributions to the D-term above the one-loop level as given in ref. [240]. The wavy line denotes a vector superfield and the solid line a chiral superfield.

We can thus summarize the results for the supersymmetric theories under consideration. Provided that $\mathrm{Tr}\, Q = 0$, there are no quadratic divergencies. The existing logarithmic divergencies correspond to wavefunction renormalizations of the chiral and vector field as well as the gauge coupling renormalization. The parameters in the superpotential (terms like $\int d^2\theta$) are not renormalized at all. The same is true for the vacuum energy reflecting the fact that supersymmetry remains well defined in perturbation theory. This fact might lead to the speculation that the naturalness problem of the cosmological constant might be solved by supersymmetry [584]. This is however not the case, at least not in the way it is meant here. We will see this in the next section when we couple supersymmetric theories to gravity. So let us for the moment put aside these considerations for the vacuum energy.

Supersymmetry (with $\mathrm{Tr}\, Q = 0$) is free of quadratic divergences. One might now ask the question whether all theories that do not have quadratic divergences are automatically supersymmetric (and let us forget for the moment about the divergences of the vacuum energy). The answer to this question is no. Supersymmetry can be broken explicitly and there are still no quadratic divergences in field-dependent quantities [277, 122, 309, 490]. The nature of these allowed breakings is however restricted. The breakings that lead to theories where quadratic divergencies are still absent are called soft breakings. They have been completely listed by Girardello and Grisaru [277] and are of the form

$$m^2\varphi\varphi^*\,, \qquad m^2(\varphi^2+\varphi^{*2})\,, \qquad \mu(\varphi^3+\varphi^{*3})\,, \qquad \mu\lambda\lambda\,, \tag{2.83}$$

where φ is a complex scalar and λ a gauge fermion. In particular one can see from this list that, of course, spontaneous breaking is soft: only the first two terms in (2.83) can be obtained there as we have seen in the earlier examples. It is striking that the term $\mu\lambda\lambda$ is soft whereas terms like $\mu\psi\psi$ (where ψ is the fermion of a chiral multiplet) are not soft. This can be explained by gauge symmetry and the fact that $\lambda\lambda$ is the "lowest" member of the $W^\alpha W_\alpha$ multiplet. The fact that a $\mu\psi\psi$ term is hard might be surprising at first sight for another reason since we are allowed to add the soft terms $m^2\varphi\varphi^*$. Consider for example again a model with superpotential

$$W = m\phi^2 + \lambda\phi^3 \tag{2.84}$$

leading to the masses $2m$ for φ $(4m^2\varphi\varphi^*)$ and ψ. Naively one would think that it should be equivalent for the ultraviolet structure of the theory whether one adds a $\mu\psi\psi$ mass term or subtracts a $\mu'\varphi\varphi^*$ term. This, however, is not the case, since it is in general the relation of the fermion mass and the coupling λ that keeps the divergencies under control. This can for example be seen if we consider the potentially quadratic divergent contributions to a term linear in a scalar field φ (fig. 2.10). The first graph contains the trilinear coupling $\varphi^2\varphi^*$ with coupling proportional to $m\lambda$ whereas the second contains the $\lambda\varphi\psi\psi$ coupling and a mass insertion m for the fermion. The two graphs cancel each other if supersymmetry is

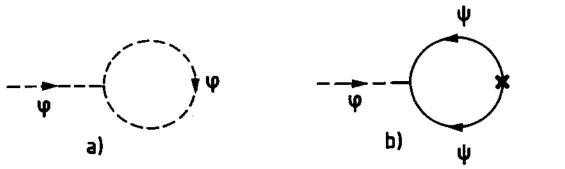

Fig. 2.10. Quadratically divergent contributions to a term linear in φ.

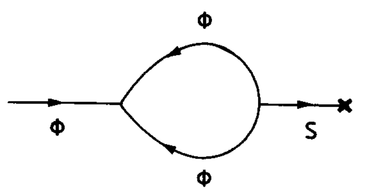

Fig. 2.11. Supergraph for a contribution $F_\phi = \int d^4\theta S^*\phi$ where S is a spurion superfield that represents a soft breakdown of supersymmetry.

exact. If, however, supersymmetry is broken by a term $\mu\psi\psi$ this cancellation ceases to hold, the first contribution is unchanged, whereas the mass insertion in the second gets changed from $2m$ to $2m+\mu$ and the resulting contribution to terms linear in φ is quadratically divergent. We have thus explicitly seen that the breaking $\mu\psi\psi$ is hard. On the other hand a term $\mu^2\varphi\varphi^*$ does not disturb the cancellation of the graphs in fig. 2.10. This does not, of course, completely prove the softness of $\varphi\varphi^*$. The proof of the softness of the terms in (2.83) is again most conveniently performed with the use of the supergraph formalism. The explicit breakdown in this context is introduced [277] by x_μ-independent but θ-dependent spurion superfields. They do not obey the translational invariance in superspace and thus break supersymmetry. An example is the spurion superfield $S=\mu^2\theta^2$. With it the breaking term $\mu^2\varphi^2$ can be introduced in the action through the term $\int d^2\theta S\phi\phi \sim \mu^2\varphi^2$. The introduction of these spurion superfields leads to new divergencies and also breaks the nonrenormalization theorems. Now an F-term in the superpotential can for example be generated in perturbation theory with the help of S. $\int d^4\theta S^*\phi \sim F_\phi$ (fig. 2.11) represented as an integration over the full superspace occurs. The nature of the newly introduced divergencies depends on the dimensionality of the spurion superfield and can be investigated with the power counting rules for superfields. This has been done explicitly by Girardello and Grisaru, with the conclusion that in general only the four terms given in (2.83) are soft. With this explicit breaking parameters new logarithmic divergencies usually appear (e.g. logarithmically divergent mass counterterms) and there is also a quadratic divergence of the vacuum energy. One should stress that the breakings in (2.83) are always soft, other breakings that are in general not soft, e.g. $\mu\psi\psi$, however, could be soft in particular models where the potentially quadratic divergent contributions are forbidden by the absence of certain couplings or by gauge invariance. The problematic graphs in fig. 2.10 for the $\mu\psi\psi$ insertion would for example not exist if the theory does not contain a scalar field that is a gauge singlet.

We will terminate here our introduction to global supersymmetry. A more detailed treatment (as well as a more complete list of references) can be found in the existing reviews [217, 560, 510]. The next chapter will serve as a short introduction to the local form of supersymmetry.

3. Supergravity

3.1. *Local supersymmetry implies (super) gravity*

In the last chapter we have discussed global (or rigid) supersymmetry in which the parameter α transformed as a two-component spinor and was space–time independent $\partial_\mu\alpha=0$. We now want to relax this condition and consider local supersymmetry in which the parameter $\alpha=\alpha(x)$ depends explicitly on x_μ. That local supersymmetry might be of particular interest can already be read off from the algebra (e.g. eq. (2.10))

$$[\alpha Q, \bar{Q}\bar{\alpha}] = 2\alpha\sigma_\mu\bar{\alpha}P^\mu\,, \tag{3.1}$$

which says that the product of two supersymmetry transformations corresponds to a translation in space–time of which the four momentum P^μ is the generator. With space–time dependent parameters $\alpha_1(x)$ and $\bar{\alpha}_2(x)$ the product of two local supersymmetry transformations leads to space–time translations $\alpha_1(x)\sigma_\mu\bar{\alpha}_2(x)\partial^\mu$ that differ from point to point, a general coordinate transformation. From this one might expect gravity to appear necessarily in any locally supersymmetric theory. We will see that this expectation will turn out to be correct. For this reason local supersymmetry is usually called supergravity.

Before we go to a localized version of supersymmetry let us recall the transition from the global to local case in ordinary symmetries. By "ordinary" we mean symmetries in which the generators are scalar objects, in contrast to supersymmetry where the generators are spinorial. As an example consider the action of a free massless Dirac spinor

$$\mathscr{L} = \tfrac{1}{2}\mathrm{i}\bar{\psi}\not{\partial}\psi\,. \tag{3.2}$$

In the last chapter we have exclusively used two-component Weyl spinors. A four-component Dirac spinor can be decomposed in two Weyl spinors $\psi_\alpha = (\chi_\alpha, \bar{\eta}^{\dot{\alpha}})$. The symbol $\bar{\psi}$ denotes $\psi^+\gamma^0$ and $\not{\partial} = \gamma^\mu\partial_\mu$ (for our notation and the used representation of the γ-matrices see the appendix). The Lagrange density in (3.2) has a global symmetry under which ψ transforms into $\mathrm{e}^{-\mathrm{i}\varepsilon}\psi$ with scalar parameter ε. We want to promote this symmetry to a local symmetry and allow $\varepsilon = \varepsilon(x)$ to depend on the space–time point. Now the action is no longer invariant. Under the local transformation $\psi \to \exp(-\mathrm{i}\varepsilon(x))\psi$ we have

$$\delta\mathscr{L} = \tfrac{1}{2}(\partial^\mu\varepsilon(x))\bar{\psi}\gamma_\mu\psi\,. \tag{3.3}$$

Since the density $\mathscr{L}$ was invariant for constant ε the change is proportional to the derivative of ε. To arrive at a locally invariant theory we have to add some terms to the action. This procedure to arrive at a locally symmetric theory is called the Noether procedure, since it uses the Noether current of the symmetry which in our case is $J_\mu = \frac{1}{2}\bar{\psi}\gamma^\mu\psi$. The variation of $\mathscr{L}$ under the local transformation is $\delta\mathscr{L} = (\partial^\mu\varepsilon)J_\mu$. To cancel this term we introduce a gauge field A^μ with transformation property $A^\mu \to A^\mu + \partial^\mu\varepsilon$ and add the term $\mathscr{L}_\mathrm{g} = -A_\mu J^\mu$ to the action. Thus we have

$$\mathscr{L} = \tfrac{1}{2}\mathrm{i}\bar{\psi}\gamma^\mu(\partial_\mu + \mathrm{i}A_\mu)\psi\,, \tag{3.4}$$

which is invariant under the local transformations. We had been lucky here, the addition of one term (plus the kinetic terms for the A-field in the usual gauge invariant way) were sufficient to arrive at a locally invariant action. This is in general not the case. The Noether procedure is applied in general for infinitesimal transformations and becomes an iterative procedure in the parameter ε. In principle it could happen that this procedure might not lead to an invariant action after a finite number of steps and different methods have to be used to promote a global to a local symmetry. But in our example the procedure has worked after one step and has led to the minimal coupling of the gauge field to the Dirac fermion.

For the local symmetry to exist it requires the introduction of new degrees of freedom, the gauge fields: here A_μ. Under the local symmetry the change of these gauge fields is given by terms which contain the derivative of the symmetry parameter: here $\partial^\mu\varepsilon$. In our example the gauge field is thus a spin 1 field because the parameter $\varepsilon(x)$ was scalar. In our minimal formulation only the gauge fields transform with terms that are proportional to $(\partial_\mu\varepsilon)$. The transformation of the other fields contains ε but not $\partial^\mu\varepsilon$. With this in mind we can now start to consider the supersymmetric case. As an exercise we will do this in a four-component spinor notation. Instead of a two-component Weyl spinor ψ_α we work with a four-component Majorana spinor $\chi = (\psi_\alpha, \bar{\psi}^{\dot{\alpha}})$. We start with the free action of a chiral multiplet

$$\mathscr{L} = (\partial^\mu \varphi^*)(\partial_\mu \varphi) + \tfrac{1}{2}\mathrm{i}\bar{\chi}\gamma^\mu \partial_\mu \chi\,, \tag{3.5}$$

where we have already eliminated the auxiliary field F via the equations of motion (compare section 2.3.). φ denotes a complex scalar field $\varphi = (a + \mathrm{i}b)/\sqrt{2}$ and χ is a Majorana spinor with $\bar{\chi} = \chi^+\gamma^0 = (\psi^\alpha, \bar{\psi}_{\dot{\alpha}})$ (for our notation see the Appendix). The action corresponding to (3.5) is invariant under the global supersymmetry transformations as given in eqs. (2.35)–(2.37). In a four-component spinor notation with $\xi = (\alpha_\alpha, \bar{\alpha}^{\dot{\alpha}})$ and using $F = 0$ they read

$$\delta a = \bar{\xi}\chi\,, \qquad \delta b = -\mathrm{i}\bar{\xi}\gamma_5\chi\,, \qquad \delta\chi = -\mathrm{i}\gamma^\mu \partial_\mu (a - \mathrm{i}\gamma_5 b)\xi\,,$$

with

$$\gamma_5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}. \tag{3.6}$$

Performing explicitly these variations of the fields in (3.5) we see that $\mathscr{L}$ changes by a total derivative

$$\delta\mathscr{L} = \partial_\mu(\tfrac{1}{2}\bar{\xi}\gamma^\mu[\not{\partial}(a - \mathrm{i}\gamma_5 b)]\chi)\,, \tag{3.7}$$

which in addition vanishes on-shell (using equations of motion). We now would like to promote the global supersymmetry to a local one and allow ξ to be x-dependent. One question arises immediately concerning the variation of the fermion field since it contains a derivative. According to the remarks made in the previous example we choose $\delta\chi = -\mathrm{i}\gamma^\mu[\partial_\mu(a - \mathrm{i}\gamma_5 b)]\xi(x)$ where the derivative only acts on the scalar fields but not on $\xi(x)$. We now compute the variation of (3.5) under these local transformations to lowest order in the parameter $\xi(x)$, and obtain

$$\mathscr{L} = \partial_\mu(\tfrac{1}{2}\bar{\xi}\gamma^\mu[\not{\partial}(a - \mathrm{i}\gamma_5 b)]\chi) + (\partial_\mu\bar{\xi})[\gamma^\mu(\not{\partial}(a - \mathrm{i}\gamma_5 b))\chi]\,, \tag{3.8}$$

which shows that $\mathscr{L}$ is not locally supersymmetric. A gauge field has to be introduced. Since ξ is a spinor and since the gauge field should transform into $\partial^\mu\xi$ the gauge field of local supersymmetry is a spinorial vector ψ^μ_α. It is called gravitino. We add to the action the Noether coupling

$$\mathscr{L}_\mathrm{N} = -k\bar{\psi}_\mu\gamma^\mu(\not{\partial}(a - \mathrm{i}\gamma_5 b))\chi\,, \tag{3.9}$$

where we have introduced the dimensionful coupling constant $k = 1/\text{mass}$ to give $\delta\mathscr{L}$ the correct dimension. As a consequence ψ^μ should transform as $\delta\psi^\mu = (1/k)\partial_\mu\xi$ under the local supersymmetry transformations. The resulting action is not yet locally supersymmetric, as can be already seen from the multiplet structure. We have introduced a spinorial gauge field $\psi_{\mu\alpha}$ and we know that supersymmetric models have the same number of fermionic and bosonic degrees of freedom. We thus expect new bosonic fields to be necessary to obtain a locally supersymmetric theory.

Explicit variation of $\mathscr{L} + \mathscr{L}_\mathrm{N}$ confirms this expectation. Consider e.g. the variation of χ in $\mathscr{L}_\mathrm{N}$. It leads to

$$\delta\mathscr{L}_\mathrm{N} \sim k\bar{\psi}_\nu\gamma_\mu T^{\mu\nu}\xi \tag{3.10}$$

where the tensor $T^{\mu\nu}$ contains two scalar fields and two derivatives (it actually corresponds to the energy momentum tensor of the scalar fields). This term can only be cancelled through the introduction of a tensor field $g_{\mu\nu}$ with local supersymmetry transformation $\delta g_{\mu\nu} \sim k\bar{\psi}_\mu \gamma_\mu \xi$ and we add to the action

$$\mathscr{L}_g = -g_{\mu\nu} T^{\mu\nu}, \tag{3.11}$$

which corresponds to the coupling of the graviton to the energy momentum tensor. We thus see that any locally supersymmetric theory has to include gravity; it includes the massless spin 2 particle (the graviton) mediating the gravitational interactions. Its partner is the massless spin 3/2 gravitino, the gauge field of local supersymmetry. Our arguments before had not uniquely told us that $\psi_{\mu\alpha}$ corresponds to a pure spin 3/2 state (it could also contain spin 1/2 states) but since $\psi_{\mu\alpha}$ is the supersymmetry partner of the spin 2 graviton, we know that $\psi_{\mu\alpha}$ corresponds to a spin 3/2 particle. The graviton and the gravitino form thus the basic multiplet of supergravity, and one expects the simplest locally supersymmetric model to contain just this multiplet. Let us therefore forget the original model we started with and try to construct the Lagrangian for pure supergravity [248, 113]. We start with the Hilbert action

$$\mathscr{L}_G = -(1/2k^2)\sqrt{|g|}R\,. \tag{3.12}$$

Since we know that there will be fermions present in our model it is necessary to use the vierbein e_μ^m instead of the metric $g_{\mu\nu}$ with the relation $g_{\mu\nu} = e_\mu^m e_\nu^n \eta_{mn}$. (We cannot give here a detailed presentation of gravity and we refer the reader who is unfamiliar with the subject to consider the standard textbooks [548].) The expression (3.12) then reads $-(1/2k^2)eR$ where the curvature R depends on the vierbein e_μ^m and the spin connection ω_μ^{mn}. The spin connection does not represent a new degree of freedom; it can be eliminated by the equations of motion. Next we need the kinetic terms for the gravitino ψ_μ, the Rarita–Schwinger action

$$\mathscr{L}_{RS} = -\tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_5 \gamma_\nu \partial_\rho \psi_\sigma\,. \tag{3.13}$$

This action has to be covariantized with respect to gravity by replacing ∂_μ in (3.13) by the covariant derivative $D_\mu = \partial_\mu + \frac{1}{2}\omega_\mu^{mn}\sigma_{mn}$ where $\sigma_{mn} = \frac{1}{4}[\gamma_m, \gamma_n]$. The sum of the two terms $\mathscr{L}_G$ and $\mathscr{L}_{RS}$ (in the covariantized form) is locally supersymmetric. The corresponding supersymmetry transformations are

$$\delta e_\mu^m = \tfrac{1}{2}k\bar{\xi}\gamma^m\psi_\mu\,, \qquad \delta\psi_\mu = \frac{1}{k}(\partial_\mu + \tfrac{1}{2}\omega_\mu^{mn}\sigma_{mn})\xi \equiv \frac{1}{k}D_\mu\xi\,, \qquad \delta\omega_\mu^{mn} = 0\,. \tag{3.14}$$

The last equation implies the use of the 1.5 order formalism [533]. We see that the first two transformation laws contain the terms that we had found earlier with the Noether procedure.

3.2. Coupling of matter and gauge fields to supergravity

In global supersymmetry the most general coupling of chiral superfield matter and vector superfield gauge fields can be written as

$$\mathrm{Re}\int d^2\theta f(S)WW + \int d^4\theta\phi(\bar{S}\,e^{2eV}, S) + \mathrm{Re}\int d^2\theta g(S)\,, \tag{3.15}$$

where $S(V)$ denote the chiral (vector) superfields, and $g(S)$ is the superpotential. $f(S)$ transforms like a chiral superfield under supersymmetry and as a symmetric product of two adjoint representations with respect to the gauge group under consideration. The theory is renormalizable only if $f(S)$ is constant. The function ϕ transforms as a real vector superfield and only leads to a renormalizable theory if $\phi = \bar{S}\,\mathrm{e}^{2eV}S$. Finally renormalizability requires $g(S)$ to be a polynomial of degree less or equal to three. We give up here the strict criterion of renormalizability since we want to couple this matter gauge system to supergravity, which in itself is a nonrenormalizable theory even if the system coupled to it is renormalizable. After the coupling to supergravity we will however demand that all the existing nonrenormalizable terms contain the gravitational coupling constant such that in the flat limit $k \to 0$ the theory becomes renormalizable. This is the best we can do since there is no known satisfactory solution of the problem of the nonrenormalizability of gravity. We just assume that this problem has no severe implications on the physics at energies low compared to the Planck mass.

The task is now to promote the global supersymmetry in (3.15) to a local symmetry which is equivalent to a symmetric coupling of the field S and V to the supergravity multiplet. There are several ways to do this. One is the Noether procedure we have sketched in the last section. Other more elegant methods using off-shell formulations are also available. All of these methods require rather tedious calculations and space and time available here do not permit us to repeat here a detailed discussion. Such a detailed discussion can be found in ref. [533]. Since the coupling of the action (3.15) to supergravity is given in the literature in its most complete form we content ourselves to give here the final form of the action in component fields. For the derivation of the action we refer to the original papers by Cremmer et al. [100, 101, 97, 98, 574] whose notation we also adopt. The members of the chiral multiplets are denoted by (z_i, χ_i) where i denotes a group index if these fields transform nontrivially under the considered gauge group. The gauge fermions are called λ^α and the field strengths $F^\alpha_{\mu\nu}$ where α labels the adjoint representation of the gauge group and μ, ν are curved space–time indices. The part of the action that contains exclusively bosonic fields is then given by

$$\begin{aligned} e^{-1}\mathcal{L}_{\mathrm{B}} = {} & \exp(-G)(3 + G_k (G^{-1})^k_l G^l) - \tfrac{1}{2}\tilde{g}^2 \operatorname{Re} f^{-1}_{\alpha\beta}(G^i T^{\alpha j}_i z_j)(G^k T^{\beta l}_k z_l) \\ & - \tfrac{1}{4}\operatorname{Re} f_{\alpha\beta} F^\alpha_{\mu\nu} F^{\mu\nu\beta} + \tfrac{1}{4}\mathrm{i}\operatorname{Im} f_{\alpha\beta} F^\alpha_{\mu\nu}\tilde{F}^{\mu\nu\beta} + G^i_j D_\mu z_i D^\mu z^{*j} - \tfrac{1}{2}R\,, \end{aligned} \tag{3.16}$$

here $\tilde{g}$ denotes the gauge coupling constant, $T^{\alpha j}_i$ the group generators in the suitable representation and $\tilde{F}_{\mu\nu}$ is the dual of $F_{\mu\nu}$. The covariant derivatives D_μ are covariant with respect to gravity and the gauge group. The function $G(z^i, z^*_i)$ is a real function of the scalar fields. In terms of the input function ϕ and g in (3.15) it can be written as

$$G = 3\log(-\phi/3) - \log(|g|^2)\,, \tag{3.17}$$

and is called the Kähler potential. The quantities $G'^i = G^i$ etc. are derivatives of G:

$$G''^i_j \equiv G^i_j = \partial^2 G/\partial z_i \partial z^{*j}\,. \tag{3.18}$$

We will discuss this function later in detail. Next we give the action for the terms containing fermions and covariant derivatives

$$e^{-1}\mathscr{L}_{\mathrm{Fkin}} = \tfrac{1}{2}\operatorname{Re} f_{\alpha\beta}(-\tfrac{1}{2}\bar\lambda^\alpha \not{D}\lambda^\beta + \tfrac{1}{2}\bar\lambda^\alpha\gamma^\mu\sigma^{\rho\sigma}\psi_\mu F^\beta_{\rho\sigma} - \tfrac{1}{2}\bar\lambda^\alpha_{\mathrm{L}}\gamma_\mu\lambda^\beta_{\mathrm{R}} G^i D^\mu z_i) - \tfrac{1}{8}\mathrm{i}\operatorname{Im} f_{\alpha\beta} e^{-1} D_\mu(e\bar\lambda^\alpha\gamma_5\gamma^\mu\lambda^\beta)$$
$$- \tfrac{1}{2} f^i_{\alpha\beta}\bar\chi_{\mathrm{L}i}\sigma^{\mu\nu}F^\alpha_{\mu\nu}\lambda^\beta_{\mathrm{L}} - \tfrac{1}{4}\bar\psi_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma\varepsilon^{\mu\nu\rho\sigma}e^{-1} + \tfrac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar\psi_\mu\gamma_\nu\psi_\rho G^i D_\sigma z_i - G^j_i\bar\psi_{\mu\mathrm{L}}\not{D}z^{*i}\gamma^\mu\chi_{\mathrm{L}j}$$
$$- \bar\chi_{\mathrm{L}i}\not{D}z_j\chi^k_{\mathrm{R}}(G^{ij}_k + \tfrac{1}{2}G^i_k G^j) + G^i_j\bar\chi_i\not{D}\chi^j_{\mathrm{R}} + \text{h.c.},$$

where $f^i_{\alpha\beta} = \partial f_{\alpha\beta}/\partial z_i$. The part of the fermionic action that does not contain covariant derivatives reads

$$e^{-1}\mathscr{L}_F = \mathrm{e}^{-G/2}\bar\psi_{\mu\mathrm{R}}\sigma^{\mu\nu}\psi_{\nu\mathrm{R}} + \tfrac{1}{4}\mathrm{e}^{-G/2}G^l(G^{-1})^k_l f^*_{\alpha\beta k}\lambda^\alpha\lambda^\beta + \mathrm{e}^{-G/2}[G^{ij} - G^iG^j - G^l(G^{-1})^k_l G^{ij}_k]\bar\chi_{\mathrm{L}i}\chi_{\mathrm{L}j}$$
$$- \tfrac{1}{2}\mathrm{i}\tilde g G^i T^{\alpha j}_i z_j\bar\psi_{\mathrm{L}\mu}\gamma^\mu\lambda^\alpha_{\mathrm{R}} - \mathrm{e}^{-G/2}G^i\bar\psi_{\mathrm{R}\mu}\gamma^\mu\chi_{\mathrm{L}i}$$
$$+ \tfrac{1}{2}\mathrm{i}(\operatorname{Re} f)^{-1}_{\alpha\beta}f^{\beta\gamma k}\tilde g G^i T^{\alpha j}_i z_j\bar\chi_{\mathrm{L}k}\lambda_{\mathrm{L}\gamma} + 2\mathrm{i}\tilde g G^i_j T^{\alpha k}_j z_k\bar\lambda_{\alpha\mathrm{R}}\chi^i_{\mathrm{R}}$$
$$+ \tfrac{1}{32}(G^{-1})^k_l f^l_{\alpha\beta}f^*_{\gamma\delta k}\bar\lambda^\alpha_{\mathrm{L}}\lambda^\beta_{\mathrm{L}}\bar\lambda^\gamma_{\mathrm{R}}\lambda^\delta_{\mathrm{R}} + \tfrac{3}{32}(\operatorname{Re} f_{\alpha\beta}\bar\lambda^\alpha_{\mathrm{L}}\gamma_m\lambda^\beta_{\mathrm{R}})^2 + \tfrac{1}{8}\operatorname{Re} f_{\alpha\beta}\bar\lambda^\alpha\gamma^\mu\sigma^{\rho\sigma}\psi_\mu\bar\psi_\rho\gamma_\sigma\lambda^\beta$$
$$+ \tfrac{1}{2}f^i_{\alpha\beta}(\bar\chi_{\mathrm{L}i}\sigma^{\mu\nu}\lambda^\alpha_{\mathrm{L}}\bar\psi_{\nu\mathrm{L}}\gamma_\mu\lambda^\beta_{\mathrm{R}} + \tfrac{1}{4}\bar\psi_{\mu\mathrm{R}}\gamma^\mu\chi_{\mathrm{L}i}\bar\lambda^\alpha_{\mathrm{L}}\lambda^\beta_{\mathrm{L}}) + \tfrac{1}{8}G^j_i\bar\chi^i_{\mathrm{R}}\gamma_d\chi_{\mathrm{L}j}(\varepsilon^{abcd}\bar\psi_a\gamma_b\psi_c - \bar\psi^a\gamma_5\gamma^d\psi_a)$$
$$+ \tfrac{1}{16}\bar\chi_{\mathrm{L}i}\gamma^\mu\chi^j_{\mathrm{R}}\bar\lambda^\delta_{\mathrm{R}}\gamma_\mu\lambda^\gamma_{\mathrm{L}}(-2G^i_j\operatorname{Re} f_{\gamma\delta} + \operatorname{Re} f^{-1}_{\alpha\beta}f^i_{\alpha\gamma}f^*_{\beta\delta j})$$
$$+ \tfrac{1}{16}\bar\chi_{\mathrm{L}i}\chi_{\mathrm{L}j}\bar\lambda^\gamma_{\mathrm{L}}\lambda^\delta_{\mathrm{L}}(-4G^{ij}_k(G^{-1})^k_e f^l_{\gamma\delta} + 4f^{ij}_{\gamma\delta} - \operatorname{Re} f^{-1}_{\alpha\beta}f^i_{\alpha\gamma}f^j_{\beta\delta})$$
$$- \tfrac{1}{16}\bar\chi_{\mathrm{L}i}\sigma_{\mu\nu}\chi_{\mathrm{L}j}\bar\lambda^\gamma_{\mathrm{L}}\sigma^{\mu\nu}\lambda^\delta_{\mathrm{L}}\operatorname{Re} f^{-1}_{\alpha\beta}f^i_{\alpha\gamma}f^j_{\beta\delta}$$
$$+ (-\tfrac{1}{2}G^{ij}_{kl} + \tfrac{1}{2}G^{ij}_m(G^{-1})^m_n G^n_{kl} - \tfrac{1}{4}G^i_k G^j_l)\bar\chi_{\mathrm{L}i}\chi_{\mathrm{L}j}\bar\chi^k_{\mathrm{R}}\chi^l_{\mathrm{R}} + \text{h.c.} \tag{3.20}$$

This completes the action. We have set the gravitational coupling constant equal to unity. It can be inserted by dimensional considerations as we will later do in special examples. It should be noted that the considered action depends only on the two functions $G(z, z^*)$ and $f_{\alpha\beta}(z)$, whereas the globally supersymmetric theory defined by (3.15) had three input functions f, ϕ and g. Coupled to supergravity the kinetic function ϕ and the superpotential g loose their independent meaning and enter only through the function $G(z, z^*)$ as given in (3.17), expressing the fact that the considered scalar field space in supergravity is a Kähler manifold [585, 574, 24, 25]. G is called the Kähler potential and the Kähler metric G^j_i defines the kinetic terms of the scalar fields as can be seen from (3.16).

The action given above is, in the given form, ill defined at points where the superpotential g vanishes. This, however, is just an artifact of our notation. The correct action in the case $g(z) = 0$ is given by replacing G everywhere by $J = 3\log(-\phi/3)$ except for the terms e^{-G}, $\mathrm{e}^{-G/2}$ which should be set equal to zero. There are some more complications if one introduces a Fayet–Iliopoulos term [42], but we will not discuss them here.

We finally give the explicit form of the supersymmetry transformations

$$\delta e^m_\mu = \tfrac{1}{2}\bar\varepsilon_{\mathrm{L}}\gamma^m\psi_{\mu\mathrm{R}} + \text{h.c.}\,,$$

$$\delta\psi_{\mu\mathrm{L}} = (\partial_\mu + \tfrac{1}{2}\omega_{\mu mn}(e, \psi))\sigma^{mn}\varepsilon_{\mathrm{L}} + \tfrac{1}{2}\sigma_{\mu\nu}\varepsilon_{\mathrm{L}}G^j_i\bar\chi_{\mathrm{R}i}\gamma^\nu\chi_{\mathrm{L}j} + \tfrac{1}{2}\gamma_\mu\varepsilon_{\mathrm{R}}\,\mathrm{e}^{-G/2}$$
$$+ \tfrac{1}{4}\psi_{\mu\mathrm{L}}(G^i\bar\varepsilon_{\mathrm{L}}\chi_{\mathrm{L}i} - G_i\bar\varepsilon_{\mathrm{R}}\chi^i_{\mathrm{R}}) - \tfrac{1}{4}\varepsilon_{\mathrm{L}}(G^iD_\mu z_i - G_iD_\mu z^{*i}) + \tfrac{1}{4}(g_{\mu\nu} - \sigma_{\mu\nu})\varepsilon_{\mathrm{L}}\bar\lambda^\alpha_{\mathrm{L}}\gamma^\nu\lambda^\beta_{\mathrm{R}}\operatorname{Re} f_{\alpha\beta}\,,$$
$$\delta B^\alpha_\mu = -\tfrac{1}{2}\bar\varepsilon_{\mathrm{L}}\gamma_\mu\lambda^\alpha_{\mathrm{R}} + \text{h.c.}\,,$$

$$\delta\lambda^\alpha_{\mathrm{R}} = \tfrac{1}{2}\sigma^{\mu\nu}\hat F^\alpha_{\mu\nu}\varepsilon_{\mathrm{R}} + \tfrac{1}{2}\mathrm{i}\varepsilon_{\mathrm{R}}\operatorname{Re} f^{-1}_{\alpha\beta}(-\tilde g G^i T^{\beta j}_i z_j + \tfrac{1}{2}\mathrm{i}f^i_{\beta\gamma}\bar\chi_{\mathrm{L}i}\lambda^\gamma_{\mathrm{L}} - \tfrac{1}{2}\mathrm{i}f^*_{\beta\gamma i}\bar\chi^i_{\mathrm{R}}\lambda^\gamma_{\mathrm{R}}) - \tfrac{1}{4}\lambda^\alpha_{\mathrm{R}}(G^i\bar\varepsilon_{\mathrm{L}}\chi_{\mathrm{L}i} - G_i\bar\varepsilon_{\mathrm{R}}\chi^i_{\mathrm{R}})\,,$$

$$\begin{aligned}
\delta z_i &= \bar{\varepsilon}_{\mathrm{L}} \chi_{\mathrm{L}i} \,, \\
\delta \chi_{\mathrm{L}i} &= \tfrac{1}{2} \hat{\not{D}} z_i \varepsilon_{\mathrm{R}} - \tfrac{1}{2} \varepsilon_{\mathrm{L}} \, \mathrm{e}^{-G/2} (G^{-1})_i^j G_j - \tfrac{1}{8} \varepsilon_{\mathrm{L}} \bar{\lambda}^\alpha \lambda^\beta (G^{-1})_i^k f^*_{\alpha\beta k} \\
&\quad + \tfrac{1}{2} \varepsilon_{\mathrm{L}} (G^{-1})_i^k G_k^{jl} \bar{\chi}_{\mathrm{L}j} \chi_{\mathrm{L}l} + \tfrac{1}{4} \chi_{\mathrm{L}i} (G_j \bar{\varepsilon}_{\mathrm{R}} \chi_{\mathrm{R}}^j - G^j \bar{\varepsilon}_{\mathrm{L}} \chi_{\mathrm{L}j}) \,.
\end{aligned} \tag{3.21}$$

3.3. Spontaneous symmetry breakdown

In global supersymmetry we had seen that a spontaneous breakdown occurs if auxiliary fields F or D receive a vacuum expectation value (compare (2.65)). The corresponding fermionic partner of the auxiliary field was identified as the Goldstone fermion. As shown, this fact had been a direct consequence of the algebra and should therefore be also valid in the case of local supersymmetry (with an appropriate interpretation of the Goldstone fermion as we will see later). The quantities to look for if one wants to find out whether supersymmetry is spontaneously broken are thus the auxiliary fields that are the partners of spin 1/2 fermions. We have not yet discussed them explicitly in the framework of local supersymmetry but we know how to find them. They are obtained by a supersymmetry transformation on the spin 1/2 fermions of the theory. More specifically they are the terms in those expressions which do not contain space–time derivatives and can be read off from eqs. (3.29). For the chiral fermions the auxiliary field is [98]

$$F_i = \exp(-G/2)(G^{-1})_i^j G_j + \tfrac{1}{4} f_{\alpha\beta k} (G^{-1})_i^k \lambda^\alpha \lambda^\beta - (G^{-1})_i^k G_k^{jl} \chi_j \chi_l - \tfrac{1}{2} \chi_i (G_j \chi^j) \,. \tag{3.22}$$

For the auxiliary partner of the gauge fermion we obtain

$$\tilde{D}_\alpha = \mathrm{i} \, \mathrm{Re} \, f^{-1}_{\alpha\beta} (-\tilde{g} G^i T_i^{\beta j} z_j + \tfrac{1}{2} \mathrm{i} f^i_{\beta\gamma} \chi_i \lambda^\gamma - \tfrac{1}{2} \mathrm{i} f_i^{*\beta\gamma} \chi^i \lambda_\gamma) - \tfrac{1}{2} \lambda_\alpha (G^i \chi_i) \,. \tag{3.23}$$

As a consequence of the algebra supersymmetry is broken if at least one of these auxiliary fields receives a vacuum expectation value, and the supersymmetry breaking scale M_{S}^2 is given by $\langle F \rangle$ or $\langle D \rangle$ in complete analogy to the situation in global supersymmetry. Notice that all the terms but the first in (3.22) and (3.23) contain fermion fields explicitly. A vacuum expectation value for these terms can thus only occur in presence of a strongly interacting gauge force that leads to a vacuum condensation of bilinear fermion–antifermion states. We will discuss these terms at a later stage and neglect them for the moment. This has the advantage that our formulae simplify substantially. The first terms in (3.22) and (3.23) contain only the scalar fields z_i. Whether F_i or D^α receive actually vacuum expectation values or not depends on the functions G and f and can only be decided after a minimalization of the scalar potential. For the moment, however, let us assume that we have done this and suppose that one auxiliary field, let us say F_i, has a vacuum expectation value $\langle 0 | \exp(-G/2)(G^{-1})_i^j G_j | 0 \rangle \neq 0$. Since the kinetic terms of the scalar fields are given by $G_j^i D_\mu z_i D^\mu z^{*j}$ we see that G_i^j has to be nonzero for the model to be well defined. In addition $\exp(-G/2)$ is in general different from zero except at the points where the superpotential vanishes. It is thus the quantity G_i that is relevant for a discussion of local supersymmetry breaking. This is also true if we would have considered a vacuum expectation value of a D-field as can be seen in (3.23).

In the case of global supersymmetry the fermionic partner of the auxiliary field with nonvanishing v.e.v is the goldstino; a massless particle. Let us now see what happens here. The relevant terms for this discussion are the first, third and fifth in eq. (3.20)

$$\exp(-G/2)\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu - \exp(-G/2)G^i\bar{\psi}_\mu\gamma^\mu\chi_i + \exp(-G/2)[G^{ij} - G^iG^j - G^l(G^{-1})^k_l G^{ij}_k]\chi_i\chi_j\,. \tag{3.24}$$

We see that with $\langle G^i\rangle \neq 0$ the fermion χ_i gets mixed with the gravitino and $\langle G^i\rangle \neq 0$ is just the criterion for supersymmetry breakdown. What happens here is the super-Higgs effect [114, 100], the analogue of the Higgs mechanism in ordinary gauge theories. The spin 1/2 Goldstone fermion $\exp(-G/2)G^i\chi_i$ combines with the massless spin 3/2 gauge particle of local supersymmetry to a massive spin 3/2 gauge particle with mass $\exp(-\langle G\rangle/2)$.

Before we make this relation of the gravitino mass to the supersymmetry breaking more explicit let us first discuss the scalar potential. It is given by the first two terms in (3.16)

$$V = -\exp(-G)[3 + G_k(G^{-1})^k_l G^l] + \tfrac{1}{2}f^{-1}_{\alpha\beta}D^\alpha D^\beta\,, \tag{3.25}$$

where

$$D^\alpha = \tilde{g}G^iT^{\alpha j}_i z_j\,. \tag{3.26}$$

Although the formula is similar to the one in global supersymmetry there is a striking difference and this is the relation of the vacuum energy to the supersymmetry breakdown. In the global case the statement $E_{\mathrm{vac}} = 0$ was equivalent with the presence of unbroken supersymmetry. This ceases to be true in supergravity. Let us first consider unbroken supersymmetry, which implies $\langle G_k\rangle = \langle D^\alpha\rangle = 0$ and we obtain $V = -3\exp(-G)$ which in general is nonvanishing. It tells us that in unbroken supergravity the vacuum energy is negative semidefinite (supergravity is always broken if $E_{\mathrm{vac}} > 0$). If supergravity is broken the situation is more complicated. Suppose we have found the minimum of V. We now make a redefinition of the fields such that the kinetic terms are canonical. We can always do such a redefinition if our model is well defined. We then have $G^k_l = -\delta^k_l$ and $f_{\alpha\beta} = \delta_{\alpha\beta}$, and the potential reads (at the minimum)

$$V_0 = -3\exp(-G_0) + \exp(-G_0)G_{k0}G^k_0 + \tfrac{1}{2}D_{\alpha 0}D^\beta_0\,. \tag{3.27}$$

The last two terms are in exact analogy to the terms in a globally supersymmetric model. With G_{0k} or D^α_0 different from zero (i.e. broken supergravity) everything can happen: the vacuum can be negative, positive or even zero. What is interesting about this point is *the possibility of broken supergravity with vanishing vacuum energy*; a situation that did not exist in the case of global supersymmetry. As we will see later this cancellation of the vacuum energy is usually obtained by an explicit fine tuning. The problem why the cosmological constant vanishes is not understood, but we have at least the possibility to fine tune it to zero, which did not exist in a globally supersymmetric model in which supersymmetry was spontaneously broken. Let us again for the moment put $D_\alpha = 0$ and assume that $V_0 = 0$. This implies that $G_kG^k = 3$ in the broken case. We can now give a relation between the gravitino mass and the scale of supersymmetry breakdown. For this relation between masses we put back in the gravitational coupling constant k

$$1/k = M = M_{\mathrm{p}}/\sqrt{8\pi} = 2.4\times 10^{18}\,\mathrm{GeV}\,. \tag{3.28}$$

The gravitino mass is (3.24)

$$m_{3/2} = M\exp(-G_0/2)\,, \tag{3.29}$$

and the supersymmetry breaking scale is

$$M_S^2 = -M^2 \exp(-G_0/2)(G_0^{-1})^i_j G_{0i}\,, \tag{3.30}$$

with $G^i_{0j} = -1$ as discussed. $V_0 = 0$ implies $G_j = \sqrt{3}$ if only one G' has a v.e.v. (if more of them have v.e.v.'s one has just to take the appropriate linear combination and redefine the fields). This gives the Deser–Zumino [114] relation

$$m_{3/2} = M_S^2/\sqrt{3}M\,, \tag{3.31}$$

in the case of vanishing cosmological constant. The mass $m_{3/2}$ represents the splitting in the supergravity multiplet (the graviton of course stays massless). The relation (3.31) is similar to those we obtained in the case of global supersymmetry (compare (2.74)) where here the coupling constant is gravitational. This implies that the mass splittings can become very small compared to M_S in the case of $M_S \ll M$. We will later use this property extensively.

Although it is not of immediate interest for applications to particle physics, let us also discuss the case of a nonvanishing cosmological constant. Formula (3.29) is valid independent of the vacuum energy. Imagine now a situation with unbroken supergravity $G_i = 0$ but $\exp(-G/2) \neq 0$: $V_0 = -3\exp(-G)$ and $m_{3/2} = \exp(-G/2)$. Unbroken supergravity with $m_{3/2} \neq 0$ is hard to understand. The resolution lies in the rather delicate concept of mass in De Sitter or anti-De Sitter space. A mass term for the gravitino in anti-De Sitter space does not necessarily imply that it has four degrees of freedom. If the cosmological constant $\Lambda = V_0$ is related to the gravitino mass by

$$\Lambda = -3M^2 m_{3/2}^2\,, \tag{3.32}$$

the gravitino has only two dynamical degrees of freedom (which corresponds to a massless particle in the case $\Lambda = 0$). This can be seen by the fact that supersymmetry is still unbroken and as such contains the "massless" spin 3/2 gauge particle with two degrees of freedom. The transformation law of the gravitino, however, changes. The term $D_\mu \xi$ becomes $(D_\mu + \frac{1}{2}m_{3/2}\gamma_\mu)\xi$ as can be explicitly seen in (3.21). This consideration of the $\gamma_\mu \xi$ term in the transformation law of the gravitino is actually an easy way to see whether $\Lambda = 0$ or not [227]. Usually this term is written as $u\gamma_\mu \xi$ where u is called the complex scalar auxiliary field of Poincaré supergravity. The mass term $m_{3/2} = M\exp(-G/2)$ can thus only in the case of $\Lambda = 0$ be considered as an unambiguous signal of supergravity breakdown. The order parameters of supergravity are the auxiliary fields given in (3.22) and (3.23).

Up to now we have discussed the scalar potential only in terms of the general function G. Let us now write these formulas more explicitly in order to compare it to the potential of globally supersymmetric models and to see the role of the superpotential. In general the Kähler potential G has to fulfill certain requirements for the resulting model to be well defined. It should for example lead to the correct sign of the kinetic terms of the scalar fields $G^j_i < 0$. The model is well defined only in those regions of field space where this inequality is satisfied. For the applications one would even prefer models where this relation holds globally. One special choice is the choice of minimal kinetic terms [100]

$$G^j_i = -\delta^j_i\,, \qquad f_{\alpha\beta} = \delta_{\alpha\beta}\,, \tag{3.33}$$

which we will adopt throughout most of our discussion. It allows us to see all essential features of these potentials in a simplified way. We thus use $G(z, z^*) = -z_i z^{*i} - \log|g(z_i)|^2$ where $g(z_i)$ is the superpotential. This leads to

$$G^i = -z^{*i} - g^i(z_i)/g(z_i)\,, \tag{3.34}$$

with $g^i(z_i) = \partial g(z_i)/\partial z_i$, and some manipulations lead to the scalar potential

$$V = \exp\left(\frac{z_i z^{*i}}{M^2}\right)\left[\left|g^i(z_i) + \frac{z^{*i}}{M^2} g(z_i)\right|^2 - \frac{3}{M^2}|g|^2\right], \tag{3.35}$$

where summation over i is understood. Supersymmetry is broken if $f^i = g^i + z^* g/M^2$ receives a vacuum expectation value. In the globally supersymmetric case this role as an order parameter had been played by $g^i(z_i)$. The addition of the term $z^* g(z)/M^2$ leads to potentially interesting consequences. Suppose we have a superpotential $g(z_i)$ that contains an intrinsic scale μ such that, e.g., at the minimum $g(z_i) = \lambda\mu^3$ where λ is a coupling constant. In a globally supersymmetric model with spontaneously broken symmetry such a superpotential would lead to a breakdown scale

$$M_S^2 \sim \lambda\mu^2\,, \tag{3.36}$$

essentially of order μ. This can be different in the local case. Suppose that supersymmetry is broken $f^i \neq 0$ and that $\Lambda = 0$:

$$M_S^2 = f_0^i = (\sqrt{3}/M) g_0(z_i) \sim \lambda\mu^3/M\,. \tag{3.37}$$

Vacuum expectation values of fields of order μ lead to a breakdown of supersymmetry at a mass scale μ^3/M which can be substantially smaller than μ as long as μ is smaller than M. Moreover the splittings of the multiplets are given in general not by μ but by $m_{3/2}$ the gravitino mass

$$m_{3/2} \sim M_S^2/M \sim \lambda\mu^3/M^2\,. \tag{3.38}$$

In the breakdown of supergravity it seems likely that huge ratios between mass scales are generated [446]. According to our discussion in Chapter 1 it is among others this behavior of supergravity that awakened the interest of applying these theories to models of high energy physics. We will come back to these questions in more detail in Chapters 6 and 7, where we will also give explicit examples.

We have here essentially limited our discussion of supergravity breakdown to the case where F^i terms receive a vacuum expectation value. This allowed us to use rather simple formulas. The inclusion of different mechanisms is straightforward. If a D^α term is at the origin of the supersymmetry breakdown it will lead to a vacuum energy [97, 98]

$$V_0 = \exp(-G_0)[G_{0k} G_0^k - 3] + \tfrac{1}{2} f_{0\alpha\beta}^{-1} D_0^\alpha D_0^\beta\,, \tag{3.39}$$

a supersymmetry breakdown scale $M_S^2 = D_0^\alpha$ and the corresponding gauge fermion λ^α will play the role of the "goldstino" which provides the two additional degrees of freedom for the massive gravitino. If the breakdown is dynamical, e.g. through a condensation of the gauge fermions $\langle\lambda\lambda\rangle \neq 0$ the potential is given by [226]

$$V=-3M^4\exp(-G)+\left[M^2\,\mathrm{e}^{-G/2}G_j+f_j\frac{\lambda\lambda}{4M}\right]\left[M^2\,\mathrm{e}^{-G/2}G_k+f_k\frac{\lambda\lambda}{4M}\right]^*(G^{-1})^j_k \tag{3.40}$$

as can be derived from (3.22). The corresponding goldstino is $f^i_{\alpha\beta}\langle\lambda^\alpha\lambda^\beta\rangle\chi_i$. We will come back to these questions in Chapter 6.

3.4. *The supertrace of the mass matrix*

We have seen in section 2.4 that in spontaneously broken globally supersymmetric models strong mass relations hold at the tree level and we will discuss this question here in the case of local supersymmetry. The relevant quantity to compute is the supertrace of the squared mass matrix $\mathrm{STr}\,\mathcal{M}^2=\Sigma_J(-1)^{2J}(2J+1)\mathcal{M}^2_J$ where J denotes the spin [227]. We will do this here in the case of N chiral superfields (z_i,χ_i) with minimal kinetic terms for a model with vacuum energy equal to zero [98, 574]. More general formulas will be given at the end of this section. The potential is given by $\exp(-G)[G_iG^i-3]$ and $\Lambda=0$ implies $G_iG^i=3$ at the minimum of the potential. The minimum condition $V_j=\partial V/\partial z^j=0$ gives

$$G_{ij}G^i=G_j\,, \tag{3.41}$$

where we have explicitly used $G_iG^i=3$ and $G^i_j=-\delta^i_j$. The mass of the gravitino is $\exp(-G/2)$ and we get a contribution of $-4\exp(-G)$ to the supertrace coming from $J=3/2$. The mass of the scalar particles is obtained through the second derivative of the potential V^j_i. In our special case this is

$$V^k_j=\exp(-G)[G_{ij}G^{ik}+G^k_iG^i_j]\,. \tag{3.42}$$

The situation for the spin 1/2 particles is more complicated. The masses of these particles can be read off from the third term in eq. (3.20) but the particle that was absorbed by the gravitino has to be removed since we already counted it in the spin 3/2 sector. The goldstino is given by $\psi=G^i\chi_i$. The mass matrix for the remaining $(N-1)$ chiral fermions becomes proportional to $G^{ij}-\frac{1}{3}G^iG^j$. Summing up these contributions gives for the supertrace

$$\mathrm{STr}\,\mathcal{M}^2=2(N-1)m^2_{3/2}\,, \tag{3.43}$$

a surprising result. Unlike in the case of spontaneously broken *global* supersymmetry the $\mathrm{STr}\,\mathcal{M}^2$ does not vanish here. If one considers supergravity in the flat limit $M\to\infty$ (but $m_{3/2}$ fixed) spontaneously broken local supersymmetry resembles more closely to explicitly than spontaneously broken global supersymmetry [98]. In (4.43) the supertrace is positive which implies that *on the average* the bosons are heavier than their fermionic partners, a property which will be useful for later model building [98, 177].

Under the given restrictions $\Lambda=0$, $G^i_j=-\delta^i_j$ (3.43) is valid independently of the chosen superpotential, of which we will now consider a special case. Suppose that $g(z)$ depends only on one of the scalar fields z_1. If this would be the only field we would have vanishing supertrace since $N=1$. With the addition of the $N-1$ fields $z_2,\ldots,z_N$ the supertrace changes to $2(N-1)m^2_{3/2}$. What happens is that all the fermions $\chi_2,\ldots,\chi_N$ remain massless and the $2(N-1)$ real scalar fields receive a common mass $m_{3/2}$ as the reader can easily convince himself through an explicit calculation. Notice that it is the gravitino mass and not the supersymmetry breaking scale M_S that corresponds to the splitting of the supersymmetry multiplets. Notice also that in this special case all the $2(N-1)$ real bosons are degenerate in mass.

Formula (3.43) has been derived under the restrictions $\Lambda = 0$ and $G^i_j = -\delta^i_j$, and we have not yet included the (spin 1/2, spin 1) multiplets. With these taken into account (still $\Lambda = 0$, $G^i_j = -\delta^i_j$) the general formula reads [98]

$$\mathrm{STr}\,\mathcal{M}^2 = (N-1)[2m^2_{3/2} - D^\alpha D_\alpha/M^2] - 2\tilde{g}_\alpha D^\alpha \,\mathrm{Tr}\, T^\alpha \,. \tag{3.44}$$

The last term corresponds to the one also present in the globally supersymmetric case. The D^2/M^2 term is new and gives a negative contribution to the supertrace. The condition of $\Lambda = 0$ requires $D^2/M^2 < 6m^2_{3/2}$ still leaving open the possibility that it might cancel or dominate the $2m^2_{3/2}$ term. No explicit model, however, is known where this happens. The results in (3.43) and (3.44) change if we relax the conditions on Λ and G^i_j. For $\Lambda = m^2_{3/2}M^2[2x-3]$ eq. (3.43) generalizes to [446]

$$\mathrm{STr}\,\mathcal{M}^2 = 4m^2_{3/2}[(N-1)(x-1) - 2(1-\tfrac{2}{3}x)^2] \,. \tag{3.45}$$

It also changes for nonminimal kinetic terms. (This is actually also true in the case of global supersymmetry, but there we did not consider nonminimal kinetic terms.)

Equation (3.43) for nonminimal kinetic terms reads ($\Lambda = 0$) [294]

$$\mathrm{STr}\,\mathcal{M}^2 = 2(N-1)m^2_{3/2} - 2R^i_j F_i F^j \,, \tag{3.46}$$

with

$$R^j_i = [\log \det(G^l_k)]^j_i \,, \tag{3.47}$$

$$F_i = \exp(-G/2)(G^{-1})^j_i G_j \,, \tag{3.48}$$

compare (3.22). The generalization of (3.44) to nonminimal kinetic terms is given in ref. [294]. We will later discuss the implications of these relations in the context of explicit models in Chapter 6.

This concludes our collection of formulae relevant in $N = 1$ supergravity. A more detailed treatment can be found in refs. [533, 97, 98, 100, 101].

4. Low energy supersymmetry: basics

By low energy we denote the 100 GeV region. In this section we introduce and discuss the ingredients that every model must possess in which supersymmetry has implications on this low energy region. We will also encounter the potential dangers in these models which could lead to inconsistencies when compared to experimental results. We start with the introduction of the standard nonsupersymmetric model.

4.1. *The standard model* [59, 280, 500, 547, 495, 281, 368]

The standard model is based on the gauge interactions of the strong and electroweak interactions with gauge group SU(3)×SU(2)×U(1). It contains 12 spin 1 gauge bosons: eight gluons of SU(3), three SU(2) weak gauge bosons and the hypercharge gauge boson of $U(1)_y$. The photon will be a particular combination of an SU(2) gauge boson and the hypercharge boson as we will see in a moment. The fermions in the theory are three generations of quarks and leptons

$$\begin{array}{llllll} \text{I.} & \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix} & \bar{\mathrm{u}} \quad \bar{\mathrm{d}} & \begin{pmatrix} \nu_{\mathrm{e}} \\ \mathrm{e} \end{pmatrix} & \bar{\mathrm{e}} \\ \text{II.} & \begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix} & \bar{\mathrm{c}} \quad \bar{\mathrm{s}} & \begin{pmatrix} \nu_{\mu} \\ \mu \end{pmatrix} & \bar{\mu} \\ \text{III.} & \begin{pmatrix} \mathrm{t} \\ \mathrm{b} \end{pmatrix} & \bar{\mathrm{t}} \quad \bar{\mathrm{b}} & \begin{pmatrix} \nu_{\tau} \\ \tau \end{pmatrix} & \bar{\tau} \end{array} \tag{4.1}$$

where we have included the top quark, which is not yet experimentally confirmed. This is a threefold repetition of a set of fermions that has the following transformation properties with respect to SU(3)×SU(2)×U(1)

$$\begin{aligned} &U^a = \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix} = (3, 2, \tfrac{1}{6}), \qquad \bar{\mathrm{u}} = (\bar{3}, 1, -\tfrac{2}{3}), \qquad \bar{\mathrm{d}} = (\bar{3}, 1, \tfrac{1}{3}), \qquad a = 1, 2 \\ &L^a = \begin{pmatrix} \nu_{\mathrm{e}} \\ \mathrm{e} \end{pmatrix} = (1, 2, -\tfrac{1}{2}), \qquad \bar{\mathrm{e}} = (1, 1, 1), \end{aligned} \tag{4.2}$$

where the first two entries in the brackets denote the dimensions of the SU(3)×SU(2) representations and the last entry denotes U(1) hypercharge. The set in (4.2) is anomaly free with respect to SU(3)×SU(2)×U(1). (This is one of the strong arguments to believe that the t quark exists.) The fermions in (4.2) denote left-handed two-component Weyl spinors. Mass terms are forbidden by the gauge interactions. They can only occur if one of the gauge symmetries is broken. The electric charge is given by $Q = T_3 + Y$ where T_3 is the diagonal generator of SU(2); e.g. we have $T_3(u) = +1/2$ and $Q(u) = 2/3$.

Let us denote the gauge fields for SU(2) and U(1) by A^i_μ and B_μ and the gauge couplings by g_2 and g_1 respectively ($i = 1, 2, 3$). The field strengths are then given by

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_2 \varepsilon^{ijk} A^j_\mu A^k_\nu , \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu , \tag{4.3}$$

and the gauge interactions of, for example, the leptons are given by

$$\mathscr{L}_{\mathrm{ge}} = \mathrm{i}\bar{e}^+ \sigma^\mu (\partial_\mu - \mathrm{i} g_1 B_\mu) \bar{e} + \mathrm{i} L^+ \sigma^\mu (\partial_\mu + \tfrac{1}{2}\mathrm{i} g_1 B_\mu - \tfrac{1}{2}\mathrm{i} g_2 \sigma^i A^i_\mu) L . \tag{4.4}$$

The SU(3) interactions with coupling g_3 are given in a similar way.

$SU(3)\times SU(2)_w \times U(1)_y$ is broken to $SU(3)\times U(1)_{em}$. This is achieved through the introduction of scalar fields: the Higgs sector. The minimal choice is one SU(2) doublet of charged scalars $h = \binom{h^0}{h^-}$ with hypercharge $Y = -1/2$, with potential

$$V = \mu^2 h^+ h + \lambda (h^+ h)^2 . \tag{4.5}$$

One also introduces Yukawa couplings for the interactions of the scalars with the fermions, e.g.,

$$\mathscr{L}_Y = g_d U h \bar{d} + g_u U h^* \bar{u} \tag{4.6}$$

in all combinations that are consistent with the SU(3) × SU(2) × U(1) gauge symmetry.

The multiplication of the doublets in (4.6) is understood as $U^a h^a \varepsilon_{ab}$ where a is the SU(2) index and ε_{ab} is antisymmetric, the second term is $U^a (h^*)_b \delta^b_a$. Next one lets μ^2 to become negative so that one component, which we choose to be the neutral component of h, develops a vacuum expectation value

$$\langle h \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad v = (-\mu^2/\lambda)^{1/2}. \tag{4.7}$$

This breaks both SU(2) and $\mathrm{U(1)_y}$ but leaves $\mathrm{U(1)_{em}}$ unbroken. The Higgs mechanism takes place and the charged bosons $W^\pm_\mu = (A^1_\mu \mp \mathrm{i} A^2_\mu)/\sqrt{2}$ receive a mass $M_W = \frac{1}{2} g v$. One of the neutral gauge bosons

$$z_\mu = (-g_2 A^3_\mu + g_1 B_\mu)/(g_1^2 + g_2^2)^{1/2}, \tag{4.8}$$

receives the mass $\frac{1}{2} v (g_1^2 + g_2^2)^{1/2}$ whereas the orthogonal combination

$$A_\mu = (g_2 B_\mu + g_1 A^3_\mu)/(g_1^2 + g_2^2)^{1/2} \tag{4.9}$$

(the photon) remains massless. There is one remaining real scalar field with mass $\sqrt{-2\mu^2}$, the Higgs particle. The three parameters v, g_1 and g_2 can be determined experimentally by the electromagnetic coupling constant e, the Fermi coupling constant G_F and a measurement of the strength of the weak neutral currents. With $\tan\theta_w = g_1/g_2$ we have the following relations

$$\begin{aligned} G_F/\sqrt{2} &= g_2^2/8M_W^2 = \tfrac{1}{2} v^2, \\ e &= g_2 \sin\theta_w = g_1 \cos\theta_w, \end{aligned} \tag{4.10}$$

where M_W is the mass of the charged weak bosons. This determines v to be of the order of 250 GeV and $M_W \approx (38\ \mathrm{GeV})/\sin\theta_w$ as well as $M_Z \approx (38\ \mathrm{GeV})/\frac{1}{2}\sin 2\theta_w$, where from the strength of the weak neutral currents one obtains $\sin^2\theta_w \approx 0.21$. The consistency of the (recently discovered) W and Z boson masses with these predictions is a sensitive test of the model. The mass of the Higgs $\sqrt{-2\mu^2}$ remains however a completely undetermined parameter.

Up to now we have seen the implication of the Higgs sector to the gauge boson sector. The Higgs sector fulfills an additional task: the mass generation for the fermions. With SU(2) × $\mathrm{U(1)_y}$ broken to $\mathrm{U(1)_{em}}$ the symmetries SU(3) × $\mathrm{U(1)_{em}}$ do no longer forbid mass terms for the quarks and leptons and the Yukawa couplings like in (4.6) provide these mass terms. The mass for the d quark is for example given by $g_d(v/\sqrt{2})$, with v as defined in (4.7). No predictions, however, arise from the model: the experimentally determined masses of quarks and leptons are obtained by adjusting the Yukawa couplings to the desired values. In particular it is not understood why the different generations of quarks and leptons have so vastly different masses. Moreover, the mass matrix is not diagonal in the weak basis. Mixings between the different generations occur as given by the Kobayashi–Maskawa [368] matrix:

$$(\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 \, e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 \, e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 \, e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 \, e^{i\delta} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix} \tag{4.11}$$

where we have chosen the (d, s, b) matrix to be diagonal. The symbols $s_i = \sin\theta_i$, $c_i = \cos\theta_i$ denote three real parameters (θ_1 is the Cabibbo angle). The phase δ is CP violating. Three generations of quarks are necessary to arrive at CP violation through this mechanism.

Much more could be said about the standard model which we cannot repeat here in detail.

4.2. *Grand unification* [271, 473, 394]

Grand unified models are models that unify the gauge interactions $SU(3)\times SU(2)\times U(1)$ in a larger group. To be successful they should reduce to the standard model at low energies. We will not attempt here to give a full account of what has been done in the context of these models (for a review see the Physics Reports of Langacker [394]) but will just introduce the simplest SU(5) model [271] both to explain the ideas and fix the notation. This minimal model might at the moment have some difficulties because of the absence of proton decay at the predicted rate [56], but we want to discuss the issues of grand unification in the simplest model.

The gauge group $SU(3)\times SU(2)\times U(1)$ is unified in the semisimple group SU(5), i.e. the three coupling constants g_3, g_2 and g_1 are given by one coupling constant g_5. Now at a hundred GeV the coupling constants g_3, g_2 and g_1 are very different from each other and this is not a very strong indication for unification. The resolution [272] of this lies in the fact that the coupling constants are not renormalization group invariant objects. Secondly a grand unified model needs a mass scale as input. This is the scale where the grand unified group breaks into the $SU(3)\times SU(2)\times U(1)$ subgroup. It is at this scale that we expect an algebraic relation between the coupling constants dictated by group theory. Since the couplings g_3 and g_2 are quite different from each other, 100 GeV, one should not be very surprised if the scale of the SU(5) breakdown (the grand unification scale M_x) is very different from 100 GeV. The discussion of these issues is somewhat model independent and we will therefore introduce the model first and then discuss these questions.

As a gauge group we have SU(5) and this implies the existence of 24 gauge bosons, 12 of which will correspond to the gluons, the $W^{\pm}$, Z and photon γ. The additional 12 gauge bosons, which under $SU(3)\times SU(2)$ transform as $(3,2)+(\bar{3},2)$ are supposed to receive a mass of order $g_5 M_x$ through the Higgs mechanism in SU(5) breakdown. The fermions of the standard model should be assigned to multiplets in the grand unified group. It is the great beauty of the SU(5) model that this can be done in a way without predicting new fermions in the low energy spectrum. In this sense the SU(5) model is the minimal model. The fifteen Weyl fermions of one generation of quarks and leptons can be assigned to two irreducible SU(5) multiplets a $\bar{5}$ (the complex conjugate of the fundamental representation) and a 10 (a two index antisymmetric representation)

$$(\bar{5})_a = (\bar{d}_1, \bar{d}_2, \bar{d}_3, e, \nu_e)\,, \qquad (10)^{\alpha\beta} = \begin{pmatrix} 0 & \bar{u} & \bar{u} & u & d \\ & 0 & \bar{u} & u & d \\ & & 0 & u & d \\ & & & 0 & \bar{e} \\ & & & & 0 \end{pmatrix} \tag{4.12}$$

where $\alpha = 1, \ldots, 5$ is an SU(5) index and $SU(3) \times SU(2) \times U(1)$ is embedded in an obvious way, the hypercharge generator Y corresponding to the generator

$$\frac{1}{3}\begin{pmatrix} -1 & & & & 0 \\ & -1 & & & \\ & & -1 & & \\ & & & \frac{3}{2} & \\ 0 & & & & \frac{3}{2} \end{pmatrix} \tag{4.13}$$

of SU(5). Formula (4.12) gives the content of one family, the additional two are obtained by repetition. The assignment of quarks and leptons tells us that both baryon (B) and lepton (L) number can no longer be conserved in this grand unified model. Quarks with $B = 1/3$, $L = 0$ and leptons with $B = 0$ and $L = 1$ are members of the same SU(5) multiplets and the SU(5) gauge bosons cause transitions. These are of course those gauge bosons that are in the coset $SU(5)/SU(3) \times SU(2) \times U(1)$ and which will become massive after SU(5) breakdown. An example for an interaction that leads to proton decay is given in fig. 4.1, where $i, j = 1, 2, 3$ denote color SU(3) indices. This graph leads to proton decay $p \to e^+ \pi^0$. There is actually a selection rule for proton decay in the simple SU(5) model. The reason for this is a nonanomalous global U(1) symmetry that occurs because one family of quarks and leptons is made up of two irreducible representation of SU(5). With respect to this symmetry the $\bar{5}$ has charge $X = -3$ whereas the 10 transforms with $X = 1$. This leads to

$$B - L = \tfrac{1}{5}(X + 4Y) \tag{4.14}$$

as a global symmetry and the difference of baryon and lepton number is conserved in the minimal SU(5) model and proton decay leads to final states that contain an antilepton. This conservation is a property of the simple model, it is no longer true in more complicated models. The property of proton decay, however, is in principle common to all grand unified models. The long lifetime of the proton $\tau \geq (10^{30}\text{–}10^{31})$ years leads to a lower bound of the grand unification scale M_x of $M_x \geq 10^{15} \sim 10^{16}$ GeV, indeed an enormous scale compared to the mass scale of the breakdown of the weak interactions.

So far we have only discussed the gauge boson and fermion spectrum of the model. Scalars are also needed to trigger the breakdown of $SU(5) \to SU(3) \times SU(2) \times U(1)$ and of $SU(2) \times U(1)$ to $U(1)_{em}$. A minimal set includes an adjoint $\phi(24)$ representation of SU(5) (to break SU(5)) and a fundamental H(5) representation which contains the scalar doublet of the standard model. The potential of the 24 is arranged in a way that ϕ gets a vacuum expectation value v of order of the grand unification scale

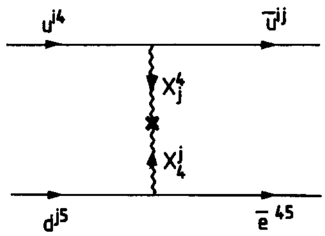

Fig. 4.1. Proton decay due to heavy SU(5) gauge bosons: $p \to e^+ \pi^0$.

$$\langle\phi^{\beta}_{\alpha}\rangle = v \begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & -\frac{3}{2} & \\ 0 & & & & -\frac{3}{2} \end{pmatrix} \tag{4.15}$$

which breaks SU(5) to the desired subgroup. As a result 12 of the 24 gauge bosons will receive a mass and the remaining scalars also have a mass of the order of M_x. The SU(2) doublet in H_5 is used to break $SU(2)\times U(1)$ at a scale of 100 GeV. The Yukawa couplings of H_5 to quarks and leptons are given through

$$g_{Yab} H^*_5 \bar{5}_a 10_b + g'_{Yab} H_5 10_a 10_b \tag{4.16}$$

$(a, b = 1, \ldots, 3)$ are generation indices. The first term in (4.16) gives masses to the electron and the d quarks, whereas the second term gives mass to the u quark through the vacuum expectation value of the SU(2) doublet in H_5 after the breakdown of $SU(2)\times U(1)$. In addition H_5 contains a color SU(3) triplet. Since through the Yukawa couplings in (4.16) this triplet leads to proton decay through graphs like the one in fig. 4.2 this color triplet has to be sufficiently heavy. The relevant Yukawa couplings in (4.16) are dictated by the masses of quarks and leptons and are smaller than the SU(5) gauge coupling. As a result the lower bound on the H_3 masses is smaller than the ones for the gauge bosons: 10^{10}–10^{11} GeV. In the minimal model they receive a mass of order of M_x through terms like

$$m^2 H_5 H^*_5 + m' H_5 \phi_{24} H^*_5 \,, \tag{4.17}$$

where m and m' are of order of M_x. Notice that two terms are necessary since (4.17) has to give a large mass to the triplet and a tiny mass of order of 100 GeV to the doublet in H_5. Replacing ϕ by its v.e.v. one needs

$$m^2 - \tfrac{3}{2} m' v \ll m^2, m'^2, v^2 \,, \tag{4.18}$$

which can be obtained by carefully adjusting the parameters. This completes the definition of the minimal SU(5) model.

SU(5) breaks to $SU(3)\times SU(2)\times U(1)$ at M_x and at M_x we have $g_5 = g_3 = g_2 = \sqrt{5/3}\, g_1$. The factor $(5/3)^{1/2}$ occurs because the hypercharge generator Y as defined in (4.13) was not normalized to $\mathrm{Tr}\, Y^2 = 1/2$ mainly for historical reasons. Below M_x the gauge coupling constants evolve according to the expressions $(\alpha_i = g_i^2/4\pi)$

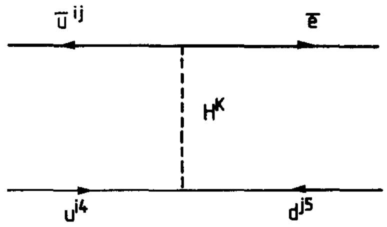

Fig. 4.2. Proton decay through Higgs scalars in the fundamental representation of SU(3).

$$1/\alpha_i(\mu) = 1/\alpha_i(M_x) + (b_i/2\pi)\log(M_x/\mu)\,, \tag{4.19}$$

with the β-function defined as $\beta_i = b_i g_i^3/16\pi^2$ at the one-loop level. The b_i are given in general by

$$b = -\tfrac{11}{3}C + \tfrac{4}{3}N_f l_f(\mathrm{R}) + \tfrac{1}{3}N_s l_s(\mathrm{R})\,, \tag{4.20}$$

where C is the quadratic Casimir operator of the adjoint representation, $N_{f,s}$ is the number of Dirac fermions (complex scalar bosons) in representation R of which $l_{f,s}(\mathrm{R})$ is the quadratic Casimir (equal to 1/2 for the fundamental representation). For the minimal SU(5) model we obtain

$$b_3 = -11 + \tfrac{4}{3}N_G\,, \qquad b_2 = -\tfrac{22}{3} + \tfrac{4}{3}N_G + \tfrac{1}{6}H\,, \qquad b_y = \tfrac{4}{3}N_G + \tfrac{1}{10}H\,, \tag{4.21}$$

where N_G is the number of generations (4.3) and H is the number of light Higgs doublets (4.1). As a result the coupling constants g_i evolve differently below M_x. It is the great success of this model that with an actual computation of this evolution one obtains the correct coupling constants $g_i(M_W)$ in the standard model provided one chooses $M_x \sim 10^{14}$–10^{15} GeV and $\alpha_5 \sim (\frac{1}{40}$–$\frac{1}{45})$ [272]. Thus the two estimates of M_x, one from the limits of lifetime of the proton, the other from the evolution of the coupling constants, lead to enormous mass scales like 10^{15} GeV. Recent upper limits on proton decay, however, seem to indicate that these two estimates seem to be inconsistent for the minimal SU(5) model [56]. We will not comment on these questions here but will come back to it in the framework of supersymmetric models where things change quantitatively.

4.3. *Supersymmetry and the mass scale of the weak interactions*

We now want to focus our attention to the sector of the standard or grand unified models in which the magnitude of the scale $v \sim 250$ GeV of the breakdown of the weak interactions is introduced. This is done in the Higgs sector which exclusively consists of scalar particles. None of these particles has been detected experimentally so that this sector is completely unchecked. In the standard model the v.e.v. of $v = 250$ GeV is inserted by hand through the parameters μ^2 and λ (compare (4.7)). The mass of the remaining Higgs $(-2\mu^2)^{1/2}$ is completely undetermined. Of course μ^2 cannot be arbitrarily large since we know that $-\mu^2/\lambda$ has to be $(250\ \mathrm{GeV})^2$ and large μ^2 would imply large λ which would mean that our perturbative small coupling constant treatment of the model would be inadequate [538, 540]. We therefore think that μ should not be much larger than let us say a TeV.

In a grand unified model the parameters μ^2 and λ are put in by hand in the same way. Actually here some subtleties with the definition of μ^2 appear because the Higgs is in a SU(5) multiplet together with a color triplet that necessarily has to be heavy. To have μ^2 of the order of $(100\ \mathrm{GeV})^2$ one actually has to "fine tune" the parameters of the model (compare (4.18)) to an accuracy of $\mu^2/\mathrm{m}^2 \sim 10^{-26}$.

Independent of this problem we are in a situation that in both models the parameters μ^2 and λ are introduced by hand and we do not understand why $v \approx 250$ GeV. In particular we do not understand why v is so tiny compared to the grand unification scale M_x or more generally the Planck mass M_p. In a more complete theory one would like to understand why this cale is so small compared to the other scales. Looking at models with gauge bosons or fermions we can imagine mechanisms that keep parameters small: gauge symmetries and chiral symmetries. These mechanisms provide in general a reason that the corresponding parameters are zero in the symmetric limit. The smallness of these parameters coming from a breakdown of the symmetries, e.g. the $\mathrm{SU(2)\times U(1)}$ breakdown of the standard model, is necessary to allow masses for the gauge bosons and fermions. Models in which extremely tiny quantities are understood in this way are often called natural [531].

In the case under consideration we have $v^2/M^2 \sim 10^{-34}$, a very small quantity which suggests that to find an understanding of v one first should try to understand the case $v = 0$ in terms of a symmetry. If we now take the standard model with $\mu = 0$ we realize that this does not increase the symmetries of the system. The model is not natural in the sense specified above. This has as one consequence that $\mu = 0$ is not stable in perturbation theory. In fact the parameter μ^2 receives quadratically divergent contributions in perturbation theory. This is no problem since μ^2 is a free input parameter and it is the renormalized quantity that is of relevance and can be chosen at will.

But let us now consider a natural model, where a symmetry keeps $\mu^2 = 0$. The radiative corrections will respect this symmetry and $\mu^2 = 0$ will be valid in any order of perturbation theory. If this symmetry is broken with parameter Δm^2 the radiative corrections will be finite and $\mu^2 \sim \alpha \Delta m^2$ will be generated and still understood. If Δm^2 becomes infinite μ^2 will become quadratically divergent and the value of the bare parameter looses its meaning.

One might now try to find a symmetry that could render the standard model natural. The quantity that should vanish in the symmetric limit is the mass term $\mu^2 hh^*$ of the Higgs doublet, and the only symmetry we know of that can do this is supersymmetry [542, 569]. This is the basic motivation for us to consider the application of supersymmetry to models of particle physics. In the framework of a supersymmetric standard model we would have $\mu^2 = 0$ and this would be stable in perturbation theory as we have seen in Chapter 2. This comes from the fact that any particular contribution to μ^2 is cancelled by another contribution involving the partner of those fields that gave rise to the original contribution. If supersymmetry is broken this cancellation is no longer exact since the masses of the supermultiplets are split by Δm and a well defined $\mu^2 \sim \alpha(\Delta m)^2$ arises in perturbation theory. What supersymmetry essentially does for these properties of the scalars is that it relates them to fermions for which we can have "reasons" why they are massless. Supersymmetry just tells us that the properties of these fermions are also valid for the bosons. In the following we will explore the possibilities of a natural standard model in the framework of supersymmetry.

4.4. *The particle content of a supersymmetric standard model* [203]

A supersymmetric model has the same number of bosonic and fermionic degrees of freedom, as we have seen in Chapter 2. The standard model with three families contains 28 bosonic degrees of freedom (12 massless gauge bosons, 2 complex scalars) and 90 fermionic degrees of freedom (45 Weyl fermions). In order to make this model supersymmetric, new degrees of freedom have to be added, one cannot just do it through relations between masses and coupling constants. Let us start with the spin 1 bosons. They are required to have fermionic partners (the gauge fermions or gauginos) with the same quantum numbers: an SU(3) octet, an SU(2) triplet and a singlet with $Y = 0$. None of the fermions in the standard model have these quantum numbers and these gauge fermions have to be added as new fields, and instead of the 12 gauge bosons in the standard model we now have 12 massless vector supermultiplets. The fermions of the standard model have to be accompanied by spin zero particles. They cannot be partners of spin 1 bosons because we have seen that the only satisfactory way to introduce spin 1 bosons is through gauge symmetries and if such bosons were partners of the fermions this would lead to an enlargement of the gauge group. Apart from that gauge fermions are always in the adjoint representations of a gauge group, which in particular is a real representation, a property not shared by the fermions of the standard model. The fermions are thus enlarged to chiral supermultiplets and every

Weyl fermion has a complex scalar as supersymmetric partner. Finally we have to discusss the Higgs doublet. Could it be that the role of the Higgs is played by the scalar partners of quarks or leptons? An inspection of the quantum numbers shows that the Higgs doublet (with $Y=-1/2$) transforms in exactly the same way as the left-handed lepton doublets, e.g. $\binom{\nu_e}{e}$ (compare (4.2)). Unfortunately this identification cannot be made. In the full model the Higgs has to receive a vacuum expectation value that breaks $SU(2)\times U(1)$ and with this identification this would be the partner of the neutrino. In this process the global lepton number symmetry would be broken and this would lead to disastrous consequences like large Maiorana neutrino masses, a massless Goldstone boson, unacceptably large neutrino oscillations, etc. We have to introduce the Higgs doublet separately and with it its fermionic partner. So we have seen that none of the particles of the standard model has a supersymmetric partner in this model. For every particle we have to introduce a supersymmetric partner. But it comes even worse. The fermionic partners of the Higgs bosons transform nontrivially under $SU(2)\times U(1)$ and render the particle content of the model anomalous. To cancel this anomaly an additional Higgs doublet chiral superfield with $Y=+1/2$ has to be introduced. A supersymmetric model with just one complex Higgs doublet cannot exist. Supersymmetry requires after the $SU(2)\times U(1)$ breakdown the existence of charged Higgs scalars, contrary to the case of the standard model which only contained a neutral Higgs scalar to survive the Higgs mehcanism.

The supersymmetric version of the standard model thus contains the vector superfields corresponding to the $SU(3)\times SU(2)\times U(1)$ gauge symmetry, three $(i=1,\dots,3)$ families of left-handed chiral superfields U_i^a, $\bar{U}_i$, $\bar{D}_i$, L_i^a, $\bar{E}_i$ ($a=1,2$ is an SU(2) index) and two Higgs left-handed chiral superfields H^a and $\bar{H}_a$. A supersymmetric action can be obtained following the construction explained in Chapter 2. This involves the $W^\alpha W_\alpha$ kinetic gauge multiplet terms for $SU(3)\times SU(2)\times U(1)$, the chiral superfield kinetic terms including the minimal gauge couplings $\phi^* e^{2gV}\phi$ and the superpotential g for the chiral superfields. This superpotential represents the supersymmetric generalization of all those terms in the action of the standard model that are not kinetic terms and that do not contain a gauge coupling. In particular g should contain the Yukawa couplings which generalize to

$$g=\lambda_{ij}U_i^a\bar{H}_a\bar{U}_j+\lambda'_{ij}U_i^aH^b\varepsilon_{ab}\bar{D}_j+\lambda''_{ij}L_i^aH^b\varepsilon_{ab}\bar{E}_j \tag{4.22}$$

with the $\int d^4x(\int d^2\theta g+\int d^2\bar{\theta}g^*)$ contribution to the action. (The symbol ε_{ab} is antisymmetric.) Notice that we cannot include a term like $U^aH^a\varepsilon_{ab}\bar{U}$ in (4.22) since this term has hypercharge $Y=-1$ and would thus explicitly break $U(1)_Y$. From (4.22) we can read off a posteriori an additional reason for the introduction of $\bar{H}$, which we can see by the consideration of the terms that are responsible for the mass terms of the quarks and leptons. Taking $\int d^2\theta g$ and choosing $H=\binom{h^0}{h^-}$, $\bar{H}=\binom{\bar{h}^0}{\bar{h}^+}$ for the scalar fields of the chiral supermultiplets we obtain

$$\mathcal{L}_Y=\lambda_{ij}u_i\bar{h}^0\bar{u}_j+\lambda'_{ij}d_ih^0\bar{d}_j+\lambda''_{ij}e_ih^0\bar{e}_j\,. \tag{4.23}$$

Without $\bar{h}^0$ we would not be able to give masses to the up quark. In the nonsupersymmetric case the mass of the u quark was obtained through the $uh^{0*}\bar{u}$ coupling, but in the supersymmetric version this is not possible. The right-handed chiral superfield H^* cannot couple to the left-handed $U,\bar{U}$ superfields in the superpotential. We simply need two left-handed superfield H and $\bar{H}$. Both h^0 and $\bar{h}^0$ have to receive vacuum expectation values to provide masses for all quarks and leptons.

The trilinear couplings in the superpotential (4.22) are not the most general terms that are consistent with the $SU(3)\times SU(2)\times U(1)$ symmetry. In fact we could add

$$\delta g = U^a L^b \varepsilon_{ab} \bar{D} + L^a \bar{E} L^b \varepsilon_{ab} + \bar{U}\bar{D}\bar{D} . \tag{4.24}$$

We have not included them in (4.22) because these interactions lead to disastrous consequences. The first two terms in (4.24) violate lepton number and the last term violates baryon number. The last two terms could combine to give proton decay via the graph shown in fig. 4.3, with a rate that is many orders of magnitude larger than the experimental limits, as long as the couplings in (4.24) are not extremely small. Another way out would be a very large mass (e.g. 10^{15} GeV) for the scalar partner of the d quark, but this would imply that supersymmetry is broken with mass splittings of order 10^{15} GeV and could be disregarded at low energies. The terms in (4.24) should therefore be absent. This can be achieved by demanding the existence of global symmetries, which can be done in different ways. One could introduce the B, L global symmetries or discrete symmetries. In supersymmetry there is in addition a possibility called R-symmetry [200, 499]. R-symmetries are symmetries that do not commute with the supersymmetry generators, unlike the case with usual global symmetries. In the usual symmetries all component fields have the same transformation properties with respect to that symmetry. This is no longer true in the case of an R-symmetry where also the superspace coordinates θ get transformed. As an example let us consider a symmetry $\theta \to e^{i\alpha}\theta$, and a chiral superfield ϕ with $\phi \to e^{i\alpha}\phi$. This implies that the scalar component of φ transforms in $\varphi\, e^{i\alpha}$ and the fermionic component ψ is not affected by this R-symmetry since $(\theta\psi) \to e^{i\alpha}(\theta\psi)$ and this is already achieved by the θ-transformation. How does this affect the terms in the superpotential? Well, we know that $\int d^2\theta g$ has to be invariant and $d^2\theta$ transforms with $e^{-2i\alpha}$, and this implies that the terms in the superpotential have to transform with $e^{2i\alpha}$. For the case under consideration this allows only a term quadratic in ϕ, $g = m\phi^2$: all other terms are forbidden by the R-symmetry.

In the supersymmetrized version of the standard model a suitable R-symmetry would be the one where all the particles in the standard model (with two Higgs doublets) like gauge bosons, quarks and leptons and Higgs scalars have charge zero whereas their supersymmetric partners – the newly introduced particles to render the model supersymmetric – transform nontrivially under the R-symmetry. Notice that gauge bosons always have $R = 0$ since they appear in the combination $\theta\sigma_\mu\bar{\theta}v^\mu$ in the vector supermultiplet and if θ picks up a phase under R-transformations $\bar{\theta}$ receives the opposite phase, and V the vector supermultiplet is real. The above mentioned R-symmetry corresponds to $R = 0$ for the vector multiplets and the Higgs superfields and $R = 1$ for the quark and lepton superfields if θ has $R = 1$. Such a symmetry forbids the terms in (4.24) and allows the terms in (4.22). It is actually not necessary to have this full continuous global symmetry to do this. In fact a smaller symmetry called R-parity could do the job as well. R-parity has a multiplicative quantum number R_p and all particles in the standard model are R even ($R_p = +1$) whereas their partners are odd. In component fields this assignment corresponds to

$$R_p = (-1)^{3B+L+2S} , \tag{4.25}$$

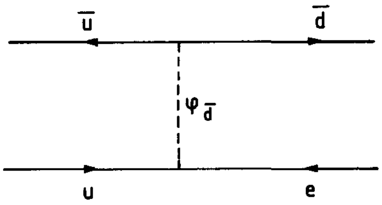

Fig. 4.3. A potentially dangerous contribution to proton decay through the exchange of the scalar partners of the right-handed d-quarks.

where B, L, S denote baryon, lepton number and spin respectively. R-parity is a subgroup of the continuous R-symmetry and could remain valid even after the continuous symmetry is broken. In addition to forbidding the terms in (4.24) these symmetries imply that the newly introduced supersymmetric partners can only be produced in pairs. This in particular means that the lightest R-odd particle is stable. We will later come back to these questions when we discuss explicit models. Notice at this point that a continuous R-symmetry of the type introduced above would forbid Majorana masses for all the gauginos whereas R-parity does not. As we will see later massless gluinos would lead to a phenomenological problem and in general the continuous R-symmetries have to be broken in a realistic model.

We have not yet discussed the Higgs potential in the supersymmetrized version of the standard model. The only renormalizable additional term in the superpotential (4.22) involving the Higgs fields that is consistent with the gauge symmetries could be $mH^a\bar{H}_a$. In the presence of a continuous R-symmetry this term would be forbidden. Even in the presence of such a term, however, this is not sufficient to induce a breakdown of $SU(2) \times U(1)$. This term leads to m^2hh^* masses for the Higgs scalars and it does not give us the opportunity to introduce $m^2 < 0$. As a result the minimum of the potential will always be supersymmetric $V = 0$ and all vacuum expectation values of the Higgs scalars vanish. It tells us that additional superfields have to be introduced in the model. One way to induce an $SU(2) \times U(1)$ breakdown would be the introduction of a singlet superfield Y with superpotential

$$g = Y(H\bar{H} - m^2)\,, \tag{4.26}$$

which would lead to $\langle h^0 \rangle = m$ with unbroken supersymmetry. The question of the breakdown of $SU(2) \times U(1)$ in the absence of supersymmetry breakdown, however, is not the relevant question to ask here. The aim of our approach is to intimately relate the breakdown of $SU(2) \times U(1)$ with the one of supersymmetry. In this respect terms like $mH\bar{H}$ in the superpotential are actually not desired since they would correspond to the explicit insertion of a mass parameter of order 100 GeV in our model that is not related to the breakdown of supersymmetry. We do not like to insert these mass parameters in the theory but would rather try to understand them from supersymmetry breaking. The question of the breakdown of $SU(2) \times U(1)$ should thus be postponed till we discuss the possible mechanisms that break supersymmetry. We have not stated it in this section, but it should be obvious that a breakdown of supersymmetry is a necessity. As it stands the model would predict for example the existence of charged scalars with the mass of the electron and if they exist they should have been detected experimentally a long time ago. Before we, however, address the questions of supersymmetry breakdown, let us introduce supersymmetric grand unified models.

4.5. *Supersymmetric grand unified models* [569, 143, 122, 449, 490, 353]

Again we limit our discussion to the SU(5) model, generalizations to other models are straightforward. To render the model supersymmetric we add gauge fermions in the adjoint representation of SU(5) to complete the vector supermultiplet. The three families of quarks and leptons receive complex scalar partners that form chiral supermultiplets $\mathrm{Y}_i(\bar{5})$ and $\mathrm{X}_i(10)$ $(i = 1, 2, 3)$. Three Higgs multiplets $\mathrm{H}(5)$, $\bar{\mathrm{H}}(\bar{5})$ and $\phi(24)$ are introduced as well. The action of this model is given by the usual kinetic and gauge coupling terms and a superpotential

$$g = g_{ij}X_iX_jH + g'_{ij}X_iY_j\bar{H} + \lambda_1 H\phi H + \lambda_2\phi^3 + M\phi^2 + M'H\bar{H}\,, \tag{4.27}$$

where SU(5) contractions are understood, and all our remarks about B and L variation in the standard model apply as well. As in the standard model we are forced to introduce H and $\bar{H}$. H alone will not do, both because of anomalies and the necessity to give masses to all quarks and leptons through the Yukawa couplings which represent the first two terms in (4.27). The remaining terms define the Higgs potential where M and M' are of the order of the grand-unification scale which will be specified later.

SU(5) should be broken to $SU(3)\times SU(2)\times U(1)$ at this scale. Supersymmetry is supposed to be broken at a scale small compared to M_x to have any implications on the low energy sector. The Higgs potential is given by

$$V_H = \left|\frac{\partial g}{\partial \phi}\right|^2 + \left|\frac{\partial g}{\partial H}\right|^2 + \left|\frac{\partial g}{\partial \bar{H}}\right|^2 , \tag{4.28}$$

and the (supersymmetric) minima of this potential are given by

$$\lambda_1 h\bar{h} + 3\lambda_2\varphi^2 + 2M\varphi = 0 , \qquad \lambda_1\varphi\bar{h} + M'\bar{h} = 0 , \qquad \lambda_1\varphi h + M'h = 0 , \tag{4.29}$$

where h, $\bar{h}$ and φ denote the scalar components of H, $\bar{H}$ and ϕ and SU(5) indices have been suppressed. There are many degenerate minima $V_H = 0$ of this potential one of which corresponds to a breakdown of SU(5) to $SU(3)\times SU(2)\times U(1)$ with $\langle h\rangle = \langle\bar{h}\rangle = 0$ and

$$\langle\varphi\rangle = v\begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & -\frac{3}{2} & \\ 0 & & & & -\frac{3}{2} \end{pmatrix} \tag{4.30}$$

where v is proportional to M/λ_2. Modifications of the model in which it is possible to have (4.30) the lowest nondegenerate minimum [146, 149] will be discussed later in the framework of explicit models. With this breakdown of SU(5) the components of H and $\bar{H}$ will in general have a mass of order of the grand unification scale. In section 4.2 we have seen that this is very desirable for the color triplets (antitriplets) in $H(\bar{H})$ in order to suppress proton decay. The SU(2) doublets in H and $\bar{H}$ should however not be supermassive since they constitute a vital ingredient of the low energy standard model. As in the ordinary SU(5) model (compare (4.18)) we have to fine tune the parameters in the superpotential. To assure the Higgs doublets to be massless we have to impose

$$M' = \tfrac{3}{2}\lambda_1 v , \tag{4.31}$$

which can be seen from the last two equations in (4.29) by inserting (4.30). Two observations should be made here. First of all we have put this fine tuning at the tree graph level and through supersymmetry we know that this relation remains exact in all orders of perturbation theory [569], it has not to be repeated like it was the case in the nonsupersymmetric model. This has led to the notion that such a fine tuning is "technically natural". This does not mean very much at the moment but one should remember that whenever one might be in a situation where one has an argument for such a quantity to be small this nonrenormalization of this quantity might be a very useful property. Secondly and more importantly relation (4.31) is an exact equality and the Higgs doublet masses are fine tuned to be exactly zero, contrary to the case in the ordinary SU(5) model where the two large terms in (4.18) conspire to produce a mass scale thirteen orders of magnitude smaller. One has the feeling that $m/M_x = 0$ might be easier to explain than $m/M_x = 10^{-13}$. Relation (4.31) might be the consequence of the Clebsch–Gordan [449, 129] coefficients of the groups under consideration whereas it is hard to imagine this to produce the number 10^{-13}.

Several suggestions have been made to obtain massless Higgs doublets without an explicit fine tuning of the parameters of the models, and we will discuss them here. The first is called the "sliding singlet" [570, 332] mechanism. One introduces a singlet superfield $Z = (z, \psi)$ and adds a term $\lambda HZ\bar{H}$ to the superpotential. The last two equations in (4.29) read now

$$\lambda_1\varphi\bar{h} + \lambda z\bar{h} + M'\bar{h} = 0\,, \qquad \lambda_1 h\varphi + \lambda hz + M'h = 0\,, \tag{4.32}$$

where the vacuum expectation value (4.30) is fixed by the first equation in (4.29). Suppose now that in a complete model supersymmetry is broken at a certain scale and that the Higgs doublets receive a vacuum expectation value of 100 GeV, by a mechanism which is not yet contained in our model. The square of the left hand side of (32) gives a contribution to the potential and in order to minimize this contribution in the case $\langle\bar{h}_2\rangle \neq 0$ the singlet would develop a vacuum expectation value (it slides) to make $-\frac{3}{2}v\lambda_1 + \lambda\langle z\rangle + M' = 0$, and this would also imply that the contribution to the Higgs mass of this term vanishes. Investigating additional terms that might give a contribution to this mass reveals that they can at most be of the order of the (small) vacuum expectation value of $\langle h\rangle$. This mechanism, however, has one drawback. One has already assumed that the Higgs v.e.v. is small and nonzero. In the context of explicit models, even if such a local minimum (with $\langle h\rangle \neq 0$ and small) exists, the global minimum of such a model might prefer to have $\langle h\rangle = 0$ or large h v.e.v.'s. It has been proven to be very difficult to make this mechanism to work in a satisfactory model exactly for this reason, as we will see later [130, 442, 148].

A second method to obtain light Higgs doublets without fine-tuning is the so-called "missing partner" mechanism [129, 291, 416, 70, 408]. It is based on purely group theoretical arguments. One likes to construct a situation where a direct mass term $mH_5\bar{H}_5$ does not exist in the superpotential and where the color triplets mix with other SU(5) representations to become massive. These other SU(5) representations should have the property that this same mass mixing is not possible for the SU(2) doublets. The simplest case in the framework of SU(5) that exhibits this mechanism [291, 416] requires the introduction of a 75 (instead of 24) and a 50 and $\overline{50}$ representation of SU(5). The 75 can be used instead of the adjoint to break SU(5) to SU(3)×SU(2)×U(1). It is a four index representation with $(75)^{ij}_{kl} = -(75)^{ji}_{kl} = -(75)^{ij}_{lk}$ and $(75)^{ij}_{ik} = 0$. With respect to the SU(2)×SU(3) subgroup it decomposes into $(1,1)+(1,3)+(2,3)+(1,\bar{3})+(2,\bar{3})+(2,\bar{6})+(2,6)+(1,8)+(3,8)$. The 50 decomposes in $(1,\bar{6})+(2,8)+(1,1)+(2,3)+(3,6)+(1,\bar{3})$ and the $\overline{50}$ is the conjugate of 50. Notice that the 50 contains a $(1,\bar{3})$ representation but no (2, 1). The superpotential reads

$$g = \lambda(75)^3 + M(75)^2 + \lambda_1 50\times 75\times\overline{50} + \lambda_2 50\times 75\times H + \lambda_3\overline{50}\times 75\times\bar{H} + \bar{M}50\times\overline{50}\,. \tag{4.33}$$

At the minimum the (1, 1) component of 75 receives a v.e.v. The terms with λ_2 and λ_3 provide then mass terms for the color triplets in H and $\bar{H}$ in mixing them with the triplets in the 50 and $\overline{50}$ leading to a mass matrix

$$\begin{pmatrix} 0 & \lambda_2 v \\ \lambda_3 v & \bar{M} \end{pmatrix}. \tag{4.34}$$

Nothing happens to the Higgs doublets since the 50 and $\overline{50}$ do not contain SU(2) doublets (which are SU(3) singlets). It was of course crucial that a term $MH\bar{H}$ was omitted in (4.33). A further complication of the model can, however, lead to the situation where such a term is forbidden by a global symmetry [291]. The missing partner mechanism requires thus (at least in SU(5)) the introduction of rather large SU(5) representations. In contrast to the "sliding singlet" mehcanism, however, it is universally applicable.

We now have to discuss the magnitude of M_x in supersymmetric grand unified models. The particle content of the low energy sector has changed compared to the ordinary SU(5) model discussed in section 4.2 and this will certainly influence the evolution of the coupling constants. It might even be that the three coupling constants g_1, g_2 and g_3 do not meet all three at the same mass scale which would be disastrous for the concept of supersymmetric grand unification. The general formula for the evolution of the coupling constants has been given earlier (see (4.19) and (4.20)) and we compute in the supersymmetric case [125, 154]

$$b_3 = -9 + 2N_G\,, \qquad b_2 = -6 + 2N_G + H/2\,, \qquad b_y = 2N_G + \tfrac{3}{10}H\,, \tag{4.35}$$

where N_G is the number of generations which we take to be 3 and H is the number of Higgs doublets. The introduction of supersymmetric partners has increased the coefficients b_i, e.g. b_3 has changed from -7 to -3. This slows down the evolution of the coupling constant g_3. Notice, however, that the additional particles have roughly a similar effect on all three of the coefficients, so that it is still possible for them to meet at the same point. To determine the grand unification scale we use the standard formulas

$$\begin{aligned} &\sin^2\theta_w = (1/d)[(b_2 - b_3) + (b_y - b_2)5\alpha/3\alpha_3]\,, && (1/2\pi)\log(M/\mu) = t = (1/d)[1/\alpha - 8/3\alpha_3]\,, \\ &1/\alpha_5 = (1/d)[((b_2 + \tfrac{5}{3}b_y)/\alpha_3) - b_3/\alpha]\,, && d = b_2 - b_3 + \tfrac{5}{3}(b_y - b_3)\,, \end{aligned} \tag{4.36}$$

where we shall use $\alpha_3(\mu) = g_3^2(\mu)/4\pi \approx 0.1$ and $\alpha^{-1}(\mu) \approx 128$ for $\mu \approx 100\,\mathrm{GeV}$. With the value of $0.2 \leq \sin^2\theta \leq 0.24$ this leads in our minimal supersymmetric SU(5) model to $M_x \approx 2\times 10^{16}\,\mathrm{GeV}$ and $g_5 \approx 1/25$. We see that M_x here is more than an order of magnitude larger than in the ordinary SU(5) model. This has, of course, consequences for the predictions of proton decay in this model. Since the proton decay rate goes with M_x^{-4} this model predicts proton decay with a lifetime of 10^{33}–10^{34} years, beyond the present experimental limits. In particular the supersymmetric minimal SU(5) model does not yet encounter the problems of the minimal SU(5) model with regard to the experimental findings. Two words of caution should be added here. The statement above is valid for the minimal model and an introduction of additional particles with masses small compared to M_x will change these results. For example with the introduction of additional 5 + 10 Higgs representations at an intermediate mass scale it was shown that M_x can be lowered [335, 415]. The statement that a given supersymmetric model leads to a larger M_x than its nonsupersymmetric counterpart (which is obtained by deleting the partners), however, remains true. Secondly our statements above are only valid for proton decay mediated by gauge bosons. We had already seen that proton decay might occur due to the exchange of Higgs bosons of mass $10^{11}\,\mathrm{GeV}$. The Higgs bosons in our model are sufficiently heavy. In the context of a supersymmetric grand unified model there is however a new mechanism: "proton decay through dimension 5 operators" [553, 493]. In ordinary SU(5) we have proton decay through a process where two quarks interact to give an antiquark and antilepton in the final state. This process involves four fermions: a dimension 6 operator, and we conclude that the amplitude is proportional to $1/M_x^2$ from dimensional considerations. These contributions also exist in the supersymmetric case but they are suppressed since M_x is larger. In supersymmetry one could in addition have a process where two quarks annihilate to the scalar partners of an antiquark and antilepton (fig. 4.4), a process that involves two fermions and two bosons: a dimension 5 operator. Consequently its amplitude is proportional to $1/M_x$. Of course, the scalars $\varphi_{\bar{q}}$ and $\varphi_{\bar{\ell}}$ have in a second step to produce their partners $\bar{q}$ and $\bar{\ell}$ but this could for example happen through the exchange of an SU(2) gaugino without involving a propagator of mass M_x. As a result this decay through dimension 5 operators is suppressed by only one power of M_x. We start our discussion with the enumeration of the dimension 5 operators

$$\begin{aligned}&(U\bar{U}L^*)_D\,, \quad (\bar{U}\bar{D}^*\bar{E})_D\,, \quad (UUUL)_F\,, \quad (UUUH)_F\,,\\ &(U\bar{U}\bar{E}H)_F\,, \quad (\bar{U}\bar{U}\bar{D}\bar{E})_F\,, \quad (LL\bar{H}\bar{H})_F\,, \quad (LH\bar{H}\bar{H})_F\,,\end{aligned} \tag{4.37}$$

in our usual notation where U, L denote the quark, lepton doublet chiral superfield. D and F denote $\int d^4\theta$ and $\int d^2\theta$ respectively. We have to recall now that in the supersymmetric model in general also dimension 4 operators occur (compare (4.24)), but they are forbidden because of either continuous or discrete R-symmetries like, e.g., R-parity. These symmetries exclude automatically also some of the operators in (4.37) and we remain with $(UUUL)_F$ and $(\bar{U}\bar{U}\bar{D}\bar{E})_F$ as the only allowed dim-5 operators. Let us now first consider $(\bar{U}\bar{U}\bar{D}\bar{E})_F$ which reads

$$(\bar{U}_{ia}\bar{U}_{jb}\bar{D}_{kc}\bar{E}_l\varepsilon^{abc})_F\,, \tag{4.38}$$

where a, b, c are SU(3) indices and i, j, k, l label the generations. The first two fields are antisymmetric in a and b and obey bose statistics which implies $i \neq j$ and expression (4.38) includes at least one charm or top quark superfield. These superfields are SU(2) singlets and can therefore not change flavor in the second step that is required for proton decay, so that (4.38) can only lead to proton decay in either a charm or a top quark. This leaves $(UUUL)_F$ as the only possibility to induce proton decay, written explicitly

$$(U^r_{ia}U^s_{jb}U^t_{kc}L^u_l\varepsilon^{abc}\varepsilon_{rs}\varepsilon_{tu})_F \tag{4.39}$$

with SU(2) indices $r, \dots, u$. U is an SU(2) doublet and contains the superfield $\hat{u}$ and $\hat{d}$, however three U's appear in (4.39) which then vanishes if $i = j = k$. Again particles of at least two generations have to enter (4.39) to assure a nonvanishing result [126]. Notice further (4.39) corresponds to a process with four ingoing left-handed fields. The simplest graph for this process can be written as shown in fig. 4.4

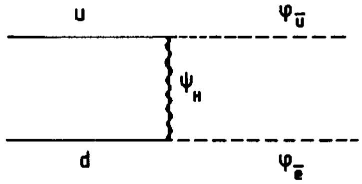

Fig. 4.4. Proton decay through dimension 5 operators. The first step involves the exchange of a heavy Higgs (gauge) fermion and transforms quarks into partners of antiquarks and leptons.

with the exchange of a fermion of mass M_x. In principle this could be either a fermionic partner of the Higgs triplets or a superheavy gauge fermion. But gauge particles have helicity conserving vertices and as such cannot lead to a process like (4.39) and we are left with the contribution of the Higgs triplet fermions (fig. 4.4).

The second step involves the transformation of the scalars to their fermionic partners through the exchange of a light particle, which again could be either gauge fermions or Higgs fermions (here the doublet). Since, however, the gauge couplings are larger than the Yukawa couplings the former give the dominant contribution, and we have to consider gluino, wino (W-ino) and bino (B-ino) exchange [126, 179, 169, 11, 492, 76]. Denoting (4.39) by $\tilde{O}_{ijkl}$ one obtains for the correspondingly induced dimension 6 operators through the gluino [76]

$$O^{(g)}_{ijkl} = -\tfrac{2}{3}[f^{(g)}_{ij}(\tilde{O}_{ijkl} + \tilde{O}_{jikl}) - f^{(g)}_{ik}\tilde{O}_{kijl} - f^{(g)}_{jk}\tilde{O}_{kjil}] , \tag{4.40}$$

the wino

$$O^{(w)}_{ijkl} = -\tfrac{1}{4}[3(f^{(w)}_{ij} + f^{(w)}_{kl})(\tilde{O}_{ijkl} + \tilde{O}_{jikl}) - (f^{(w)}_{ik} + f^{(w)}_{jl})(2\tilde{O}_{ikjl} + \tilde{O}_{kijl}) - (f^{(w)}_{jk} + f^{(w)}_{il})(2\tilde{O}_{jkil} + \tilde{O}_{kjil})] , \tag{4.41}$$

and the bino

$$O^{(b)}_{ijkl} = \tfrac{1}{36}[(f^{(b)}_{ij} - 3f^{(b)}_{kl})(\tilde{O}_{ijkl} + \tilde{O}_{jikl}) - (f^{(b)}_{ik} - 3f^{(b)}_{jl})\tilde{O}_{kijl} - (f^{(b)}_{jk} - 3f^{(b)}_{il})\tilde{O}_{kjil}] , \tag{4.42}$$

where g is the gauge coupling and m the Majorana mass of the gauge fermion under consideration. generation. This assumption allows us to write O_{ijkl} in terms of the $SU(3)\times SU(2)\times U(1)$ invariant operators $\tilde{O}$. The functions f depend explicitly on the masses involved in the process

$$f_{ij} = \frac{g^2 m}{32\pi^2(m_i^2 - m_j^2)}\left[\frac{m_i^2}{m^2 - m_i^2}\log\left(\frac{m^2}{m_i^2}\right) - \frac{m_j^2}{m^2 - m_j^2}\log\left(\frac{m^2}{m_j^2}\right)\right] , \tag{4.43}$$

where g is the gauge coupling and m the Majorana mass of the gauge fermion under consideration. Notice that the assumption of a mass degeneracy of all partners of quarks and leptons (even for different generations) would lead to vanishing contributions in (4.40) and (4.42). Proton decay through gluino and bino exchange is proportional to the mass differences of the scalar partners of quarks and leptons in different generations.

Let us now try to find the decay modes of proton decay via this mechanism and try to estimate the rate. In a first step we would like to neglect the Kobayashi–Maskawa mixings (4.11). We had already seen that in $\tilde{O}_{ijkl}$ not all i, j and k can be equal, particles outside the first generation have to enter the process. The c quark is ruled out since it is heavier than the proton.

What remains are processes that include the u, d, s quarks and the ν_μ neutrino. The muon cannot appear because of the absence of the c quark. This gives as dominant proton decay mode [126]

$$p \to K^+ \bar{\nu}_\mu \tag{4.44}$$

for example through the graph in fig. 4.5. The rate is determined by the Yukawa couplings responsible for the masses of the u and s quarks, two gauge couplings, the mass M_x of the Higgs fermion and the

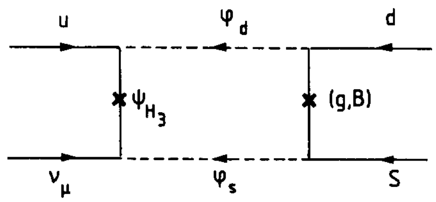

Fig. 4.5. Diagram for $p \to K^+ \bar{\nu}_{\bar{\mu}}$.

Majorana mass of the gauge fermion which we assume to be of the order of 100 GeV. This leads to a rate of order 10^{30} years for the process in (4.44). Up to now we have neglected the Cabbibo mixings between the generations. If we include them and assume that the mixings between the third and the first generation are not too small, the dominant contribution might come from graphs where scalars of the third family appear in the loop. This could be dominant because in the first step of the process the Yukawa couplings enter. They are adjusted to give the quarks and leptons their masses and since the particles in the third generation are heavy these couplings are large. It could thus be that the mode $p \to K^+ \bar{\nu}_\tau$ is dominant [179]. This, however, depends on the mixing angles between the families which we do know only partially. In addition it seems not to be too easy to distinguish between the modes $K\bar{\nu}_\mu$ and $K\bar{\nu}_\tau$ experimentally. Nonetheless the $K\bar{\nu}$ mode is the dominant proton decay mode in this mechanism. This is interesting because there is no other model which predicts $K\bar{\nu}$ as the dominant mode. An observation of this mode in absence of other modes would therefore favor a supersymmetric interpretation. Observe however that this proton decay through dim-5 operators does not necessarily happen in a supersymmetric grand unified model. It might very well be (and happens in a lot of specific models) that there exist accidental global symmetries which exclude the dim-5 operators just as the dim-4 operators are forbidden. In this case the dominant decay modes would be given by gauge boson exchange which predict $e^+\pi^0$ as the dominant mode in this model. In addition one could have proton decay via Higgs exchange with mass of 10^{11} GeV and dim-6 operators which seems to favor the $K\mu$ modes. Here, however, one has then to forbid the dim-5 operators.

Let us finally mention two results in connection with this discussion. The contributions to proton decay via dim-5 operators we have discussed so far vanish if supersymmetry is unbroken [393]. Secondly there are additional mechanisms of this type that involve the auxiliary D-fields of the gauge super-multiplets [111] which we cannot discuss here in detail.

4.6. *Constraints from rare processes*

If supersymmetry is relevant for the solution of our theoretical problems in the mass range of 100 GeV, we should find deviations from the ordinary models in this energy range. We have not yet direct experimental access to this region in the sense that the particles of highest mass produced in the laboratories so far are lighter than 100 GeV. But we know already something about this energy region and this knowledge comes from extremely accurate experiments at lower energies. One of these experiments is for example the measurement of proton decay which in principle could give us information about an energy region of 10^{15} GeV. But there are also other experiments that can tell us what might happen or not in the region above hundred GeV. These include limits on flavor changing neutral currents, $(g-2)$ of the muon, K_L–K_S mass difference, $e \to \mu\gamma$ measurements and similar processes, We consider here models where new particles exist in the region beyond 100 GeV and in this chapter we want to explore the constraints on the masses and couplings of these new particles imposed by the knowledge of the results of the above mentioned rare processes. This we will do in the framework of the most general model that satisfies our theoretical requirements and as we have explained this is a model with soft explicit breakdown of supersymmetry [122, 490]. This model includes supersymmetry breaking Majorana mass terms for the gauge fermions, mass terms for the scalar partners of quarks and leptons and the allowed soft trilinear couplings. We also include an explicit mass term for the Higgs fermions which in the context of the models we consider is soft, although it is not soft in general. Such a version of a "supersymmetric" standard model or grand unified model can lead (with certain restrictions on the parameters) to a satisfactory model with no phenomenological problems, but it has of course many arbitrary breaking parameters the origin of which is not understood. We will here just consider this model to derive the constraints on this model that can make it phenomenological acceptable given the present experimental information, and with that we will cover the most general case that can occur in the context of spontaneously broken supersymmetry.

The bounds on the existence of charged scalars in e^+e^- annihilation experiments tell us that all the partners of quarks and leptons (with exception of the neutrinos) must be heavier than 20 GeV [529, 4, 32, 52] and this bound, of course, also applies in the case of the charged Higgs fermions. Available lower limits on gaugino masses are more model dependent, since they involve indirect measurements like beam dump experiments [27]. For the gluino this leads to $m_{\tilde{g}} > (1\text{–}5)$ GeV depending on the masses of the quark partners. No direct limits on Majorana wino, zino and photino masses are yet known.

Additional indirect limits on the parameters can be obtained from the above mentioned rare processes [176, 44, 337, 524]. We will restrict our discussion here to the processes that give the strongest constraints. These are $\mu \to e\gamma$ for the lepton sector and the K_L–K_S system for the quark sector. The limits on $K_L \to \mu\mu$, $K_L \to \mu e$, $\mu \to e\bar{e}e$ or $\mu N \to eN$ are in general not violated if the parameters of the models fulfill the requirements imposed by $\mu \to e\gamma$ and K_L–K_S. The quantity $(g-2)$ vanishes in the supersymmetric limit [232] and hardly gives constraints in the broken case.

We start with the K_L–K_S system. In the nonsupersymmetric model this receives a contribution from diagrams as in fig. 4.6, and one has to sum over all possible intermediate quark states. Independent of the KM angles (as long as the matrix stays unitary), we know that this contribution cancels if the quarks under consideration have the same mass. In a more general context flavor changing neutral currents vanish if quarks with same isospin and same third component of isospin are degenerate in mass. In the supersymmetric model there are now additional contributions corresponding to the same box diagrams where the quarks in the box are substituted by their partners and the gauge bosons by the gauge fermions (fig. 4.7). Since the partners of u, c, t have the same quantum numbers as u, c, t again this contribution cancels if φ_u, φ_c, φ_t have the same masses. The smallness of the K_L–K_S mass difference will thus not give constraints on the magnitude of the masses of the scalars but will constrain the intergenerational mass differences. Estimates of this process can easily be obtained in the limiting cases

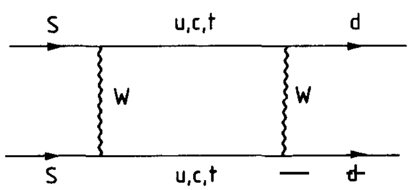

Fig. 4.6. K_L–K_S mixing through W-exchange.

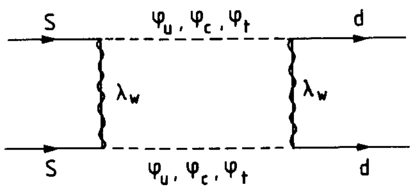

Fig. 4.7. K_L–K_S mixing through the exchange of charged gauge fermions.

$m_\lambda \ll m_s$ or $m_s \ll m_\lambda$, where m_λ (m_s) denote the mass of the gauge fermions (scalar partners of quarks). One obtains for $m_s \ll m_\lambda$ contributions of the form

$$(\alpha_w^2 \sin^2\theta/m_\lambda^2)(\Delta m_s^2/m_s^2)^2\,, \tag{4.45}$$

and for $m_\lambda \ll m_s$

$$\alpha_w^2 \sin^2\theta(\Delta m_s^2)^2/m_s^6\,, \tag{4.46}$$

where α_w is the weak gauge coupling, $\sin^2\theta$ denotes the relevant KM angles and Δm_s^2 denotes the difference of the squared masses of the scalars under consideration. The general formulae are given in refs. [176, 44, 144, 390] and we will not repeat them here. Using the experimental value of the real part of the K_1–K_2 particles in connection with the usual values for the Cabbibo angles one obtains

$$\frac{1}{m_\lambda^2 \text{ or } m_s^2}\left(\frac{\Delta m_s^2}{m_s^2}\right)^2 \lesssim 10^{-7}\,\text{GeV}^{-2}\,, \tag{4.47}$$

a rather strong constraint on the mass difference of the scalars. Notice, however, that this bound only applies to the mass differences of the u, c, t scalars. The mass difference of, for example, the u and d scalars is not constrained. The bound given in (4.47) can be strengthened since in general there are contributions to the $\Delta S = 2$ process through gluino exchange as shown in fig. 4.8. A nonvanishing result of this contribution requires the mass matrices of the scalars to have off diagonal elements in the basis where the corresponding quark mass matrix is diagonal, a situation which, however, is possible in general and which will actually be present in various models. This gives constraints on the mass differences of the down (d, s, b) scalars. Since here the strong coupling constant α_s is involved and $\alpha_s^2 \approx 10\alpha_w^2$ at 100 GeV one arrives at a bound like (4.47) with $10^{-8}\,\text{GeV}^{-2}$ at the right hand side. Assuming for the moment m_λ to be 100 GeV this leads to $\Delta m_s^2/m_s^2 \leq 10^{-2}$, as a constraint from the real part of the K_1–K_2 mass matrix. The supersymmetric partners of the quarks (separately in the up and the down sector) should therefore be highly degenerate in all realistic models. In addition we have a constraint from the imaginary part of the K_1–K_2 mass matrix. Potentially this bound is stronger by 3 orders in magnitude corresponding to the one in (4.47). It requires the mass matrix of the scalars to be complex in the basis where the corresponding mass matrix of the quarks is real. This can happen in general but it is not likely to happen in the models that have been considered so far in the literature. We will see this in more detail in the following chapters. So far our discussion of the constraints on the mass differences of the partners of quarks. As we have said, the constraints from the K_S–K_L system give stronger bounds than those of other flavor changing neutral current processes like $K_L \to \mu^+\mu^-$.

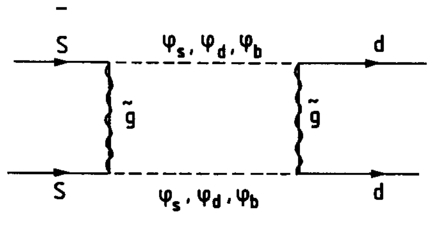

Fig. 4.8. $\Delta S = 2$ process through gluino exchange.

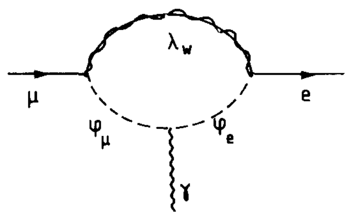

Fig. 4.9. Supersymmetric contribution to $\mu \to e\gamma$.

Next we come to $\mu \to e\gamma$ which will give us a constraint on the mass^2 differences of the partners of leptons. A contribution to this process in the framework of a supersymmetric model is shown in fig. 4.9. A nonvanishing contribution of this diagram again requires the mass matrix of the e, μ, τ scalar mass matrix to have off-diagonal elements, which of course can happen in the most general case. Assuming these mixing angles to be of the order of the Cabbibo angle one can derive a bound like the one in (4.47) now for the lepton sector. With the bound of the $\mu \to e\gamma$ branching ratio 1.9×10^{-10} one obtains [176]

$$\frac{1}{m_s^2 \text{ or } m_\lambda^2}\left(\frac{\Delta m_s^2}{m_s^2}\right) < 10^{-7}\,\text{GeV}^{-2}\,, \tag{4.48}$$

which leads to $\Delta m_s^2 \leq 10^{-3} m_s^2$ and thus also requires a nearly perfect degeneracy in mass of the partners of the charged leptons. We have thus seen that existing experimental information gives strong constraints on the mass differences of the scalar partners of quarks and leptons. The separate sectors should be sufficiently degenerate in mass to allow for a GIM mechanism to suppress flavor changing neutral currents.

This concludes our discussion of the rather model independent constraints on supersymmetric models. Other constraints like the one of CP violation and the electric dipole moment of the neutron cannot be discussed in a model-independent way and will be later discussed in the framework of specific models. Up to now our discussion has been independent of a specific breakdown of supersymmetry which has to be incorporated in the models. Our framework till now has been an explicit soft breakdown of supersymmetry with arbitrary breaking parameters which represents the most general possibility. The discussion in this subsection has, however, shown that these arbitrary parameters have to be constrained because of already existing experimental data. It is hoped that such restrictions could be explained by a more constrained way of the supersymmetry breakdown, like a spontaneous breakdown. One would of course also like to understand a relation between the breakdown of supersymmetry and the breakdown of the weak interactions as a final goal.

5. Global supersymmetry: models

In this section we will discuss supersymmetric extensions of the standard model and grand unified models in which supersymmetry is broken spontaneously. Theoretically this is considered to be more satisfactory than just the introduction of explicit breaking parameters. Here one starts with manifestly supersymmetric actions. This form of the breakdown, however, is much more restrictive than an explicit breakdown as one can already see from the results on $\mathrm{STr}\,\mathcal{M}^2$ in Chapter 2.

In our discussion we will roughly follow the historical development of the field. This implies that we start with models in which supersymmetry is broken at a scale $M_S \sim 100\,\mathrm{GeV}$ and where all dominant contributions to the masses (and coupling constants) are already given at the tree graph level. We will finally end up to discuss models in which $M_S \sim 10^{10}\,\mathrm{GeV}$ and where the splittings of, let us say, the quark and lepton supermultiplets arise through radiative corrections.

5.1. Fayet – $\tilde{U}(1)$

These models are due to the pioneering work of Fayet [201–210] and are the first attempt to give the standard model a supersymmetric extension. Many of the model-independent results which we have described in the last section were discovered in the framework of this model. Specifically this model tries to incorporate a spontaneous breakdown of supersymmetry at a scale of the order of a hundred GeV in which the scalar partners are all heavier than the corresponding quark and lepton masses at the tree graph level. Given these requirements the possible mechanisms of the spontaneous breakdown are already limited. The reason for this is the mass formula [227] (2.75)

$$\mathrm{STr}\,\mathcal{M}^2 = \sum_J (-1)^{2J}(2J+1)\mathcal{M}_J^2 = 2\,\mathrm{Tr}\,Q_\alpha D^\alpha\,, \tag{5.1}$$

which holds at the tree graph level in the case of a spontaneous breakdown of supersymmetry. Models in which supersymmetry is broken by the vacuum expectation values of the auxiliary fields (F) of chiral superfields thus always have a vanishing right hand side in eq. (5.1). This implies that *on average* the masses of the bosons are the same as the masses of the fermions in the theory. In the simple cases we discussed in Chapter 2 this relation was even true separately for the different supermultiplets. The two (real) scalar partners of (let us say) the electron satisfied the equality

$$m_A^2 + m_B^2 = 2m_e^2\,, \tag{5.2}$$

which of course cannot lead to a satisfactory model since m_A and m_B have both to be heavier than 20 GeV as discussed in the last chapter. In general $\mathrm{STr}\,\mathcal{M}^2 = 0$ does on the other hand not directly imply relations like (5.2) for every multiplet separately; it only tells us that this is true on the average and one could in principle think of a situation in which the scalar partners of quarks and leptons as well as the Higgs fermions are heavy such that $\mathrm{STr}\,\mathcal{M}^2 = 0$ and there are no problems. This, however, does not work; at least I am not aware of any model with the particle content of the standard model that exhibits these properties. One reason for this is the fact that the spectrum of the (chiral) fermions is not affected by the spontaneous breakdown of supersymmetry at the tree level: there is no $F\psi\psi$ term, and fermions feel the supersymmetry breakdown through radiative corrections. The mass spectrum of the scalars depends on the coupling of the superfields to the goldstino superfield through $F_G\varphi\varphi$ (where F_G is the auxiliary field of the goldstino multiplet and $\langle F_G\rangle \neq 0$). For such a model to be satisfactory all scalar partners of quarks and leptons have to receive a large mass cancelled by negative m^2 contributions to the Higgses. There is no known model that fulfills these criteria.

Guided by these arguments Fayet considered models with nonvanishing supertrace. Such a situation can only occur if supersymmetry is broken by v.e.v.'s of the auxiliary D-fields of the gauge supermultiplets. More specifically this implies that (before or after gauge symmetry breakdown) a U(1) gauge group has to be present in the theory since D^α carries the index of the adjoint representation of the gauge group and $\langle D^\alpha \rangle \neq 0$ leads to a gauge symmetry breakdown except in the case where D corresponds to a U(1) group. In the standard model the candidates for these U(1) groups are $\mathrm{U}(1)_{\mathrm{y}}$ and $\mathrm{U}(1)_{\mathrm{em}}$ and one could try to break supersymmetry through v.e.v.'s of the D-fields of these groups. The corresponding goldstinos would be the bino or the photino. We know now from our discussion in Chapter 2 that the mass splittings of the scalar particles are given through the gauge coupling to the goldstino (recall (2.74)). In the case of the photino the partners of the e^- for example would then receive a negative $(\text{mass})^2$ because the charge of e^- is negative. This would induce a breakdown of $\mathrm{U}(1)_{\mathrm{em}}$ incompatible with present observations. Similar problems arise if the goldstino were the bino since for example $\bar{u}$ has a negative hypercharge. Thus the D-terms of $\mathrm{U}(1)_{\mathrm{y}}$ and $\mathrm{U}(1)_{\mathrm{em}}$ cannot be used to break supersymmetry. In general one could try to have a linear combination of chiral and these U(1) gauge fermions as the goldstino, but since this does not lead to a satisfactory model in either of the limiting cases it will also not work in any linear combination. As a result of this discussion we conclude that under the given assumptions a new $\tilde{\mathrm{U}}(1)$ gauge symmetry has to be introduced. The simplest choice is to assign positive charges ($\tilde{Q} = +1$) to all quark and lepton superfields and negative charges ($\tilde{Q} = -2$) to the H, $\bar{H}$ superfields. One wants now to find a potential in which supersymmetry and $\mathrm{SU}(2) \times \mathrm{U}(1)$ is broken. To break supersymmetry via a v.e.v. of $\tilde{D}$ of $\tilde{\mathrm{U}}(1)$ this requires the introduction of a Fayet–Iliopoulos [218] term $\int \mathrm{d}^4\theta \xi \tilde{V} \sim \xi \tilde{D}$. To discuss the scalar potential we assume for a moment that the Yukawa couplings are zero and we have only to consider the Higgses. Later on one must of course check whether the so-found minimum of the potential remains still a minimum after the Yukawa couplings have been reintroduced. The simplest starting point would then be to have a vanishing superpotential $g = 0$ and just the $\xi \tilde{D}$ term which leads to the scalar potential

$$V = \tfrac{1}{2}\tilde{D}^2 = \tfrac{1}{2}|\xi - 2\tilde{g}|h|^2 - 2\tilde{g}|\bar{h}|^2|^2 \tag{5.3}$$

(compare (2.72)) where h, $\bar{h}$ denote the complex scalars of H, $\bar{H}$ and $\tilde{g}$ is the coupling constant of $\tilde{\mathrm{U}}(1)$ ($\tilde{g} > 0$). We have now two choices for ξ. It can be positive implying $V = 0$ at the minimum and thus unbroken supersymmetry, or it can be negative which leads to $V_m = \frac{1}{2}\xi^2$ with unbroken gauge symmetries. Thus the potential in (5.3) is too simple to achieve what we want. In a next step one might add a term $mH\bar{H}$ in the superpotential leading to additional terms $m^2|h|^2 + m^2|\bar{h}|^2$ in the scalar potential. For the range of parameters $\xi > m^2/\tilde{g}$ the desired minimum is obtained with $V_0 = (m^2/2\tilde{g})\xi$, $\langle D \rangle = m^2/\tilde{g}$ and $\langle |h|^2 + |\bar{h}|^2 \rangle = (1/2\tilde{g})(\xi - m^2/g)$. The goldstino is a linear combination of the $\tilde{\mathrm{U}}(1)$ gaugino and the chiral Higgs fermions with weight $m^2/\tilde{g}$ and $[m^2/2g(\xi - m^2/\tilde{g})]^{1/2}$ respectively. The scalar partners of quarks and leptons receive $\varphi\varphi^*$ mass terms proportional to m^2 and off-diagonal terms $(\varphi^2 + \varphi^{*2})$ with coefficient $\frac{1}{4}\tilde{g}m(1/\tilde{g})(\xi - m^2/\tilde{g})$. To make sure that this leads to mass matrices with positive determinant one sees here explicitly that the $\tilde{\mathrm{U}}(1)$ gaugino has to be the dominant component of the goldstino. The gauge bosons of the broken gauge symmetries become massive through the Higgs mechanism and the corresponding gauge fermions combine with the Higgs fermions to massive Dirac particles (with exception of the goldstino), as has been shown explicitly in the simpler example of Chapter 2. One potential problem of the model is the existence of the $\tilde{\mathrm{U}}(1)$ gauge interactions which might disturb the usual $\mathrm{SU}(2) \times \mathrm{U}(1)$ weak interaction phenomenology. Difficulties can however be avoided by special choices of the parameters in the model, such as using a small $\tilde{g}$. We will not show this here in detail, since the model as it stands is not complete because of triangle anomalies involving the $\tilde{\mathrm{U}}(1)$ current. Before we are allowed to gauge $\tilde{\mathrm{U}}(1)$ we must make sure that the fermion representations we consider are anomaly free. One might even like to impose an additional constraint by observing that in the present model the D-term (i.e. ξ) receives quadratic divergent contributions in perturbation theory. A ξ of order of 100 GeV would therefore not be natural. This second constraint would just require that $\mathrm{Tr}\,\tilde{Q}=0$ for the representation under consideration [240]. The cancellation of the anomalies is much more complicated [553, 13, 391, 40]. First of all it is not just enough to have $\mathrm{Tr}\,\tilde{Q}^3=0$ since there are also anomalies involving, e.g., two SU(3) currents and the $\tilde{\mathrm{U}}(1)$ current. All the quarks have $\tilde{Q}=+1$ and to cancel these anomalies one has to introduce representations which transform nontrivially under $\mathrm{SU(3)_c}$ and have $\tilde{Q}<0$. Supersymmetry now broken by the v.e.v. of $\tilde{D}$ would give negative m^2 contributions to the scalars of these superfields which indicates the danger of a possible spontaneous $\mathrm{SU(3)_c}$ breakdown. One might try to avoid this problem by giving sufficiently large mass terms to these additional fields. But this is not possible in a gauge invariant way. The additional fields are supposed to cancel anomalies of the original model. They thus form an anomalous (reducible) representation and mass terms would explicitly break the gauge symmetry. As a result in a wide class of models it has been shown that the absolute minimum of the potential breaks $\mathrm{SU(3)_c}$ spontaneously [13, 553].

To give you a feeling what is required to cancel the anomalies let us consider the simplest example. In addition to the quarks and leptons of one family and the H, $\bar{H}$ one introduces superfields with the following $\mathrm{SU(3)\times SU(2)\times U(1)\times \tilde{U}(1)}$ quantum numbers

$$K(3,1,\tfrac{1}{3},-2)\,,\qquad \bar{K}(\bar{3},1,-\tfrac{1}{3},-2)\,,\qquad X(1,1,0,4)\,,\qquad Z(1,1,0,1)\,. \tag{5.4}$$

Actually the representations in (5.4) together with H, $\bar{H}$ and one family of quarks and leptons are just a decomposition of a 27 representation in $\mathrm{E_6}$ and thus anomaly free. $\tilde{\mathrm{U}}(1)$ corresponds to the coset $\mathrm{E_6/SO(10)}$ [129, 449]. In fact a model with three 27 representations would be a candidate for a grand unified model for the unification of $\mathrm{SU(3)\times SU(2)\times U(1)\times \tilde{U}(1)}$ but in such a model one has to make sure that K and $\bar{\mathrm{K}}$ are superheavy to avoid proton decay at a disastrous rate. Such a $\mathrm{K\bar{K}}$ mass term requires a breakdown of $\tilde{\mathrm{U}}(1)$ in agreement with our earlier expectations and implies that $\tilde{\mathrm{U}}(1)$ gauge interactions can no longer be relevant in the 100 GeV energy range. Independent of this embedding in $\mathrm{E_6}$ it has been shown that models with the addition of the fields in (5.4) do not lead to the desired vacua [40, 391], which should break supersymmetry and $\mathrm{SU(2)\times U_y(1)\times \tilde{U}(1)}$ but not $\mathrm{SU(3)\times U_{em}(1)}$.

Other simple choices of anomaly cancellation like the one with a $\tilde{Q}=-2$ color octet superfield [553] and singlets were shown to have the same disease. Very often supersymmetric minima with broken color SU(3) occur. As a result of this more and more fields have to be introduced. This, of course, also increases the potential danger that supersymmetric minima exist since the equation

$$\tilde{D}=\left|\,\tilde{\xi}+\tilde{g}\sum_i Q_i|\phi_i|^2\right|=0 \tag{5.5}$$

is easier to solve if one has a lot of fields ϕ_i some of which have positive and some of which have negative charge. The by now very complicated scalar potential has to be minimized and a judicious choice of parameters has to be imposed to make sure that supersymmetry is broken and that the desired minimum is obtained. Nonetheless there exist models in the literature that claim to have solved all these problems [305, 463, 256]. They have for example a particle content like [463] eq. (5.6) which contains thirteen new $\mathrm{SU(3)\times SU(2)}$ singlet superfields. With all of this in mind, there might arise the idea that this way of breaking supersymmetry at $M_\mathrm{S}\sim 100$ GeV and where all scalars are massive at the tree graph level is perhaps not the most economical method to arrive at a supersymmetric version of the standard model. At the origin of all these complications is of course the supertrace formula which turned out to be so restrictive. It is however only valid at the tree graph level [279].

	$SU_c(3)$	$SU_w(2)$	$U_y(1)$	$U'(1)$		$SU_c(3)$	$SU_w(2)$	$U_y(1)$	$U'(1)$
Q_i	3	2	1/6	1	P'	1	1	1	−2
U_i^c	$\bar{3}$	1	−2/3	1	P'^c	1	1	−1	−2
D_i^c	$\bar{3}$	1	1/3	1	J	1	1	0	4
L_i	1	2	−1/2	1	J^c	1	1	0	−4
E_i^c	1	1	1	1	J_1	1	1	0	4
N_i^c	1	1	0	1	J_2	1	1	0	4
H	1	2	−1/2	−2	J_3	1	1	0	4
H'	1	2	1/2	−2	R_a	1	1	0	−2
K	8	1	0	−2	R	1	1	0	−2
T	1	3	0	−2	X	1	1	0	0
P	1	1	1	−2	Y	1	1	0	0
P^c	1	1	−1	−2					

(5.6)

5.2. *Supercolor* [123, 569, 137, 140]

Since the supertrace formula (5.1) ceases to be valid beyond the tree graph level it is logical to use this fact to circumvent the restrictions. Models that are based on this observation usually contain a special sector that is responsible for the breakdown of supersymmetry and it is only this sector that feels the breakdown at the tree graph level. The partners of quarks and leptons remain degenerate with quarks and leptons at the tree graph level; they have no tree level couplings to the goldstino, and they will receive their masses through radiative corrections. This might require the supersymmetry breaking scale M_S to be somewhat higher than in the models previously discussed. A typical scale (which of course depends on the coupling constant) is $M_S = 1\,\mathrm{TeV}$. Historically the first of this class of models was the supercolor model by Dimopoulos and Raby [123] and Dine, Fischler and Srednicki [137]. One might have some reservations with respect to the mechanism of supersymmetry breaking in these models (compare our discussion in section 2.4) but these models are very well suited to describe the mechanism by which quark and lepton scalars receive their mass through radiative corrections.

The model is based on a gauge group $SU(N)_{sc} \times SU(3) \times SU(2) \times U(1)$. The sector responsible for the supersymmetry breakdown consists of the particles that transform nontrivially under $SU(N)$ supercolor [140]

$$S = (N, 3, 1, -1/3)\,, \qquad \bar{S} = (\bar{N}, \bar{3}, 1, 1/3)\,, \qquad T = (N, 1, 2, 1/2)\,, \qquad \bar{T} = (\bar{N}, 1, 2, -1/2)\,, \tag{5.7}$$

all chosen to be left-handed chiral superfields. In addition to the H, $\bar{H}$ and quark and lepton superfields one introduces a singlet superfield $N = (1, 1, 1, 0)$. The superpotential is chosen to be

$$g = \lambda_1 S\bar{S}N + \lambda_1' T\bar{T}N + \lambda_2 H\bar{H}N + \tfrac{1}{3}\lambda_3 N^3 + \sum_{i,j} (g_{ij} U_i \bar{H} \bar{U}_j + g'_{ij} U_i H \bar{D}_j + g''_{ij} L_i H \bar{E}_j)\,. \tag{5.8}$$

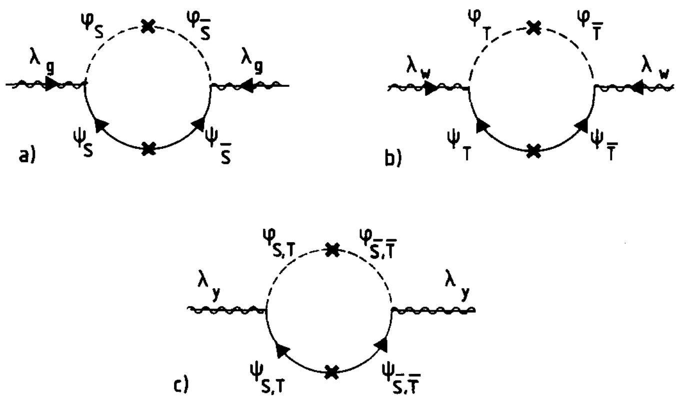

Fig. 5.1. Contributions to gauge fermion masses in supercolor models.

This is the most general superpotential consistent with baryon and lepton number conservation, gauge symmetries and an R-invariance under which all superfields transform with the same phase. Observe that up to now no mass parameter has been inserted in the theory. It is the R-symmetry that forbids $S\bar{S}$, $T\bar{T}$, $H\bar{H}$ and N^2 terms in the superpotential.

Supercolor SU(N) is supposed to become strong at a scale $\Lambda \sim 1\,\mathrm{TeV}$ and lead to condensation

$$\langle \varphi_S \varphi_{\bar{S}} \rangle = \langle \varphi_T \varphi_{\bar{T}} \rangle = \Lambda^2 \,, \qquad \langle \psi_S \psi_{\bar{S}} \rangle = \langle \psi_T \psi_{\bar{T}} \rangle = \Lambda^3 \,, \tag{5.9}$$

where $\varphi(\psi)$ denote the bosonic (fermionic) components of the respective superfields. Supersymmetry is broken by the $\psi\psi$ condensates (compare section 2.4) and the corresponding goldstino is a combination of the composite fermions $\varphi_S \psi_{\bar{S}}$, $\varphi_{\bar{S}} \psi_S$, $\varphi_T \psi_{\bar{T}}$ and $\varphi_{\bar{T}} \psi_T$. None of the gauge symmetries, however, is broken since the quantities in (5.9) are SU(N) $\times$ SU(3) $\times$ SU(2) $\times$ U(1) singlets. The mass splittings of the supermultiplets of the model are determined by the coupling to the goldstino. At the tree graph level only N couples to the goldstino. For the other multiplets this coupling occurs in higher order of perturbation theory. The SU(3) $\times$ SU(2) $\times$ U(1) gauge fermions receive a mass through the diagrams shown in fig. 5.1. With $\Lambda = 2.5\,\mathrm{TeV}$ this leads to [140]

$$m_{\lambda_g} = 1.2\,\mathrm{TeV} \,, \qquad m_{\lambda_w} = 490\,\mathrm{GeV} \,, \qquad m_{\lambda_y} = 250\,\mathrm{GeV} \,, \tag{5.10}$$

where we have used the usual values for the gauge coupling constants. The scalar partners of quarks and leptons as well as the Higgses receive a mass by the diagrams in fig. 5.2. If supersymmetry were exact the contributions of these 4 graphs would cancel. This cancellation is incomplete here since the gauginos are massive but the gauge bosons not. This then leads to masses

$$m_\varphi^2 = \tfrac{3}{4} C^2 (190\,\mathrm{GeV})^2 + \tfrac{4}{3} T^2 (54\,\mathrm{GeV})^2 + 4\,Y^2 (10\,\mathrm{GeV})^2 \,, \tag{5.11}$$

where we have cut off the integrals at Λ. $C^2 = \frac{4}{3}$ for a color triplet and $T^2 = \frac{3}{4}$ for a weak doublet. Notice that these contributions to the masses of the scalars arise effectively at the two-loop level: first the gauge fermions gain a mass at the one-loop level and then they enter the contribution to the scalar masses which would be zero if m_λ would be zero. Notice also from (5.11) that scalars with the same quantum numbers receive the same contribution to their masses avoiding potential problems with flavor changing

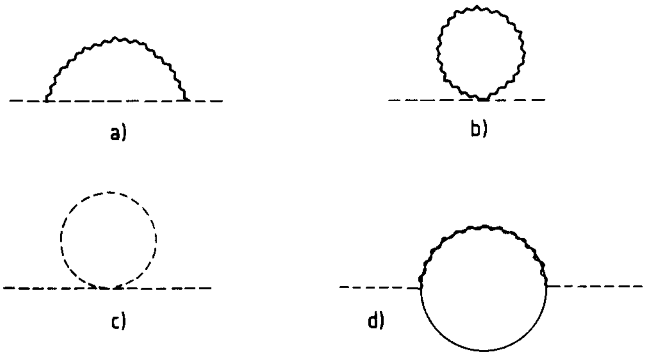

Fig. 5.2. Various contributions to scalar masses. Dashed (solid, wavy, solid–wavy) lines correspond to scalars (chiral fermions, gauge bosons, gauge fermions). In the limit of exact supersymmetry these four contributions cancel.

neutral currents as discussed in section 4.6. Because of the strong coupling constant g_3 colored scalars receive the highest masses. The scalar partner of the right-handed electron (μ, τ) receives the smallest mass, since it only couples to hypercharge, here its mass is 20 GeV and we now see the reason why Λ has to be larger than 2.5 TeV in this model.

We have not yet discussed the breakdown of SU(2)$\times$U(1) in this model. The Higgs scalars have received positive m^2 contributions from (5.11) and this certainly does not lead to such a breakdown. To discuss this question we have to consider the superpotential (5.8) and we will find the reason why the superfield N has been introduced. The scalar potential obtained from (5.8) has to be treated carefully. The supercolor bilinears are replaced by their vacuum expectation values to arrive at a low energy effective potential and to the tree level potential we must add the radiative contributions from (5.11). The qualitative properties of this potential can already be seen from an inspection of the auxiliary field of N,

$$F_N^* \sim \lambda_1 \Lambda^2 + \lambda_2 h\bar{h} + \lambda_3 \varphi_N^2 \,. \tag{5.12}$$

The term $|F_N|^2$ is a contribution to the potential and $F_N = 0$ would favor a vacuum expectation value for h, $\bar{h}$ or φ_N such that one would expect a breakdown of SU(2)$\times$U(1) to be possible. The full potential, however, has to be considered. Neglecting the Yukawa couplings this corresponds to

$$V = |\lambda_1 \Lambda^2 + \lambda_2 h^0 \bar{h}^0 + \lambda_3 \varphi_N^2|^2 + 2\lambda_1^2 |\varphi_N|^2 + \lambda_2^2 |\varphi_N|^2 [|h^0|^2 + |\bar{h}^0|^2] + m^2(|h^0|^2 + |\bar{h}^0|^2) - \lambda_1 \Lambda^3 (\varphi_N + \varphi_N^*) \,, \tag{5.13}$$

where we have already minimized the $\frac{1}{2}D^2$ contribution by assuming that only the neutral components of the h, $\bar{h}$ doublets receive a v.e.v. The last term arises from the $S\bar{S}N$ term where $\langle \psi_S \psi_{\bar{S}} \rangle$ is replaced by Λ^3 and m^2 denotes the mass contribution to the Higgses from (5.11). Potential (5.13) has to be minimized which is hard to do analytically, but special solutions (numerical) have been found by Dine and Srednicki [140]. For $\lambda_1 = 0.0013$, $\lambda_2 = 0.2$ and $\lambda_3 = 0.31$ they found the minimum to be at $\langle \varphi_N \rangle \sim 500$ GeV and $\langle h^0 \rangle = \langle \bar{h}^0 \rangle = 120$ GeV which gives the correct values of the W and Z masses. In a next step it has to be checked that this minimum remains the correct one if one includes the Yukawa couplings of the Higgses to the quarks and leptons and to check whether the estimate (5.11) for their masses remains valid. This corresponds to an inspection of the auxiliary fields of the H, $\bar{\mathrm{H}}$ multiplets

$$-F_{\mathrm{H}}^{*} = \lambda_2 \bar{h}\varphi_N + \sum_{i,j} (g'_{ij} U_i \bar{D}_j + g''_{ij} L_i \bar{E}_j)\,, \qquad -F_{\bar{\mathrm{H}}}^{*} = \lambda_2 h \varphi_N + \sum_{i,j} U_i \bar{U}_j\,, \tag{5.14}$$

and it was found that these additional terms neither destabilize the minimum nor change the estimates in (5.11) substantially for the given choice of parameters. The bosonic spectrum is thus phenomenologically acceptable and it remains to discuss the fermionic spectrum. Charged fermions are the λ^+, λ^- gauge fermions and the Higgs fermions ψ_{h}^- and $\psi_{\bar{\mathrm{h}}}^+$, which combine to two Dirac fermions the lightest of which comes out to be 85 GeV. The neutral fermions ψ_{h}^0, $\psi_{\bar{\mathrm{h}}}^0$, ψ_N, λ^0 and λ^y also become heavy partly because of the radiative contributions in (5.10). The lightest of these neutral fermions has a mass of 35 GeV with our choice of parameters [140].

The model thus leads to an acceptable spectrum. The lightest new boson has a mass of 20 GeV; it is shortlived and decays into its partner (e, μ, τ) and the goldstino. The goldstino itself is sufficiently decoupled to avoid phenomenological problems. The next lightest fermion has a mass of 35 GeV which decays preferably to the heaviest lepton and the scalar partner of its right-handed component. Care has to be taken also of the pseudo-Goldstone bosons that occur in the process of the breakdown of global symmetries through the supercolor condensations. The lightest of those are in the region of 40 GeV. These are the ones that do not receive masses through SU(3) × SU(2) × U(1) gauge interactions but only through the coupling constants (e.g. λ_1) in the superpotential which also explicitly break some global symmetries otherwise unbroken at the supercolor scale. Mass formula (5.11) assures the suppression of flavor changing neutral currents, and there are hence no phenomenological problems with this model at the moment. The model can be straightforwardly extended to a grand unified model based on $\mathrm{SU}(N)_{\mathrm{sc}} \times \mathrm{SU}(5)$ in the way discussed in the last chapter.

Notice that the only dimensionful parameter in the theory Λ is introduced through the supercolor coupling constant. This mass scale sets the scale $M_{\mathrm{S}} = \Lambda$ of supersymmetry and for $\Lambda = 2.5$ TeV; this induces the correct scale of SU(2) × U(1) breakdown (with the choices for λ_i as given above). One might now ask: what are the allowed values of Λ? We have already seen that with the given gauge couplings g_3, g_2 and g_1 and $\Lambda = 2.5$ TeV the lightest bosons in the theory (the partners of the right-handed leptons) receive a mass of 20 GeV, so $\Lambda \geq 2.5$ TeV. Raising Λ would raise the masses of these scalars. It would however also raise the radiative contribution to the mass^2 of the Higgses and thus make it more difficult to induce an SU(2) × U(1) breakdown through the terms in the superpotential, which in principle would give us an upper bound on Λ. For a sufficiently large Λ the coupling constants λ_i in the superpotential have to be carefully adjusted to have the $\langle h \rangle$, $\langle \bar{h} \rangle$ now small compared to Λ. If one would insist that the correct pattern of SU(2) × U(1) breakdown is obtained for a large range of the parameters in the superpotential one would be forced to use a Λ which is not much larger than 10 TeV. It should be stressed again that Λ is the only dimensionful parameter in this model which determines all these breakdown scales. Another potentially possible term of this type would be a Fayet–Iliopoulos term with parameter ξ. Unlike for Λ (which we obtain through dimensional transmutation) a natural value of ξ would be of the order of the grand unification or the Planck scale. ξ was chosen to be equal to zero and since $\mathrm{Tr}\, Y = 0$ it does not receive contribution in perturbation theory [240]. A possible explanation of the absence of ξ would be the embedding of $\mathrm{U}_Y(1)$ in a semi-simple group [569] (like SU(5)) for which a gauge invariant ξD-term cannot exist. Other possible explicit mass terms like $mH\bar{H}$ were forbidden by an R-symmetry under which θ transforms with charge $R = +3/2$ and all the chiral superfields with charge $R = +1$. This R-symmetry is broken by the condensates in (5.9) spontaneously and this breakdown was necessary for example to allow Majorana masses for the gluinos. But a spontaneously broken global symmetry usually implies the existence of a Goldstone boson. The R-symmetry under consideration (though not explicitly broken by terms in the superpotential) is explicitly broken by a supercolor anomaly and this would be Goldstone boson receives a mass of order Λ through this anomaly (just like the η' receives a mass contribution from the axial QCD anomaly). The model thus does not give rise to any phenomenological problems.

5.3. O'Raifeartaigh models and the connection between supersymmetry and weak interaction breakdown

The previously discussed supercolor models are based on the assumption that a formation of fermion–antifermion condensates appears as a result of the strong supercolor force. This then leads to a breakdown of supersymmetry: the reason supercolor was invented. The assumption of this condensation has been seriously questioned by Witten [573] on the basis of his index argument [75]. This argument, however, is strictly valid only in the case in which one includes masses for the nontrivial supercolor representations. Indeed, results obtained in an effective Lagrangian approach [543, 530] have recently given back the hope that supersymmetry might be broken in the massless limit, leaving this important question still under discussion [475a]. At the moment we do now know whether supercolor breaks supersymmetry or not [104a, 448, 273].

As a result of this ambiguity one has tried to construct models similar to the supercolor models in which the sector that was responsible for the supersymmetry breakdown was replaced by an O'Raifeartaigh model [461]. By similar we mean that this supersymmetry breakdown sector is somewhat hidden from the normal particles such as to forbid masses at the tree graph level for the partners of quarks and leptons. In the supercolor model this hidden sector consisted of the strongly interacting particles. The normal particles, of course, did not participate in these strong interactions and in addition they were not coupled to this sector in the superpotential to avoid a direct coupling to the goldstino. Such a procedure can also be realized with a hidden O'Raifeartaigh sector and models along these lines have been constructed by Dine and Fischler [132], Nappi and Ovrut [437], Alvarez-Gaumé, Claudson and Wise [13]. The construction of the hidden sector requires the introduction of several new chiral superfields. In the model of Dine and Fischler [132], for example, the following fields have been added:

$$\begin{aligned} &S, S' = (1, 2, 1/2)\,, \qquad \bar{S}, \bar{S}' = (1, 2, -1/2)\,, \qquad T, T' = (3, 1, -1/3)\,, \\ &\bar{T}, \bar{T}' = (\bar{3}, 1, 1/3)\,, \qquad X, Y = (1, 1, 0)\,. \end{aligned} \tag{5.15}$$

These are all left-handed chiral superfields and the brackets give their $SU(3)\times SU(2)\times U(1)$ representation content. In the context of a grand unified model this would correspond to (at least) the addition of two 5 and two $\bar{5}$ representations [133]. The superpotential of this model should contain an O'Raifeartaigh sector in which one of the auxiliary fields of the singlets (F_x or F_y) receives a vacuum expectation value. One can write

$$\begin{aligned} g = {} & m_1(\bar{S}'S + S'\bar{S}) + m_2(\bar{T}'T + T'\bar{T}) + m_3 S\bar{S} + m_4 T\bar{T} + Y(\lambda_1 S\bar{S} + \lambda_2 T\bar{T} - \mu_1^2) \\ & + \lambda_3 X(H\bar{H} - \mu_2^2) + g_{ij}U_i\bar{H}\bar{U}_j + g'_{ij}U_iH\bar{D}_j + g''_{ij}L_iH\bar{E}_j\,, \end{aligned} \tag{5.16}$$

and the model possesses six independent dimensionful parameters which are supposed to be in the region 100 GeV–10 TeV. This model has no R-invariance. In addition to lepton and baryon number conservation the only other conserved quantities are the number of S-type and T-type superfields. Actually superpotential (5.16) is not the most general one could write consistent with these symmetries, which would, e.g., allow terms quadratic and trilinear in Y. These couplings are left out since their presence in (5.16) would prevent a breakdown of supersymmetry [132]. We know that such terms are not generated in perturbation theory and it is thus "technically natural" to leave them out. One could of course forbid such terms by imposing an R-symmetry but such a symmetry in the context of these models poses problems. The R-symmetry forbids for example a Majorana mass for the gluinos and we conclude that it is spontaneously broken. A spontaneous breakdown would lead to an unobserved Goldstone boson. To generate a mass for this particle one would have to break the R-symmetry explicitly. In the supercolor models this occurred because of a supercolor anomaly, here we can in principle have a color anomaly but this would make an unwanted Peccei–Quinn–Weinberg–Wilczek axion [474, 552, 566] out of the unwanted Goldstone boson. Such an R-symmetry thus has to be explicitly broken in the superpotential, equivalent with the statement that g is not allowed to possess an R-symmetry. This implies that such a symmetry cannot be used to forbid the mentioned Y^3 and Y^2 terms in the superpotential. Certain terms in g that are consistent with all the symmetries have to be omitted. This unsatisfactory property is common to all the models we will discuss in this section [198].

Apart from these questions g in (5.16) has the desired properties. The hidden sector, given by the first five terms in (5.16) does not couple to the observable sector at the tree graph level and the restrictions of the $\mathrm{STr}\,\mathcal{M}^2$ formula are avoided. Mass splittings in the observable sector will be generated in perturbation theory and since terms in the superpotential do not receive radiative contributions this mass generation will proceed through gauge interactions which in turn assures the absence of problems with flavor changing neutral currents.

Supersymmetry is broken in the hidden sector since there is no common solution to $F_X = F_S = F_{\bar{S}} = F_{S'} = \cdots = F_{\bar{T}'} = 0$, the minimum is obtained for $F_Y = \mu_1^2$ and all scalar fields have vanishing v.e.v. except for φ_Y whose v.e.v is undetermined. In the observable sector the equation $F_X = 0$ forces $h\bar{h} = \mu_2^2$ and implies the breakdown of SU(2) × U(1). Observe that in this model SU(2) × U(1) is broken by hand through the introduction of the term $-\mu_2^2 X$ in the superpotential. In particular the breakdown of supersymmetry and weak interactions is no longer related. One must actually be careful that the radiative contribution to the Higgs masses from the hidden sector are not too large to restore supersymmetry in the effective potential. In this model this implies an upper bound of order of 100 TeV on the parameters $m_1, \ldots, m_4$ and μ_1 and also gives an upper bound of 100 GeV for the lightest partners of quarks and leptons. But nonetheless supersymmetry and weak breakdown are not related. If μ_1 would be zero supersymmetry would be unbroken and similarly $\mu_2 = 0$ implies unbroken SU(2) × U(1) independent of the value of μ_1. Recalling our discussion about the motivation to consider supersymmetric models we would, however, prefer models in which the supersymmetry breakdown induces the breakdown of the weak interactions. This would mean that SU(2) × U(1) gets restored once the supersymmetry breaking parameter M_S vanishes and the breakdown of the two symmetries is intimately related. In the context of the models discussed here such a possibility exists as has been observed by several groups [332, 13, 133, 124]. We will give a simplified version of this mechanism here. The full calculation would have to include renormalization group considerations of the effective potential which we will not give here since we will treat this subject in Chapter 7 in detail. The essentials of the mechanism can already be seen in the simplified version.

Suppose that supersymmetry is broken in a hidden sector such that there are no tree level couplings of the ordinary sector to the goldstino. The ordinary sector will receive the information of the supersymmetry breakdown through radiative corrections due to the gauge interactions. Suppose now (which is normally the case as you can see from the examples given so far) that at the one loop level the SU(3) × SU(2) × U(1) gauginos receive a mass. The scalar partners of quarks and leptons receive their

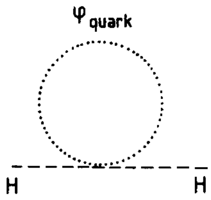

Fig. 5.3. Contribution to the mass of the Higgs boson through its coupling to the partners of quarks.

mass because of the presence of this nonvanishing gaugino mass from the graphs in fig. 5.2, the sum of which would vanish if m_λ would be zero. The graph containing λ gives a negative contribution to m_φ^2 and since this graph becomes suppressed because of m_λ the full contribution to m_φ^2 is positive if supersymmetry is broken. As a result scalar partners of quarks and leptons as well as the Higgses receive positive m^2 contributions. With $m_\lambda = 1$ TeV this would lead to a mass of order 100 GeV for the partners of quarks and a mass of order 10–50 GeV for the Higgses and the partners of leptons (compare (5.11)). Including these masses we now reconsider the graphs in fig. 5.2. The graph with quadrilinear scalar coupling includes now a massive propagator and since it corresponds to a positive contribution to m^2 this contribution is reduced once the scalars are massive. The effect is strongest if the particle in the loop is a partner of a quark since they are heaviest. Such a situation can either occur if the particle under consideration is a partner of a quark or a Higgs (it cannot occur for the partners of leptons) as shown in fig. 5.3. This two-loop effect of course cannot drive the already heavy partners of quarks to negative m^2 but it can have an influence on m_{Higgs}^2 provided that there is a large quadrilinear coupling between Higgses and quarks partners. A candidate for such a large coupling is the Yukawa coupling responsible for the mass of the top quark. Whether or not at the end the Higgses receive negative m^2 depends now on the balance between the suppression of the graphs c and d in fig. 5.2. The contribution to the m_{Higgs}^2 can approximately be written as

$$m_{\text{Higgs}}^2 \simeq \alpha_2 m_{\lambda_2}^2 - g_t m_{\varphi_q}^2 \simeq \alpha_2 m_{\lambda_2}^2 - g_t \alpha_3 m_{\lambda_3}^2 \tag{5.17}$$

which can become negative if g_t (the top quark Yukawa coupling) is large enough. The actual values needed for g_t to lead to negative m_{Higgs}^2 and induce an SU(2) × U(1) breakdown are model dependent and can only be found after a correct treatment of the two-loop effective potential. In some cases a top quark mass larger than 25 GeV is sufficient to drive the breakdown of the weak interactions [172]. (We will later come back to this point.) Observe that there is no danger to induce negative (mass)^2 for the partners of leptons since they do not couple to the partner of quarks and secondly there is no negative m^2 for the partners of quarks since they have received a large mass at the first stage.

This concludes this section on the hidden sector O'Raifeartaigh models with $M_S \sim 1$ TeV. They require the introduction of new fields in the hidden sector with the introduction in general of several mass parameters. Care should be taken to avoid an R-symmetry in the superpotential, which implies that the superpotential has not the most general form consistent with its symmetries. A sufficiently large value of the top quark mass can induce SU(2) × U(1) breakdown as a result of the supersymmetry breakdown.

5.4. *The inverse hierarchy*

In this section we want to discuss a class of O'Raifeartaigh models invented by Witten [570] which are different in spirit than the one we have discussed so far. In these models the scale M_S is supposed to be the fundamental scale and the large scales M_x or M_p are generated in perturbation theory. This approach makes use of a common property of all O'Raifeartaigh models that the potential has a flat direction at the minimum. This comes from the fact that broken supersymmetry requires the equations $F_i = \partial W/\partial\phi_i = 0$ for the auxiliary fields have no common solution. This algebraic inconsistency implies the F_i to be independent of one combination of the scalar field φ^i whose vacuum expectation value is in turn undetermined at the tree graph level and leads to a vacuum degeneracy along this direction. Let us consider as an example a model with superpotential [570]

$$g(A, X, Y) = \lambda X(A^2 - M^2) + gYA\,, \tag{5.18}$$

and let us furthermore assume that $g/\lambda M \gg 1$. The scalar potential reads

$$V = g^2|a|^2 + \lambda^2|a^2 - M^2|^2 + |gy + 2\lambda xa|^2\,. \tag{5.19}$$

For the given range of parameters the minimum is at $\langle a\rangle = 0$ and $\langle F_x\rangle = \lambda M^2 \neq 0$ as well as $\langle y\rangle = 0$. The vacuum expectation value of x (the scalar whose auxiliary field has a v.e.v.) is undetermined at the tree graph level. This degeneracy of the vacuum will disappear if one goes beyond the tree level. Actually this calculation has to be done since the v.e.v. of x determines the mass of the a particle. These corrections beyond the tree graph level have been discussed in detail by Huq [321], Clark, Piguet and Sibold [85]. The one-loop correction to the effective potential is determined by the supertrace of the fourth power of the mass matrix

$$\delta V = \frac{1}{64\pi^2}\sum_J (-1)^{2J} M_J^4(\phi)\log(M_J^2/\mu^2)\,, \tag{5.20}$$

where μ is a renormalization mass. Evaluating the leading terms for the model given above gives

$$V(x) = \lambda^2 M^4[1 + (\lambda^2/8\pi^2)\log(|x|^2/\mu^2) + \cdots]\,. \tag{5.21}$$

The logarithm in (5.21) can be understood as a replacement of λ by an effective coupling $\lambda(x)$. In theories with spin 0 and spin 1/2 fields only, this coupling always increases with increasing x. As a result the coefficient of the logarithm is positive and the one-loop corrections determine the vacuum expectation value of x to be zero in the model under consideration.

In theories with gauge interactions, however, the sign of the logarithm can become negative. The potential then might look like

$$V(x) = \mathrm{d}\lambda^2 M^4[1 + (b\lambda^2 - ce^2)\log(|x|^2/\mu^2)]\,, \tag{5.22}$$

where d, b, c are positive numerical constants and e is a gauge coupling. One might now have a situation where at small values of $\langle x\rangle b\lambda^2 < ce^2$. If the gauge interactions are asymptotically free the effective coupling $\bar{e}$ will decrease at large x whereas $\bar{\lambda}$ increases. As a result the effective coefficient $(b\lambda^2 - ce^2)$ of the logarithm changes sign at some x, and the one-loop effective potential will look like that shown in fig. 5.4. x will receive a nonvanishing v.e.v. Since the evolution of the coupling constants with the logarithm is rather slow; the vacuum expectation value of x will be large compared to the scale

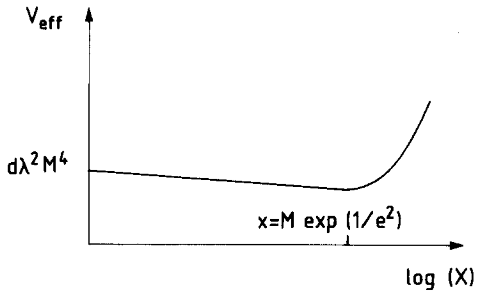

Fig. 5.4. One-loop effective potential for a model based on the inverse hierarchy.

M, the intrinsic scale of the model and will typically be of the order $M\exp(+1/e^2)$, the exact value depending on the parameters of the model. The model will thus contain two scales M and $\langle x\rangle$ with the latter obtained by "dimensional transmutation" and $\langle x\rangle \gg M$. This is the typical situation described by Coleman and Weinberg [93]. The nice thing here is that the flat direction of the potential need not be obtained by a fine tuning of parameters but necessarily exists in any O'Raifeartaigh type model.

Having made this observation Witten constructed a model in which M characterizes the scale of the breakdown of the weak interactions whereas $\langle x\rangle$ corresponds to a grand unification scale M_X. The fundamental scale of this model is no longer the big scale but the small scale.

The O'Raifeartaigh sector now contains two adjoint representation A^i_j and Y^i_j of SU(5) together with a singlet X,

$$g_R = \lambda \operatorname{Tr} A^2 Y + gX(\operatorname{Tr} A^2 - M^2)\,, \tag{5.23}$$

which leads to broken supersymmetry and

$$\langle a\rangle = gM(\lambda^2+30g^2)^{-1/2}\begin{pmatrix} 2 & & & & 0\\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3\end{pmatrix}. \tag{5.24}$$

One combination of $\langle x\rangle$ and $\langle y\rangle$ is undetermined and

$$\langle y\rangle = \frac{g}{\lambda}\langle x\rangle\begin{pmatrix} 2 & & & & 0\\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ 0 & & & & -3\end{pmatrix}. \tag{5.25}$$

The one-loop effective potential that determines $\langle x\rangle$ and $\langle y\rangle$ reads

$$V(x) = \frac{M^4 g^2\lambda^2}{\lambda^2+30g^2}\left(1+\frac{30g^2(29\lambda^2-50e^2)}{80\pi^2(\lambda^2+30g^2)}\log\frac{|x|^2}{\mu^2}\right), \tag{5.26}$$

which leads to large $\langle x\rangle$ and $\langle y\rangle$ if $29\lambda^2 < 50e^2$, where e is the SU(5) gauge coupling. The potential decreases for increasing x till the region where the approximate formula (5.26) is no longer valid. Asymptotic freedom, however, suggests that the potential turns over at some point and a stable minimum develops. The alternative would be that the potential decreases monotonically. Since our model is supersymmetric the potential is bounded from below by zero and one could still have a cosmological interpretation of such a model in which the ground state becomes supersymmetric asymptotically.

The fields $\langle x\rangle$ and $\langle y\rangle$ now receive large vacuum expectation values and SU(5) is broken at a scale characterized by $M^2\exp(1/g^2)$. In addition to Y we need Higgses in the $5+\bar{5}$ representation of SU(5) and we also add a singlet Z that will play the role of a sliding singlet (see our discussion in Chapter 4). We add to g_{R}

$$g_{\mathrm{S}} = f\bar{H}YH + hZ(H\bar{H} - M'^2)\,, \tag{5.27}$$

where M' is a mass comparable to M. Its purpose is to induce the breakdown of the weak interactions. As a result the scalars of H and $\bar{H}$ will receive a v.e.v. of order M'. The sliding singlet Z will adjust its v.e.v. to leave the color triplets in H and $\bar{H}$ heavy but keep the masses of the other components small in the range of M, M'. A complete inspection of the model shows that these v.e.v.'s of $H, \bar{H}$ disturb the vacuum expectation values of A and more importantly Y which leads to the following breaking pattern. $\langle y\rangle$ breaks $\mathrm{SU}(5)\to\mathrm{SU}(3)\times\mathrm{U}(1)\times\mathrm{U}(1)$ at the large scale and at the small scale H and $\bar{H}$ break $\mathrm{SU}(3)\times\mathrm{U}(1)\times\mathrm{U}(1)$ to $\mathrm{SU}(3)\times\mathrm{U}(1)_{\mathrm{em}}$. This is a desease of the simplified model which arises because the Y^i_j couples both to A^i_j and $H^i\bar{H}_j$. The problem can be cured by replacing A^i_j by a 75 representation of SU(5) and the correct pattern $\mathrm{SU}(5)\to\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\to\mathrm{SU}(3)\times\mathrm{U}(1)$ can be obtained [268].

Independent of these questions such a model with $M\sim 100\,\mathrm{GeV}$ is problematic because it contains too many light fields and if one adds three families of quarks and leptons $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ is no longer asymptotically free and the couplings constant become large before the grand unification scale [570, 268]. It is not clear whether unification still makes sense under these circumstances. But let us postpone the discussion of this question for the moment and add the quark and lepton superfields to the model with the usual Yukawa couplings to $H, \bar{H}$ in the superpotential. We want to discuss the mass splittings in these multiplets that arise through the breakdown of supersymmetry. This breakdown occurred in the sector described by g_{R} in (5.23) in a manner that both F_x and F_y (the "hypercharge" component) received a vacuum expectation value. The goldstino is then the appropriate linear combination of the fermions in the X and Y superfields. Again the splittings of the multiplets are determined through the coupling to the goldstino and at the level of the given effective potential only the scalars of the A-multiplet receive corrections due to the breakdown of supersymmetry. The A-scalars are very heavy due to the large v.e.v. of x and y of order M_{x} whereas the supersymmetry splitting is characterized by M. The ordinary field quarks, leptons and Higgses now couple to a^i_j via gauge couplings and their splittings arise in higher order. Their couplings to the goldstino are mediated through loops involving the superheavy a-fields whose mass is much larger than the splittings due to supersymmetry breakdown. As a result the splittings in the light sector are small and of order M^2/M_{x} multiplied by the appropriate coupling constants. With $M\sim 10^3\,\mathrm{GeV}$ and $M_{\mathrm{x}}\sim 10^{16}\,\mathrm{GeV}$ this implies for Δm_{L} the splittings in the light sector to be of order $10^{-10}\,\mathrm{GeV}$, a ridiculously small number which, e.g., represents the splitting of the electron from its scalar partner. The only way out is to raise M compared to M' which represents the weak scale. In fact M^2/M_{x} should be of the order of M' to obtain a reasonable model [30, 124, 571]. With the usual uncertainties of the coupling constants of the model this would correspond to a breakdown of supersymmetry at a scale of 10^9–10^{11} GeV. It is a remarkable property of supersymmetry that even with such a large breakdown scale the partners of quarks and leptons are kept in the 100 GeV range. A price of course has been paid to achieve this, the introduction of a small parameter M'/M. With the now large scale M of course also the problems with asymptotic freedom of $SU(3)\times SU(2)\times U(1)$ in these Witten-type models disappear.

5.5. *How to hide the hidden sector*

Several groups have independently observed this possibility to have $M_S \sim 10^9$–10^{11} GeV in the context of various models and we could call this the $B^2D^2EF^2IKNPR^2SW$ [30, 40, 124, 133, 171, 480, 571] observation. In the model such a situation can usually be realized by inserting appropriate small parameters that render the coupling of the goldstino to the observable sector small. However, in the different models this could be achieved in different ways and we will in the following briefly review the possibilities.

In the framework of the Fayet $\tilde{U}(1)$ models this situation has been realized by Barbieri, Ferrara and Nanopoulos [40]. In addition to the quark, lepton and Higgs superfields this model contains five singlets with $SU(3)\times SU(2)\times U(1)\times \tilde{U}(1)$ transformations

$$S(1,1,0,4)\,,\qquad R(1,1,0,4)\,,\qquad \bar{R}(1,1,0,-4)\,,\qquad P(1,1,0,1)\,,\qquad N(1,1,0,0)\,, \tag{5.28}$$

and a superpotential

$$g = g_Y + g_H + g_N\,, \tag{5.29}$$

with

$$\begin{aligned} g_Y &= g_{ij}U_i\bar{U}_jH + g'_{ij}U_i\bar{D}_jH + g''_{ij}L_i\bar{E}_jH + \delta LP\bar{H}\,,\\ g_H &= \lambda H\bar{H}S + \mu R\bar{R}\,, \qquad g_N = \eta NS\bar{R} + mN(\rho m + N)\,. \end{aligned} \tag{5.30}$$

Observe that g has no R-symmetry and that it is not the most general superpotential consistent with its symmetries. This property is necessary to allow gluino masses without the appearance of a Goldstone boson (or axion) of spontaneously broken R-symmetry. Notice also that the theory is neither anomaly free nor is $\mathrm{Tr}\,\tilde{Y}=0$. This might not be serious in this case since we will see that $\tilde{U}(1)$ is broken at very high energies (M_p) and one might argue that anomalies at high energy do not necessarily lead to inconsistencies in the low energy approximation. This argumentation, however, is not necessarily convincing.

The potential of the model is discussed in the range of patameters

$$g_2^2 > 2\lambda^2\,,\qquad 4\tilde{g}^2\tilde{\xi} > \mu^2(1+8\tilde{g}^2/\lambda^2)\,, \tag{5.31}$$

and the absolute minimum is found at

$$\langle h^0\rangle = \langle \bar{h}^0\rangle = \frac{\mu}{\sqrt{2}\lambda}\,,\qquad \langle \bar{r}\rangle^2 = \frac{4\tilde{g}^2\lambda^2\tilde{\xi} - \mu^2(8\tilde{g}^2+\lambda^2)}{16\tilde{g}^2\lambda^2}\,,\qquad \langle\tilde{D}\rangle = \mu^2/4\tilde{g}^2\,, \tag{5.32}$$

where $\tilde{\xi}(\tilde{g})$ denote the $\tilde{U}(1)$ Fayet–Iliopoulos parameter (coupling constant) respectively. Without making the model inconsistent one can now assume that $\tilde{\xi}$ is very large (or order $M_{\rm P}$) whereas μ is small $\mu \sim 100\,\text{GeV}$, such that $\tilde{\xi}/\mu^2 \sim 10^{34}$. With $\tilde{\xi} \sim M_{\rm P}^2$ there are also no problems with $\operatorname{Tr} \tilde{Y} \neq 0$ since we can hardly imagine the quadratic divergences to contribute more than $M_{\rm P}^2$ to $\tilde{\xi}$. $\tilde{\xi}$ now determines the breakdown of $\tilde{\rm U}(1)$ and this group is broken at $M_{\rm P}$ such that the corresponding gauge boson will certainly not cause phenomenological problems in the low energy theory. The Goldstone fermion is given by

$$\psi_g = \tfrac{1}{2}\tilde{g}\tilde{D}\tilde{\lambda} + (\partial g/\partial\varphi_i)\chi_i\,. \tag{5.33}$$

With $\xi/\mu^2 \gg 1$ the dominant component of the goldstino will be the fermionic component of R whose auxiliary field has received a v.e.v. of order $\mu\sqrt{\xi}$. The vacuum energy is given by

$$V_0 = \tfrac{1}{4}\mu^2\tilde{\xi} - \mu^4(\lambda^2 + 8\tilde{g}^2)/32\tilde{g}^2\lambda^2 \simeq \tfrac{1}{4}\mu^2\tilde{\xi}\,. \tag{5.34}$$

The observable sector couples only to the (tiny) $\tilde{\lambda}$-component of the goldstino and since $\langle\tilde{D}\rangle = \mu^2/4\tilde{g}^2$ the mass splittings in the low energy sector will be of order μ, the scale of the breakdown of the weak interactions. Thus with the two scales $M_{\rm P}$ and μ one has a model where the supersymmetry breakdown occurs at $M_{\rm S}^2 = \mu M_{\rm P} \sim (10^{10}\,\text{GeV})^2$. The light sector receives mass splittings of order μ since it only couples to a tiny component in the goldstino. As it is always the case in the $\tilde{\rm U}(1)$ models all the masses and splittings are already given at the tree graph level. These contributions are the dominant ones and are not upset by radiative corrections as we will see later.

In the framework of the O'Raifeartaigh models discussed in section 5.3 a grand unified model with $M_{\rm S} \sim 10^{10}\,\text{GeV}$ has been given by Dine and Fischler [133]. They start with the two input scales $M_{\rm x} \sim 10^{17}\,\text{GeV}$ and $\mu \sim 10^{10}\,\text{GeV}$. The scale $M_{\rm x}$ is responsible for the breakdown of SU(5) whereas μ governs the breakdown of supersymmetry. The structure of the model is similar to the ones discussed in section 5.3. Again an R-symmetry has to be avoided for the well-known reasons and the superpotential is at most technically natural. The difference is now the appearance of the small parameter $\mu/M_{\rm x}$. The goldstino is not coupled to the light sector directly and in higher orders such a coupling can only occur through heavy intermediate states. As a result a small scale of order $\mu^2/M_{\rm x} \sim 10^3\,\text{GeV}$ is induced by the radiative corrections which governs the mass splittings in the light sector. These radiative corrections also induce a breakdown of SU(2)×U(1) in this model at scale $\mu^2/M_{\rm x}$ very much in spirit of the mechanism discussed in section 5.3. Here the two scales μ and $M_{\rm x}$ conspire to give a scale $\mu^2/M_{\rm x}$ to the low energy sector. The essential ingredient of the model is $\mu/M_{\rm x} \sim 10^{-7}$, and the light mass scale including the breakdown of SU(2)×U(1) is induced radiatively. We will discuss these radiative corrections in a moment.

A different mechanism to raise the breakdown scale of supersymmetry in the context of these O'Raifeartaigh has been proposed by Ellis, Ibanez and Ross [171]. Instead of introducing two mass scales μ and M with a tiny ratio μ/M they introduce explicit small coupling constants that communicate between the goldstino and the light sector. Instead of one small coupling, as they propose, one could also use a succession of hidden sectors with not so small coupling constants. Instead of $\lambda_1 \ll 1$ it is through $(\lambda_i)^n \ll 1$ that the light sector receives the information about the supersymmetry breakdown and the separate λ_i need not be as small as λ_1. The model is constructed in such a way that the hidden sector closest to the light sector contains particles that have SU(3)×SU(2)×U(1) gauge interactions and induce a mass for the gauginos in the light sector and the induction of the weak interaction breakdown can then proceed by the mechanism described in section 5.3. The supersymmetry breakdown scale in such a model is actually arbitrary. It could be even large compared to the Planck mass provided the appropriate couplings are made small enough (or the appropriate number of intermediate hidden sectors is large enough). We will however see later that the coupling to gravity, an interaction in which all the fields in all the hidden sectors participate, provides us with an upper limit on M_S if we require that the light sector still contains supersymmetric partners.

In the context of models based on the inverted hierarchy the necessity of a large breakdown scale has been observed by Banks, Kaplunovsky [30], Witten [571], Dimopoulos and Raby [124]. Unlike in the other models the large scale $M_S \sim 10^{10}$ GeV is a necessity rather than a possibility. Along these lines a grand unified model based upon the inverse hierarchy has been constructed. It has the nice feature that only one scale $\mu \sim 10^{10}$ GeV is introduced, out of which the large scale M_x is induced out of the flat direction. Finally the scale in the light sector is induced as μ^2/M_x. This model, like the other ones discussed so far also has a superpotential that is not the most general consistent with its symmetries. Also care has to be taken with the color Higgs triplets which in this model turn out to have a mass of order μ. They could introduce proton decay via dimension five operators at an unacceptable rate. In the model this problem is solved by doubling the number of 5 and $\bar{5}$'s and introducing some singlets. The model uses the sliding singlet mechanism to keep the Higgs doublets massless. Here, however, a complete treatment of the effective potential has to be given to make sure that the theory has the correct minimum that breaks $SU(2)\times U(1)$. Investigations of this effective potential have shown that there are potential problems with the use of the sliding singlet [130, 442]. These are even more pronounced when the supersymmetry breaking is very large, especially in the context of models based on the inverted hierarchy. It might therefore be very difficult for such a model to induce the breakdown of the weak interactions at the small scale. To answer all these questions a careful examination of the two- (or even three-) loop effective potential of the low energy theory is necessary. Given the complexity of such models the search for the absolute minimum of the potential is not an easy task.

5.6. *The stability of the small scale* [480, 124, 133, 41]

We still have to discuss explicitly the perturbative corrections in models with a large supersymmetry breaking scale. It seems, at least to me, highly nontrivial that all the radiative corrections that appear in the low energy sector correspond to a mass scale that is so small compared to M_S. A full treatment of these questions has been given by Polchinski and Susskind [480] in a toy model that has all the essential ingredients of the models discussed in the last section. We cannot repeat their discussion here in full completeness but will follow it closely also in notation, so that the interested reader can complete his knowledge through a study of the original paper. The model contains four chiral superfields with superpotential

$$g(B, C, X, L) = gX(B^2 - \mu^2/g) + MB^2 + M'BC + gB^3 + gB^2L + gBL^2 + gL^3\,, \tag{5.35}$$

where all dimensionless coupling constants have been called g. X will be the superfield that contains the goldstino, B, C describe heavy fields and L the light fields. We assume the parameters of the model to be arranged in such a form that at the minimum $b = c = l = x = 0$ and $F_x = \mu^2$. $M_S = \mu$ is assumed to be of the order of 10^{10} GeV whereas M, M' correspond to the large scale M_x or M_p of order 10^{16}–10^{18} GeV. The light field L will be massless at the tree graph level and it will not be split since it does not couple to the goldstino in X at this stage. The light field L couples only to the heavy field B. Our task is now to see how L is affected by radiative corrections. The scalar potential at the tree graph level is given by

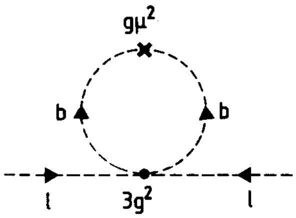

Fig. 5.5. Contribution to the mass of a light scalar l through its coupling to the heavy scalar b. The cross denotes the supersymmetry breaking in the heavy sector.

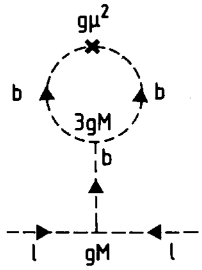

Fig. 5.6. Contribution to m^2l^2 that cancels the one in fig. 5.5.

$$V = |gb^2 - \mu^2|^2 + M'^2|b|^2 + |2gxb + Mb + M'c + 3gb^2 + 2gbl + gl^2|^2 + |gb^2 + 2gbl + 3gl^2|^2 . \qquad (5.36)$$

The heavy field B couples to the goldstino and there is a splitting due to the breakdown of supersymmetry. In addition to the supersymmetric mass term $(M^2 + M'^2)bb^*$ there is a term $g\mu^2(b^2 + b^{*2})$ which splits the two real scalars. It is this splitting that is the source for all splittings in the light sector in perturbation theory since L couples to B. A mass contribution for the light fields of the form m^2l^2 could now be induced in the effective potential through a graph like the one shown in fig. 5.5, which gives a contribution of order $g^3\mu^2$ resulting in a mass of 10^{10} GeV for the scalar particles in the "light" sector. We have however to be more careful: in addition to the graph in fig. 5.5, there is a contribution from the graph in fig. 5.6. Again the leading contribution of this graph to l^2 is $g^3\mu^2$ but this contribution cancels exactly the former one as an explicit calculation of the graphs will show. The reason is that these two graphs are contained in the one supergraph shown in fig. 5.7. The supersymmetry breaking is represented by the v.e.v. of the auxiliary field of X. The contribution to l^2 is obtained as a $(LLX)_F = \int d^2\theta LLX$ contribution of this supergraph. We have seen in Chapter 2 that such an F-contribution cannot be generated in perturbation theory; here we have an explicit example where in the component language two graphs cancel. To obtain a nonvanishing contribution to l^2 we have to consider a supergraph with at least two X field insertions to construct a D or $\int d^4\theta$ contribution as for example $(LLX^*X)_D$ (see fig. 5.8). This graph, however, has now two μ^2 insertions and an additional heavy propagator and it results in a contribution $(g^4\mu^4/M^2)l^2$ which is much smaller than μ^2. In analogy contributions to ll^*-mass terms can be discussed. The most dangerous contributions come from $(LL^*D)_D$ as shown in fig. 5.9. It gives $g^3\mu^2/M(LL^*)_F$ as the scale in the light sector. Polchinski and Susskind have given a complete consideration of all the possible contributions in perturbation theory.

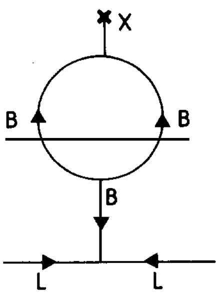

Fig. 5.7. Supergraph that contains the contributions given in figs. 5.5 and 5.6.

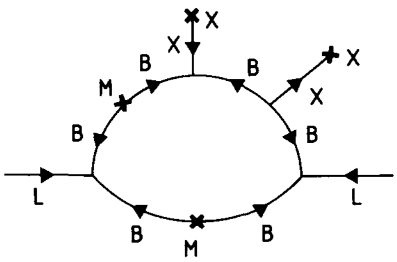

Fig. 5.8. Supergraph which gives a contribution to $(LLX^*X)_D$ allowed by the nonrenormalization theorems.

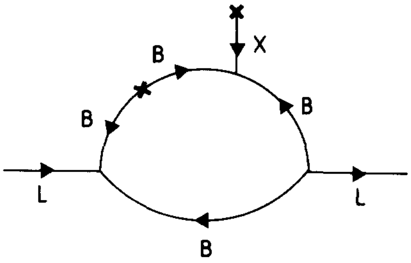

Fig. 5.9. Contribution to $(LL^*X)_D$.

With the exception of one case they have shown that the highest scale that could be induced in the light sector is given by $g^3\mu^2/M$. Moreover they have verified that the low energy sector looks as an explicitly (but soft) broken supersymmetric theory. All the soft breaking terms are in general generated in perturbation theory and not the hard ones. Masses for chiral fermions can also be generated but only in a supersymmetric way. This means that in models with $M_S \sim 10^{10}$ GeV there can still exist a light sector characterized by a scale small compared to M_S and that this separation remains stable in perturbation theory. Supersymmetry can thus protect particle masses even at a scale small compared to M_S, without fine tunings in any order of perturbation theory. There is, however, an exception to this case and this occurs if the theory contains a light singlet. Such a singlet superfield will mix with the goldstino superfield and its typical mass scale as well as the splittings will be of order $\mu \sim M_S$ [480, 451, 389]. If it does not yet exist at the tree graph level such a mass will be induced by radiative corrections. If the singlet then couples to light as well as heavy fields it will communicate the mass splittings of the heavy sector to the light sector and masses of order μ will also be induced for the other nonsinglet scalars in the light sector. The separation between the light and the heavy sector is thus potentially unstable in the presence of a light singlet. This problem with the light singlet is a typical problem of naturalness. One can build models that include a light singlet and where there is still a strict separation of the light and heavy sector at the expense of inserting small parameters in the theory. If the singlet for example only couples with Yukawa couplings of order 10^{-8} to the other particles of the theory it does of course not cause any problems. In most cases, however, the introduction of such an additional small parameter in the theory is at least unsatisfactory. We will discuss these problems with the light singlet in more detail in the context of the $N = 1$ supergravity models in the next chapter.

We shall stop here our discussion of the globally supersymmetric models. The reader who has followed the discussion up to this point will now be able to construct his own model and discover (or not) the potential problems it might contain.

6. Low energy supergravity: basics

In the last chapter we have seen a variety of models based on global supersymmetry. Supersymmetry was spontaneously broken in the so-called hidden sector which did not couple directly to the so-called observable sector. In fact the most promising models were those in which the supersymmetry breaking scale was quite large $M_S \gtrsim 10^{10}$ GeV which of course implied that the hidden sector was actually only very weakly coupled. We were then in a situation in which although supersymmetry was broken at

$M_S \geq 10^{10}$ GeV it could still protect the scalar masses of the observable sector to stay in the 100 GeV region. In this section we will try to couple these models to supergravity. In principle one has finally to couple models to gravity, it is just a question whether gravity has an influence at low energies. In the case of the standard $SU(3) \times SU(2) \times U(1)$ model gravity has no influence on the physics at 100 GeV and it is believed that these effects can be neglected up to energy scales that are comparable to the Planck mass of 10^{19} GeV. In the case of supersymmetric models one has also neglected gravitational effects up to now. To include them it is natural to do it in the framework of local supersymmetry which already includes gravity. Here as in the nonsupersymmetric case one has of course now to deal with a nonrenormalizable theory. Although models based on $N = 1$ supergravity have a better ultraviolet behaviour than usual gravity, it is still nonrenormalizable. To be able to work with such a theory we define the theory with a cutoff Λ of the order of the Planck mass. This explicitly assumes that there exists a meaningful theory of supergravity (which we do not know yet) to which $N = 1$ supergravity is an approximation at scales smaller than M_P and that the effects that finally render the complete theory meaningful have no direct influence at energies small compared to M_P. It is sometimes argued that such a mechanism might be provided by extended supergravity.

Having stated these assumptions, as in the usual theories the question remains whether (and under which circumstances) (super) gravity can have influence at all on the low energy sector we want to discuss.

6.1. *The gravitino mass*

A quantity relevant for this discussion is the gravitino mass. In supergravity the graviton is accompanied by a spin 3/2 fermion, the gravitino. In the unbroken case graviton and gravitino are degenerate and massless. We have seen in Chapter 3 that the discussion of the gravitino mass term is quite subtle if the cosmological constant is different from zero. We will for the moment assume that $\Lambda \equiv 0$ to simplify the discussion. If supersymmetry is broken this manifests itself in a splitting of the supergravity multiplet: the graviton will stay massless and the gravitino will receive a mass. As usual the gravitino mass is not what gives directly the amount of supersymmetry breakdown but we have the relation (3.31) [114, 101]

$$m_{3/2} = M_S^2/M\sqrt{3}\,, \tag{6.1}$$

in which $M = M_P/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV and M_S denotes the supersymmetry breakdown scale obtained through the vacuum expectation values of auxiliary fields. Let us now assume, as happens in some globally supersymmetric models, that $M_x = 10^{10}$ GeV. Through (1) this leads to a gravitino mass of 100 GeV. *It is this fact that could tell us that gravity effects have to be included if one wants to discusss supersymmetric models with a large breakdown scale* [42, 444, 445]. If one would have, e.g., $M_S =$ 100 GeV the gravitino mass would be tiny and as in the case of the usual models we would safely assume that gravitational effects can be neglected in the discussion of the models at 100 GeV. But in models with $M_S \geq 10^{10}$ GeV such effects might become important. This is actually not such a big surprise if one looks at the models with large M_S. The hidden and the observable sectors in such models are only coupled very weakly, but all the particles under consideration are believed to participate in the gravitational interactions. It is then just a question at which point the gravitational interactions dominate the other interactions between the hidden and the observable sector. Equation (6.1) might indicate that this happens at $M_S = 10^{10}$ GeV. It is still the question whether the splitting of the supergravity multiplet is representative for the supergravity induced splittings in the observable sector. It might very well be that the observable sector is still unaffected. Indeed the first mass formulas for the coupling of one chiral superfield to supergravity gave us [227, 97, 98]

$$\mathrm{STr}\,\mathcal{M}^2 = 0\,, \tag{6.2}$$

even in the presence of supergravity breaking (at the tree graph level), which if it generalizes to more fields would indicate that the observable sector might not be affected at all by this type of supergravity breaking at the tree graph level. Let us now, however, couple more chiral superfields to supergravity and let us for the sake of simplicity assume that we have minimal kinetic terms. Formula (6.2) then generalizes to formula (3.43) [100, 101]

$$\mathrm{STr}\,\mathcal{M}^2 = 2(N-1)m_{3/2}^2\,. \tag{6.3}$$

We could now treat one of the chiral fields as the hidden sector responsible for supergravity breakdown and the other $(N-1)$ fields as the observable sector (usual quark, lepton and Higgs superfields) [101]. Both sectors are only coupled through gravitational interactions. The hidden sector including one chiral superfield as well as the graviton and the gravitino could then satisfy a mass formula like (6.2) and the $(N-1)$ chiral fields in the observable sector would have a nonvanishing $\mathrm{STr}\,\mathcal{M}^2$ at the tree level. The $\mathrm{STr}\,\mathcal{M}^2$ is positive indicating that the bosons are on average heavier than their fermionic partners by an amount of order of the gravitino mass. In this case the observable sector has mass splittings as large as the supergravity multiplets confirming our suspicion that in models with $M_S \geq 10^{10}$ GeV gravitational effects could influence the observable sector and cannot be neglected in general. In addition formula (6.3) indicates that these gravitational effects do everything in an easy way, what we were trying so hard to realize in globally supersymmetric models. They lift the masses of the bosonic partners of quarks and leptons by $m_{3/2}$ already at the tree level, violating the supertrace formulas of global supersymmetry. The conditions under which (6.3) was derived, however, are not the most general ones, nonminimal kinetic terms and gauge multiplets might be included which we will discuss in following sections. Having, however, some experience with the supertrace formulas from the last chapter we expect gravitational effects to play a role in the observable sector if M_S is large, barring some accidental cancellations. We could even imagine that in the case of $M_S \gg 10^{10}$ GeV the supersymmetric partners of quarks and leptons are no longer protected to have masses in the 100 GeV range and gravitational effects raise the masses to scales which render these models uninteresting for our discussion in which we want to relate the supersymmetry breaking scale and the breakdown scale of the weak interactions. In principle, these gravitational effects will then give an upper limit on M_S for the models under consideration, above a certain value the models will no longer contain supersymmetric partners in the 100 GeV range. Before we discuss these questions in detail, let us mention some other improvements that occur in the context of supergravity.

First of all the theory now contains all the interactions we know of. This includes now also the Planck mass, believed to be a very important mass scale in physics. One motivation for us to consider supersymmetric models was an attempt to understand relations between mass scales like, e.g., M_W and M_x. Maybe such scales can only be understood if the Planck mass is also included. It might very well be possible that we can understand the magnitude M_W not alone in terms of M_x but only in terms of both M_x and M_p.

Secondly supergravity has the nice feature that the Goldstone fermion which arises in globally supersymmetric models now disappears via the super-Higgs effect. There is no longer a massless essentially decoupled fermion in the theory.

Our last comment in this section concerns the cosmological constant. In global supersymmetry the vacuum energy was an order parameter for supersymmetry breakdown. Since we know that supersymmetry has to be broken in nature this leads in the context of spontaneously broken global supersymmetry to a nonvanishing positive vacuum energy. If this would be true also after the coupling to supergravity we would know that these models are not relevant for a description of nature. Fortunately this is not true as we have seen in Chapter 3. Unbroken supergravity can occur not only with vanishing but also with negative cosmological constant. Spontaneously broken supergravity can occur for any value of the cosmological constant. In particular we can have spontaneously broken supergravity with $\Lambda = 0$. In all cases we know this is obtained through very special choices of the parameters of the model. We do not know why Λ is zero but at least we can fine tune it to any value we want.

6.2. *The hidden sector*

The models we consider contain two sectors: a hidden sector responsible for the spontaneous breakdown of supergravity and the observable sector containing quarks, leptons and Higgs superfields the gauge multiplets and in case of a grand unified model also the corresponding additional particles. These two sectors only communicate through the gravitational interactions. This implies that the fields in the hidden sector are singlets with respect to all gauge interactions in the observable sector and also do not have Yukawa couplings to this sector. Of course one could have other very weak interactions between the two sectors but since there are no indications and no reasons why they should exist we do not consider them. In this section we want to discuss the hidden sector and the possibilities for a spontaneous supergravity breakdown. We will first do this in the context of what was called minimal kinetic terms in Chapter 3 which implies for the Kahler metric $G^i_j = -1$ everywhere. If the hidden sector consists solely of chiral superfields in addition to the supergravity multiplet the relevant quantity to study is the scalar potential (compare (3.25))

$$V = \exp(-G)[G_i G^i - 3]\,, \tag{6.4}$$

and supersymmetry is broken if G_i has a nonvanishing v.e.v., the supersymmetry breaking scale is given by (3.30)

$$M_S^2 = M^2 \exp(-G/2) G_i\,, \tag{6.5}$$

with a gravitino mass term

$$m_{3/2} = M \exp(-G/2)\,. \tag{6.6}$$

For $E_{\mathrm{vac}} = 0$ one has $|G_i|^2 = 3$ and one obtains relation (6.1) between M_S and $m_{3/2}$. With these minimal kinetic terms the Kähler potential G for one superfield (z, χ, F) is given by

$$G(z, z^*) = -zz^*/M^2 - \log(|g(z)|^2/M^6)\,, \tag{6.7}$$

and we have still the freedom to choose the superpotential $g(z)$ at our will. In (6.7) we have indicated that g has the dimension of (mass)^3 to indicate the connection with the global limit, in which V is of dimension (mass)^4 and $V = |g_{1z}|^2$. For this special case (6.7) the potential (6.4) can now be written as

$$V = \exp\left(\frac{zz^*}{M^2}\right)\left[\left|\frac{\partial g}{\partial z} + \frac{z^*}{M^2} g\right|^2 - \frac{3}{M^2}|g|^2\right], \tag{6.8}$$

and supersymmetry is broken if $F_z = g_{1z} + (z^*/M^2)g$ has a nonvanishing v.e.v. To become acquainted with this potential let us first discuss the simplest case [324]: a constant superpotential $g = m^3$. The potential

$$V = m^6 \exp(zz^*/M^2)[|z|^2/M^4 - 3/M^2] \tag{6.9}$$

has as stationary points $z = 0$ and $|z| = M\sqrt{2}$ out of which only $z = 0$ corresponds to unbroken supersymmetry. For $|z| = M\sqrt{2}$ we have $|F_z| = \sqrt{2}(m^3/M)$ and supersymmetry is broken. The form of this potential is shown in fig. 6.1. The supersymmetric stationary point is a local maximum and the absolute minimum is obtained for $|z| = M\sqrt{2}$ with a vacuum energy $E_{\text{vac}} = -e^2 m^6/M^2$ and broken supersymmetry. Observe that here in contrast to O'Raifeartaigh models in renormalizable global supersymmetry, supergravity can be broken in the presence of one field only. Observe further that supergravity breaking minima can be the lowest ones even if supersymmetric stationary points exist, again in contrast to the globally supersymmetric case where the existence of a supersymmetric stationary point implied unbroken supersymmetry with $E_{\text{vac}} = 0$. One might now ask the question how one could arrive at $E_{\text{vac}} = 0$ in the model given above. The only way to do this is by putting $m = 0$. In this case the potential however is identically zero and no well-defined vacuum state exists.

We are interested in a case where a nonsupersymmetric minimum with $E_{\text{vac}} = 0$ exists. The simplest example of such a case can be realized in this context through the Polonyi superpotential [483] $g = m^2(z + \beta)$ in which β is a dimensionful parameter used to adjust E_{vac} to vanish. Since this model is a widely used hidden sector in the construction of models we will discuss it in detail. The order parameter of supergravity breaking is now given by $F_z = m^2 + (m^2/M^2)z^*(z + \beta)$. Solutions to the equation $F_z = 0$ do not exist if $|\beta| < 2M$ and we are sure that supergravity is broken in this range of β. We want now to investigate whether in this range there exist stationary points with $V = 0$. These are the solutions of the equation

$$(\beta + z + z^*)[M^2 + z^*(z + \beta)] = 3M^2(z^* + \beta). \tag{6.10}$$

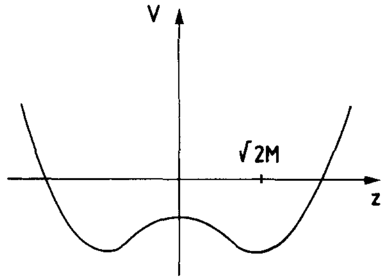

Fig. 6.1. Scalar potential (6.8) in the presence of a constant superpotential.

Solutions to these equations only exist for the four values $\beta = (\pm 2 \pm \sqrt{3})M$. Only the values $(2-\sqrt{3})M$ and $(-2+\sqrt{3})M$ lie in the range where no supersymmetric solutions exist. They correspond to v.e.v. of the z-field, $z = (\sqrt{3}-1)M$ and $z = (1-\sqrt{3})M$ respectively at the minimum of the potential. The other solutions $\beta = \pm(2+\sqrt{3})M$ lie outside the range $|\beta| < 2M$. An inspection of the second derivatives of the potential shows that these latter stationary points with $V = 0$ are not local minima but saddle points and can therefore not correspond to ground states of the model. For $|\beta| < (2-\sqrt{3})M$ the potential, however, is positive definite, and for $|\beta| = (2-\sqrt{3})M$ the potential has an absolute minimum at $V = 0$ and supergravity is broken since $|F_z| = m^2\sqrt{3} \neq 0$. The resulting gravitino mass is

$$m_{3/2} = (m^2/M)\exp((\sqrt{3}-1)^2/2)\,. \tag{6.11}$$

The model with $g = m^2(z+\beta)$ shows thus the desired behavior. The real scalars have (masses)^2 $2\sqrt{3}m_{3/2}^2$ and $2(2-\sqrt{3})m_{3/2}^2$ and the formula (6.3) is satisfied [43]. The fermionic partner of z is absorbed by the gravitino.

A hidden sector with one chiral superfield and superpotential $g = m^2(z + (2-\sqrt{3})M)$ is thus acceptable. Supergravity is broken at a scale m^2 through v.e.v.'s of the z-field which are of order of the Planck mass and the vacuum energy can be fine tuned to vanish for $\beta = (2-\sqrt{3})M$. Observe, however, that the v.e.v. of the scalar is of order of M_p and this is in the range in which our approximation at the tree graph level might break down. Notice also that the superpotential $g = m^2(z+\beta)$ is not natural in the strict sense, it is not the most general consistent with its symmetries since the terms z^2 and z^3 have been omitted. A superpotential $g = m^2 z$ would be natural since it has an R-symmetry that forbids these higher-order terms. The introduction of β destroys this symmetry. In general introducing terms z^2 and z^3 in the superpotential destroys the nice feature of $g = m^2(z+\beta)$ to have supergravity broken and to have the *absolute* minimum at $V = 0$. In the more general cases such a behavior can only be obtained for local minima at $V = 0$. There are, however, also exceptions to this rule, and with some effort other acceptable solutions can be found. These investigations require a minimalization of very complicate potentials and usually numerical methods have to be used to find the minima [378].

Let us come back to the question of naturalness. Does there exist at least one example of a hidden sector that is the most general consistent with its symmetries and that has its absolute supersymmetry breaking minimum at $V = 0$. The answer is yes but the example I know of has nonminimal kinetic terms. It is based on a Kähler potential [87]

$$G = -\frac{zz^*}{M^2} - \frac{\alpha}{2}\frac{(zz^*)^2}{M^4} - \log(|m^2 z|^2)\,, \tag{6.12}$$

and the vacuum energy (at the absolute minimum) vanishes if

$$\alpha = \tfrac{1}{4} + \tfrac{3}{16}[(2+\sqrt{3})^{1/3} + (2-\sqrt{3})^{1/3}]\,. \tag{6.13}$$

The superpotential $g = m^2 z$ has an R-symmetry that forbids additional terms.

After the discussion of these special examples let us discuss some more general properties of the supergravity scalar potential [555, 446]. Our first remark concerns the role of supersymmetric stationary points. Let us consider the general case with

$$G = K(z_i, z_i^*) - \log|g(z_i)|^2\,, \tag{6.14}$$

where the index i labels the various scalar fields. The resulting scalar potential is

$$V = \exp(K/M^2)[F_i(K^{-1})^i_j F^{*j} - (3/M^2)|g|^2]\,, \tag{6.15}$$

with

$$F_i = g_i + (1/M^2)K_i g\,; \qquad g_i \equiv \partial g/\partial z^i\,. \tag{6.16}$$

Supersymmetric solutions exist if $F_i = 0$. An inspection of the derivative of the potential (6.15) shows that points with $F_i = 0$ are always stationary points. We also know that such supersymmetric solutions correspond to negative potential $V = -3\exp(K/M^2)(|g|^2/M^2)$. This then implies that the true vacuum of the theory whether supersymmetric or not has to have negative vacuum energy, with, of course, the exception that for $E_{\rm vac} = 0$ there is a degeneracy of a supersymmetric and a nonsupersymmetric ground state. With this one exception one should look for potentials in which the $F_i = 0$ have no common solution in order to construct the desired situation where the absolute minimum breaks supersymmetry and has $E_{\rm vac} = 0$. One might of course also consider situations in which the theory lives in a false vacuum which is stable (enough). In this case many of the restrictions quoted above do not apply.

Let us now also discuss a special relation between different supersymmetric minima. Recall that in a globally supersymmetric model different supersymmetric minima are always degenerate with $E_{\rm vac} = 0$. Such a situation occurred, e.g., in many SU(5) grand unified models where different supersymmetric minima with SU(5), SU(4) × U(1) or SU(3) × SU(2) × U(1) or others were present. This property is no longer true in the context of supergravity [555, 77]; different supersymmetric minima have $V_0 = -3(|g|^2/M^2)\exp(K/M^2)$ and are only degenerate if they have the same value of the superpotential which is in general not the case. The highest of these minima is the one with $g_0 = 0$ and hence $V_0 = 0$. The other supersymmetric minima have in general $V_0 < 0$. Nonetheless there are arguments that the highest minimum might be stable [555, 91]. In general, however, these considerations only apply to the so-called observable sector. In the hidden sector supersymmetry is broken and the question of different SU(5) minima can only be answered in the context of specific models. There are models where only one minimum with the correct breakdown of SU(5) → SU(3) × SU(2) × U(1) exists [438, 149].

Up to now we have discussed only the mechanism which in the context of globally supersymmetric models was called F-type breaking. One could also consider D-type breaking. This requires the existence of a U(1) factor in the gauge group before or after the gauge symmetry breakdown, and acceptable models are only obtained of this U(1) cannot be embedded in a grand unified group as we have discussed in Chapters 2 and 5. In the global case such models have usually problems with anomalies. In the case of supergravity there is a further restriction on these models because they are only consistent in the presence of an exact R-symmetry [42]. All these restrictions do not really rule out the possibility of D-type breaking but they are so strong that nobody has really tried to construct a satisfactory model.

Another possibility is a dynamical breakdown of supersymmetry due to condensation processes in strongly interacting gauge theories in the hidden sector [444–446]. This breakdown has certain nice features but is of course very difficult to compute quantitatively apart from the fact that some dynamical assumptions have to be made that cannot really be proven rigorously. We have already discussed the $\langle\psi\bar{\psi}\rangle$ condensation in the context of supercolor, which of course could also be used here in the hidden sector. But there are also other possibilities, like gauge fermion condensation. From all our discussion we are quite confident that such a $\lambda\lambda$-condensation does not break global supersymmetry [443, 543]. In local supersymmetry we do not know. A special example based on an effective Lagrangian approach [543], however, has shown that such a breakdown of supergravity might be possible [444]. This is quite interesting in connection with the discussion of the mass scales. For this remember that in the examples with a scalar hidden sector, the supersymmetry breaking scale of let us say $M_S = 10^{10}$ GeV has been put in by hand, e.g. by choosing $m = 10^{10}$ GeV in the Polonyi case and the v.e.v.'s of the scalar fields were typically of the order of M_p. Here the scale μ^3 of the $\langle\lambda\lambda\rangle$ condensate is determined through the strong gauge coupling in the hidden sector. Knowing that this process does not break global supersymmetry we deduce that $M_S^2 = f(\mu, M)$ has to vanish in the limit $M \to \infty$ in which we switch off the gravitational interactions. The largest supersymmetry breaking scale which can arise in this case is then $M_S^2 = \mu^3/M$ which is also realized in a specific model. This leads to a gravitino mass of order $m_{3/2} = \mu^3/M^2$. A gravitino mass of the desired magnitude can thus be obtained even for $\mu \gg 10^{10}$ GeV. Such a model then has the two input scales μ and M both large compared to M_S. Nonetheless they conspire to produce the tiny scale $m_{3/2}$ and could provide us with an understanding of the small scale of the breakdown of the weak interactions [446]. Of course such a model is very hard to discuss quantitatively.

The problem with all the hidden sectors is of course that from their very nature they are not (easily) accessible through experiment and are subject to theoretical speculations and taste. The only way in which they can be restricted is through their influence on the observable sector. This, however, is in general not very restrictive, since widely different hidden sectors could still have the same implications on the observable sector. Our task therefore has to be to isolate the parameters induced by the hidden sector into the observable sector in a model independent way.

6.3. *The coupling of the hidden sector to the observable sector*

There are several ways to couple these two sectors and we will first discuss the one most commonly used [43, 77]. Let us denote the fields in the hidden sector by z_i, the fields in the observable sector by y_a and let us write for the superpotential

$$\tilde{g}(z_i, y_a) = h(z_i) + g(y_a), \tag{6.17}$$

and let us use for this first discussion minimal kinetic terms and postpone the discussion of the most general case. We assume that supersymmetry is broken in the hidden sector with vacuum expectation values

$$\langle z_i \rangle = b_i M, \qquad \langle h_i \rangle = \langle \partial h/\partial z_i \rangle = a_i^* mM, \qquad \langle h \rangle = mM^2, \tag{6.18}$$

and that the minimum has $E_{\text{vac}} = 0$. We also assume that the superpotential of the observable sector does not contain the large scale M and that all the vacuum expectation values in the observable sector are small compared to M. The scalar potential of such a model is then given by

$$V = \exp\left(\frac{|z_i|^2 + |y_a|^2}{M^2}\right)\left[\left|h_i + \frac{z_i^*}{M^2}\tilde{g}\right|^2 + \left|g_a + \frac{y_a^*}{M^2}\tilde{g}\right|^2 - \frac{3}{M^2}|\tilde{g}|^2\right] + \tfrac{1}{2}D_\alpha D^\alpha . \tag{6.19}$$

This is now a very complicated expression. Since, however, the scales in $g(y_a)$ are all small compared to M we can simplify our discussion by using a leading order approximation in μ/M where μ is a characteristic scale of $g(y_a)$. If the μ is of order of 100 GeV this approximation is fantastically good. If the model also includes the grand unification some corrections could occur. They, however, affect only the quantitative but not the qualitative result of our approximation as we will see later. This approximation is obtained in the so-called flat limit [43] $M \to \infty$ but $m_{3/2}$ fixed. The gravitino mass $m_{3/2}$ through (6.18) is of order m in our model. The low-energy effective potential is then obtained by replacing z_i, h_i and h by their v.e.v.'s and keeping only those terms which do not vanish when $M \to \infty$ (m fixed). It can be obtained from (6.19) by a straightforward calculation. With

$$m_{3/2} = \exp(\tfrac{1}{2}|b_i|^2)m\,, \tag{6.20}$$

and the rescaled superpotential

$$\hat{g}(y_a) = \exp(\tfrac{1}{2}|b_i|^2)g(y_a)\,, \tag{6.21}$$

we obtain [43]

$$V = |\hat{g}_a|^2 + m_{3/2}^2|y_a|^2 + m_{3/2}[y_a\hat{g}_a + (A-3)\hat{g} + \text{h.c.}]\,, \tag{6.22}$$

where $y_a\hat{g}_a = \Sigma_a\, y_a(\partial\hat{g}/\partial y_a)$ and A is given by [450]

$$A = b_i^*(a_i + b_i)\,, \tag{6.23}$$

with a_i and b_i defined in (6.18). For $V = 0$ at the minimum these quantities are restricted to obey

$$\sum_i |a_i + b_i|^2 = 3\,. \tag{6.24}$$

The first term in (6.22) is the usual scalar potential of a globally supersymmetric model with superpotential $\hat{g}$. The second term is a common mass term for all the scalar particles in the observable sector and is equal to the gravitino mass. In such a model all the scalar partners of quarks and leptons receive a common mass of order $m_{3/2}$. This is a very desired property and it confirms our expectations from the discussion of the supertrace formula that $m_{3/2}$ is the scale relevant for the low energy sector. That they receive all a common $m_{3/2}^2|y_a|^2$ contribution is not true in the general case. It is due to the choice of minimal kinetic terms in the observable sector. Nonetheless this splitting of the scalars from the corresponding fermionic partners (already at the tree graph level) is exactly what we were looking for in all the attempts of supersymmetric model building.

The third term in (6.22) gives us new trilinear bilinear and linear couplings between the scalars. It contains a single numerical parameter A which depends on the choice of the hidden sector [43, 450]. For our example $h(z) = m^2(z + \beta)$ with $\beta = (2 - \sqrt{3})M$ and $\langle z\rangle = (\sqrt{3} - 1)M$ one would obtain $A = 3 - \sqrt{3}$. Observe that this last term in (6.22) breaks all the R-symmetries of the superpotential $\hat{g}$. This is because the superpotential $\hat{g}(z)$ necessarily transforms nontrivially under any R-symmetry since $\int d^2\theta\hat{g}(z)$ has to be invariant. The models we can now consider are thus allowed to have R-symmetries (which e.g. could forbid some unwanted terms in $\hat{g}$) and we have not to be afraid of Goldstone bosons through the necessary spontaneous breakdown of such symmetries, necessary to allow gaugino masses. This unsatisfactory feature of most of the globally supersymmetric models is here absent because these

R-symmetries are explicitly broken through the coupling to supergravity. One such R-symmetry would be the one where all superfields in the standard model have a common charge $R = 1$ and where θ-transforms with $R = 3/2$. In this case $\hat{g}(y_a)$ would only contain terms trilinear in the fields. In particular this superpotential then does not contain explicit mass parameters of lets say 100 GeV arbitrarily put in by hand. Such an R-symmetry has been used as a naturalness condition for models in which the scale of the breakdown of the weak interactions can be understood in terms of the gravitino mass alone and where $SU(2) \times U(1)$ breakdown at the right scale is induced through the breakdown of supergravity. With such an R-symmetry the potential in (6.22) simplifies to [450]

$$V = |\hat{g}_a|^2 + Am_{3/2}(\hat{g} + \hat{g}^*) + m_{3/2}^2|y_a|^2 , \tag{6.25}$$

and the R-symmetry is explicitly broken by the term proportional to A in the process of spontaneous breakdown of supergravity.

The potential (6.22) has the form of the potential in an explicitly (but softly) broken globally supersymmetric model. The soft breaking terms are the $y_a y_a^*$ mass terms of the scalars and the scalar couplings proportional to A. Spontaneously broken supergravity thus manifests itself in the low energy theory as soft explicitly broken global supersymmetry [43], a property very convenient for model building as we have seen already from our discussion of $\mathrm{STr}\,\mathcal{M}^2$ in the beginning of this chapter.

The potential in (6.22) does not represent the most general case and we will discuss more general cases in the next section. Before we do this, let us add some comments to (6.22). The parameters A and $m^2|y_a|^2$ are defined at a very large scale M. If one now considers the supersymmetric $SU(3) \times SU(2) \times U(1)$ model the values of these parameters at a small scale of 100 GeV have to be obtained by a renormalization group calculations [324]. To render our discussion more transparent, we will postpone these considerations to the next chapter, but this fact should be kept in mind. If the considered model is a grand unified model the same statement applies. In addition to that, if one wants to construct the low energy potential in the region of $m_{3/2}$ the heavy grand unification fields have to be integrated out [307] and this changes somewhat the parameters of the low energy potential. If one has, e.g., a low energy superpotential $\hat{g} = \hat{g}^3 + \hat{g}^2 + \hat{g}^1$ where $\hat{g}^i$ denotes the terms that are i-linear in the fields the A terms in (22) would read

$$A\hat{g}^3 + (A-1)\hat{g}^2 + (A-2)\hat{g}^1 . \tag{6.26}$$

In the case with heavy fields, this ratio $A:(A-1):(A-2)$ will change [307]. These effects have to be computed in the explicit models under consideration and will of course depend strongly on the grand unified model used.

Before we discuss more general cases, let us mention that the used coupling of the hidden and the observable sector as given in (6.17) is not unique. An interesting alternative [93] is to add the logarithms of the superpotential instead of the superpotentials h and g themselves. The Kähler potential of the hidden and the observable sector are added

$$G(z_i, z_{ij}^* y_a, y_a^*) = -\frac{z_i z_i^*}{M^2} - \frac{y_a y_a^*}{M^2} - \log\frac{|h(z)|^2}{M^6} - \log\frac{|g(y)|^2}{M^6} , \tag{6.27}$$

i.e., the resulting superpotential is a product of the separate superpotentials. This is as good a coupling of the two sectors as the other one. In a special case one could choose $h(z) = m^2(z + 2 - \sqrt{3})$ and

$$\tilde{g}(z, y) = h(z)(1 + g(y)/M^2 m_{3/2}) . \tag{6.28}$$

The low energy superpotential that arises in this coupling is

$$V = |\hat{g}_a + m_{3/2} y_a^*|^2 + \tfrac{1}{2} D^2 . \tag{6.29}$$

Comparing this to (6.22) we see that both potentials coincide for $A = 3$. The coupling of the two sectors like (6.27) always gives $A = 3$ independent of the special form of the hidden sector. We will later see that $A = 3$ is a very special value.

6.4. *The most general case*

In the last section we have seen that spontaneously broken supergravity manifests itself at low energies as a theory of soft explicitly broken global supersymmetry. The soft breaking parameters present in (6.22) where scalar masses $m_{3/2}^2 |y_a|^2$ and the terms $m_{3/2} A \hat{g}$. As we have seen in (2.83) there are more possibilities for soft breaking terms and we will discuss in this section whether they can occur here or not.

We start with a gaugino mass term. In a grand unified model with heavy particles that transform nontrivial under $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ masses for the gluinos, winos, zino and photino will be induced through radiative corrections. This happens already at the one-loop level since supersymmetry is broken at the tree graph level for these heavy particles. The graph in fig. 6.2 gives a finite contribution to the gaugino masses. In the case of $m_{\mathrm{Q}} \gg m_{3/2}$ this contribution is [252, 45, 112]

$$m_i = (\alpha_i / 2\pi) m_{3/2} \sum_\alpha T(R_\alpha) , \tag{6.30}$$

where $T(R_\alpha)$ is the quadratic Casimir operator of the heavy fields in representation R_α. The contribution of light particles $m_{\mathrm{Q}} \ll m_{3/2}$ is of order $\alpha m_{\mathrm{Q}}^2 / m_{3/2}$ and small. In addition to this one-loop contribution there are logarithmically divergent [252] two-loop contributions from the graphs in fig. 6.3, which can however in general be neglected compared to the former ones. The induced radiative gaugino masses thus depend strongly on the particle content of a grand unified sector. In SU(5) we have $T(5) = 1/2$, $T(24) = 5$, $T(50) = 35/2$ and $T(75) = 25$ and the induced gaugino masses might reach the magnitude of the gravitino masses (compare (6.30)). In general one does expect these gaugino masses to be present [112].

Apart from these radiative contribution there is also the possibility to have nonzero gaugino masses at the three graph level [98, 324], which requires the presence of nonminimal gauge kinetic terms. To see this remember the term $\int \mathrm{d}^2\theta f_{\alpha\beta} W^\alpha W^\beta$ in (3.15). The gauge kinetic terms are nonminimal if

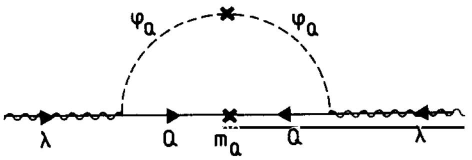

Fig. 6.2. Contribution to gauge fermion (λ) masses at the one-loop level. Q denotes a quark and φ_{Q} its scalar partner.

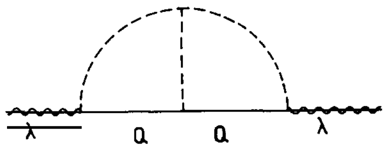

Fig. 6.3. Logarithmically divergent contribution to gauge fermion masses at the two-loop level.

$f_{\alpha\beta} \neq \delta_{ab}$ where f is a function of the chiral superfields in the theory. The general formula of the $N = 1$ supergravity action contains a term (second term in (3.20))

$$\tfrac{1}{2}\exp(-G/2)G^i(G^{-1})_i^k \frac{\partial f^*_{\alpha\beta}}{\partial z^*_k}\lambda^\alpha\lambda^\beta\,, \tag{6.31}$$

a potential gaugino mass term. This mass can of course only be different from zero if supersymmetry is broken, comparing (3.22) we can write (6.31) as $\frac{1}{4}M_S^2 f^{*\prime}\lambda\lambda$, but in addition one needs a nontrivial function $f(z_i)$. Since f is dimensionless f' has dimension $(\text{mass})^{-1}$ and since in the scalar sector the z fields have v.e.v.'s of the order of M such terms will lead to gaugino masses of order $M_S^2/M \sim m_{3/2}$. In addition to the two types of soft breakings discussed in the last section $m_{3/2}^2|y_a|^2$ and $m_{3/2}A\hat{g}$ a common mass of order $m_{3/2}$ for the gauginos thus constitutes an additional soft breaking term of the effective action of the observable sector, which will be present in the general case.

More complications arise if we also allow nonminimal kinetic terms for the chiral superfields [516, 79]. We then have to consider the potential (6.15)

$$V = \exp\left(\frac{K}{M^2}\right)\left[F_i(K^{-1})^i_j F^{*j} - \frac{3}{M^2}|\tilde{g}|^2\right], \tag{6.15}$$

where $\tilde{g}$ is to be understood as $\tilde{g}(z_i, y_a)$. In general we could write

$$\tilde{g} = \sum_{n=0}^{\infty} M^n f_n(z_i, y_a)\,, \tag{6.32}$$

but a necessary condition for the y-fields not to be as heavy as M reduces this form to

$$\tilde{g}(z_i, y_a) = M^2 h_2(z_i) + M h_1(z_i) + f_0(z_i, y_a)\,, \tag{6.33}$$

where only the last term depends also on the observable fields. By the same arguments we can parametrize the most general form of the kinetic terms

$$K = M^2 K_2(z_i, z_i^*) + M K_1(z_i, z_i^*) + K_0(z_i, z_i^*, y_a, y_a^*)\,. \tag{6.34}$$

One, however, would like to constrain the theory in such a way that the observable sector in the flat limit is renormalizable which requires

$$K_0 = \Lambda_{ab}(z_i, z_i^*) y_a y_b^* + \Gamma(z_i, z_i^*, y_a) + \Gamma^*(z_i, z_i^*, y_a^*)\,. \tag{6.35}$$

Inserting (6.33) and (6.34), (6.35) in (6.15) gives in the flat limit

$$V = |g_a|^2 + m_{3/2}(h(y) + h^*(y^*)) + m_{3/2}^2 S_{ab} y_a y_b^*\,. \tag{6.36}$$

with g defined through

$$\exp\left(\frac{K}{2M^2}\right) f_0 = g(y_a) + f(z_i, y_a)\,, \tag{6.37}$$

and $h(y_a)$ is an arbitrary additional function contained in $f(z_i, y_a)$ completely independent of $g(y_a)$. S_{ab} finally is a complicated function of Λ_{ab}, h_2 and K_2 [516]. The appearance of such a nontrivial $S_{ab} \neq \delta_{ab}$ implies a nondegeneracy of the $m_{3/2}^2|y_a|^2$ mass terms and it is due to the nonminimality of the kinetic terms in the observable sector, and its existence confirms the expectations originating from the general $\mathrm{STr}\,\mathcal{M}^2$ formula (3.46) [294].

The terms in the potential (6.36) constitute (together with the gaugino masses) the most general soft breaking that can be induced through a spontaneous breakdown of supergravity, consistent with some necessary conditions (6.33) and (6.34) arising from the requirement that the y_a-fields do not acquire masses of order M. Whether or not these conditions are also sufficient is not yet proven and is in fact very difficult to decide. The problem lies in the fact that before one considers the potential one decides which fields are "heavy" z_i and which are "light" y_a. In general this question can only be decided after the minimization of the potential. In the derivation of (6.36) the necessary requirements (6.33) and (6.34) make sure that there are not obvious $M^2|y_a|^2$ terms. Inspecting, however, the first term in (6.35) we see that z-fields and y-fields mix in the kinetic terms which makes it of course difficult and ambiguous to decide who is z_i and who is y_a, and additional criteria are apparently necessary. Only in very special cases of $\tilde{g}(z_i, y_a)$ the separation of z_i and y_a at this level seems to be consistent after the minimization of the potential. It is not known that these general cases exist at all; no explicit model has been constructed in which the general soft breaking in (6.36) are realized after a minimization of the complete potential, nor is it proven that these more general function $h(y)$ and $S_{ab}(z_i)$ cannot occur in a nontrivial form. To resolve this question is very important and it seems to me that further study is absolutely necessary.

Of course the choice of the hidden sector at this stage is arbitrary; one would, however, in the absence of an explicit example that induces the general terms in (6.36) be tempted to assume that not (6.36) but (6.22) constitutes the most general form of the scalar potential induced by a spontaneous breakdown of supergravity. This situation would actually be preferable for several reasons. First of all (6.22) is less general and if it is the most general this would imply that these theories have a predictive power independent of the special choice of the hidden sector, which of course would be lost if (6.36) has to be considered. Secondly the universality of the $m_{3/2}^2|y_a|^2$ terms is a very desired property in (6.22), which everybody will agree with if he has followed our discussion concerning flavor changing neutral currents in section 4.6. If all the general soft breaking terms in (6.36) could be induced we actually would be forced to adjust a number of these terms in order to avoid phenomenological problems and the general approach alone would, e.g., not explain the absence of flavor changing neutral currents. We do not know for certain at the moment how to answer this question. We will, however, in the following assume that $S_{ab} = \delta_{ab}$ and work with scalar potentials of the form of (6.22) and hope that this will be close to the most general form that can be obtained. Actually a slight generalization of (6.22) has been found in a special case [226]. There the $m^2|y_a|^2$ terms are still universal but are not necessarily equal to $m_{3/2}^2$. For $m^2 > 0$ such a change, however, can in general be absorbed in a redefinition of A.

We should stress again that the parameters contained in (6.22) as well as the value of the gaugino masses (the other possible soft breaking term) are defined at a very high mass scale and the values at low energies have to be obtained through renormalization group improved perturbation theory. They will be different at low energies just in the same way the masses of the bottom quark and τ-lepton masses in grand unified models differ in value at low energies even if they were identical at M_x.

In the following chapter we will discuss the implications of (6.22) in the framework of various models and investigate if the three parameters $m_{3/2}$, A and a common gaugino mass are sufficient to render these models phenomenologically acceptable.

7. Supergravity: models

7.1. Early attempts

The first attempt to discuss a low energy supersymmetric standard model in the framework of supergravity was undertaken by Ovrut and Wess [470]. Their model did not contain a hidden sector to break supergravity spontaneously but just consisted of the low energy observable fields with a mass scale m of the order of 100 GeV. Such a model then in general has a nonvanishing cosmological constant Λ. Ovrut and Wess now cancelled this cosmological constant by adding a term to the action and then took the flat limit. The additional term breaks supergravity explicitly, it corresponds to the introduction of an x-independent spurion superfield of the type discussed in section 2.5. As a consequence the multiplets in the low energy theory are split. The low energy theory in the flat limit looks the same as an explicitly (*but softly*) broken globally supersymmetric model. To have the splittings inside the supermultiplets to be of the order m^2 one requires the introduction of a term $\Lambda' = m^3 M$; and this is exactly the order of magnitude of the cosmological constant in these models. One can see this for example in our toy model with constant superpotential discussed in the last chapter, where one obtained a vacuum energy of order M_S^6/M^2. Introducing $\Lambda' \sim m^3 M$ then corresponds to $M_S^2 \sim mM$, a quantity which represents the supersymmetry breaking scale in spontaneously broken supergravity. As in this case here the relevant scale is $M_S \sim 10^{10}$ GeV.

The soft breakings in the low energy theory induced through the nonsupersymmetric cancellation of the cosmological constant are of the type

$$\varphi,\ \varphi^2,\ \varphi^3,\ \varphi^2\varphi^*,\ \varphi\varphi^{*2} + \text{h.c.}\,, \tag{7.1}$$

all with coefficients of order m. The terms linear in $\varphi(\varphi^*)$ are irrelevant and can be absorbed in a redefinition of the scalar fields. Observe that the list in (7.1) does not include "diagonal" mass terms $m^2\varphi\varphi^*$, these terms are not generated through the introduction of Λ'. As a result, these models always lead to $\mathrm{STr}\,\mathcal{M}^2 = 0$ at the tree graph level, a property which always has made it quite difficult to have it as a basis for a realistic model. The scalars of the supermultiplets of mass m exhibit the splittings $m^2 + \Delta m^2$, and more structure has to be added to the models to make them acceptable. Apart from that the fact that supersymmetry is explicitly broken is rather unsatisfactory and one would like to replace this by spontaneous breakdown for the usual reasons. It is, however, interesting to see how the introduction of one breakdown parameter leads to so much structure in the low energy theory. This type of model has in addition the nice feature that the usual vacuum degeneracy of globally supersymmetric grand unified models is lifted exactly in the same way as discussed in the last chapter.

The first attempt to investigate the implications of spontaneously broken supergravity to models of particle physics was made in ref. [444]. Supergravity was broken in a hidden sector here consisting of a supersymmetric pure Yang–Mills theory. For the description of such a theory an effective Lagrangian approach, constructed in globally supersymmetric context [543], was coupled to supergravity. The basic property of this Lagrangian was the occurrence of gauge fermion condensation $\langle\lambda\lambda\rangle$ due to the strong interactions in the hidden sector. Whereas in the globally supersymmetric case this condensation does not lead to a breakdown of supersymmetry in the local case an explicit example shows the possibility of a supergravity breakdown. As a consequence of these property the breakdown scale M_S^2 can be much smaller than the scale $\langle\lambda\lambda\rangle = \mu^3$ where condensation occurs since supersymmetry is restored in the global limit $M \to \infty$. In fact in the specific model one obtains

$$M_S^2 \simeq \mu^3/M\,, \tag{7.2}$$

and consequently

$$m_{3/2} \simeq \mu^3/M^2\,. \tag{7.3}$$

The scale μ is put in through the gauge coupling in the hidden sector and a gravitino mass as low as 10^3 GeV can be obtained from a μ as large as 10^{14} GeV by this mechanism [444–446].

The next step to discuss is the communication of the hidden sector and the observable sector. At this stage the results of Cremmer, Ferrara, Girardello and Von Proyen [97, 98] were not yet available and with it the most general form of the breaking terms in the low energy theory.

One obvious way in which the breakdown of supergravity affects the observable sector is a splitting of the gauge fermions from the corresponding gauge bosons in this sector. It was first speculated that such contributions to the low energy gaugino masses comes from a quadratically divergent graph [42] like the one shown in fig. 7.1. Later it was however shown that this quadratically divergent contribution is exactly cancelled [45] by the one from fig. 7.2. But apart from these contributions to the low energy gaugino masses there are the finite ones we have discussed in section 6.4. With these we expect the gaugino masses to become of order $\alpha m_{3/2}$ or $m_{3/2}$ depending on the heavy particles in the theory [252, 45].

In the flat limit $M \to \infty$, $m_{3/2}$ fixed, the observable sector was globally supersymmetric except for these soft breaking terms represented by the gaugino masses. We are thus exactly in the situation which we have discussed in section 5.3. From the gaugino masses also masses for the scalar partners of quarks and leptons and Higgses are radiatively induced,

$$m_Q^2 \simeq \alpha_3 m_{g3}^2\,, \qquad m_L^2,\, m_H^2 \simeq \alpha_2 m_{g2}^2\,, \tag{7.4}$$

where α_3 denotes the SU(3) coupling constant, etc. A breakdown of SU(2) × U(1) can be induced if there exist large Yukawa couplings in the theory. In a model in which the top-quark mass is larger than 25 GeV a negative (mass)^2 will be induced for the Higgs scalar that couples to this heavy quark, while all other scalars of the theory remain at positive (mass)^2. In this way the model has all the desired properties [444]. The input scales M and $g_s(M)$ lead to a gravitino mass of order 1 TeV and in the presence of a moderately large Yukawa coupling the breakdown of SU(2) × U(1) is induced through the breakdown of supergravity. If one would restore supersymmetry by force also the SU(2) × U(1) breakdown would disappear. There are also no problems with flavor changing neutral currents since the appropriate scalar particles are degenerate in mass as indicated in (7.4). We will see later that such an induction of the SU(2) × U(1) breakdown by supergravity through gaugino masses can be realized in many specific models and is also the one which is theoretically most convincing. Of course the value

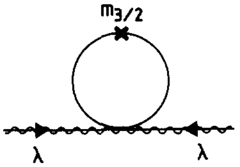

Fig. 7.1. Contribution to m_λ through a massive gravitino (solid line).

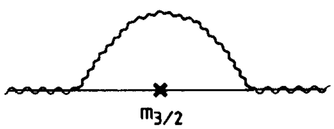

Fig. 7.2. Contribution to m_λ that cancels the quadratically divergent part of fig. 7.1 as observed in ref. [45].

$m_t \sim 25\,\text{GeV}$ indicates only the scale and the actual value is model dependent. It will later be determined through the renormalization group equations of the relevant parameters.

In the original model only the effects due to the nonvanishing gaugino masses had been considered. After the existence of ref. [98] a more general evaluation of the soft breaking terms including those in the scalar potential in the flat limit was possible. After the compensation of a possible cosmological constant one obtains [226]

$$V = |\hat{g}_a|^2 + Bm_{3/2}^2|y_a|^2 + m_{3/2}[y_a\hat{g}_a + (A-3)\hat{g} + \text{h.c.}]\,, \tag{7.5}$$

where A and B depend on the special form of the hidden sector. In various explicit examples it can be shown that B can be positive negative or even zero. A model with negative B does not make sense since in general this will also imply a breakdown of $SU(3)_c \times U(1)_{em}$, and for positive B its effects can be absorbed in a redefinition of A. The relevance of this additional parameter B in (7.5) compared to the case described in (6.22) can only be seen in relation to the gaugino masses, the other soft breakdown parameter. With the possibility $B < 1$ gaugino masses of order $m_{3/2}$ could be larger than the coefficient of $|y_a|^2$. In such a situation it is usually easier to obtain the induced breakdown of $SU(2) \times U(1)$ in the sense that this breakdown can happen without the need to choose large Yukawa couplings corresponding to large top-quark masses. We will later study these questions in detail.

7.2. Models with SU(2) × U(1) breaking at the tree level

Before explicit models with a radiative breakdown of $SU(2) \times U(1)$ had been investigated in detail several models with a breakdown of the weak interactions at the three graph level had been constructed. Such models were first proposed independently by Arnowitt, Chamseddine and Nath [77] and Barbieri, Ferrara and Savoy [43]. Both models relied on a hidden sector with $h(z) = m^2(z+\beta)$ where m was chosen to be of order $10^{10}\,\text{GeV}$ to arrive at a gravitino mass in the $100\,\text{GeV}$ range. Consequently the low energy effective potential [43] was given by (6.22)

$$V = |\hat{g}_a|^2 + m_{3/2}^2|y_a|^2 + m_{3/2}[y_a\hat{g}_a + (A-3)\hat{g} + \text{h.c.}]\,, \tag{6.22}$$

with $A = 3 - \sqrt{3}$ and it was implicitly assumed that the induced $SU(3) \times SU(2) \times U(1)$ gaugino masses are small compared to $m_{3/2}$. Reference [77] discussed a grand unified model and ref. [43] a supersymmetrized version of a low energy standard model. We will start with a discussion of the low energy sector and later add the grand unified sector.

Apart from the graviton, the gravitino and the scalars of the hidden sector with masses

$$m_{3/2} = (m^2/M)\exp(\tfrac{1}{2}(\sqrt{3}-1)^2)\,, \qquad m_1^2 = 2\sqrt{3}m_{3/2}^2\,, \qquad m_2^2 = 2(2-\sqrt{3})m_{3/2}^2\,, \tag{7.6}$$

this model should then contain the usual Higgs and quark and lepton superfields. With these fields alone the most general superpotential according to our prejudices would be (compare (4.22))

$$g = \mu H\bar{H} + \lambda_{ij}U_i\bar{H}U_j + \lambda'_{ij}U_iH\bar{D}_j + \lambda''_{ij}L_iH\bar{E}_j\,. \tag{7.7}$$

The quantity $\hat{g}$ that enters formula (6.22) is obtained from g via (6.21) and we will in general drop the exponential factor which is irrelevant for our discussion. The particle content in (7.7) is now not sufficient to give a breakdown of $SU(2)\times U(1)$ at the tree graph level. The $\mu H\bar{H}$ term will lead to $\mu^2(hh^* + \bar{h}\bar{h}^*)$ masses in the scalar potential and also the additional terms in (6.22) cannot lead to a negative (mass)^2 of a Higgs scalar. It is thus necessary to introduce additional fields. The simplest choice is to add an $SU(3)\times SU(2)\times U(1)$ singlet superfield Y and a possible choice of the superpotential is then [43]

$$g = \lambda Y(H\bar{H} - \mu^2) + \text{Yukawa terms}\,. \tag{7.8}$$

The scalar potential of the Higgs sector is given by

$$\begin{aligned} V = {} & m_{3/2}^2(|y|^2 + |h|^2 + |\bar{h}|^2) + \lambda^2|h\bar{h} - \mu^2|^2 + \lambda^2|yh|^2 + \lambda^2|y\bar{h}|^2 + m_{3/2}(3-\sqrt{3})\lambda(yh\bar{h} + y^*h^*\bar{h}^*) \\ & - m_{3/2}(1-\sqrt{3})\lambda\mu^2(y + y^*) + \tfrac{1}{2}D^2\,, \end{aligned} \tag{7.9}$$

where y, h, $\bar{h}$ denote the scalar components of the superfields Y, H and $\bar{H}$. We have dropped all factors $\exp(\sqrt{3}-1)^2$ which implies that we also use $m_{3/2} = m^2/M$ instead of (7.6). The correct masses in the theory can then be obtained at the end by multiplying all masses with this factor. In a range of parameters $\mu > m_{3/2}/\lambda$ the minimum of (7.9) is obtained at [43]

$$\langle y\rangle = \frac{m_{3/2}q}{\lambda}\,; \qquad \langle h\rangle = \begin{pmatrix}\mu p/\lambda \\ 0\end{pmatrix}; \qquad \langle\bar{h}\rangle = \begin{pmatrix}0 \\ \mu p/\lambda\end{pmatrix}, \tag{7.10}$$

where p and q are determined through the equations

$$(2p^2+1)q + (\sqrt{3}-1)(\sqrt{3}p^2 + \mu^2\lambda^2/m_{3/2}^2) = 0\,, \qquad q^2 + (3-\sqrt{3})q + 1 + p^2 - \mu^2\lambda^2/m_{3/2}^2 = 0\,. \tag{7.11}$$

The v.e.v.'s of h and $\bar{h}$ align and are equal which minimizes the $\frac{1}{2}D^2$ contribution in (7.9). At this minimum $SU(2)\times U(1)$ is broken and we have gauge boson masses

$$M_W^2 = 2g_2^2m_{3/2}^2(p/\lambda)^2\,, \qquad M_z^2 = 2(g_2^2 + g_1^2)m_{3/2}^2(p/\lambda)^2\,, \qquad M_\gamma^2 = 0\,. \tag{7.12}$$

The $\mathcal{M}^2$ matrix of the scalars is in general given by

$$\mathcal{M}_{ab}^2 = \begin{pmatrix} g_{ac}^*g^{cb} + D_aD^b + m_{3/2}^2\delta_a^b & g_{abc}g^{c*} + D_aD_b + m_{3/2}h_{ab}^* \\ g^{abc}g_c^* + D^aD^b + m_{3/2}h^{ab} & g^{ac}g_{cb}^* + D^aD_b + m_{3/2}^2\delta_b^a \end{pmatrix} \tag{7.13}$$

where

$$h^{ab} = \frac{\partial^2}{\partial y_a\partial y_b}\left[y_c\frac{\partial g}{\partial y_c} + (A-3)g\right]. \tag{7.14}$$

For the scalar masses in the Y, H, $\bar{H}$ sector this implies in our case for the two charged ones

$$m_\pm^2 = 2(1+q^2)m_{3/2}^2 + M_W^2\,, \tag{7.15}$$

and the five neutral ones

$$
\begin{aligned}
&m_0^2 = 2(1+q^2)m_{3/2}^2 + M_z^2\,,\\
&m_{1,2}^2 = (2p^2 + \tfrac{1}{2} \pm [\tfrac{1}{4} + 2p^2(3-\sqrt{3}+2q)^2]^{1/2})m_{3/2}^2\,,\\
&m_{3,4}^2 = (2p^2 + \tfrac{3}{2} + q^2 \pm [(\tfrac{1}{2}+q^2)^2 + 2(3-\sqrt{3})^2p^2]^{1/2})m_{3/2}^2\,.
\end{aligned}
\tag{7.16}
$$

The scalar partners of quarks and leptons have separate mass matrices of the form

$$
\begin{pmatrix} m_{3/2}^2 + (\sigma m_{3/2}p/\lambda)^2 & \sigma A m_{3/2}^2 p/\lambda \\ \sigma A m_{3/2}^2 p/\lambda & m_{3/2}^2 + (\sigma m_{3/2}p/\lambda)^2 \end{pmatrix}
\tag{7.17}
$$

where σ denotes the respective Yukawa coupling. Since the masses of the quarks and leptons are given by $m_Q = \sigma(m_{3/2}p/\lambda)$ the mass eigenstates of (7.17) are given by

$$
m_{a,b}^2 = m_{3/2}^2 + m_Q^2 \pm A m_{3/2} m_Q\,,
\tag{7.18}
$$

where a, b denote the two (real) scalar partners of the fermion of mass m_Q and $A = 3-\sqrt{3}$ in the case under consideration. The fermion masses of the Y, H, $\bar{H}$ sector can be obtained in the same way as in a globally supersymmetric model. They are not affected by the soft breakings since we here have assumed that the Majorana gaugino masses vanish at the tree graph level.

The above model is an example of a phenomenologically acceptable model in the 100 GeV range provided that $m_{3/2}$ is chosen to be larger than something like 20 GeV. In the presence of a very heavy top quark one has of course also to require $m_{3/2}^2 + m_t^2 - (3-\sqrt{3})m_{3/2}m_t$ to be larger than 20 GeV. To be at the correct minimum we had also to require $\mu > m_{3/2}/\lambda$. From this we see that it is the introduction of the parameter μ in the low energy theory that causes the breakdown of the weak interactions. Would we choose $\mu = 0$ SU(2) × U(1) would be left unbroken. The SU(2) × U(1) breakdown is thus not related to the breakdown of supersymmetry and our goal to explain the one through the other is not achieved. Theoretically this seems to be unsatisfactory and we will later come back to this point.

Let us first discuss the inclusion of grand unification in this context first proposed in refs. [77, 324] in which the model contained $\Sigma(24)$, $H(5)$, $\bar{H}(\bar{5})$ and three generations of quarks and leptons $Y_i(\bar{5})$ and $X_i(10)$ where the numbers in the brackets denote the dimension of the SU(5) representations. The superpotential was chosen to be [77] (compare (4.27))

$$
g = \lambda_1(\tfrac{1}{3}\operatorname{Tr}\Sigma^3 + \tfrac{1}{2}\operatorname{Tr}\Sigma^2) + \lambda_2\bar{H}(\Sigma + 3M')H + \lambda_3 U H\bar{H} + g_{ij}X_iX_jH + g'_{ij}X_iY_j\bar{H}\,,
\tag{7.19}
$$

where U is an SU(5) singlet chiral superfield. In the globally supersymmetric case this leads at the grand scale to a potential with several degenerate supersymmetric minima in which X, Y, H, $\bar{H}$, U have vanishing v.e.v.'s and

$$
\begin{aligned}
&\text{(i)}\quad \Sigma = 0\,,\\
&\text{(ii)}\quad \Sigma_a^b = \tfrac{1}{2}\tilde{M}[\delta_a^b - 5\delta_5^b\delta_a^5]\,,\\
&\text{(iii)}\quad \Sigma_a^b = \tilde{M}[2\delta_a^b - 5(\delta_5^b\delta_a^5 + \delta_4^b\delta_a^4)]\,.
\end{aligned}
\tag{7.20}
$$

Solution (iii) corresponds to the desired breakdown to SU(3)×SU(2)×U(1). To arrive at massless Higgs doublets for the low energy theory in global supersymmetry one has to choose $\tilde{M} = M'$, which gives a zero coefficient in the second term of (7.19) for the 4, 5 components of H and $\bar{H}$.

This system is now coupled to supergravity including a hidden sector $h(z) = m^2(z+\beta)$ which breaks supergravity spontaneously. The supersymmetric solutions (7.20) obtained through the equations $\partial g/\partial\Sigma = 0$ are affected by the coupling to supergravity. Forgetting the light fields for a moment and just considering Σ the stationary points of the potential coupled to supergravity are now obtained not by $\partial g/\partial\Sigma = 0$ but by $F_\Sigma = \partial g/\partial\Sigma + (\Sigma^*/M^2)[h(z) + g(\Sigma)] = 0$ since supergravity is only broken in the hidden sector, as we have explicitly seen in the last chapter in our discussion following eq. (6.16). As a result of this the coupling to supergravity with hidden sector $h(z)$ induces a change of the v.e.v.'s in (7.20). We have now to solve

$$F_\Sigma = \partial g/\partial\Sigma + (\Sigma^*/M^2)(m^2M + g(\Sigma)) = 0\,, \tag{7.21}$$

where we have used $\beta = (2-\sqrt{3})M$ and have inserted the v.e.v. $\langle z\rangle = (\sqrt{3}-1)M$ of the scalar in the hidden sector. Notice that the second term in (7.21) corresponds to $\Sigma^* m_{3/2}$ and as a consequence of this the v.e.v.'s given in (7.20) will receive a shift of order $m_{3/2}$ through the coupling to supergravity [447]. This is the leading contribution if $\tilde{M} \ll M$. In the case of $\tilde{M} \approx M$ this shift will even be larger.

In the model given by (7.19) there exist now three solutions with $F_\Sigma = 0$. They will be nondegenerate since the contribution to the potential is given by $(F_\Sigma)^2 - 3|g|^2/M^2$ and $g(\Sigma)$ has different values for these three solutions. In general this term induces a negative cosmological constant which, however, can be cancelled by a slight redefinition of β. One would like to do this in a way that the SU(3)×SU(2)×U(1) minimum is at $E_{\text{vac}} = 0$. In the model under consideration this then implies that the other minima have $E_{\text{vac}} < 0$ and the former is not the absolute minimum. One now either argues that the local minimum with $E_{\text{vac}} = 0$ is stable [555], or one complicates $g(\Sigma)$ (e.g. by introducing additional 24-representations) and arrives at a situation in which the SU(3)×SU(2)×U(1) minimum with $E_{\text{vac}} = 0$ is the absolute minimum. An example of such a case is given in ref. [438] through the introduction of an additional adjoint representation and a singlet.

Suppose now that we are in the SU(3)×SU(2)×U(1) minimum obtained through superpotential (7.19). With the choice $\tilde{M} = M'$ to assure vanishing masses of the Higgs doublets in the globally supersymmetric case, we now have the following effective superpotential relevant for a discussion in the 100 GeV range $g_{\text{eff}} = \lambda_2\mu\bar{H}H + \lambda_3 UH\bar{H}$ + Yukawa terms (7.22) where $H, \bar{H}$ now denote the SU(2) doublets and no heavy fields enter g_{eff}. The parameter μ is of order $m_{3/2}$ and appears as a result of a fine tuning in the global limit. It is a free parameter in the theory since a slightly different fine tuning taking into account the shift of the Σ v.e.v. due to the coupling to supergravity (compare (7.21)) would allow us to choose $\mu = 0$ [447].

It can now be shown as in the example discussed earlier that the scalar potential derived from (7.22) including the $m_{3/2}^2|y|^2$ and trilinear scalar couplings leads to a breakdown of SU(2)×U(1) for a range of parameters of the theory. The existence of a nonvanishing μ in (7.22) is crucial for this behaviour. The low energy spectrum is similar to the one in the example given earlier and there are no phenomenological problems. This also includes the gauge fermion sector (e.g. gluino masses) provided that there exist some heavy particles in the theory that have nontrivial SU(3)×SU(2)×U(1) transformation properties, as we have discussed in section 6.4.

A theoretically somewhat unsatisfactory property of the two models discussed in this section is the ad hoc introduction of a mass parameter μ of order of 100 GeV in the theory that is independent of the gravitino mass. In the first example this parameter was introduced directly, whereas in the second one a particular choice of fine tuning led to a nonvanishing μ, and as a result supersymmetry and $SU(2)\times U(1)$ breakdown are not really related, especially since the value of this parameter μ was crucial for $SU(2)\times U(1)$ breakdown.

Of course, the grand unified model of this section was a special example and one might ask the question whether in more general cases the situation is different. In all cases where the coefficient of $H\bar{H}$ results from a fine tuning the value of μ is arbitrary and unrelated to $m_{3/2}$. In models where $\mu=0$ results from group theory, like the example with 75, 50 and $\overline{50}$ representations [416, 291] discussed in section 4.5, however, the situation might be different, since such models need no fine tuning to make the Higgs doublets light. In such models we have $\mu=0$ since the Higgs doublets do not couple to fields with large vacuum expectation values. As a result $\mu=0$ will remain true at the three graph level after the coupling to supergravity which slightly shifts these v.e.v.'s, and a nonvanishing μ to induce $SU(2)\times U(1)$ breakdown has to be put in by hand.

7.3. *Can we avoid the introduction of a small mass parameter?*

The motivation to consider supersymmetric models was purely theoretical and related to some naturalness problems with the mass scale of the breakdown of the weak interactions. The explicit introduction of a parameter μ (in $\mu H\bar{H}$) with the sole reason to introduce $SU(2)\times U(1)$ breakdown is thus a procedure which leads to similar problems that we actually would require to be solved.

One is thus tempted to consider models where all the low energy mass parameters really arise from the breakdown of supersymmetry. In such models $SU(2)\times U(1)$ would remain unbroken in the limit $m_{3/2}=0$. Such models in particular are not allowed to contain explicit small renormalizable mass parameters in the superpotential, as stressed in ref. [450]. The only way to achieve this is through the choice of a low energy superpotential only trilinear in the fields,

$$3g(y_a)=\sum_a y_a(\partial g/\partial y_a)\,. \tag{7.22}$$

In a first step we require this only for parameters of order 100 GeV. If one discusses grand unified models one would allow explicit mass parameters of order M_x and postpone attempts to understand M_x in terms of M_P. For our discussion of $SU(2)\times U(1)$ breakdown we disregard the inclusion of grand unification for the moment.

A superpotential that obeys (7.22) automatically possesses an R-symmetry, according to which all superfields transform with the same charge. Such a superpotential has the nice feature that it is strictly natural in the sense that we can write the most general superpotential consistent with its symmetries and still are not forced to choose mass parameters to be zero that could in principle be nonvanishing. The R-symmetry forbids the existence of these parameters. A superpotential with property (7.22) leads to a scalar potential of the form [450] (compare (6.22))

$$V=|g_a|^2+Am_{3/2}(g+g^*)+m_{3/2}^2|y_a|^2\,. \tag{7.23}$$

What is remarkable about this form, is the fact that it allows a model independent statement concerning the breakdown of $SU(2)\times U(1)$. Notice first that $V=0$ at the point where all the light fields have vanishing v.e.v. To have a breakdown of $SU(2)\times U(1)$ at the tree graph level this must appear at minima with $V<0$. This actually will induce a cosmological constant but in the full model this can be cancelled by a different fine tuning of the parameters in the hidden sector, e.g. for β slightly different from $(2-\sqrt{3})M$. Since the cosmological constant has to be fine tuned in all cases this poses no additional problems.

The first and the third term in (7.23) are positive definite. Only the second term can give a negative contribution and the requirement $V\leq 0$ at an $SU(2)\times U(1)$ breaking minimum will give a lower bound on A. To see this we take (7.23), use (7.22) and write for $A=3$

$$V=|g_a+m_{3/2}y_a^*|^2\,, \tag{7.24}$$

which is still positive definite. For $|A|<3$ $SU(2)\times U(1)$ breakdown cannot occur at the absolute minimum since the absolute minimum at $V=0$ corresponds to vanishing v.e.v.'s for all the scalar fields. For a potential of form (7.23) breakdown of $SU(2)\times U(1)$ can only occur for [450]

$$|A|\gtrsim 3\,. \tag{7.25}$$

We now see explicitly that in the models discussed in the last section the introduction of the parameter μ was crucial for the breakdown of the weak interactions since these models were based on a hidden sector that gave $A=3-\sqrt{3}<3$. With $\mu=0$ $SU(2)\times U(1)$ would not break.

The question remains whether even with $|A|>3$ $SU(2)\times U(1)$ can break. We will study this now in a simple example. We have already seen in the last section that the Higgs sector has to be enlarged in order to have $SU(2)\times U(1)$ breaking. The simplest enlargement is the inclusion of a singlet superfield Y. Without it the only possible term in the Higgs sector of the superpotential would be $\mu H\bar{H}$ and this is forbidden by our naturalness condition (7.22), and the complete superpotential would just contain the Yukawa coupling terms. Such a superpotential has too many symmetries, there would, e.g., exist a Peccei Quinn type symmetry which after the breakdown of the weak interaction would lead to a phenomenological unacceptable axion, another reason to enlarge the Higgs sector. With the inclusion of Y the only choice for a superpotential that fulfills (7.22) is

$$g=\lambda YH\bar{H}+\tfrac{1}{3}\sigma Y^3\,. \tag{7.26}$$

With $A>3$ such a superpotential will lead to a scalar potential where $SU(2)\times U(1)$ is broken at the absolute minimum. Such a minimum with nonvanishing v.e.v.'s for y and h and $\bar{h}$ exist provided that the relation [450]

$$1\leq\sigma/\lambda\leq\tfrac{1}{16}[A+(A^2-8)^{1/2}]^2 \tag{7.27}$$

is fulfilled. Thus for $A>3$ a breakdown of the weak interactions can occur at the tree graph level [450, 265].

At the moment where these results were first derived there existed a problem since the only known acceptable example of a hidden sector leads to $A=3-\sqrt{3}$ and it was not clear whether $A\geq 3$ could be possible [519]. In the mean time it has turned out that this problem is not severe, and we have already seen in section 6.3 that examples with $A\geq 3$ have been constructed [96, 378].

The real problem of this approach turns up when we include the Yukawa couplings in the superpotential as has been first pointed out in ref. [252]. To see this consider just the addition of the electron Yukawa coupling to the superpotential in (7.26)

$$g = \lambda YH\bar{H} + \tfrac{1}{3}\sigma Y^3 + \rho LH\bar{E}\,, \tag{7.28}$$

where ρ is a coupling constant of order 10^{-6}, responsible for the electron mass after the breakdown of $SU(2)\times U(1)$. In the case of ρ equal to zero the vacuum expectation values at the $SU(2)\times U(1)$ breaking minimum for $A>3$ are in general of order $(Am_{3/2}/\lambda')$ and the value of the potential at this point was (with $\lambda' = \lambda(\lambda/\sigma)^{1/2}$)

$$V_0 \simeq -(m_{3/2}^4/\lambda'^2)A^4\,. \tag{7.29}$$

With ρ different from zero, additional minima can occur. They correspond to

$$\langle h^0\rangle = \langle\varphi_e\rangle = \langle\varphi_{\bar{e}}\rangle = m_{3/2}u/\rho\,, \tag{7.30}$$

with

$$u = \tfrac{1}{4}[|A| + (A^2-8)^{1/2}] \tag{7.31}$$

(compare (7.27)). The value of the potential at this minimum is

$$V_0 = -(m_{3/2}^4/\rho^2)u^2(u^2-1)\,. \tag{7.32}$$

Since ρ is so small, the v.e.v.'s in this case are very large compared to $m_{3/2}$ and the minimum in (7.32) is very deep. The minimum in (7.32) is of course unacceptable since the scalar partners of the electron have a vacuum expectation value and since they carry electric charge $U(1)_{em}$ would be broken. Notice that minimum (7.32) remains unchanged even if one adds the $\frac{1}{2}D^2$ contributions to the potential, since (7.30) leads to $\frac{1}{2}D^2 = 0$. We must now look for a situation where the minimum in (7.29) is deeper than the one in (7.32). This can be either obtained by a fine tuning $u \approx 1$ (which corresponds to $A \approx 3$) or by choosing $\lambda' < \rho$. The latter choice, however, leads to more problems [252]. With a very tiny λ' the breakdown scale of the weak interactions of order of 100 GeV is very large compared to $m_{3/2}$ since $\langle h\rangle \sim m_{3/2}/\lambda'$ and with $\lambda' \sim 10^{-6} < \rho$, $m_{3/2}$ would have to be of the order of the electron mass and this is of course not sufficient to describe the required 20 GeV splittings between quarks (leptons) and their scalar partners. In a potential of the form (7.23) the absence of absolute minima that break electric charge thus requires

$$|A| \lesssim 3\,, \tag{7.33}$$

which together with (7.25) poses severe problems for models of this type. The case $A=3$ is of course possible, where all these possible minima are degenerate at $V_0 = 0$, and where we are free to choose our favorite minimum. There is, however, no reason to assume A to be stable in perturbation theory and the adjustment $A=3$ is spoiled by radiative corrections as we will later see explicitly [112]. This inclusion of radiative corrections gives, however, also the key to solve the problems of (7.25) and (7.33). The quantity A in (7.23) will after the inclusion of radiative corrections no longer be universal for all the terms in the superpotential. One might construct cases in which the effective A_e multiplying the third term if (7.28) in the potential is smaller than 3 whereas other A constants are larger than 3 and the dangerous $U(1)_{em}$ breaking minima are avoided. Before we, however, discuss these effects of radiative corrections let us explain another potential problem in models where $SU(2)\times U(1)$ is broken at the tree graph level.

7.4. Light singlets

This problem has to do with the necessity of enlarging the Higgs sector in models where the breakdown of the weak interactions is achieved at the tree graph level as we have seen in the last two sections. The simplest enlargement is given by the introduction of a singlet Y with, e.g., superpotential

$$g = \tfrac{1}{3}\sigma Y^3 + \lambda Y H\bar{H} . \tag{7.34}$$

One problem with singlets is the absence of a protection of a possible mass term through the gauge interactions. This is not a problem in (7.34) since there exists an R-symmetry that forbids, e.g., a mY^2 term. This symmetry is broken in the course of the breakdown of supergravity if A is different from zero. Since supersymmetry is broken at a large scale there is the potential danger of induced large mass terms if this singlet couples to heavy particles. In the framework of a grand unified model the simplest extension of (7.34) is to assume that Y is also an SU(5) singlet and as such it would couple to the heavy Higgs triplets. As in section 5.6 for globally supersymmetric models, the stability of the small mass scale of order $m_{3/2}$ has to be checked here, and as has been pointed out in refs. [480, 451, 389, 14] the existence of a light singlet poses potential problems for this stability.

Let us discuss these questions in a toy model containing a heavy chiral superfield B and a light L with superpotential [451]

$$g = \tilde{M}B^2 + \mu L^2 + \lambda_1 B^3 + \lambda_2 B^2 L + \lambda_3 L^3 + \lambda_4 B L^2 , \tag{7.35}$$

where μ is comparable to $m_{3/2}$ and $\tilde{M}$ is large, let us say $\tilde{M}\sim M_x$ the grand unification scale. Coupled to spontaneously broken supergravity this gives rise to a scalar potential

$$\begin{aligned} V = |g_b|^2 + |g_l|^2 + m_{3/2}^2(|b|^2 + |l|^2) + A m_{3/2}(\lambda_1 b^3 + \lambda_2 b^2 l + \lambda_3 l^3 + \lambda_4 b l^2 + \text{h.c.}) \\ + (A-1)m_{3/2}(\tilde{M}b^2 + \mu l^2 + \text{h.c.}) . \end{aligned} \tag{7.36}$$

Notice that all mass splittings are small (of order $m_{3/2}$) except for the one in the last term which is of order $(A-1)m_{3/2}\tilde{M}$ for the heavy B scalar. The heavy scalars are thus split by an amount of order of the supersymmetry breakdown scale and one has to worry that this might induce large splittings in the low energy sector through radiative corrections. The light fields are coupled to the heavy ones through the couplings λ_2 and λ_4. Let us discuss the two terms separately.

The effects of λ_4 can be seen in the tadpole of fig. 7.3, which gives a contribution $\tilde{M}m_{3/2}(l^2 + l^{*2})$ in leading order. This is however cancelled by the graph in fig. 7.4. The two graphs are the components of the supergraph shown in fig. 7.5, where the supersymmetry breaking mass splittings in the heavy sector are represented by a spurion superfield $S = \tilde{M}m_{3/2}(A-1)\theta^2$. The contribution to the $(l^2 + l^{*2})$ terms of this supergraph is a $\int d^2\theta LLS$ contribution (F-term) and as such is not generated in perturbation theory. In (7.36) there are, however, additional breaking terms corresponding to $Am_{3/2}\lambda_1$ and $Am_{3/2}\lambda_4$ trilinear

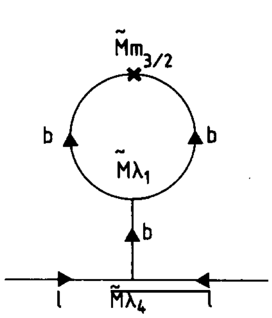

Fig. 7.3. Tadpole contribution to the mass of the light scalar.

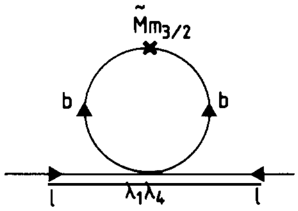

Fig. 7.4. Contribution that cancels the one in fig. 7.3.

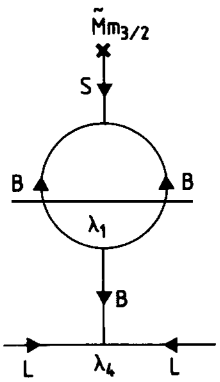

Fig. 7.5. Supergraph that contains the component graphs given in figs. 7.3 and 7.4.

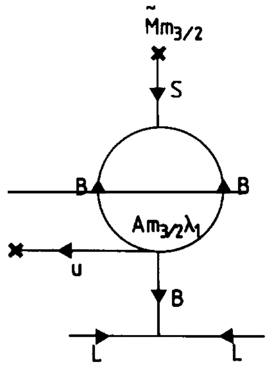

Fig. 7.6. Supergraph with two supersymmetry breaking insertions.

couplings. They also can be represented by spurion superfields, e.g., $U = Am_{3/2}\lambda_i\theta^2$ leading to a $\int d^4\theta LLU^*S$ contribution (D-term) to $(l^2 + l^{*2})$ as shown in the supergraph in fig. 7.6, a contribution not bound to vanish by the nonrenormalization theorem. In fact these and similar contribution as shown in the component graphs in fig. 7.7 do not vanish and also do not cancel each other. Compared to the $\tilde{M}m_{3/2}$ contribution before they are however suppressed since at least one of the trilinear couplings, e.g. $\tilde{M}\lambda_1$ is replaced by $Am_{3/2}\lambda_1$, and the leading noncancelled contributions to $(l^2 + l^{*2})$ are therefore of order $m^2_{3/2}$. They do not spoil the separation of the mass scales but they show that the light mass scale is defined by $m_{3/2}$. Even if we would try to define the light mass scale to be much smaller than $m_{3/2}$ at the tree level this adjustment would be spoiled by radiative corrections and the light masses would be raised

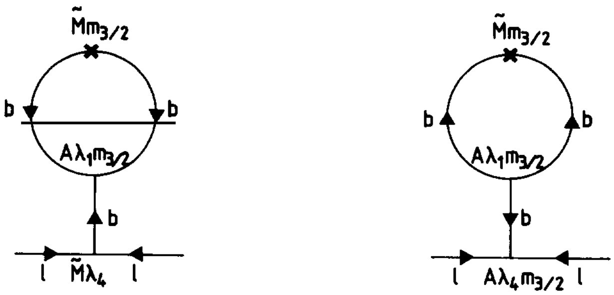

Fig. 7.7. Component graphs that give a nonvanishing uncancelled contribution to $(l^2 + l^{*2})$.

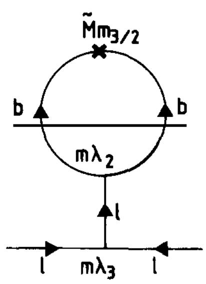

Fig. 7.8. Tadpole similar to fig. 7.3 but now with light particle exchange.

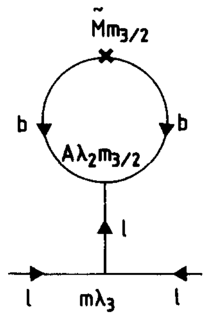

Fig. 7.9. Tadpole with l-exchange and two supersymmetry breaking insertions.

to $m_{3/2}$. All the graphs we have discussed so far are logarithmically divergent and it is understood that they are treated via the usual renormalization procedure.

We now turn to a discussion of the contributions that are proportional to λ_2, from the $\lambda_2 B^2 L$ coupling of light and heavy fields. Again the most dangerous contributions are given through the tadpole diagrams, of which an example is shown in fig. 7.8, and all the contributions that only contain one supersymmetry breaking parameter cancel. As in the former case nonvanishing and noncancelled contributions arise from diagrams with at least two supersymmetry breaking insertions. The graph in fig. 7.9 corresponds to such a leading contribution. Contrary to the former case this contribution is not suppressed by the heavy b-propagator that mediates between the light particles and the loop and also the supersymmetric coupling $m\lambda_2$ is comparable to the supersymmetry breaking trilinear coupling $Am_{3/2}\lambda_2$. As a result fig. 7.9 gives a noncancelled contribution of order $\tilde{M}m_{3/2}$ to $(l^2 + l^{*2})$. Since $\tilde{M}m_{3/2}$ is much larger than $m^2_{3/2}$ this spoils the stability of the light mass scale.

Is this a serious problem? Actually the contribution to $(l^2 + l^{*2})$ is given by $(A-1)\tilde{M}m_{3/2}\lambda_2\lambda_3$. Choosing small values for λ_2 and λ_3 this contribution could be made small. The choice $A = 1$ would imply a vanishing one-loop contribution. Since A is renormalized such a contribution would then only exist in higher loops. The whole problem constitutes what is in general called a naturalness problem. A large value of l^2 can be avoided by special choices (fine tuned) parameters of the theory, but for a wide range of parameters the induced l^2-term will turn out to be too large. Since this is a problem of the type that motivated our study of supersymmetric models one would require it to be absent in the framework of these models.

The problem has been shown to exist in the toy model given by superpotential (7.35) and next one has to see if it is relevant for realistic models. Of course, if the model under consideration contains no heavy fields the problem does not exist. But in general such models are just a low energy description which disregard the existence of heavy particles. In grand unified models for example such heavy fields exist. A way out here would be to forbid the coupling of the heavy and light fields ($\lambda_2 = 0$) which, however, in general is present, but we have to be more specific. The dangerous contribution arises from tadpole diagrams in which a light field connects the light fields with the loop, and such a contribution can only exist if this exchanged particle is a gauge singlet. Consequently the problem occurs only in the presence of a light singlet. *Requiring the absence of such a light singlet would solve the problem.* However, in the models discussed so far we had seen that an enlargement of the Higgs sector is necessary and a light singlet was the simplest choice. This light singlet had typically a coupling $YH_2\bar{H}_2$ to the Higgs doublets. The extension to a usual grand unified model would proceed by choosing Y to be an SU(5) singlet with a term $YH_5\bar{H}_5$ in the superpotential. The light singlet would then also couple to

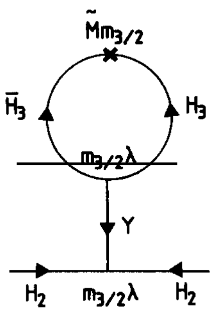

Fig. 7.10. Graph giving rise to the light singlet problem in a simple SU(5) model.

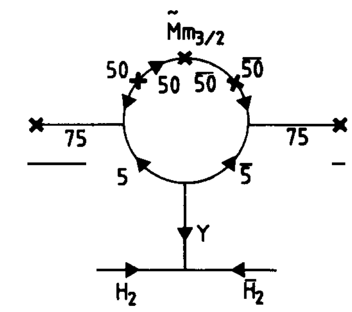

Fig. 7.11. Graph resulting in a light singlet problem in a more complicated SU(5) model.

the heavy Higgs triplets and as a result the contribution of graph in fig. 7.10 would destroy the small mass scale, provided that a direct $\tilde{M}H_3\bar{H}_3$ mass term exists. In grand unified models with 75, 50, $\overline{50}$ representations such a direct mass term does not exist, the mass of H_3 arises from off-diagonal terms with the corresponding triplet in the 50. But again there is a problem as shown in fig. 7.11. It can only be avoided by introducing additional representations and a fine tuning of the parameters of the model [230]. Even if this is done the problem comes back at the two-loop level. It just a normal problem of naturalness.

Another way out would be that Y, although an SU(3) × SU(2) × U(1) singlet, is not an SU(5) singlet but a component of larger SU(5) representation. An inspection of this possibility reveals that really large SU(5) representations are necessary and then there is the additional problem to give masses to all the members of this representation but the singlet, which leads to a really bizarre SU(5) sector [418]. The cleanest way to solve this problem is to refrain from light singlets. But in a model where SU(2) × U(1) breaking occurs at the tree graph level we have to enlarge the Higgs sector. What about an SU(2) triplet instead and a superpotential

$$g = \mathrm{Tr}\, T^3 + HT\bar{H}\,. \tag{7.37}$$

In this case we have to make sure that this triplet receives a v.e.v. small compared to the v.e.v.'s of the doublets in order not to spoil the phenomenological constraint that the ρ-parameter should be close to one [541]. It has been shown that the superpotential (7.37) does not lead to models that satisfy this constraint [14, 190]. Additional terms like $m\, \mathrm{Tr}\, T^2$ are needed which in turn violate our naturalness condition (7.22). Apart from this, the extension of such a model to a grand unified theory requires an enormous enlargement of the grand unified sector. The triplet has to be imbedded in an SU(5) representation and then one has to give large masses to all the other particles. In principal this can be done by repeating the 75, 50, $\overline{50}$ trick designed for the doublets now for the triplets but this requires the introduction of several SU(5) representations with dimensions of order of thousand [418].

To conclude this discussion it seems that all these possible enlargements of the Higgs sector lead to difficulties. It seems to be worth while to reexamine again models with a minimal Higgs sector. Of course in such models, SU(2) × U(1) breakdown cannot happen at the tree graph level.

7.5. Breakdown through radiative corrections

The mechanism in which a SU(2) × U(1) breakdown can occur through radiative corrections has already been explained in section 7.1. The driving terms for this mechanism were large gaugino masses and a large top quark mass. In the context of the supergravity models such a mechanism had been proposed in refs. [444, 324, 180], without giving calculations in a complete model. The renormalization group equations for a general explicitly broken supersymmetric model had been given in ref. [341]. Explicit models in the context of spontaneously broken supergravity have been first given in refs. [329, 14, 170]. The interest in studying this mechanism focuses of course first on a study of a minimal model, i.e. a model with minimal particle content in which the Higgs sector consists only of H and $\bar{H}$. The superpotential of such a minimal model is given by

$$g = \mu H\bar{H} + g_{\mathrm{E}}^{ij}\bar{E}_i L_j H + g_{\mathrm{D}}^{ij}\bar{D}_i Q_j H + g_{\mathrm{u}}^{ij}\bar{U}_i Q_j \bar{H} \tag{7.38}$$

where Q denotes the left-handed quark doublets and $i, j = 1, \ldots, 3$ label the three generations. The parameter μ is needed to avoid the presence of an axion. It is of course unnatural to insert such a small mass parameter in the theory, but we will postpone this discussion. Consider now the model described by (7.38) coupled to a hidden sector with spontaneously broken supergravity. This leads to a scalar potential given by [14]

$$V = |g_a|^2 + m_a^2|y_a|^2 + m_{3/2}(A_{\mathrm{E}}^{ij} g_{\mathrm{E}}^{ij}\bar{E}_i L_j H + A_{\mathrm{D}}^{ij} g_{\mathrm{D}}^{ij}\bar{D}_i Q_j H + A_U^{ij} g_U^{ij}\bar{U}_i Q_j \bar{H} + \text{h.c.}) + B\mu m_{3/2} H\bar{H} + \text{h.c.}\,, \tag{7.39}$$

where we have used the same symbols for the chiral superfields and its scalar components. Additional soft breaking terms could be present for the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ gauginos $\tilde{m}_\alpha\lambda_\alpha\lambda_\alpha$, $\alpha = 1, 2, 3$. At the Planck mass these parameters are related given a potential like (6.22). We have

$$m_a^2 = m_{3/2}^2\,, \qquad A = A_{\mathrm{E}} = A_{\mathrm{D}} = A_{\mathrm{U}}\,; \qquad B = A - 1\,, \qquad \tilde{m}_\alpha = m_0\,. \tag{7.40}$$

To define these parameters at lower energies, renormalization effects have to be included. The change of these parameters in the range between M_{p} and M_{x} is very model dependent. For definiteness we assume that (7.40) holds at M_{x} and we suppose that the heavy particles have been integrated out and study the variation of the parameters taking into account the degrees of freedom of the minimal low energy supersymmetric model. Integrating out these heavy particles will, e.g., change relation $B = A - 1$ but we have to make this assumption in order to do definite calculations. The grand unification scale of this model is $M_{\mathrm{x}} \sim 3\times 10^{16}\,\mathrm{GeV}$ with an SU(5) coupling constant $\alpha_5 = e_5^2/4\pi \approx 1/24$ and for the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ coupling constants at M_{x} we have the relation $e_3 = e_2 = (5/3)^{1/2} e_1 = e_5$. The renormalization group equations [341, 14] which govern the evolution of the gauge couplings at energies below M_{x} are given by (compare section 4.5)

$$\dot{\mu} e_3 \equiv \mu\frac{\partial}{\partial\mu} e_3 = -\frac{3}{16\pi^2} e_3^3\,, \qquad \dot{\mu} e_2 = \frac{1}{16\pi^2} e_2^3\,, \qquad \dot{\mu} e_1 = \frac{11}{16\pi^2} e_1^3\,. \tag{7.41}$$

Integrating the evolution down to energies of order $M_{\mathrm{W}} \sim 100\,\mathrm{GeV}$ this leads to $\alpha_{\mathrm{em}}^{-1} = 128$, $\alpha_3 \approx 0.12$ and $\sin^2\theta_{\mathrm{W}} \approx 0.23$ at M_{W}. The gauge fermion masses evolve similarly

$$\dot{\mu}(\tilde{m}_\alpha/e_\alpha^2) = 0\,. \tag{7.42}$$

For the Yukawa couplings we have

$$\begin{aligned}
\dot{\mu} g_{\mathrm{u}}^{ii} &= \frac{3+3\delta_{i3}}{16\pi^2}(g_{\mathrm{u}}^{33})^3 - \frac{1}{8\pi^2} g_{\mathrm{u}}^{ii}\left(\tfrac{8}{3}e_3^2 + \tfrac{3}{2}e_2^2 + \tfrac{13}{18}e_1^2\right), \\
\dot{\mu} g_{\mathrm{D}}^{jj} &= \delta_{j3}\frac{1}{16\pi^2}(g_{\mathrm{u}}^{33})^2 g_{\mathrm{D}}^{jj} - \frac{1}{8\pi^2} g_{\mathrm{D}}^{jj}\left(\tfrac{8}{3}e_3^2 + \tfrac{3}{2}e_2^2 + \tfrac{7}{18}e_1^2\right), \qquad (7.43) \\
\dot{\mu} g_{\mathrm{E}}^{ii} &= -\frac{1}{8\pi^2} g_{\mathrm{E}}^{ii}\left(\tfrac{3}{2}e_2^2 + \tfrac{3}{2}e_1^2\right),
\end{aligned}$$

where only the top quark Yukawa coupling g_{u}^{33} is large enough to give large effects. The A-parameters obey the equations

$$\begin{aligned}
\dot{\mu} A_{\mathrm{u}}^{ii} &= (3+3\delta_{i3})\frac{A_{\mathrm{u}}^{33}}{8\pi^2}(g_{\mathrm{u}}^{33})^2 - \frac{1}{4\pi^2 m_{3/2}}\left(\tfrac{8}{3}e_3^2\tilde{m}_3 + \tfrac{3}{2}e_2^2\tilde{m}_2 + \tfrac{13}{18}e_1^2\tilde{m}_1\right), \\
\dot{\mu} A_{\mathrm{D}}^{jj} &= \delta_{j3}\frac{A_{\mathrm{u}}^{33}}{8\pi^2}(g_{\mathrm{u}}^{33})^2 - \frac{1}{4\pi^2 m_{3/2}}\left(\tfrac{8}{3}e_3^2\tilde{m}_3 + \tfrac{3}{2}e_2^2\tilde{m}_2 + \tfrac{7}{18}e_1^2\tilde{m}_1\right), \qquad (7.44) \\
\dot{\mu} A_{\mathrm{E}}^{ii} &= -\frac{1}{4\pi^2 m_{3/2}}\left(\tfrac{3}{2}e_2^2\tilde{m}_2 + \tfrac{3}{2}e_1^2\tilde{m}_1\right),
\end{aligned}$$

where all Yukawa couplings except g_{u}^{33} have been neglected. The parameter μ changes according to

$$\dot{\mu}\mu = \frac{\mu}{16\pi^2}\left[3(g_{\mathrm{u}}^{33})^2 - e_1^2 - 3e_2^2\right], \qquad (7.45)$$

$$\dot{\mu} B = -\frac{3}{8\pi^2}(g_{\mathrm{u}}^{33})^2 A_{\mathrm{u}}^{33} - \frac{1}{4\pi^2 m_{3/2}}\left(\tfrac{3}{2}e_2^2\tilde{m}_2 + \tfrac{1}{2}e_1^2\tilde{m}_1\right). \qquad (7.46)$$

For the scalar (mass)^2 terms we have

$$\dot{\mu} m_{\mathrm{H}}^2 = \frac{1}{8\pi^2}\left[3(g_{\mathrm{u}}^{33})^2(m_{\bar{\mathrm{H}}}^2 + m_{\bar{\mathrm{u}}_3}^2 + m_{\mathrm{Q}_3}^2 + m_{3/2}^2|A_{\mathrm{u}}^{33}|^2)\right] - \frac{1}{2\pi^2}\left[\tfrac{3}{4}|\tilde{m}_2|^2 e_2^2 + \tfrac{1}{4}|\tilde{m}_1|^2 e_1^2\right], \qquad (7.47)$$

$$\dot{\mu} m_{\bar{\mathrm{u}}_3}^2 = \frac{1}{8\pi^2}\left[2(g_{\mathrm{u}}^{33})^2(m_{\bar{\mathrm{H}}}^2 + m_{\bar{\mathrm{u}}_3}^2 + m_{\mathrm{Q}_3}^2 + m_{3/2}^2|A_{\mathrm{u}}^{33}|^2)\right] - \frac{1}{2\pi^2}\left[\tfrac{4}{3}|\tilde{m}_3|^2 e_3^2 + \tfrac{4}{9}|\tilde{m}_1|^2 e_1^2\right], \qquad (7.48)$$

$$\dot{\mu} m_{\mathrm{Q}_3}^2 = \frac{1}{8\pi^2}\left[(g_{\mathrm{u}}^{33})^2(m_{\bar{\mathrm{H}}}^2 + m_{\bar{\mathrm{u}}_3}^2 + m_{\mathrm{Q}_3}^2 + m_{3/2}^2|A_{\mathrm{u}}^{33}|^2)\right] - \frac{1}{2\pi^2}\left[\tfrac{4}{3}|\tilde{m}_3|^2 e_3^2 + \tfrac{3}{4}|\tilde{m}_2|^2 e_2^2 + \tfrac{1}{36}|\tilde{m}_1|^2 e_1^2\right]. \qquad (7.49)$$

We neglect the influence of the Yukawa couplings except for (g_{u}^{33}) and all other scalar masses evolve according to

$$\dot{\mu} m_i^2 = -\frac{1}{2\pi^2}\sum_{\alpha} e_\alpha^2 |\tilde{m}_\alpha|^2 C_{\alpha i} \qquad (7.50)$$

as a consequence of the gaugino mass terms which in (7.47)–(7.49) correspond to the second terms. The first terms in (7.47)–(7.49) are given by the top quark Yukawa coupling and it is important to note here that the coefficients of these terms behave like 3 : 2 : 1 for the three different masses. This is important since the sign of these first terms is positive which implies decreasing m^2 with decreasing μ. *The fact that $m^2_{\bar{\mathrm{H}}}$ has the largest coefficient implies that it decreases faster* and this is exactly the behavior we are looking for, since $m^2_{\bar{\mathrm{H}}}$ should become negative at low energies to induce SU(2) × U(1) breakdown while $m^2_{\bar{\mathrm{u}}_3}$ and $m^2_{\mathrm{Q}3}$ should stay positive. The second terms in (7.47)–(7.49) have negative sign and they do not help directly. But this is already clear from our earlier discussion. The gaugino masses induce first positive (mass)^2 for the scalar partners of quarks and leptons and Higgses and especially the partners of quarks receive a large contribution. These masses then enter the first terms in (7.47)–(7.49) and speed up the desired evolution provided that g^{33}_{u} is large enough.

From the renormalization group equations it is clear that if radiative corrections drive a (mass)^2 of a scalar to negative values this will first be the scalar of $\bar{H}$. To identify the conditions for an induced SU(2) × U(1) breakdown consider the Higgs potential

$$V = m_1^2|h|^2 + m_2^2|\bar{h}|^2 + m_3^2(h\bar{h} + h^*\bar{h}^*) + \tfrac{1}{8}(e_1^2 + e_2^2)[|h|^2 - |\bar{h}|^2]^2 , \tag{7.51}$$

where the last term is the contribution from the $\frac{1}{2}D^2$ terms of the gauge interactions. This term is important since it is the only term quartic in the Higgs fields, which is needed to prevent infinite Higgs v.e.v.'s in case of negative (mass)^2 for h or $\bar{h}$. From (7.39) we see that at the grand unification scale the parameters m_i^2 are given by

$$m_1^2 = m_2^2 = m_{3/2}^2 + \mu^2 , \qquad m_3^2 = B\mu m_{3/2} , \tag{7.52}$$

and below M_x these quantities will vary with m_2^2 the candidate to become negative. Suppose that this happens. The last term in (7.51) vanishes for $\langle h\rangle = \langle\bar{h}\rangle$ and the quartic terms are absent in this direction. To make sure that the potential is still well behaved and does not allow the v.e.v.'s to run to infinity the inequality

$$m_1^2 + m_2^2 \geq 2|m_3|^2 \tag{7.53}$$

has to be satisfied. In addition a necessary condition for SU(2) × U(1) breakdown is given by [341]

$$m_1^2 m_2^2 < |m_3|^4 , \tag{7.54}$$

which in general is fulfilled for $m_2^2 < 0$ and m_1^2 still positive.

Let us now discuss the behavior of m_2^2 according to (7.47) at low energies. To isolate the influence of the various terms on this evolution we consider first the case $\mu = 0$ and vanishing gaugino masses: $m_0 = 0$ in (7.40). We know that $\mu = 0$ is problematic since the model contains an axion but we will discuss these questions later. The evolution of m_2^2 is then given by

$$\mu \frac{\partial}{\partial\mu} m^2_{\bar{\mathrm{H}}} = \frac{3(g^{33}_{\mathrm{u}})^2}{8\pi^2} [m^2_{\bar{\mathrm{H}}} + m^2_{\bar{\mathrm{u}}3} + m^2_{\mathrm{Q}3} + m^2_{3/2}|A^{33}_{\mathrm{u}}|^2] , \tag{7.55}$$

where also (7.43), (7.44) and (7.48), (7.49) have to be taken into account for the parameters that appear in the right hand side of (7.55). The evolution of these parameters has to be determined numerically. Given (7.40) we have in addition to $m_{3/2}$ the two input parameters g_u^{33} and A, and large g_u^{33} and A can lead to a fast evolution of $m_{\bar{H}}^2$ to negative values. From (7.55) one might even think that a large A alone could drive the mechanism even for small top quark masses (given by g_u^{33}) but we have to be careful. From our discussion in section 7.3 we have seen (7.33) that for $A > 3$ potentially dangerous minima with broken $U(1)_{em}$ can occur, where scalar partners of the leptons receive large vacuum expectation values. We have to avoid them here as well. In the example discussed there $\langle h \rangle$, $\langle \bar{E} \rangle$ and $\langle L \rangle$ had vacuum expectation values of order $m_{3/2}/g_E$ and we had to choose $A < 3$ to make sure that such a minimum was not the absolute minimum of the model. Here A is no longer universal and constraints have to be obtained for A_E, A_u, A_D separately. In the simplified case under consideration the masses of the particles with small Yukawa couplings are not much affected and to avoid these undesired minima we have to choose $|A_E|$, $|A_D| < 3$. The general constraints give [112]

$$\begin{aligned} &m_H^2 + m_E^2 + m_L^2 > (|A_E|^2/3) m_{3/2}^2 \,, \\ &m_H^2 + m_D^2 + m_Q^2 > (|A_D|^2/3) m_{3/2}^2 \,, \\ &m_u^2 + m_Q^2 - |m_{\bar{H}}|^2 > (|A_u|^2/3) m_{3/2}^2 \,, \end{aligned} \tag{7.56}$$

where the relevant parameters in (7.56) are those at the breakdown scale of $SU(2)\times U(1)$. From (7.44) we see that in the absence of gaugino masses A_u and A_D decrease with decreasing energy whereas A_E stays constant. The value of A at M_x is thus bound to obey $|A| < 3$ to avoid the $U(1)_{em}$ breaking minima.

The second parameter in (7.55) is the top-quark Yukawa coupling. It evolves according to (7.43) and we see that for large g_u^{33} it decreases at smaller energies. Actually if we would choose an arbitrarily large g_u^{33} at M_x this decrease is proportional to $(g_u^{33})^3$ and thus arbitrarily large, and g_u^{33} at low energies might still be small. With (7.43) one obtains an upper limit on the Yukawa coupling at low energies, corresponding to $m_{top} \sim 200$ GeV. Of course (7.43) is no longer valid for arbitrarily large g_u^{33} at M_x and we note this upper bound just for curiosity.

The results of a numerical analysis of (7.55) and (7.43)–(7.49) are shown in fig. 7.12. This result holds for $\mu = 0$ and $m_0 = 0$, a grand unification scale of 3×10^{16} GeV with $\alpha_5 \sim 1/24$. To obtain the correct magnitude of the $SU(2)\times U(1)$ breakdown the condition

$$m_2^2(Q^2 \simeq (100\ \text{GeV})^2) = -\tfrac{1}{2} M_z^2 \tag{7.57}$$

(with M_z the mass of intermediate Z-boson) has to be fulfilled. Notice that in this case only a v.e.v. for $\bar{h}$ is induced and we have

$$\langle \bar{h} \rangle^2 = -\frac{4m_2^2}{(e_1^2 + e_2^2)}\,, \qquad m_2^2 = -M_W^2/2\cos^2\theta_w = -\tfrac{1}{2}M_z^2\,, \tag{7.58}$$

but the down quarks and leptons will stay massless because of $\langle h \rangle = 0$. This will change as soon as $\mu \neq 0$ which we have to choose later in any case to avoid an axion. Diagram 7.12 shows the results for a choice $M_z \sim 90$ GeV in (7.57), as a function of A, $m_{3/2}$ and g_u^{33}. The quantity α_y denotes $(g_u^{33})^2/4\pi$ at the grand unification scale. The dashed line gives the limit on A from (7.56) here

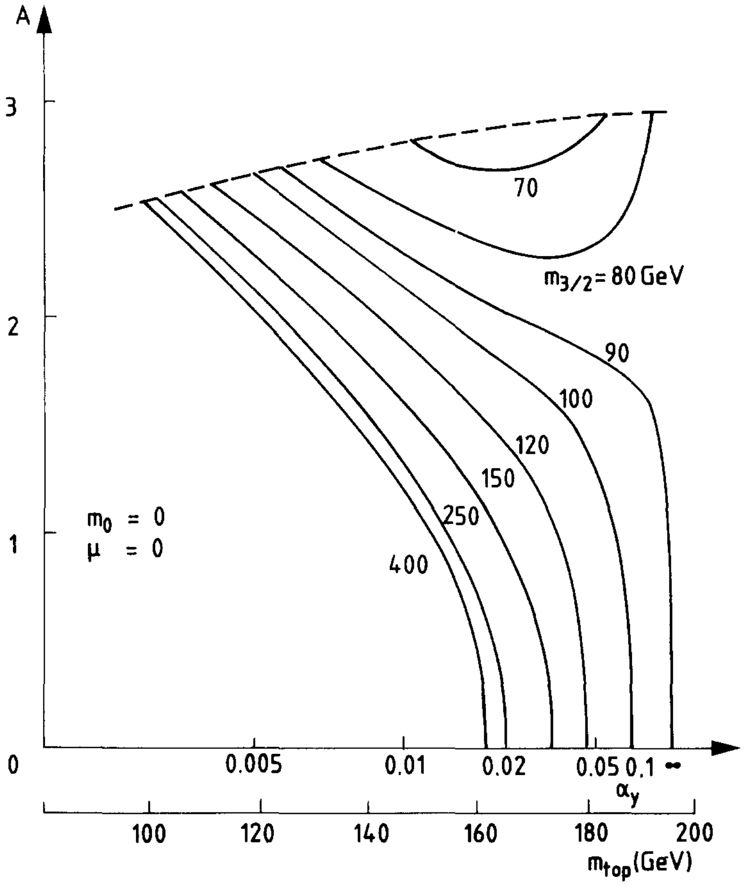

Fig. 7.12. Relations between A, $m_{3/2}$ and $m_{\rm top}$ for a model with radiatively induced breakdown of SU(2)×U(1) as given in ref. [14]. This result holds in the case $m_0 = \mu = 0$.

$$m_{\tilde{u}}^2 + m_{\tilde{Q}}^2 - \tfrac{1}{2}M_Z^2 > (|A_u|^2/3)m_{3/2}^2\,, \tag{7.59}$$

which in our special case implies

$$|A_u| < \tfrac{5}{2} - M_Z^2/4m_{3/2}^2\,. \tag{7.60}$$

With this bound on A the mechanism needs large top quark masses

$$100\,\mathrm{GeV} \le m_{\rm top} \le 190\,\mathrm{GeV}\,, \tag{7.61}$$

where we have to remember that for $\alpha_y \sim 1$ our approximation breaks down. The value of the gravitino mass $m_{3/2}$ can lie anywhere between 70 GeV and infinity but we see that for most of the parameter space it lies between 80 and 400 GeV. For typical values like $A = 3 - \sqrt{3}$ and $m_{3/2} = 100$ GeV one needs a top quark mass as large as 180 GeV to induce the breakdown of SU(2)×U(1). These are the results for our special case $\mu = m_0 = 0$. In general we will see later that nonvanishing μ and m_0 will allow an SU(2)×U(1) breakdown even in the presence of smaller top quark masses than given by (7.61). $\mu = 0$

will also lead to an axion if $\langle h\rangle$ would be nonvanishing but with the prescription (7.57) we have $\langle h\rangle = 0$ and therefore no down quark and lepton masses. The latter problem can be removed by demanding

$$(m_1^2 + m_2^2)|_{Q^2 \simeq (100\ \mathrm{GeV})^2} = 0 \tag{7.62}$$

instead of (7.57) as proposed in ref. [170]. In general if we define $\langle \bar{h}\rangle/\langle h\rangle = \tan\theta$ we obtain

$$\sin 2\theta = 2m_3^2/(m_1^2 + m_2^2)\,, \tag{7.63}$$

and we see that for $m_3 = 0$ ($\mu = 0$) (7.62) has to be satisfied to obtain a nonvanishing v.e.v. for h. Models based on (7.62) instead of (7.57) lead to similar bounds on m_{top} as given in (7.61). In this case the breakdown scale of the weak interactions is related to the gravitino mass through dimensional transmutation obtained through the flat direction in the potential implied through constraint (7.62).

The bound (7.61) on m_{top} requires very large top quark masses but before we explore the various possibilities of lowering this bound, let us discuss the spectrum of the models described above. Before we do this we have to include $\mu \neq 0$ and we will assume for the moment that this inclusion does not change the results of fig. 7.12 drastically. We will later see that for a large range of μ this assumption is correct. Equation (7.53) now gives $m_1^2 + m_2^2 > 2\mu B m_{3/2}$ for the stability of the minimum. With this inclusion and m_2^2 ((100 GeV)^2) $= -M_Z^2/2$ not only $\bar{h}$ but also h will receive a v.e.v.

$$\langle h^0\rangle \simeq \frac{B\mu m_{3/2}}{m_1^2 - |m_2|^2}\langle \bar{h}^0\rangle + \mathrm{O}(\mu^3)\,, \tag{7.64}$$

where we work in the limit $\mu < m_{3/2}$. μ should not be too small because otherwise one would have $\langle h^0\rangle \ll \langle \bar{h}^0\rangle$ which would imply the presence of a very large Yukawa coupling of the bottom quark to give it a mass of 4.5 GeV and this certainly had to be taken into account in the renormalization group equations. We assume that μ is large enough to avoid these problems.

The spectrum of the scalar particles of the model is given in table 7.1, for $\mu = 0$. The scalar h^0 does not receive a v.e.v. and the Peccei–Quinn symmetry remains unbroken. With $\mu \neq 0$ h^0 will receive a v.e.v. as shown in (7.64). In this case one of the neutral Higgs particles (a pseudoscalar) will have a mass of order μ. In the absence of the breaking term $B\mu m_{3/2}$ one would have the mass of this particle of order $\mu^2 m_{3/2}$ but the soft trilinear supersymmetry breaking term raises this mass of the "axion" to μ.

The gluinos and photinos receive masses through radiative corrections as discussed in section 6.4. Depending on the parameters of the model we expect these in the ranges

$$5\ \mathrm{GeV} \le \tilde{m}_3 \le 100\ \mathrm{GeV}\,, \qquad 0.5\ \mathrm{GeV} \le \tilde{m}_\gamma \le 15\ \mathrm{GeV}\,. \tag{7.65}$$

The remaining gauginos mix with the Higgs fermions. We have for the charged ones:

$$\begin{array}{c|cc} & \tilde{\mathrm{w}}^+ & \tilde{\bar{\mathrm{h}}}^+ \\ \hline \tilde{\mathrm{w}}^- & |\tilde{m}_2| & \sqrt{2}M_W \\ \tilde{\mathrm{h}}^- & \sqrt{2}M_W|x| & \mu \end{array} \tag{7.66}$$

where x denotes the ratio of h to $\bar{h}$ v.e.v.'s which is of order $\mu/m_{3/2}$. With $\langle h\rangle$ as given in (7.64) the lightest of these fermions will have a mass

Table 7.1

The spectrum of the scalar particles in the model with $\mu = 0$ as given in ref. [14]. Partners of quarks and leptons are denoted by φ_{q}, φ_{ℓ} respectively. The off-diagonal terms $Am_{3/2}/m_{\mathrm{Q}}$ are explicitly given for the partners of the top quark and neglected in the other cases.

$\varphi_{\mathrm{u}}, \varphi_{\mathrm{c}}$	$m_{3/2}^2 + m_{\mathrm{z}}^2(\frac{2}{3}\sin^2\theta_{\mathrm{w}} - \frac{1}{2})$
$\varphi_{\mathrm{d}}, \varphi_{\mathrm{s}}$	$m_{3/2}^2 + m_{\mathrm{z}}^2(\frac{1}{2} - \frac{1}{3}\sin^2\theta_{\mathrm{w}})$
$\varphi_{\bar{\mathrm{u}}}, \varphi_{\bar{\mathrm{c}}}$	$m_{3/2}^2 - m_{\mathrm{z}}^2(\frac{2}{3}\sin^2\theta_{\mathrm{w}})$
$\varphi_{\bar{\mathrm{d}}}, \varphi_{\bar{\mathrm{s}}}, \varphi_{\bar{\mathrm{b}}}$	$m_{3/2}^2 + m_{\mathrm{z}}^2(\frac{1}{3}\sin^2\theta_{\mathrm{w}})$
φ_{b}	$\frac{2}{3}m_{3/2}^2 + m_{\mathrm{z}}^2(\frac{1}{3} - \frac{1}{3}\sin^2\theta_{\mathrm{w}})$
$\left.\begin{matrix}\varphi_{\mathrm{t}}\\ \varphi_{\bar{\mathrm{t}}}\end{matrix}\right\}$	$\begin{pmatrix} \frac{2}{3}m_{3/2}^2 + m_{\mathrm{z}}^2(\frac{2}{3}\sin^2\theta_{\mathrm{w}} - \frac{2}{3}) + m_{\mathrm{top}}^2 & A_{\mathrm{u}}^{33}m_{3/2}m_{\mathrm{top}} \\ A_{\mathrm{u}}^{33}m_{3/2}m_{\mathrm{top}} & \frac{1}{3}m_{3/2}^2 + m_{\mathrm{z}}^2(-\frac{1}{3} - \frac{2}{3}\sin^2\theta_{\mathrm{w}}) + m_{\mathrm{top}}^2 \end{pmatrix}$
$\varphi_{\mathrm{e}}, \varphi_{\mu}, \varphi_{\tau}$	$m_{3/2}^2 + (\frac{1}{2} - \sin^2\theta_{\mathrm{w}})m_{\mathrm{z}}^2$
$\varphi_{\bar{\mathrm{e}}}, \varphi_{\bar{\mu}}, \varphi_{\bar{\tau}}$	$m_{3/2}^2 + \sin^2\theta_{\mathrm{w}} m_{\mathrm{z}}^2$
$\varphi_{\nu_{\mathrm{e},\mu,\tau}}$	$m_{3/2}^2 - \frac{1}{2}m_{\mathrm{z}}^2$
$\sqrt{2}\,\mathrm{Re}\,\bar{h}^0$	m_{z}^2
h^0	$m_{3/2}^2 - \frac{1}{2}m_{\mathrm{z}}^2$
h^+	$m_{3/2}^2 + m_{\mathrm{z}}^2(\frac{1}{2} - \sin^2\theta_{\mathrm{w}})$

$$\sqrt{2}M_{\mathrm{W}}\mu|B|m_{3/2}/(m_{3/2}^2 - \tfrac{1}{2}M_z^2) + \mathcal{O}(\mu^3/m_{3/2}^2)\,. \tag{7.67}$$

For this particle to have a mass larger than 20 GeV one must have $|x| > 0.17$ which implies $\mu|B| \sim 0.05$–0.17 still consistent with our assumption that μ is small. We have here such a light particle because of the asymmetry of the h and $\bar{h}$ vacuum expectation values. Its left-handed pieces are predominantly ψ_{w^+} and ψ_{h^-}.

The mass matrix of the neutral fermions of the Higgs sector is given by

$$\begin{array}{c} \tilde{\mathrm{w}}^0 \\ \tilde{\mathrm{B}}^0 \\ \tilde{\mathrm{h}}^0 \\ \tilde{\bar{\mathrm{h}}}^0 \end{array} \begin{pmatrix} \tilde{m}_2 & 0 & \mathrm{i}M_z x\cos\theta_{\mathrm{w}} & \mathrm{i}M_z\cos\theta_{\mathrm{w}} \\ 0 & \tilde{m}_1 & \mathrm{i}M_z x\sin\theta_{\mathrm{w}} & \mathrm{i}M_z\sin\theta_{\mathrm{w}} \\ \mathrm{i}M_z x\cos\theta_{\mathrm{w}} & \mathrm{i}M_z x\sin\theta_{\mathrm{w}} & 0 & \mu \\ \mathrm{i}M_z\cos\theta_{\mathrm{w}} & \mathrm{i}M_z\sin\theta_{\mathrm{w}} & \mu & 0 \end{pmatrix} \tag{7.68}$$

This matrix has two large eigenvalues of order M_z and two small ones. One of the light particles is the already discussed photino

$$\tilde{\gamma} = \sin\theta_{\mathrm{w}}\tilde{W}^0 - \cos\theta_{\mathrm{w}}\tilde{B}^0\,, \tag{7.69}$$

and the second one is the partner of the "pseudoscalar axion" discussed earlier, its main component is ψ_{h} and its mass is proportional to μ. Which of the two is the lightest particle depends on $\tilde{m}_\alpha$ and μ. If one considers models with $m_0 = 0$ as we did in this section one in general expects the photino to be lighter than ψ_{h} since μ has to be chosen large enough to satisfy the lower bound on the charged fermion mass given in (7.67).

The light supersymmetric particles in this class of models with $m_0 = 0$ and small μ are the photino whose mass is small because of $m_0 = 0$ and a charged and neutral fermion as well as a pseudoscalar boson with mass proportional to μ. The other supersymmetric R-odd particles have masses typically of order of the mass of the weak gauge bosons. The lightest of the scalar partners of quarks and leptons is one of the partners of the top quark provided that A_{u}^{33} is not to small. The mechanism of induced SU(2) × U(1) breaking requires the existence of a large g_{u}^{33} Yukawa coupling and thus a large mass of the top quark $100\,\mathrm{GeV} \leq m_{\mathrm{top}} \leq 190\,\mathrm{GeV}$ with either choice (7.57) or (7.62) as boundary conditions for m_1^2 and m_2^2 at energy scale $Q^2 \sim (100\,\mathrm{GeV})^2$.

The case $\mu = 0$ is not possible. It either has an axion or only one of the Higgs fields receives a v.e.v. and leptons and down quarks have to stay massless. To avoid this a nonzero μ has to be put in by hand. A priori this parameter is unrelated to the gravitino mass and it violates the naturalness condition (22). It should not be too small to avoid the prediction of a charged fermion with mass smaller than 20 GeV.

7.6. Su(2) × U(1) breakdown with smaller top quark mass

In the last section we have considered a special class of models with $m_0 = 0$ and small μ which required a large top quark Yukawa coupling to induce the SU(2) × U(1) breakdown. We now want to address the question whether this also works with smaller top quark masses.

One way to do this is the choice $m_0 \neq 0$. At first sight this is not obvious as an inspection of (7.47) shows. Nonzero gaugino masses $\tilde{m}_2$ and $\tilde{m}_1$ give a negative contribution to the right hand side and with just this contribution $m_{\bar{\mathrm{h}}}^2$ would increase with decreasing energy. But we know already from our discussion in section 7.1 that the most important influence of the gaugino masses is indirect. It first induces large masses for the scalar partners of the quarks via the $e_3^2|\tilde{m}_3|^2$ contributions in (7.48) and (7.49). These large masses then enter the first term in (7.47) and this accelerates the evolution of $m_{\bar{\mathrm{h}}}^2$ if (g_{u}^{33}) is large enough to dominate the second term (which does not contain $e_3^2|\tilde{m}_3|^2$). Our earlier discussion indicates that in such a case the induction of the weak breakdown might be possible with smaller values of m_{top} than the range given by (7.61) and here we can check this explicitly.

There are two ways in which a nonzero m_0 can appear in the theory as we have discussed in section 6.4. The first possibility is to have $m_0 \sim m_{3/2}$ at the tree level and at the grand unification scale the gluinos, winos and bino have the same mass. Below M_{x} these masses evolve differently according to (7.42) and this will result in

$$\tilde{m}_a = (\alpha_a/\alpha_5)m_0 \sim (\alpha_a/\alpha_5)m_{3/2}\,, \tag{7.70}$$

where $a = 1, 2, 3$, $\alpha_a = e_a^2/4\pi$ the running coupling constants and α_5 the grand unified coupling constant at M_{x}.

A second possibility for an appearance of gaugino masses is through radiative corrections in the presence of heavy particles. At M_{x} this will result in

$$\tilde{m}_a = C\alpha_5 m_{3/2} \tag{7.71}$$

following our discussion in section 6.4. The factor C depends on the spectrum of the heavy particles. The masses in (7.71) evolve according to (7.42)

$$\tilde{m}_a = C\alpha_a m_{3/2}\,. \tag{7.72}$$

Thus in both cases we expect the gluinos to be the heaviest gauginos at low energies and the photino will have a much smaller Majorana mass

$$\tilde{m}_\gamma/\tilde{m}_3 \simeq 8\alpha_{\mathrm{em}}/3\alpha_3\,. \tag{7.73}$$

With either (7.70) or (7.71) we can now compute the evolution of the various parameters of the model. We still use small μ and demand boundary condition (7.57) at low energies. The alternative choice (7.62) will give similar results. The models now depend on the three parameters A, $m_{3/2}$ and m_0. Let us first consider the standard value $A = 3-\sqrt{3}$ to investigate the influence of m_0. The calculation of the evolution according to (7.41)–(7.50) has to be done numerically and we use the same M_x, α_5 input parameters as in the last section. The result is given in fig. 7.13. For $m_0 = 0$ we obtain the old results in the case of $A = 3-\sqrt{3}$. The lower bound on m_{top} is given by 135 GeV and for $m_{3/2} \lesssim 70$ GeV the $SU(2)\times U(1)$ breakdown will not be induced at all. For $m_0 \neq 0$ this changes and m_{top} can be as low as 60 GeV. In the case with general A and $m_0 = m_{3/2}$ one obtains a lower bound $m_{\text{top}} \geq 65$ GeV. A strict lower bound [329, 14, 170] is obtained in the limit $m_0 \gg m_{3/2}$,

$$m_{\text{top}} \geq 55\ \text{GeV} \tag{7.74}$$

which, however, is hard to reach since we expect in general $m_0 \sim m_{3/2}$. In models with a low energy potential like the one in (7.5) one could have $m_0^2 \gg Bm_{3/2}^2$ for small B, but in these cases one has to be careful to make sure that $A/\sqrt{B} < 3$ to avoid unwanted minima [226].

In any case the presence of $m_0 \neq 0$ makes it easier to induce the breakdown of the weak interactions as we have expected. The smallest value for the top quark mass is obtained for $m_{3/2}/m_0 = 0$. The spectrum of these models with small μ and nonvanishing m_0 is similar to the one discussed in the last section. Observe that with $m_0 \neq 0$ not only the value of m_{top} but also the value of $m_{3/2}$ can be lower

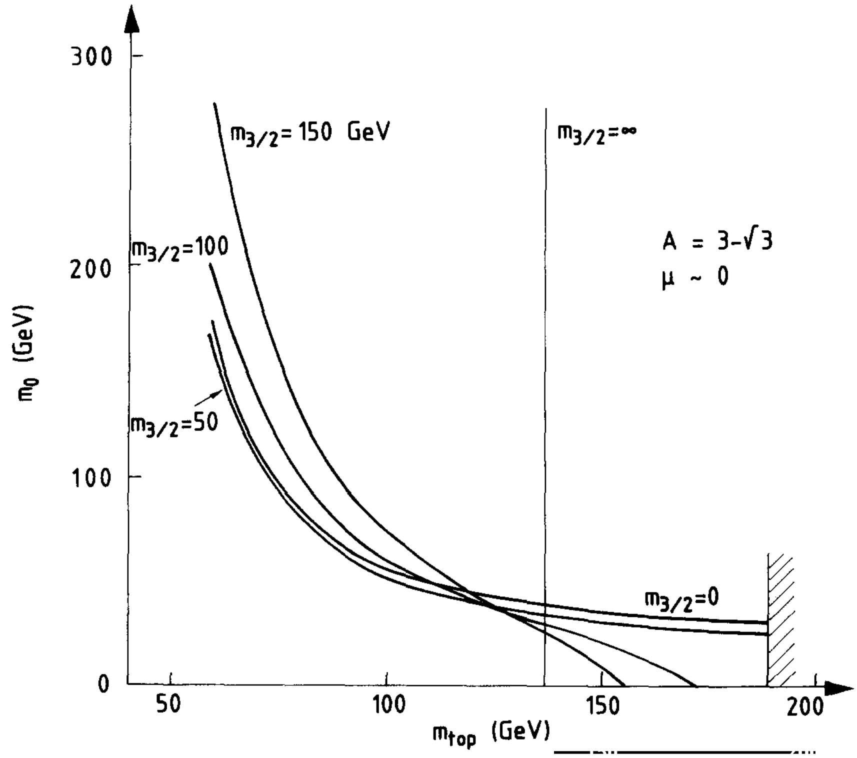

Fig. 7.13. Influence of nonvanishing gaugino masses m_0 on the radiatively induced $SU(2)\times U(1)$ breakdown as given in ref. [329].

than in the previous case (where we needed $m_{3/2} \gtrsim 70$ GeV). Here one can in principle have arbitrarily low values for $m_{3/2}$ and as a result the scalar partners of quarks and leptons could be lighter than those given in table 7.1. The photino has still a fair chance to be the lightest R-odd particle since the influence of m_0 is not very large (7.73) as long as m_0 is of the order $m_{3/2}$. It might however happen that one of the particles with mass proportional to μ is lighter than the photino. The masses of the scalar partners of quarks and leptons are affected by the presence of the gaugino masses (7.50). A large contribution is given to the partners of quarks because of the presence of $e_3^2|\tilde{m}_3|^2$ in the right hand side of (7.50) (compare (7.48) and (7.49)). The partners of the right-handed leptons receive the smallest contribution since they only couple to $\mathrm{U(1)_y}$. In general one would expect in this class of models these particles to be the lightest of the partners of quarks and leptons, and they could very well be in the mass range of 20 GeV. Only for $m_t > m_{3/2}$, m_0 and not too small A one of the partners of the top quark could be lighter. Observe that these contributions to the masses of quark/lepton partners do not create problems with flavor changing neutral currents, particles of same charge and same helicity receive the same contribution. Mass contributions that cause flavor changing neutral currents can only come from those terms in the renormalization group equations which contain Yukawa couplings. We will discuss these effects later in detail.

Gaugino masses also influence the evolution of A. With $m_0 = 0$ the sign and phase of A was actually unphysical. It could be absorbed by a redefinition of the fields. The equations (7.44) contain only terms proportional to A itself and the Yukawa couplings and the sign of the term coincides with the sign of A. As a result the absolute value of A decreases with decreasing energy and starting with $A < 3$ at M_x one was sure that all A's remained smaller than 3 at low energies. With $m_0 \neq 0$ the relative sign and phase of A and m_0 become important. In (7.44) the gaugino masses contribute to a negative term on the right hand side which corresponds to increasing A at low energies. This, however, is in general not very dangerous since the masses of the scalar partners of quarks and leptons also increase such that the upper bound on $|A|$ is in general not violated by radiative corrections, in some cases one might choose A negative to avoid potential problems. A complex phase between m_0 and A is in general a dangerous source for CP violation which we will discuss later.

We have now included nonvanishing m_0 and this leads to the lower bound (7.74) on m_{top}, which is still large. There are additional possibilities to lower this bound. One of them is to consider larger A. As we have pointed out earlier models with $A > 3$ have several problematic properties. First of all the desired $\mathrm{SU(2)} \times \mathrm{U(1)}$ breaking minimum in which only h^0 and $\bar{h}^0$ have nonvanishing v.e.v. is no longer the absolute minimum (compare our discussion in section 7.3). The absolute minimum corresponds to broken $\mathrm{U(1)_{em}}$ and there are other minima with broken $\mathrm{SU(3)_c}$ still lower than the desired minimum. The minima are very deep because fields get v.e.v.'s of order $m_{3/2}/\rho$ where ρ is a small Yukawa coupling constant. On the other hand these small couplings also imply that these different local minima are separated by a large barrier of order $m_{3/2}^4/\rho^2$ from the desired minimum and it might in fact be that once the desired minimum was chosen at a certain time for certain reasons the theory might sit in such a minimum for a long time. The question of the stability of such a false vacuum was examined in ref. [86], and it was found that for $A > 3$ there exist certain ranges of the parameters of the model in which the false vacuum has a lifetime larger than the age of the universe. In particular this range of parameters included the case in which g_{u}^{33} is small. In fact it turned out that under these circumstances the $\mathrm{SU(2)} \times \mathrm{U(1)}$ breakdown could be induced for arbitrarily small top quark masses (for very small g_{u}^{33} this is compensated by A since only the product $g_{\mathrm{u}}^{33}A$ enters (7.47)). In this case a lower bound on $m_{\mathrm{top}} \gtrsim 20$ GeV comes solely from the nonobservation of a toponium resonance below 40 GeV in e^+e^- collisions. A second problem with large A concerns the masses of the scalar partners of the top quark.

They are in general given by a $(\text{mass})^2$ matrix of the type

$$\begin{pmatrix} m_{3/2}^2 + m_{\text{top}}^2 & Am_{\text{top}}m_{3/2} \\ Am_{\text{top}}m_{3/2} & m_{3/2}^2 + m_{\text{top}}^2 \end{pmatrix} \tag{7.75}$$

where we have neglected contributions from radiative corrections and just consider the simplest model. For $|A| < 2$ the $(\text{mass})^2$ eigenvalues

$$m^2 = m_{3/2}^2 + m_{\text{top}}^2 \pm Am_{\text{top}}m_{3/2} \tag{7.76}$$

are both positive independent of the values of m_{top} and $m_{3/2}$. For larger A both eigenvalues are only positive if m_{top} and $m_{3/2}$ have sufficiently different values. One of the possibilities is to have small g_{u}^{33} in case of large A, exactly the situation described above, and as shown this is possible in a cosmologically stable false vacuum. Equations (7.75)–(7.76) show that for large A one of the partners of the top quark might be the lightest scalar partner of quarks and leptons. If large gaugino masses are present, however, these masses receive additional contributions and in such a case it might also be that the partners of right-handed leptons are lightest.

The introduction of large A thus lowers the bound on m_{top} to its phenomenological limit of 20 GeV provided one is willing to live long enough in a false vacuum.

Another possibility to lower the bound on m_{top} is given by choosing a larger grand unification scale M_{x}. This in general requires the introduction of new light (1 TeV) fields and one has to depart from the so-called minimal models. One favorite choice is the introduction of new superfields in the $(8,1,0) + (1,3,0) + (1,1,0)$ $\text{SU}(3)\times\text{SU}(2)\times\text{U}(1)$ representations, which might also be required for cosmological reasons as we will discuss later [377, 378, 327]. The inclusion of these particles raises M_{x} to the order of M_{P} and still allows an acceptable value for $\sin^2\theta_{\text{w}}$. In the framework of SU(5) the above particles are components of an adjoint representation. With M_{x} as large as M_{P} the evolution of the parameters of the theory proceeds over seventeen orders of magnitude in energy instead of fourteen as before. To reach (7.57) and induce $\text{SU}(2)\times\text{U}(1)$ breakdown smaller values of g_{u}^{33} are sufficient. With the particle content sketched above one obtains $m_{\text{top}} \geq 43$ GeV instead of 55 GeV in (7.74) for the minimal model. With this departure from the minimal model one has of course opened the door for many more possibilities which we will not discuss here in detail. It seems however possible that with some effort any value of g_{u}^{33} can induce $\text{SU}(2)\times\text{U}(1)$ breakdown in a model especially constructed for this value of g_{u}^{33}.

Let us come back to the minimal models. We have not discussed yet the case of larger μ. An inspection of the condition (7.54) for the breakdown of $\text{SU}(2)\times\text{U}(1)$

$$m_1^2 m_2^2 < |m_3|^4 \tag{7.54}$$

with values at M_{x}

$$\begin{aligned} m_1^2 &= m_2^2 = m_{3/2}^2 + \mu^2 \\ m_3^2 &= B\mu m_{3/2} \end{aligned} \tag{7.52}$$

reveals that in the presence of a μ of order of $m_{3/2}$ the $\text{SU}(2)\times\text{U}(1)$ breakdown might be easier to obtain since for $\mu \approx 0$ we needed to have $m_1^2 m_2^2 < 0$. Of course we cannot choose arbitrarily large B on

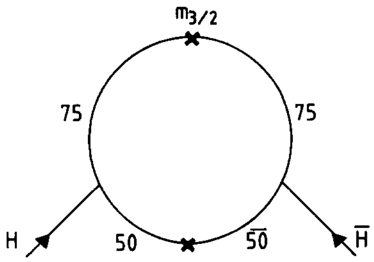

Fig. 7.14. A possible contribution to μ through radiative corrections.

one hand since B is naively related to $(A-1)$ and also because we have to satisfy (7.53),

$$m_1^2 + m_2^2 > 2|m_3|^2\,, \tag{7.53}$$

the condition on the potential to be bounded from below, and this condition has to be fulfilled at all mass scales, which in particular at M_x implies $|B| < 2$ for $\mu = m_{3/2}$.

The insertion of the parameter μ violates the naturalness condition (7.22) and one would like to have a justification for its existence. It would be very desirable to construct a connection between μ and $m_{3/2}$ and to understand a nonvanishing μ in terms of $m_{3/2}$. Remember that a lot of effort has been devoted in grand unified models to explain why μ is zero and not of the order of M_x, and finally $\mu = 0$ was achieved at the tree graph level. After the breakdown of supersymmetry a nonvanishing μ is generated in perturbation theory, for example [330], through a graph like the one in fig. 7.14. These possible contributions to μ are however in general small compared to $m_{3/2}$. To obtain $\mu \sim m_{3/2}$ new singlets or new "nonrenormalizable" couplings have to be introduced in the theory only for the purpose to generate such a large μ. These mechanisms are no more and no less artificial than introducing μ by hand and an understanding of the origin of μ is not yet obtained, except for values of μ that are small compared to $m_{3/2}$. Nonetheless the phenomenological consequences of models with $\mu \sim m_{3/2}$ should be analyzed.

To start this let us first repeat some kinematics. The Higgs potential in (7.51) for nonzero m_3^2 is in general minimized for

$$v^2 = \langle h\rangle^2 + \langle \bar h\rangle^2 = 2[m_1^2 - m_2^2 - (m_1^2 + m_2^2)\cos 2\theta]/(e_1^2 + e_2^2)\cos 2\theta\,, \tag{7.77}$$

where $\tan\theta = \langle \bar h\rangle/\langle h\rangle$. In the case of vanishing μ we had instead (7.58) $\langle \bar h\rangle^2 = 4m_2^2/(e_1^2 + e_2^2)$. Let us now write [330]

$$\cos 2\theta = (\omega^2 - 1)/(\omega^2 + 1)\,. \tag{7.78}$$

The earlier condition (7.57) or (7.62) in the case of $\mu = 0$ is now replaced by

$$(m_1^2 - \omega^2 m_2^2)/(\omega^2 - 1)|_{Q^2 \simeq M_W^2} = \tfrac{1}{2}M_z^2 \tag{7.79}$$

to have $SU(2)\times U(1)$ breakdown at the right scale. The previous case is obtained from this more general one for $\omega \to \infty$. In the present case with $\mu \neq 0$ one does not expect such an asymmetry between the v.e.v.'s of h and $\bar{h}$ and ω should be closer to one. It has been observed in refs. [330, 374, 343] that the case $\omega \approx 1$ is very special in that it admits the induction of SU(2)×U(1) breakdown with small top quark Yukawa coupling. We follow here the discussion of the excellent paper of Ibanez and Lopez [330], in which also the effects of g_{D}^{33} in the evolution are taken into account.

In a model with small Yukawa couplings (including g_{u}^{33}) one expects according to (7.47) and (7.50) a similar evolution of m_{H}^2 and $m_{\bar{\mathrm{H}}}^2$. There will be a slight difference because g_{u}^{33} has to be larger than g_{D}^{33} but this will not be significant if g_{u}^{33} is sufficiently small. In the cases discussed previously a large g_{u}^{33} was necessary to arrive at $m_{\bar{\mathrm{H}}}^2 < 0$ at low energies and now this will no longer be possible and one obtains $m_1^2 \approx m_2^2 > 0$ at low energies provided that we have chosen $m_1^2 = m_2^2$ at M_{x} which is the usual case. But even with $m_1^2 \approx m_2^2$ it is possible to satisfy (7.79), the condition for the breakdown of SU(2)×U(1). This, however, requires ω, the ratio of the v.e.v.'s of $\bar{h}$ and h to be close to one or according to (7.78) $\cos 2\theta \approx 0$. At first sight one might think that to arrive at such a situation at energies $Q^2 \sim (100\ \mathrm{GeV})^2$ one has to fine tune the parameters of the theory at M_{x} but this is not necessarily the case. To see this recall

$$\sin 2\theta = 2m_3^2/(m_1^2 + m_2^2)\,, \tag{7.63}$$

which gives us the evolution of θ in terms of the m_i^2. From the renormalization group equations (7.45) to (7.50) one obtains for small g_{u}^{33}

$$\mu \frac{\partial}{\partial\mu}(m_1^2 + m_2^2) \simeq -12\alpha_2 \tilde{m}_2^2 - 4\alpha_1 \tilde{m}_1^2\,, \qquad \mu \frac{\partial}{\partial\mu} m_3^2 \simeq (-3\alpha_2 - \alpha_1) m_3^2\,. \tag{7.80}$$

A calculation of the evolution of these quantities shows that for a large range of m_3^2 at M_{x} it will evolve to $2m_3^2 \approx m_1^2 + m_2^2$ at low energies. For $0.6 \leq 2m_3^2/(m_1^2 + m_2^2) \leq 1$ at M_{x} one will actually obtain $\sin 2\theta \approx 1$ at M_{W}, and the input parameters have not been fine tuned to a large accuracy to obtain the behavior. If therefore $m_3^2 = B\mu m_{3/2}$ is of comparable size as $m_{3/2}^2$ at the grand unification scale the breakdown of SU(2)×U(1) can be induced in absence of a large top quark mass and the only lower bound on m_{top} is given by the phenomenological bound of 20 GeV. Thus even in a minimal model with $A < 3$ the bounds in (7.61) and (7.74) on m_{top} do apply only in the special circumstances they were derived ($\mu \approx 0$).

The relation between $\sin 2\theta(M_{\mathrm{x}})$ and the mass of the top quark is shown in fig. 7.15. The usual input parameters have been used for the integration of the renormalization group equation and the figure shows the result for those cases in which (7.79) is fulfilled at $Q^2 \sim (100\ \mathrm{GeV})^2$. We see that for a large enough $\sin 2\theta$ at M_{x} the breakdown of SU(2)×U(1) occurs for small values of the top quark mass provided that the gaugino masses m_0 at M_{x} are not to large. Very large gaugino masses (compared to $m_{3/2}$) will through (7.80) induce large values of $m_1^2 + m_2^2$ at low energies and this will make it impossible to have $\sin 2\theta$ close to one at $Q^2 \approx (100\ \mathrm{GeV})^2$. A different view of the relations between the relevant parameters is shown in fig. 7.16. It shows that for a choice of $0.6 \leq \sin 2\theta(M_{\mathrm{x}}) \leq 0.7$ the SU(2)×U(1) breakdown with small m_{top} and $m_0 = 0$ can be obtained. For large values of m_0 the same behavior can only occur if $\sin 2\theta(M_{\mathrm{x}})$ is chosen to be closer to one.

The phenomenology of the models with $\mu \sim m_{3/2}$ is different from that of the models discussed previously. Of course the spectrum of the scalar partners of quarks and leptons is in general not very much influenced by μ. It is mainly given by $m_{3/2}$ and m_0. The gaugino masses are essentially determined by m_0 and if this quantity is not too large the photino is a good candidate for the lightest R-odd particle. Differences occur in the Higgs sector. In models with $\mu \approx 0$ we had identified three states with mass

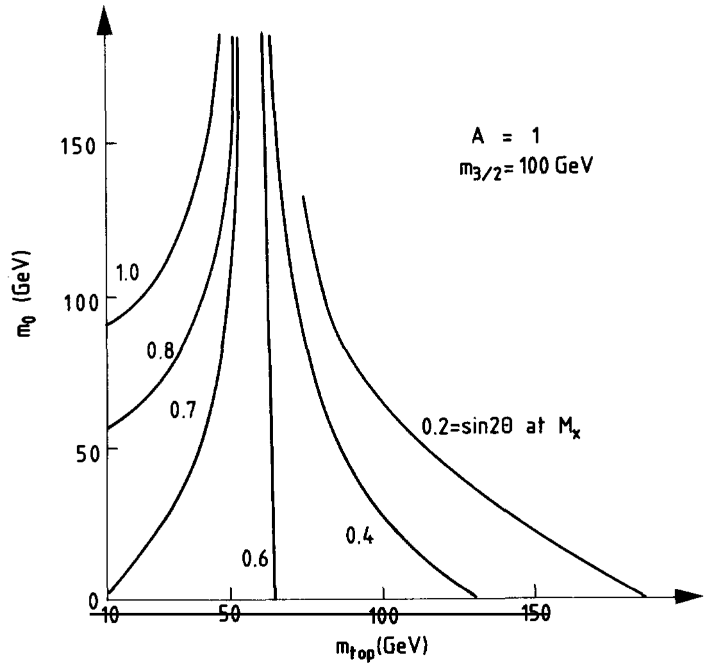

Fig. 7.15. The relation between m_0 and m_{top} for various input values of $\sin 2\theta$ (defined in (7.64)) as given in ref. [330]. For $\sin 2\theta \gtrsim 0.65$ the breakdown of SU(2) × U(1) can even be induced for small values of m_{top}.

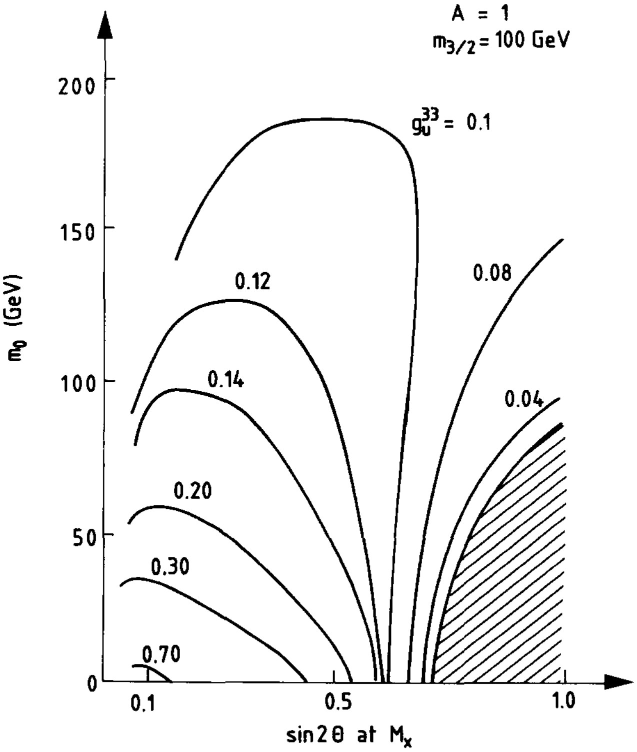

Fig. 7.16. Relation between $\sin 2\theta(M_x)$ and gaugino masses m_0 as given in ref. [330].

proportional to $\mu + \mathcal{O}(\mu^2/m_{3/2})$; they will no longer stay light in the present case since $\mu \sim m_{3/2}$. There are, however, other states which might be light in these models. To see this, let us consider the case $\sin 2\theta(M_W) = 1$. We know that this cannot be exactly true since it implies $m_1^2 = m_2^2$ at M_W and this can only happen if $g_u^{33} = g_D^{33}$ exactly which in this case implies also $m_{\text{bottom}} = m_{\text{top}}$ since $\langle h\rangle = \langle \bar{h}\rangle$, but in a realistic model $\sin 2\theta$ can be very close to one and the mechanism is easier to explain in this special limit. With $\sin 2\theta = 1$ we have $m_1^2 + m_2^2 = 2|m_3|^2$ (compare (7.53)) and since $m_1^2 = m_2^2$

$$V = m^2[|h|^2 + |\bar{h}|^2 + (h\bar{h} + h^*\bar{h}^*)] + \tfrac{1}{8}(e_1^2 + e_2^2)[|h|^2 - |\bar{h}|^2]^2 . \qquad (7.81)$$

Inequality (7.53) was a condition for the potential to be bounded from below. Here the equality holds and this implies that this potential has a flat direction and this in turn implies the existence of a massless scalar particle. Inspection of (7.81) shows that this particle is the real part of $h^0 + \bar{h}^0$. It is a scalar particle and should not be confused with the pseudoscalar "axion" we have encountered in models with small μ. For the above-mentioned reasons we have seen that $\sin 2\theta(M_W)$ cannot be exactly equal to one, but in cases where it is close to one and where consequently $\langle h\rangle \approx \langle \bar{h}\rangle$ we expect the existence of a light neutral scalar particle. This particle has similar properties as the single Higgs in the nonsupersymmetric standard model that survives the $SU(2) \times U(1)$ breakdown.

In table 7.2 we give seven examples of the low energy spectrum of models with moderately large μ and various values of m_{top}, $m_{3/2}$, m_0 and A. The previously discussed neutral scalar Higgs corresponds to H_b and it is only very light (as in models 1 and 2) if $\sin 2\theta(M_W)$ is close to one. The interested reader is invited to study this table carefully to see how the various possible input parameters influence the low energy particle spectrum. We only note here in addition that models of the type 6 have a tendency to predict light scalar partners of the neutrinos [330]. These particles are neutral and unlike in the case of the other partners of quarks and leptons they can be lighter than 20 GeV. In model 6 actually the

Table 7.2
Low energy spectrum of seven models for various ranges of the input parameters as given in ref. [330]. The masses are given in units of GeV. $H_{a,b,c}$ denote the mass eigenstates of the three neutral Higgses that survive the Higgs mechanism.

	1	2	3	4	5	6	7
m_{top}	29	29	30	63	111	111	160
$m_{3/2}$	25	50	100	50	200	50	150
m_0	17	5	71	141	108	49	6
A	1	1	1	1	0.5	1	1
$\sin 2\theta(M_x)$	0.787	0.679	0.779	0.76	0.195	0.038	0.187
$\sin 2\theta(M_W)$	0.999	0.999	0.996	0.42	0.43	0.43	0.45
Partners of quarks	50–52	51–52	215–220	380–396	300–363	120–150	143–160
Partners of leptons	26–28	49–50	105–114	84–122	207–219	67–75	138–156
						$\tilde{\nu}_L \sim 16$	
Gluino	50	15	214	423	324	147	19
Majorana wino mass	14	4	58	115	88	40	5
Bino	7	2	29	57	44	20	3
Charged Higgses	86	104	170	136	230	81	163
Neutral Higgses H_a	96	112	173	118	219	89	147
Neutral Higgses H_b	5	7	23	80	83	23	81
Neutral Higgses H_c	37	69	151	112	217	24	143

partners of the neutrinos are the lightest R-odd particles, since with the choice of $m_0 \approx m_{3/2} \approx 50$ GeV the photino is relatively heavy. The reason for the appearance of these light particles can be traced back to a negative contribution of the $\frac{1}{2}D^2$-term (compare (7.13)) to the mass matrix proportional to $m_z^2/2 \cos 2\theta$ which only occurs if $\langle h \rangle$ and $\langle \bar{h} \rangle$ are sufficiently different. In such a situation the upper components of the SU(2) doublets (like φ_ν, φ_u, φ_c, φ_t) have smaller masses than the lower components (like φ_e, φ_d) and the singlets (like $\varphi_{\bar{e}}$, $\varphi_{\bar{d}}$) and the partners of the neutrinos are the lightest of these since they receive the smallest radiative contribution from the gauginos. The presence of nonvanishing m_0 is necessary in such a model to avoid a charged scalar (e.g. the partner of the u-quark) to have a mass less than 20 GeV.

We will stop here for a moment our discussion of minimal models (i.e. models with minimal particle content in the low energy sector). In addition to the input parameters already needed in the nonsupersymmetric case four new parameters enter here which replace the parameters of the Higgs sector in ordinary theories. These are the gravitino mass $m_{3/2}$, gaugino masses m_0, the A parameter for the trilinear couplings and the quantity μ for the $H\bar{H}$-term in the superpotential. The reader might now take his own personal choice for these parameters and construct his favorite model. SU(2)×U(1) breakdown is induced through radiative corrections and in some cases a large mass of the top quark is needed to achieve this. The parameters $m_{3/2}$ and A result directly from the choice of the hidden sector. Nonvanishing m_0 can only occur in the case of broken supersymmetry and as a result the natural range of m_0 is given by $m_{3/2}$. Natural values for μ are either M_P or zero (i.e. $\mu \ll m_{3/2}$) and we still have not found a satisfactory mechanism that explains $\mu \sim m_{3/2}$. The necessity of $\mu \neq 0$ in a minimal model is still a somewhat unsatisfactory feature. The alternative to an explicit insertion of μ in the superpotential is the choice of a nonminimal model including a light singlet superfield. Such a model has also naturalness problems as discussed in section 7.4. In addition we had the apparent contradiction that for a breakdown at SU(2)×U(1) at the tree level one needed $A > 3$ but this also implied the existence of deeper $U(1)_{em}$ breaking minima after the inclusion of the Yukawa couplings. However, there we had only considered the parameters as they were given at M_x and the relevant parameters at low energies still had to be determined. Such a calculation has been done by Derendinger and Savoy [112] in a model based on a superpotential

$$g = \lambda Y H\bar{H} + \sigma Y^3 + \text{Yukawa terms}. \tag{7.82}$$

In addition to the renormalization group equations given before we now have additional equations for λ, σ, A_λ and A_σ. Starting with a universal A at M_x one has now to compute the evolution of these parameters and one has to find a situation where the inequalities in (7.56) like

$$m_H^2 + m_E^2 + m_L^2 > (|A_E|^2/3) m_{3/2}^2 \tag{7.56}$$

are satisfied for A_E, A_U and A_D at low energies to avoid the undesired minima. To have SU(2)×U(1) breakdown, however, the corresponding constraints on A_λ and A_σ should not be satisfied and one requires

$$3m_Y^2 < (|A_\sigma|^2/3) m_{3/2}^2, \qquad m_Y^2 + m_H^2 + m_{\bar{H}}^2 < (|A_\lambda|^2/3) m_{3/2}^2. \tag{7.83}$$

It has been shown that such a situation can be realized for a large range of parameters in these models. In general it seems that a simultaneous validity of (7.56) and (7.83) requires the initial A to be chosen to be less than three. Needless to say that this breakdown of $SU(2)\times U(1)$ is independent of the value of the top quark mass.

7.7. Rare processes

We have already discussed possible additional contributions to flavor changing neutral currents in supersymmetric models in section 4.6. They were shown to be absent if all the masses of the scalar partners of quarks and leptons with same helicity and same charge were degenerate. In the supergravity models discussed so far this has been the case: all these particles had equal mass: but only at M_x. At lower energies they differed, however, according to the renormalization group equations (7.48) to (7.50) and one has to face this problem again. The contributions to the scalar masses that arise from the gaugino masses are not dangerous: the scalars with same quantum numbers with respect to $SU(3)\times SU(2)\times U(1)$ receive the same contributions and we have only to worry about those terms in the renormalization group equations that contain Yukawa couplings. Usually these Yukawa couplings are quite small and one might think that these effects could be neglected. Some models have a large Yukawa coupling to the top quark and this case has to be investigated [151a, 144].

As usual the most sensitive bounds are provided by the experimental information on the K_L–K_S mass matrix. The most important contribution from supersymmetry in this context is given by gluino exchange diagrams as discussed earlier simply because α_3 is much larger than α_2 and α_1 at low energies. It turns out now that such a contribution to the K_L–K_S mass matrix is present in the earlier discussed supergravity models as has been pointed out in ref. [144]. To see this consider the mass matrices M_U for the u, c, t quarks and M_D for the d, s, b quarks and let us choose M_U to be diagonal. M_D will then be off diagonal and contain the usual Kobayashi–Maskawa angles. In a supergravity model with parameters defined at M_X the mass matrices of the scalars will then be

$$\mathcal{M}_U^2 = m_{3/2}^2 \mathbb{1} + M_U^2\,, \qquad \mathcal{M}_D^2 = m_{3/2}^2 \mathbb{1} + M_D^2\,, \tag{7.84}$$

where we have used $A = 0$ to render our formulas simpler. Nonzero A might also have an influence on flavor changing neutral currents but these contributions are small compared to those we will discuss in a moment. At M_x now we see that $\mathcal{M}_U^2$ is diagonal and $\mathcal{M}_D^2$ contains the angles. Both M_U, $\mathcal{M}_U^2$ and M_D, $\mathcal{M}_D^2$ can be diagonalized simultaneously (they are in phase) and there is no possibility to induce flavor changing neutral currents via gluino exchange. This changes after the evolution of $\mathcal{M}_U^2$ and $\mathcal{M}_D^2$ down to lower energies. Apart from contributions that are diagonal and universal in flavor space we will get additional contributions from the Yukawa couplings. In particular this will give an additional contribution to $\mathcal{M}_U^2$ that is proportional to M_D^2 and whose coefficient is of order one, (the small Yukawa couplings are already absorbed in M_D^2) and we will also have an additional contribution M_U^2 to $\mathcal{M}_D^2$ again with coefficient of order one. As a result of this $\mathcal{M}_U^2$ and $\mathcal{M}_D^2$ are in phase. To compute the angles the first term proportional to $m_{3/2}^2$ in (7.84) is irrelevant. What counts is the ratio of diagonal elements and the *differences* of the diagonal elements. Since these differences in M_U are much larger than those in M_D we thus arrive at a situation in which M_U, $\mathcal{M}_U^2$ and $\mathcal{M}_D^2$ are essentially diagonal matrices. What is important here is the fact that M_D and $\mathcal{M}_D^2$ can no longer be diagonalized simultaneously. This then implies the existence of flavor changing gluino vertices like the one shown in fig. 7.17, with coupling constants $e_3 \sin\theta$ where θ is the corresponding Kobayashi–Maskawa angle. As a result we obtain a contribution to the real part of the K_L–K_S mass matrix due to the usual box graph with gluino exchange. The magnitude of this contribution depends on the value of the gluino mass and the masses of the scalar

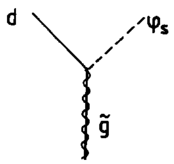

Fig. 7.17. Flavor changing gluino quark scalar vertex.

partners of quarks. The former is determined by m_0 and the magnitude of the latter ones is determined predominantly by the diagonal contributions from $m_{3/2}$ directly and m_0 through the evolution. The scalar partners of quarks are therefore still pretty degenerate at least for the first two generations and we can assume $\Delta m^2 = m_i^2 - m_j^2 \ll m_i^2$. To give you a feeling about the magnitude of the gluino exchange contribution to the K_L–K_S mass matrix let us consider a model where the gluino mass is equal to the masses of the partners of quarks. This is approximately true for a wide class of models and in models where these masses differ one in general obtains similar results. We now consider the effect of the scalars of the first two generations where we can draw conclusions because of the knowledge of the Cabibbo angle. One obtains

$$(M_W^2/\tilde{m}_3^6)(\Delta m^2)^2 \sin^2\theta_c \cos^2\theta_c \lesssim 2\times 10^{-6}\,, \tag{7.85}$$

where $\sin^2\theta_c \sim 1/20$. The quantity Δm^2 has to be estimated using the renormalization group equation. For typical models with small Yukawa couplings the coefficient of the additional contribution proportional to M_U^2 in $\mathcal{M}_D^2$ turns out to be somewhat larger than one. For the first two generation the dominant mass term comes from $m_{\rm charm}$ and we estimate

$$\Delta m^2 \simeq (2\ \mathrm{GeV})^2\,. \tag{7.86}$$

From (7.85) we then obtain the bound

$$\tilde{m}_3 \gtrsim 35\ \mathrm{GeV} \tag{7.87}$$

which in general holds if the mass m^2 of the partners of the d, s quarks is less or equal to $\tilde{m}_3$. The bound in (7.87) can only be lowered if m is larger than $\tilde{m}_3 \sim 40$ GeV. To estimate the influence of a large top quark mass is more difficult because we do not know the K–M angles between the first two and the third generation. If one uses $\sin^2\theta_c$, $\sin\theta_c$ for the angles between first and third, second and third generation and estimates $m_{\rm top}^2/\tilde{m}_3^2 \sim 1/3$ for a large top quark mass one obtains

$$\tilde{m}_3 \gtrsim 200\ \mathrm{GeV}\,, \tag{7.88}$$

a rather dramatic number. One should however use the information from the K_L–K_S mass matrix to obtain restrictions on the mixing angles instead of deriving (7.88).

In the nonsupersymmetric theory one can use the simultaneous knowledge of the K_L–K_S mass matrix and the bound on $K_L \to \mu\mu$ to compute an upper bound on the $m_{\rm top}$ [67]. This bound changes if one also takes into account the contributions of newly introduced supersymmetric particles [390]. A full calculation of these restrictions on $m_{\rm top}$ including the gluino exchange contributions is however not yet available. For the various aspects see refs. [44, 72, 144, 176, 247, 337, 390].

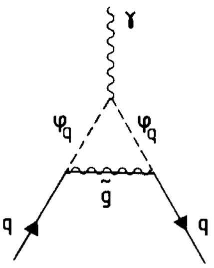

Fig. 7.18. Contribution to the electric dipole moment of the neutron through gluino exchange. q and φ_q denote a quark and its partner, respectively.

More restrictions on the parameters in the theory could come from the imaginary part of the K_L–K_S mass matrix and the question of CP violation. In this context, however, the information on the electric dipole moment of the neutron (except for the considerations [250] of ε'/ε) gives stronger constraints and we prefer to discuss this process.

The effects of supersymmetry on the value of the electric dipole moment d_n of the neutron have been estimated [161] and computed [66] in the context of a general model with soft supersymmetry breakdown. The most dangerous contributions to d_n come from gluino exchange diagrams as shown in fig. 7.18 and they could be as large as $d_n/e \sim 4\times 10^{-22}$ cm compared to the experimental limit $d_n/e < 6\times 10^{-25}$ cm. Gluino exchange could however be avoided if in the context of supergravity models A and m_0 have the same phase. In addition to this source of CP violation there exist other sources like a phase in the K–M matrix or a contribution of θ_{QCD} (the strong CP problem) [28, 103]. If one puts all these phases to zero by hand, problems with d_n can be avoided. The influence of θ_{QCD} can be naturally suppressed in the presence of an invisible axion [360, 138, 568, 449] which in the context of supersymmetry works in the same way as in ordinary models. In a supergravity model one might now start with such a prescription to have all these phases to be equal to zero. On the other hand we know that there is CP violation in the K_L–K_S system. To describe this one includes a phase δ in the K–M matrix. In addition one would like to have CP violation for a cosmological baryon number generation. This usually involves heavy fields and as such it gives no dangerous contribution to d_n. The only way a sizable contribution to d_n can occur is through the renormalization of θ if one has a phase in the mass matrix of these heavy particles. This, however, can be avoided by the choice of an invisible axion or a generation of the baryon asymmetry in higher loops.

As a result of all this one has to insert the phase δ in a supergravity model at the grand unification scale and one chooses $\theta = 0$ (either explicitly or through a Peccei–Quinn symmetry) as well as a vanishing phase between A and m_0. In a next step one has to determine these phases at lower energy with the help of the renormalization group equations. Since CP violation is present through δ a phase between A and m_0 will be generated perturbatively and at low energies one has a gluino contribution to d_n. For gluino masses of order of 100 GeV dangerous contributions to $d_n \sim 10^{-22} e$ cm × CP-violating phases will then be generated at least for a large range of parameters [66, 482]. To avoid problems with the experimental bound the parameters of the model have to be carefully chosen. First of all we have the choice of either putting the K–M phase in the up or the down quark sector. In the nonsupersymmetric case these two choices were equivalent but here with the appearance of the new phase between A and m_0 this is no longer true. Because of the difference of the masses in the two sectors one obtains a smaller contribution to d_n if CP violation originates in the down quark (d, s, b) mass matrix and where M_U is real and there is no phase between A and m_0 at M_x. In such cases the contribution to d_n can be pushed somewhat below the experimental bound even in the presence of gluino masses of order of

50 GeV [432]. With CP violation in the up quark sector this is somewhat harder to achieve and at least an axion is needed to suppress θ. CP violation in the context of supersymmetry is expected to give contributions to d_n not far from the present experimental limit, at least one expects these contributions to be larger than in the ordinary case. For additional discussions see refs. [8, 55, 66, 83, 105, 106, 151, 161, 247, 250, 266, 432, 456, 458, 482]. Other rare processes or experimentally exactly determined quantities lead to a similar situation. In the context of supersymmetry one can avoid real problems with the constraints but sometimes the contributions are expected not very far from the present limits. In some models we have seen that a large value of the mass of the top quark is required. This might also cause some potential problems in the low energy phenomenology like a deviation of the ρ-parameter [537] ($\rho = M_W/M_Z \cos\theta$) from one but they occur only for $m_{top} \gtrsim 300$ GeV [14, 47]. Strong restrictions on the validity of a large class of model will however occur if the mass of the top quark is experimentally determined. The restrictions from $g-2$ on the considered models are in general not severe [46, 166, 232, 289, 373]. Constraints from proton decay have already been discussed earlier [11, 54, 76, 108, 111, 126, 169, 179, 334, 393, 406, 436, 487, 491, 492, 493, 501]. They are essentially the same as in the globally supersymmetric case.

7.8. Beyond the minimal models

Up to now we have mainly discussed the structure of the low energy sector of models with minimal (or close to minimal) particle content and have only occasionally discussed more complicated models. The low energy parameters arise from the ones at higher energy and the effective low energy potential has to be computed. These parameters $m_{3/2}$, m_0, A and μ depend on the structure of the model at higher energies and through these parameters we could learn something about the high energy sector. There is not much more that constrains this high energy sector except from cosmological considerations which we discuss in the next chapter. The only constraints from particle physics are given through the proton decay experiments and they just tell us that $M_x > 10^{15}$ GeV. Consequently there is much freedom in constructing various models for the high energy and hidden sector and constraints come only from theoretical considerations, prejudices and taste. One has of course always to make sure that these various high energy scenarios do not lead to wrong predictions for low energy experiments. There are very many suggestions concerning the high energy properties of these theories and we will attempt to describe the major directions of research (see refs. [61, 81, 142, 149, 184, 185, 186, 187, 260, 264, 327, 333, 375, 377, 378, 379, 397, 399, 401, 426, 428, 433, 452, 457]).

We start with a discussion of M_x. In a model with minimal particle content from $M_W \sim 100$ GeV up to M_x this quantity is fixed to be somewhat above 10^{16} GeV [125, 154]. In principle, however, it could be larger and for example as large as M_P. Such an increase of M_x could be achieved by adding new degrees of freedom to the theory between M_W and M_x such that the gauge coupling constants e_1, e_2, e_3 join at higher energies. This requires some care to preserve the prediction for $\sin^2\theta_w$ but this is in general not a very strong constraint. Such models have the nice theoretical property that instead of the two scales M_x and M_P they now contain only one. On the other hand with $M_P = M_x$ we have to face the problem that in principle we have to understand gravity in order to make definite predictions from the grand unified sector contrary to the case $M_x \ll M_P$ where we can neglect gravitational corrections. Such models do in general not need a separate hidden sector since the fields that break SU(5) have already v.e.v.'s of order M_P and they can induce the breakdown of supergravity as well. In a model where this breakdown is achieved through an adjoint representation the goldstino (absorbed by the gravitino) will then correspond to the fermionic $SU(3)\times SU(2)\times U(1)$ singlet in this representation. To have a supersymmetry breakdown scale at $M_S \sim 10^{11}$ GeV one has to introduce a small mass parameter in the grand unified superpotential (just as one does usually in the hidden sector), but such a situation might be actually desirable from cosmological considerations as we will see later [428]. It also implies in general the existence of new low energy fields in the TeV range but they are usually phenomenologically unproblematic and even needed to raise M_x to M_P.

A similar situation can arise with inverted hierarchy models [456–467, 131, 519]. Here one starts with the input scale $M_x \sim 10^{11}$ GeV and generates a large scale via the mechanism discussed in Chapter 5. In principle one could even imagine that in such a model the scale M_P is generated (with $M_x = M_P$). It seems however hard to induce in such a way gravitational interactions as well. In absence of such a mechanism one couples these models to supergravity and also inserts M_P. The slowly decreasing directions in the potential will then be turned up through gravitational effects and the natural scale for the vacuum expectation values will be M_P [131]. These models have of course some additional particles in the region of 10^{11} GeV (e.g. more than two Higgs in the $(5, \bar{5})$ representation to avoid proton decay via dimension five operators) but again one needs this to raise M_x to M_P. A variant of such models can also be obtained if one just starts with the two scales M_P and $m_{3/2} \sim 100$ GeV in a supergravity model [264, 397]. Taking such a model in the flat limit certain flat directions in the potential might occur out of which the grand unification scale M_x (not necessarily equal to M_P) could be induced via dimensional transmutation. Here, however, it seems in general to be somewhat difficult to keep those particles light (e.g. the Higgs doublets) that should remain light. They in fact might even receive large v.e.v.'s. Apart from this not much more can be said in this context as the things that were already discussed in the framework of models based on global supersymmetry and the cosmological considerations which will be postponed until the next chapter.

Apart from these considerations of various grand unified sectors additional possibilities can be obtained through more general terms in the superpotential: so-called "nonrenormalizable" terms [180]. Up to now we have only discussed superpotentials that are at most trilinear in the superfields to assure the renormalizability of the theory in the flat limit. With nonrenormalizable gravitational interactions at least in the framework of $N = 1$ supergravity one could now consider a more general situation and add terms to the superpotential that are quartic and higher in the fields. With our discussion of the $N = 1$ supergravity potential at the tree graph level one might even argue that we describe the gravity sector by an effective potential and that one should include all terms consistent with the symmetries of the theory since otherwise they would be generated in perturbation theory. Now we do not really know what happens in such theories beyond the tree graph level and especially if F-terms are generated; we do not even know a satisfactory theory of gravity. Nonetheless such nonrenormalizable terms could exist and they should be investigated. One should however require that the dimensionful coupling constants of these nonrenormalizable term are given in powers of $\kappa = 1/M$. This is obvious from our discussion before and also required by renormalizibility in the flat limit. In the case where all fields have at most vacuum expectation values of order M_W such terms are irrelevant. If, however, we include grand unification such terms might become important, since they can induce renormalizable couplings in the low energy theory with coupling constants $(M_x/M)^n$ in terms where n heavy fields are replaced by their v.e.v.'s of order M_x. Such couplings are comparable to the Yukawa couplings in the low energy theory and might play a significant role. For $M = M_x$ this discussion becomes of course problematic. Let us now discuss some examples. In a grand unified model with matter in the $\bar{5}$ and 10 representations of SU(5) we could add a term [180]

$$(\lambda/M)(10 \times 10 \times 10 \times \bar{5}) \tag{7.89}$$

in the superpotential. Such a term violates baryon number and causes proton decay in a similar way as the earlier discussed dimension 5 operators. In fact these terms are dangerous and to be compatible with the experimental limit on the proton lifetime λ has to be smaller than 10^{-6} were we would have naively expected λ to be of order one [169]. We do however not know how to determine λ theoretically.

But nonrenormalizable terms do not always cause problems, they sometimes can do nice things. They for example can lead to a simplification of grand unified models in which the Higgs doublets remain massless at the grand unification scale for group theoretical reasons [379]. In the usual case this requires large SU(5) representations like 75, 50 and $\overline{50}$. With nonrenormalizable couplings we can do without the 75. Consider

$$g = M'(50\times\overline{50}) + \overline{50}\bar{H}\left(\frac{\lambda_1}{M}\Sigma^2 + \frac{\lambda^2}{M^2}\Sigma^3 + \cdots\right) + 50H\left(\frac{\lambda_1'}{M}\Sigma^2 + \frac{\lambda_2'}{M^2}\Sigma^3 + \cdots\right) \tag{7.90}$$

where Σ is an adjoint representation of SU(5). The usual vacuum expectation value of Σ at M_x breaks SU(5) to SU(3)$\times$SU(2)$\times$U(1). Nonetheless the Higgs doublets remain massless whereas the Higgs triplets become heavy.

Nonrenormalizable terms can also be used to generate the small Yukawa couplings and to construct a hierarchy between the masses of fermions in different generations [431]. One assumes that the masses of the third generation occur directly through the usual Yukawa couplings and that the masses of the second and first family only proceed through nonrenormalizable terms

$$(\lambda/M^n)(24)^n \times \bar{\mathrm{H}} \times \bar{5} \times 10\,, \tag{7.91}$$

with $n=1$ for the second and $n=2$ for the first generation to arrive at a mass ratio $\lambda : \lambda'(M_x/M) : \lambda''(M_x/M)^2$ for the three generations. The hard problem remains an explanation of the absence of renormalizable Yukawa couplings for the first and second generation. Investigations of the problem of the fermion masses have also been done by inducing light fermion masses through radiative corrections [323, 387, 392, 414, 472a].

An additional application of nonrenormalizable terms is a relation between the various mass scales M, M_x, M_S and $m_{3/2}$. One could for example choose as high energy superpotential [428]

$$g = \frac{\lambda_1}{M}X^4 + \frac{\lambda_2}{M^2}X^2\,\mathrm{Tr}(\Sigma^3) + m^2(Z + \Delta) \tag{7.92}$$

with singlets X and Z and Σ in the adjoint representation of SU(5). Δ is a constant used to adjust the cosmological constant. In such a model one obtains $M_x^2 \sim Mm$ and $m_{3/2} \sim m^2/M$ out of the two input scales M and m. Using various powers of X in the first two terms any grand unified scale $m \leq M_x \leq M$ can be obtained. For such a mechanism to work, however, renormalizable terms (like $\mathrm{Tr}(\Sigma^3)$) have to be absent in the full superpotential.

Finally one could use nonrenormalizable terms to generate a $\mu H\bar{H}$ term in the low energy superpotential with $\mu \sim m_{3/2}$ [365]. One starts with a Peccei–Quinn symmetry [474] to forbid such a term for the renormalizable part of the superpotential to avoid μ to be as large as M_x. This Peccei–Quinn symmetry is broken spontaneously at $M_{PQ} = 10^{10}$ GeV to produce an acceptable invisible axion [118, 2, 125, 485] and solve the strong CP problem. Nonrenormalizable terms allowed by the

Peccei–Quinn symmetry then induce a mass term $m_3^2(h\bar{h} + h^*\bar{h}^*)$ in the low energy scalar potential with

$$m_3^2 = \lambda A m_{3/2}(M_{\mathrm{PQ}}^2/M) = \mathcal{O}(m_{3/2}^2)\,, \tag{7.93}$$

after the $\mathrm{U}(1)_{\mathrm{PQ}}$ breaking fields have been replaced by their v.e.v.'s. $M_{\mathrm{PQ}} \sim M_{\mathrm{S}}$ is crucial to obtain this result. Actual models that incorporate this mechanism, however, have a tendency to require a complicated grand unified sector. This seems inevitable in models with an unproblematic invisible axion [363].

The sector even less constrained than the grand unified sector is the hidden sector, as the name already indicates. More general cases than the ones we have discussed earlier like for example [378]

$$g(z) = \lambda(b + z)\exp(cz) \tag{7.94}$$

have been proposed. λ, b, c are constants used to obtain the desired cosmological constant and supersymmetry breaking scale. One common property of all these hidden sectors is the fact that in general z will receive a v.e.v. of order of M_{p} and the appearance of the desired small quantity $m_{3/2}/M_{\mathrm{p}}$ enters through the explicit choice of input parameters. This small quantity is obtained either by explicit introduction or by the use of the exponential function that allows the generation of this extremely small ratio $m_{3/2}/M_{\mathrm{p}}$ out of a moderately small parameter [378, 184, 61].

It is crucial for our approach to get an idea why M_{S} is so small compared to M_{p}. But this seems to be a difficult task. It might come from a dynamical mechanism or from properties of extended supergravities, and we would have to understand gravity to solve this problem. This actually seems inevitable if the fields in the hidden sector receive v.e.v.'s which are as large as M_{p}. But maybe one could even understand M_{S} in absence of a complete knowledge of the gravity theory. In such a situation one would think that the v.e.v.'s of the hidden fields are small compared to M_{p} and one could for example imagine that M_{S} and $m_{3/2}$ are generated from M_{p} and M_x. This ratio $M_{\mathrm{S}}/M_{\mathrm{p}}$ as well as the value of the cosmological constant remain a puzzle for the time being. One possibility to explore is the question whether they are related.

Some attempts have been made to construct classes of models in which the cosmological constant is bound to vanish [99]. They are based on the observation that the Kähler potential

$$G(z, z^*) = -\tfrac{3}{2}\log(f(z) + f(z^*))^2 \tag{7.95}$$

leads to a scalar potential

$$V = 9\exp(\tfrac{4}{3}G)\left(\frac{\partial^2 G}{\partial z\partial z^*}\right)^{-1}\frac{\partial^2}{\partial z\partial z^*}\exp(-\tfrac{1}{3}G) \tag{7.96}$$

that vanishes identically [79]. Observe that in this case the Kähler manifold is no longer flat ($\partial^2 R/\partial z\partial z^* = 0$) (compare (3.48)), but obeys

$$R_{zz^*} = \tfrac{2}{3}G_{zz^*}\,, \tag{7.97}$$

and according to the usual terminology might be called an Einstein–Kähler manifold. There is a huge vacuum degeneracy and the choice of different points leads to different gravitino masses

$$m_{3/2}^2 = \exp(-G) = 1/|f(z) + f(z^*)|^3 . \tag{7.98}$$

In order to preserve the vanishing cosmological constant at the tree graph level after the introduction of the observable sector one adds the Kähler potential of the two sectors (compare (6.27))

$$G = G(z, z^*) + G(y_a, y_a^*) \tag{7.99}$$

where the y_a denote the observable fields and one in general chooses minimal kinetic terms for this sector to obtain the usual positive $\mathrm{STr}\,\mathcal{M}^2$. As shown in section 6.3 one then arrives at a low energy effective potential with $A = 3$ [96], and the cosmological constant vanishes at the tree graph level. Radiative corrections change A and induce a cosmological constant in perturbation theory. One might actually try to obtain the gravitino mass via dimensional transmutation from radiative corrections along one of the flat directions of the scalar potential. Taking into account only nongravitational interactions a small gravitino mass could be obtained in this way [173]. If, however, there exist also gravitational radiative corrections (i.e. if they are not forbidden by new nonrenormalization theorems) one would rather expect $m_{3/2}$ to be of order of M_{P} if it is nonzero, especially since these models tend to contain fields with v.e.v. large compared to M_{P} [361].

The Kähler potential in (7.95) is equivalent to

$$G = -\tfrac{3}{2}\log(z + z^*)^2 \tag{7.100}$$

up to field redefinitions $z \to f(z)$. The scalar Lagrangian corresponding to (7.100) is given by

$$3\sqrt{-g}\,g^{\mu\nu}[\partial_\mu z \partial^\mu z^*/(z + z^*)^2] , \tag{7.101}$$

and coincides with the scalar Lagrangian of pure $N = 4$ supergravity. In particular it has an SU(1, 1) noncompact symmetry

$$z \to (\alpha z + \mathrm{i}\beta)/(\mathrm{i}\gamma z + \delta) . \tag{7.102}$$

The complete $N = 4$ supergravity Lagrangian is fully SU(1, 1) symmetric [102]. In the framework of $N = 1$ supergravity, however, it is hard to imagine how this SU(1, 1) symmetry can survive especially after the introduction of the observable sector. But in principle it could be such a symmetry which requires the cosmological constant to be absent. It probably has to be broken in the breakdown of extended supergravities to simple $N = 1$ supergravity. But even in this case one could imagine a solution of the problem of the cosmological constant in the same way as a spontaneously broken Peccei–Quinn symmetry solves the strong CP problem.

The consideration of extended supergravities might give us some hints how to construct hidden sectors with certain nice properties. But the big problem of embedding $N = 1$ supergravity with complex matter representations in extended supergravity is of course not yet addressed in this context.

To close this section let us mention again one additional assumption made so far in all our discussions. We have implicitly assumed explicit baryon and lepton number conservation in the superpotential of the low energy (100 GeV) sector (compare section 4.4). This led inevitably to a conservation of R-parity (4.25) in the low energy theory, and as a consequence the lightest R-odd particle (e.g. the photino) is absolutely stable. In principle this requirement could be given up as well but one has to be careful. Having baryon violation at this level will usually lead to disastrous proton decay rates. It is easier to relax the requirement of exact lepton number conservation, but also there constraints from Majorana neutrino masses, neutrino oscillations etc. have to be taken into account as discussed in ref. [308]. These questions and the stability of the lightest R-odd particle will be important in the context of cosmological considerations to which we will devote the next chapter.

At the time this report is written the discussion of these nonminimal models is still in progress. We cannot discuss all the possibilities here. One of them is the enlargement of the grand unification group [23, 70, 84, 246, 322, 362, 398, 407, 408, 582, 583] but there are many others. For further information we refer the reader to the papers cited at the beginning of this section or to the existing review articles refs. [33, 34, 35, 38, 130, 157, 158, 159, 160, 193, 207, 212, 214, 220, 221, 222, 223, 326, 347, 425, 440, 445, 447, 460, 464, 479, 486, 506, 588].

8. Constraints from cosmological considerations

In this chapter we want to discuss possible constraints on the presented models that might arise from cosmology. This will include constraints on the mass of the lightest R-odd particle predicted by supersymmetry, the mass of the gravitino, the nature of the SU(5) phase transition in the context of supersymmetry, the cosmological baryon asymmetry, monopole abundances and related problems. We cannot give here a review on standard hot big bang cosmology and refer the reader unfamiliar to the subject to consult the standard reviews. We will mainly concentrate on those issues that are directly related to the presence of supersymmetry.

8.1. *Constraints on (almost) stable particles*

In a wide class of the discussed supersymmetric models one expects the presence of an additional stable particle, the lightest R-odd particle. In the globally supersymmetric models this is in general the exactly massless goldstino. In local supersymmetry it will be either the gravitino (in case of $M_S \ll 10^{10}$ GeV) or the photino, fermionic partners of Higgses, scalar partners of neutrinos or may be even other particles. In general we expect those particles to be electrically neutral. If they were charged they should be heavier than 20 GeV for the well-known reasons [4, 32, 52, 53, 58, 529]. The fact that the stable particle is electrically neutral is also favored cosmologically. Charged stable particles would condense in conventional matter with a density far above experimental upper limits on stable exotic relics from the big bang [514, 575].

Fortunately we have not to face these problems here but only the milder restrictions on stable neutral particles. Such restrictions arise from the upper limit on the mass density $2\times10^{-29}\ \mathrm{g/cm^3}$ of the universe derived from estimates of the Hubble constant and the deceleration parameter. Given the cosmological abundance and the mass of a neutral stable particle one has to check whether this bound on the mass density is violated or not. Such considerations have first been applied to obtain bounds on the masses of heavy neutrinos [396]. In the standard big band cosmology we expect the number density of each kind of neutrinos to be 6/11 the number density of photons in the 3K black body background radiation, hence $300\ \mathrm{cm^{-3}}$. The upper bound on the mass density of the universe then gives an upper bound on the neutrino masses: 40 eV.

This bound is true as long as our estimate of the present cosmological abundance of neutrinos is correct. It has been pointed out by Lee and Weinberg [396] that the estimate on the present neutrino abundance is incorrect if the neutrinos are sufficiently heavy. We will present this argument here since it can later be carried over to our discussion of the stable R-odd particle. These particles will drop out of thermal equilibrium when the temperature decreases to a point at which the stable particle collision rates become comparable to the expansion rate of the universe which in the case of neutrinos is 10^{10}K. If the neutrinos are light (≤ 40 eV) this gives an abundance of $300\,\mathrm{cm}^{-3}$ as quoted above. If, however, the neutrinos are heavy ($m_\nu > 1\,\mathrm{MeV} \approx 10^{11}$K) we expect this number to be smaller by the value of the Boltzman factor $\exp(-m_\nu/[1\,\mathrm{MeV}])$ at the time the heavy neutrinos drop out of thermal equilibrium. One would then require

$$m_\nu \exp(-m_\nu/[1\,\mathrm{MeV}]) \leq 40\,\mathrm{eV}\,, \tag{8.1}$$

which leads to the bounds

$$m_\nu \leq 40\,\mathrm{eV}\,, \qquad \text{or} \qquad m\nu \geq 13\,\mathrm{MeV}\,, \tag{8.2}$$

according to this naive argument. But with these heavy particles we have to be more careful. These particles usually carry a conserved quantum number (R-parity or here neutrino number) which is of course the origin of their stability. As a result their number density can relax to the equilibrium value only through particle-antiparticle annihilation. At the temperature of 1 MeV the heavy neutrinos are so rare that their annihilation rate is already much less than the cosmic expansion rate, although there energy distribution is still thermalized. Consequently the heavy neutrinos go out of chemical equilibrium (their number density exceeds the equilibrium value) at a temperature T_f much larger than 10^{10}K. The condition (8.1) has then to be replaced by

$$m_\nu \exp(-m_\nu/kT_\mathrm{f}) \leq 40\,\mathrm{eV}\,. \tag{8.3}$$

Since $kT_\mathrm{f} > 1$ MeV the naive lower bound of 13 MeV is raised if one takes into account this fact. T_f now depends on the properties of the particles under consideration. For the heavy neutrinos it has been estimated to be about 5% of the heavy neutrino mass, and one obtains

$$m_\nu \leq 40\,\mathrm{eV}\,, \qquad \text{or} \qquad m_\nu \gtrsim 2\,\mathrm{GeV}\,, \tag{8.4}$$

considerably larger than the bound of 13 MeV obtained from the naive estimate. Such stable particles are therefore cosmologically unproblematic as long as their masses are not in the forbidden mass "window": here $40\,\mathrm{eV} \leq m_\nu \lesssim 2\,\mathrm{GeV}$.

We can now apply this cosmological constraint to our supersymmetric models. In the global case the lightest R-odd particle is the massless goldstino and no constraint arises provided that the other R-odd particles decay sufficiently fast into goldstinos and R-even particles. But even these models have eventually to be coupled to gravity and in the context of supersymmetry one would expect the super-Higgs effect to occur in which this goldstino is absorbed by the now massive gravitino $m_{3/2} \sim M_\mathrm{S}^2/M$ where M_S denotes the supersymmetry breakdown scale. As before we now expect a "window" of forbidden gravitino masses and consequently a range of M_S that is not allowed. Taking into account the expected primordial gravitino abundance and assuming it to survive until present times for light gravitinos one obtains $m_{3/2} \lesssim 1$ keV as was pointed out in ref. [472], which in turn implies $M_\mathrm{S} \lesssim 2 \times 10^6$ GeV. This is the bound that corresponds to $m_\nu < 40$ eV in the case of neutrinos.

This bound does not apply if the gravitino mass is heavy enough, as in the neutrino case where above 2 GeV no problem occurred. This lower bound now depends on the decoupling temperature and the gravitino annihilation rate: it here implies $m_{3/2} > 0.3 M_{\mathrm{p}}$, and the window of forbidden gravitino masses is quite large [554]. Can we now conclude that values $10^6\,\mathrm{GeV} \leq M_{\mathrm{S}} \lesssim M_{\mathrm{p}}$ are cosmologically forbidden? There are two reasons to believe that the answer to this question is in general no. First of all we have seen that in the considered models with large $m_{3/2}$ (think of $m_{3/2} \sim 100\,\mathrm{GeV}$) we no longer expect the gravitino to be the lightest R-odd particle and consequently it will decay and this will decrease its cosmological abundance. In these cases we will of course have another stable R-odd particle whose mass bounds we will discuss later. The decay of the gravitino in lighter R-odd particles will typically proceed with a rate $\Gamma \sim m_{3/2}^3/M_{\mathrm{p}}^2$ and the question remains how heavy the gravitino has to be to decay fast enough such that no cosmological problems arise. If they decay they will give rise to an increase of the cosmic entropy density. If cosmic nucleosynthesis were over by the time this occurred this would have led to too much helium production. It seems therefore necessary to require $m_{3/2}$ to be large enough to have them decay before nucleosynthesis, or that after there decay the reheating of the universe is large enough (>0.4 MeV). These considerations require $m_{3/2} \gtrsim 10^4\,\mathrm{GeV}$ as estimated in ref. [554]. This is still large and corresponds to a supersymmetry breaking scale $M_{\mathrm{S}} \gtrsim 10^{11}\,\mathrm{GeV}$.

But there is another way out of the problem, which has to do with the fact that on the one hand gravitinos are produced at temperatures $\kappa T \sim M_{\mathrm{p}}$ in the very early universe with the normal abundance, but on the other hand the constraints on $m_{3/2}$ arise because the gravitino decay rate is much less than the expansion rate of the universe until the temperature has dropped way below $m_{3/2}$. This is a problem which typically can be solved by inflation [298]. If the universe would have undergone an inflationary epoch this would have diluted the primordial gravitino density [174]. Above $m_{3/2}$ the production rate of the gravitinos is also very small and the primordial density will therefore not be recreated and hence no problems with the cosmic mass density will occur. An actual estimate of the amount of inflation needed to solve the "gravitino mass problem" shows that one needs less inflation than required for the solution of the "monopole" and "flatness" problem [298]. Taking into account these considerations we can conclude that no serious cosmological problems constrain the gravitino mass. In the case $1\,\mathrm{keV} \leq m_{3/2} \leq 10^4\,\mathrm{GeV}$ some amount of inflation is needed to dilute the primordial gravitino density [174].

In the conventional supergravity models the gravitino is not the lightest R-odd particle. In a large class of models the photino plays this role. In some cases this lightest particle can also be a neutral Higgs fermion or a scalar neutrino as we have discussed in the last chapter. Potential cosmological problems can in these cases not be cured by inflation in the very early universe, since after inflation their equilibrium density can be recreated (before they drop out of equilibrium) due to their stronger interactions.

Let us therefore first assume that the photino is the lightest R-odd particle and derive cosmological constraints on its mass. Such a photino shows similar behavior as a neutrino, but there are some differences. First of all its interactions are not so well known as those of the neutrino, which are characterized by the $\mathrm{SU}(2) \times \mathrm{U}(1)$ coupling constants and the masses M_{W} and M_{Z}. The interactions of the photino depend on the way supersymmetry is broken. A crucial parameter is the mass of the scalar partners of quarks and leptons since $\tilde{\gamma}\tilde{\gamma}$-creation and annihilation will proceed through exchange of these particles. The cosmological bound on $m_{\tilde{\gamma}}$ will therefore depend on these masses. Secondly, unlike the discussed heavy neutrino, $\tilde{\gamma}$ is not a Dirac but a Majorana fermion and as a result $\tilde{\gamma}\tilde{\gamma}$-annihilation is suppressed by p-wave phase space at low temperatures. Consequently the bounds on heavy neutrinos cannot be directly carried over to the photino but the whole calculation has to be reexamined. Such a calculation has been done in ref. [286]. The photinos cannot be as light as 40 eV, since they receive at

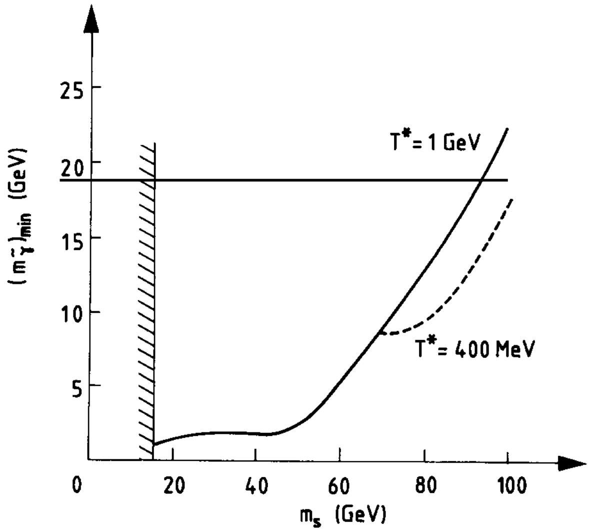

Fig. 8.1. Lower bound on photino mass ($m_{\tilde{\gamma}}$) if the photino is the stable R-odd particle as given in ref. [286]. The bound increases with increasing mass m_s of the scalar partners of quarks and leptons.

least radiative contributions from the quarks and leptons to their mass and as discussed earlier even in the absence of superheavy particles they receive a mass of at least 100 MeV. Consequently we have only to concentrate on the second cosmological (lower) bound on the photino mass. The result of the calculation is given in fig. 8.1, where the lower bound on the photino mass is plotted against m the mass of the scalar partners of quarks and leptons here assumed to be degenerate. For small m (the experimental limit is given approximately by 20 GeV) $m_{\tilde{\gamma}}$ has to be larger than 2 GeV, and this bound stays constant up to $m \sim 50$ GeV. This is due to the fact the s-wave annihilation of photinos into heavy fermions (like τ and c) is effective for $m \lesssim 50$ GeV whereas the annihilation into massless fermions is p-wave suppressed. For masses $m > 50$ GeV this annihilation of the cool photinos becomes more and more incomplete and the lower bound on $m_{\tilde{\gamma}}$ increases linearly with m. Above $m = 60$ GeV the bound depends somewhat on the choice of T^* related to the freeze-out temperature of the photinos [286, 381]. For $m = 100$ GeV the lower bound on $m_{\tilde{\gamma}}$ is of the order of 20 GeV which is a relatively large value. Looking back to the presented models, however, we see that in general these bounds on $m_{\tilde{\gamma}}$ can be easily fulfilled. So far our discussion on the photino mass.

Another candidate for the lightest R-odd particle is a massive neutral Dirac fermion formed as a combination of a left-handed Higgs fermion and a gaugino of SU(2) (compare our discussion in section 7.5). The cosmological lower bound on the mass of this fermion (if it is the lightest R-odd particle) is usually higher than the one on $m_{\tilde{\gamma}}$. This comes from the fact that the gauge couplings are usually larger than the Higgs couplings and as a result the cool gauginos annihilate more efficiently. These questions have been studied in detail in ref. [168]. In a typical model with small μ and $m_{3/2}$ and a light photino and Higgs gaugino ($\tilde{\mathrm{H}}$) the results can be presented in diagrams like given in fig. 8.2. $m_{\tilde{\gamma}}$ is plotted against $m_{\tilde{\mathrm{H}}}$ and the cosmologically allowed region is the straded area. The missing edge in the upper left hand corner is forbidden because of e^+e^- annihilation experimental constraints. In every model under consideration it should be checked whether these cosmological constraints are fulfilled. Usually they can be satisfied easily if the photino is the lightest R-odd particle. Since in most of the discussed models the photino is the lightest R-odd particle these cosmological constraints from the mass density of the

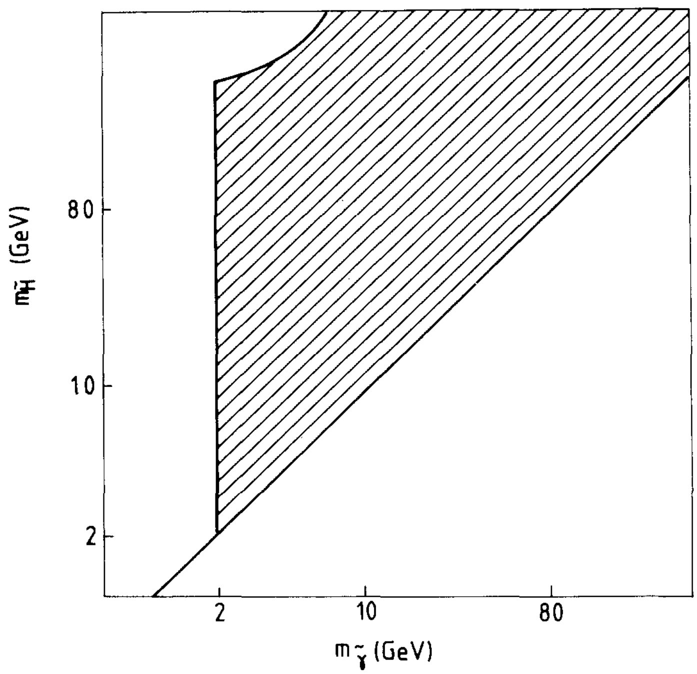

Fig. 8.2. The shaded area gives the cosmological allowed region in the "mass plane" of the photino ($m_{\tilde{\gamma}}$) and Higgsino ($m_{\tilde{H}}$) as given in ref. [168]. It is assumed that the lightest of these particles is stable.

universe are not very strong. In addition they could become completely irrelevant in models where R-parity is not an exact symmetry and hence the formerly stable R-odd particles could decay [308]. The case in which the scalar partner of the neutrino is the lightest R-odd particle has also been discussed in the literature – with similar conclusions [328]. Finally in models where the strong CP problem is solved with the introduction of an invisible axion, we expect the partner of this axion [528] (the axino; not to be confused with the saxino [363]) to be the lightest R-odd particle [361]. Since this particle is very light no cosmological problems will arise, provided that the photino decays fast enough. This lowers the bound on $m_{\tilde{\gamma}}$ compared to the bound given above [361].

8.2. The grand unified phase transition

In the first section we have considered processes in which the relevant freeze-out temperatures were of the order of $\kappa T \sim$ MeV or GeV. We will now discuss processes that happened at earlier times and higher temperatures. In the framework of grand unified models this implies a discussion of the SU(5) phase transition, the creation of a baryon asymmetry [494, 578, 127] and the production of magnetic monopoles [298].

Let us begin with an SU(5) gauge theory with one chiral superfield A in the adjoint representation and a superpotential

$$g = \mathrm{Tr}(\tfrac{1}{2}mA^2 + \tfrac{1}{3}\lambda A^3)\,. \tag{8.5}$$

In a globally supersymmetric model this leads to a scalar potential

$$V = \mathrm{Tr}(F^*F) + \tfrac{1}{2}D^\alpha D_\alpha\,, \tag{8.6}$$

with

$$F^* = -\partial g/\partial A\,. \tag{8.7}$$

The peculiar properties of such supersymmetric potentials is the existence of several degenerate vacuum states with $V = 0$, here

$$A = 0\,,\qquad A = (m/3\lambda)\,\mathrm{diag}(1,1,1,1,-4)\,,\qquad A = (m/\lambda)\,\mathrm{diag}(2,2,2,-3,-3)\,, \tag{8.8}$$

corresponding to unbroken gauge groups SU(5), SU(4) × U(1), SU(3) × SU(2) × U(1) respectively. m is a mass comparable to the grand unification scale M_X. In a locally supersymmetric model these states will get split but they will still remain approximately degenerate since their difference in energy will be very small compared to m^4. Let us now discusss this potential at high temperatures as done by Srednicki [517] and Nanopoulos and Tamvakis [434]. At temperatures large compared to m one has to add a temperature dependent term to the scalar potential [141]

$$\Delta V(T) = \tfrac{1}{8}T^2|\partial^2 g/\partial A_i\partial A_j|^2 + \tfrac{1}{4}T^2 g^2 C(R_i) A_i^* A_i\,, \tag{8.9}$$

where g is the gauge coupling and $C(R_i)$ is the eigenvalue of the quadratic Casimir operator for the representations under consideration. In our case this corresponds to [517, 434]

$$\Delta V(T) = \tfrac{1}{20}T^2(21\lambda^2 + 25g^2)\,\mathrm{Tr}(A^*A)\,. \tag{8.10}$$

In each of the three ground states (8.8) one has $F = D = 0$. At finite temperature these states split as a consequence of the additional term (8.10) in the potential. $\Delta V(T)$ is obviously minimized by $\langle A\rangle = 0$ and at high temperature the theory will favor the SU(5) symmetric phase. The approximation leading to (8.9) is only valid when T is large compared to m. At $T \sim m$ we cannot make definite statements but at low temperatures $T \ll m$ we can use the massless quantum gas approximation. The effective potential is given by the free energy of the system and one writes [434, 517]

$$V = -(\pi^2/90)T^4(N_B + \tfrac{7}{8}N_F) + \mathrm{Tr}(F^*F) + \tfrac{1}{2}D^2\,, \tag{8.11}$$

where $N_B(N_F)$ is the number of massless bosonic (fermionic) degrees of freedom at energies κT. From the three possible phases the most symmetric one (here with SU(5) symmetry) has the most massless degrees of freedom. Since the first term in (8.11) comes with a negative sign again the SU(5) phase has lowest energy as sketched in fig. 8.3. As for $T \gg m$ also for $T \ll m$ the symmetric phase is favored. This is different from the case of standard nonsupersymmetric SU(5) where the critical temperature for the SU(5) phase transition is of the order of m [298].

In the supersymmetric case the universe stays in the SU(5) symmetric phase still for temperatures

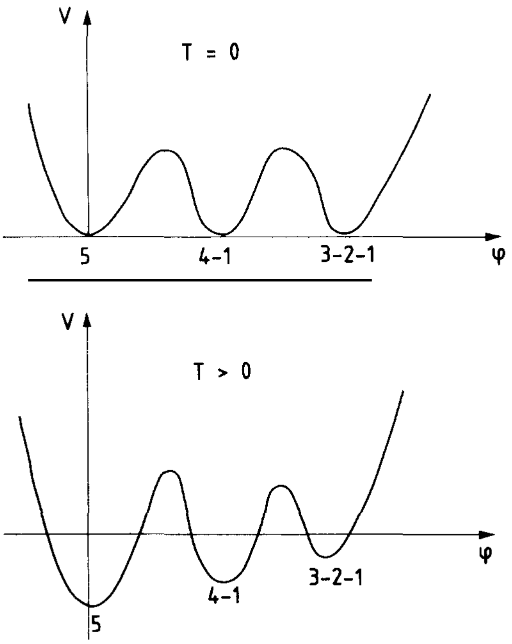

Fig. 8.3. Sketch of the scalar potential in a supersymmetric SU(5) model at zero and finite temperature.

that are small compared to M_x. One might now ask whether there is a possibility at all for a phase transition to the $SU(3)\times SU(2)\times U(1)$ phase at lower temperatures. According to (8.11) this can only happen if the numbers N_B and N_F of massless degrees of freedom change in the SU(5) phase if we go to lower energies and there is in fact such a possibility. To see this consider the running gauge coupling constants in each of the three phases. The SU(5) phase has the most massless degrees of freedom and the SU(5) coupling constant will therefore increase faster with decreasing energy as the respective coupling constants in the other phases. Let us now consider a model with A, three generations of quarks and leptons and two light Higgs doublets. With $\alpha_5 \sim 1/24$ and $M_x \sim 10^{16}$ GeV α_5 increases fast in the SU(5) phase and becomes of order unity at $\Lambda \sim 10^9$–10^{10} GeV while in the other phases the gauge coupling constants are still small. At Λ we expect confinement effects to occur in the SU(5) phase. If the strong interactions break supersymmetry one would expect an additional contribution of order Λ^4 to the vacuum energy density. If they do not break supersymmetry we would still expect the temperature independent part to have zero vacuum energy. The confinement effects do however decrease the number of the massless degrees of freedom at $T \sim \Lambda$. Massless particles have to be SU(5) singlet bound states and there are not very many of those one would expect. As a result of this the temperature dependent part of the effective potential in the SU(5) phase changes dramatically at Λ. Due to the confinement effects the vacuum energy in the SU(5) phase increases and will in general be higher than those of the other phases which fulfills thus a necessary criterion for a phase transition from SU(5) to $SU(3)\times SU(2)\times U(1)$ [517, 434].

With the specific model under consideration at $\Lambda \sim 10^9$ GeV, however, the phase with lowest vacuum energy is the $SU(4)\times U(1)$ phase. It has the most massless degrees of freedom. Consequently the SU(4) coupling constant increases fast down to lower energies and will become of order unity $\alpha_4 \sim 1$ at $\Lambda' \sim 10^6$ GeV and confinement effects will also occur in this phase while the coupling constant in the

$SU(3)\times SU(2)\times U(1)$ phase are still small. Below $\Lambda' \sim 10^6$ GeV we thus expect the $SU(3)\times SU(2)\times U(1)$ phase to have the lowest vacuum energy. The last part of our argument is model dependent. One could very well have situations in which the $SU(3)\times SU(2)\times U(1)$ phase is the lowest at $\Lambda \sim 10^9$ GeV such that a direct phase transition from SU(5) to $SU(3)\times SU(2)\times U(1)$ could occur there. For this to happen it is of course necessary to include more degrees of freedom in the theory [517]. This, however, is usually the case for other reasons as we will see later. Independently of this argument we have seen that at a certain critical temperature the effective potential evolves in a way that the desired $SU(3)\times SU(2)\times U(1)$ phase is energetically favored. This critical temperature lies somewhere between 10^6 and 10^{10} GeV and is therefore much smaller than in the nonsupersymmetric case where it is usually of the order of M_x.

This of course has certain implications, two of which we want to discuss now. The first one is the production of magnetic monopoles. These grand unified monopoles have a mass of order M_x/α_5 and are expected to be produced in a phase transition where a semisimple group breaks into a group with explicit U(1) factors [484].

Let us now suppose that the phase transition from SU(5) happens directly to $SU(3)\times SU(2)\times U(1)$ at $T_{crit} < 10^9$ GeV and let us suppose that the bubble nucleation rate is sufficient for percolation and a rapid phase transition, an assumption that is supported by the standard estimates if T_{crit} is so small [517]. The different bubbles are initially uncorrelated and arbitrary orientations of the Higgs fields in the adjoint representation occur. The bubble collision will then produce topological knots which become monopoles. The number density of the monopoles so produced is expected to be within a few orders of magnitude of the number density of bubbles and strongly depends on the unknown bubble nucleation rate. In the standard nonsupersymmetric grand unified models there appeared a problem since no bubble nucleation rate was compatible with both percolation and a sufficient suppression of monopole production to satisfy the observational bounds. This changes in the supersymmetric case and the main reason for that is the low critical temperature. If $T_{crit} < 10^9$ GeV for a large range of nucleation rates the transition is completed quickly and bubbles percolate. The thermal production of monopoles is heavily suppressed by the Boltzmann factor $\exp(-M_x/\alpha_5 T_{crit})$ and is compatible with observational bounds for the small values $T_{crit} < 10^9$ GeV. This is actually a nice property of the supersymmetric scenario [517, 434]. The SU(5) confinement scale leads to an upper bound on $T_{crit} \sim 10^9$ GeV and this is sufficient to suppress monopole production. In the nonsupersymmetric model one usually has to impose inflation to solve this problem [298].

But such a low critical temperature also has its problems and they show up if one considers the possible creation of a baryon asymmetry. Usually such a baryon–antibaryon asymmetry is generated by B and CP-violating decays of superheavy ($\sim M_x$) particles that go out of thermal equilibrium as the temperature drops below their mass [494]. In the supersymmetric case the universe stays very long in the SU(5) symmetric phase and these "heavy" particles are usually massless (e.g. SU(5) gauge fields) and stay in thermal equilibrium. The energy released during the phase transition is of order Λ^4 and this is not enough to create a sufficiently large number of these superheavy particles (in the $SU(3)\times SU(2)\times U(1)$ phase) to create an acceptable baryon–antibaryon asymmetry through their "out of equilibrium" decays.

To create an acceptable baryon asymmetry new particles with masses of order 10^{10} GeV (in the $SU(3)\times SU(2)\times U(1)$ phase) have to be introduced [436]. Some of them could be the Higgs triplets in the $(5+\bar{5})$ representations of SU(5). The absence of proton decay requires them to be heavy but since they only couple with small Yukawa couplings to the particles that participate in proton decay the present observational bounds on the proton lifetime do not really require their masses to be as large as

M_x. The bound is somewhere in the range of 10^{10} GeV and it is amazing how all these scales coincide in this range of 10^9 to 10^{11} GeV (including the supersymmetry breakdown scale). In a model with such small Higgs triplet masses one would then also expect proton decay to, e.g., μK because of the Yukawa couplings. One must however watch out in such a model with $M_H \sim 10^{10}$ GeV for proton decay via dimension 5 operators since this would lead to a far too small proton lifetime. This decay has to be forbidden and this can usually be achieved by the introduction of additional Higgs $(5+\bar{5})$ multiplets and some additional symmetries. One usually should make sure that there is only one set of light Higgs doublets ($\sim$100 GeV) in order to keep an acceptable value for $\sin^2\theta_w$ (although this also could be achieved in different ways). This increase in the number of Higgs particles is also necessary to introduce the required CP violation in this sector. This CP violation is usually put in through the couplings in this sector and not directly in the mass matrix [436]. If one would introduce it already at the level of the mass matrix this would lead to nonnegligible radiative contributions to the θ-angle which in turn would give unacceptable values for the electric dipole moment of the neutron as discussed earlier, unless an invisible axion is present.

During the phase transition at 10^9 GeV a sufficient number of these particles of 10^{10} GeV can now be produced. They are not thermalized and decay out of equilibrium. With a correct insertion of the CP-violating phases one can then obtain an acceptable baryon–antibaryon asymmetry. We should mention here that one can also construct other mechanisms to create a baryon asymmetry but the one discussed above seems to be the one which is the most straightforward and like the one above also the other ones need the introduction of additional particles whose only reason of existence is the need for the creation of the baryon asymmetry [436, 371].

Let us now look more closely at the SU(5) phase transition. Up to now we have always assumed that it happens but a closer inspection of the effective potential shows that this is not necessarily true [518, 175]. The intrinsic scale of the grand unified potential is given by M_x. Let us now consider the effective potential at 10^9 GeV and assume that the $SU(3)\times SU(2)\times U(1)$ phase has the lowest energy. The energies between the different phases are of order of 10^9 GeV, but there is a barrier between the different minima still given in magnitude by $M_x \sim 10^{17}$ GeV and huge. It is now the question whether such a high barrier will not prevent the phase transition from SU(5) to $SU(3)\times SU(2)\times U(1)$, and to decide this we have to estimate the bubble nucleation rate. In the semiclassical approximation this nucleation rate is given by [90, 71, 5, 403, 300]

$$\lambda(T) = K\exp(-S_0)\,, \tag{8.12}$$

where K is a determinantal factor and S_0 is the action of the "bounce" solution for the adjoint representation of the Higgs field

$$S_0 = \int_0^{1/T} \mathrm{d}t \int \mathrm{d}^3x [|\partial_\mu A|^2 + V(A)]\,, \tag{8.13}$$

and $V(A)$ is the free energy. An estimate of λ in the thin wall approximation gives [518]

$$\lambda(T_{\rm crit} = 10^9\,\mathrm{GeV}) = 10^{-10^{62}}\,(\mathrm{GeV})^4\,. \tag{8.14}$$

This number should be compared to the fourth power of the expansion rate of the universe at the same temperature T which is of the order of $(1\,\mathrm{GeV})^4$ at $T_{\mathrm{crit}} = 10^9\,\mathrm{GeV}$. One might now argue that the estimate in (8.14) was obtained under special assumptions which might be questionable but one can hardly imagine that it is wrong by 10^{62} orders of magnitude (observe that this is not 62 but 10^{62} orders of magnitude). Consequently a rapid phase transition in which many bubbles form and percolate is impossible in the present model. The universe will supercool in the SU(5) phase and will stay there essentially for ever [518].

One might now argue that in models based on supergravity the temperature independent part of the potential has already nondegenerate vacua and that the splitting of the vacua in the different phases might be larger than just the thermal splitting at $T \sim 10^9\,\mathrm{GeV}$, but these splittings are proportional to $\Delta E^4_{\mathrm{vac}} \sim (M_{\mathrm{x}} m_{3/2})^2$ and hence too small to enhance λ in (8.14) sufficiently. One might also consider examples for grand unified models in which the phase with unbroken gauge symmetry does not exist. But there in addition one has to make sure that the $\mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)}$ phase we want to end up with has the most massless degrees of freedom at all temperatures below M_{x}. This then also implies the absence of the $\mathrm{SU(4)} \times \mathrm{U(1)}$ phase and other phases with larger gauge symmetry than $\mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)}$ and this seems to be in general impossible (at least no example is known up to now). This means that we have no example where we understand why the $\mathrm{SU(3)} \times \mathrm{SU(1)} \times \mathrm{U(1)}$ phase might be chosen above M_{x}.

The only way to get out of this problem is to decrease the barrier that separates the more symmetric phases from the $\mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)}$ phase [429]. Unfortunately this always requires a fine tuning of the parameters of the theory or the insertion of very small parameters in the theory. Let us look at potential (8.6) to see what one can do:

$$V = \mathrm{Tr}(|mA + \lambda A^2|^2) + \tfrac{1}{2} g^2 (A^* T^{\mathrm{a}} A)^2 , \tag{8.15}$$

where T^{a} are the group generators. The A's should receive v.e.v.'s of order M_{x} which in general are given by the ratio m/λ. Thus we cannot change m/λ but we can change m and λ separately provided m/λ stays fixed. The second term $(\frac{1}{2}D^2)$ is fixed by the gauge coupling and we cannot do anything. The simplest way to suppress the barrier is to multiply the first term by a coupling constant σ and to adjust its value in a way that the phase transition can occur. The second term is still large, but there are directions in which $D = 0$ and tunnelling can occur there. The phase transition can occur at $T_{\mathrm{crit}} \sim 10^9\,\mathrm{GeV}$ if the barrier is not larger than let us say $10^{10}\,\mathrm{GeV}$. With $M_{\mathrm{x}} \sim 10^{16}\,\mathrm{GeV}$ we have thus to choose $\sigma \approx 10^{-12}$, indeed a very small number which is introduced by hand artificially. Because of this small σ one might also argue that the masses of the whole adjoint representations are small (although they have large v.e.v.'s) and have to be included in the computation of the running coupling constants. Such effects will be discussed in a moment. The introduction of $\sigma \sim 10^{-12}$ seems very artificial. One has therefore enlarged the model and has introduced additional singlet fields and more complicated superpotentials to suppress the barrier [429], but in these cases this needs some adjustments to incredible accuracy in the coupling constants of these singlets. Finally one could try to use nonrenormalizable terms as discussed in the last chapter to suppress the barrier [428]. This, however, in general only works with superpotentials in which all terms that contain particles with large v.e.v. are nonrenormalizable. The also possible renormalizable terms (like those in (8.5)) have to be suppressed by extremely small coupling constant like σ in our previous discussion and a solution to this fine-tuning problem can only be found if we find a reason why these renormalizable terms are small.

But independently of the way one achieves this fine tuning it has consequences for the low energy theory since it predicts new particles with masses in the TeV range. The potential (to be specific let us take (8.15) in which the first term is multiplied by σ) has relatively flat directions where the $\frac{1}{2}D^2$ term vanishes. This corresponds to particle masses of order σM_x if we consider the third solution of (8.8) corresponding to the SU(3) × SU(2) × U(1) phase [428]. Since σ had to be smaller than 10^{-12} to suppress the barrier sufficiently these masses have to be smaller than 10 TeV. Not all of the directions of the potential are flat only those where $\frac{1}{2}D^2$ vanishes and as a consequence not all of the components of A have to be that light. In the case in which A is the adjoint representation of SU(5) the light particles include an SU(3) octet, an SU(2) triplet and a singlet (exactly those we have already discussed in section 7.6). The presence of these light particles changes the value of M_x (it increases M_x by two orders of magnitude) and also the prediction for $\sin^2\theta_w$. Additional light fields have usually to be introduced and with these complications the model can be made to work.

This situation changes if the representation A is larger. One common choice for A is the 75 representation of SU(5). With such a choice the above described cosmological scenario becomes inconsistent [428]. To suppress the barrier many of the members of the 75 have to remain light. These light particles have to be included in the calculation of the evolution of α_5 in the SU(5) phase. The coupling α_5 had to become strong at energies below M_x to allow the SU(3) × SU(1) × U(1) phase to be energetically favored. With $A \triangleq 75$ this no longer works: α_5 decreases below M_x and the only possible choice for A remains the adjoint representation. As we have discussed in the last chapter one can also achieve a satisfactory splitting of the Higgs triplets and doublets by introducing nonrenormalizable terms. For further aspects of the SU(5) phase transition see refs. [375, 377, 378, 379, 383, 425, 428, 460].

8.3. *The cosmological scenario and models*

Most cosmological scenarios of the early universe today involve inflation, i.e., an epoch of exponential expansion [298]. For inflation in the context of supersymmetry see refs. [10, 174, 178, 178b, 181, 263, 267, 298, 313, 383, 404, 405, 427, 469, 535]. Such a mechanism allows us to explain the so-called flatness, isotropy, rotation and monopole density probelms. To introduce inflation some parameters in the potentials under consideration have to be choosen with extreme accuracy to have the inflationary epoch long enough to solve all these problems in a satisfactory way. Such a choice has also to be done in the supersymmetric case, with the difference that here one might claim such choices to be technically natural [178] (i.e. not upset by radiative corrections). But there is also another difference. In the usual nonsupersymmetric case inflation is supposed to solve the grand unified monopole density problem. Since these monopoles are produced in the grand unified phase transition the inflationary period to dilute the monopole density has to happen at a time where the temperature of the universe was smaller than M_x. This is not necessary in the supersymmetric cosmological scenario since there the SU(5) transition is supposed to happen at $T \lesssim 10^9$ GeV (with only minor reheating) and the production of monopoles is suppressed by a Boltzmann factor. In this case inflation could occur as well at times where the universe was still hotter than M_x. Such a primordial inflation [178b] solves all the other problems as well including the earlier discussed gravitino density problem (provided that inflation takes place below M_p). Primordial inflation has some additional nice properties, the fine tuning of the parameters in the potential becomes less acute and it might also be easier to avoid [178a] an overproduction of density perturbations [310, 579, 48, 299, 312, 520] $\delta\rho/\rho$ in connection with galaxy formation. As a result this primordial inflation is usually used in the context of supersymmetirc cosmological scenarios.

Such a scenario then starts with a singlet field ϕ (the inflaton) [178b] whose only role in life is to produce inflation. The parameters in the potential are carefully adjusted to have an inflationary period and to have it last long enough to solve the mentioned problems. In the context of supersymmetry one has in this case to rely on nonrenormalizable interactions in the superpotential. The mass scale m of the scalar potential of the inflation field should be of order $10^{-2}M_{\mathrm{P}}$ to have sufficient inflation and acceptable density perturbations. This then leads to a vacuum energy density in the false vacuum

$$V(\phi = 0) = \lambda(m^6/M^2) \simeq 10^{-14}M^4 \tag{8.16}$$

for a special choice of λ. During the rollover of the ϕ to its global minimum at $\langle\phi\rangle \sim M$ the universe inflates. The vacuum energy density at the completion of the phase transition is converted into the production of inflatons which dominate the universe. The mass of the inflaton field in this model is given approximately by

$$m_\varphi \simeq m^3/M^2 \simeq 10^{13}\,\mathrm{GeV}\,. \tag{8.17}$$

It is actually crucial for the model that m_φ has this value [426]. A smaller value would cause problems with the generation of a baryon–antibaryon symmetry whereas a larger value would cause a problem with too large a gravitino production after reheating. The inflatons eventually decay with rate proportional to m_φ^3/M^2 preferably with $\phi \to 3Y$ where Y is a newly introduced singlet. This happens through gravitational interactions and takes place at temperatures of the order of 10^{10} GeV. The singlet field Y was introduced to serve as a decay product for ϕ. It will also be coupled to the Higgs triplets and decay to these fields eventually. Y is supposed to have a mass of 10^{10} GeV and the parameters in the model are arranged in such a way that the $Y \to H_3\bar{H}_3$ decay happens at 10^7 GeV [460]. The universe is meanwhile in the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ phase and the out of equilibrium decay of the Higgs triplets with mass of order 10^{10} GeV produces the baryon asymmetry and this completes the cosmological scenario [460]. The SU(5) phase transition occurred as described in the last section and one had to suppress the potential barrier to make it possible which in turn implies the existence of additional light particles. As we have seen this scenario needs careful adjustments of the parameters in various stages to make it work. In addition also the introduction of the singlets ϕ and Y seems ad hoc only justified by the reasons they were invented for. Potentially additional problems could occur before primordial inflation [433] but they do not seem to be very serious. Recently also a potential entropy "crisis" [94] of supersymmetric theories has been found. This "crisis" is related to the existence of a simple hidden sector with just one field (like the Polonyi example). The crisis can be avoided by minor complications of the hidden sector [136]. We stop here our description of possible cosmological scenarios and their problems. This field is still in progress and it is by now premature to draw definite conclusions. We have not discussed cosmology in models based on the inverted hierarchy. Such questions are addressed in refs. [9, 29, 320, 367, 476].

At the end let us investigate the influence of cosmology on the low energy sector in the framework of the supergravity models. We have seen in section 8.1 that cosmological constraints from the mass density of the universe favor the photino to be the lightest R-odd particle although other possibilities are not ruled out. The main constraints from the grand unified phase transition in the scenario as described in section 8.2 imply the existence of new particles in the TeV range. They might however still be heavy enough and therefore have no immediate consequences for the physics at 100 GeV. This set of particles contain usually a light singlet. Since it is essentially decoupled from the other particles in the light sector this poses no additional naturalness problems as the one described earlier.

Other influences of the cosmological scenarios on the low energy sector of models are less direct and depend on the parameters chosen to obtain a cosmological acceptable scenario [375]. They can only be

Table 8.1
Light particle spectrum (in units of GeV) for cosmologically acceptable models (CAM) and minimal models (MIM) for various values of the top quark mass as given in ref. 375).

	CAM	MIM	CAM	MIM	CAM	MIM
A	3	3	2.8	2.8	2.0	1.6
$m_{3/2}$	15	15	15	15	15	15
m_0	42	33	48	47	53	29
μ	15	17	16	18	11	7
m_{top}	25	25	35	35	50	50
All families						
φ_ℓ	29	27	32	36	35	25
$\varphi_{\bar\ell}$	21	20	22	23	23	19
First two families						
φ_q	58	77	66	108	72	67
$\varphi_{\bar q}$	54	74	61	104	67	65
			60	103	66	
Third family						
φ_b	58	76	64	106	68	66
$\varphi_{\bar b}$	54	74	60	104	66	65
φ_t	81	96	95	132	112	106
$\varphi_{\bar t}$	26	54	23	78	21	37
Charged Higgses	96	93	95	94	88	83
Neutral Higgses	106	104	105	105	100	95
	3	3	4.4	5.3	6	5
	51	46	49	48	35	19
Gluinos	42	84	47	118	52	72
Photino	11	4.6	9.4	7.3	4	3
Winos, zinos Higgsinos	89	87	87	94	84	90
	78	82	79	79	82	75
	99	108	99	116	101	106
	92	85	91	80	88	83
	26	23	24	24	17	9

revealed after the solution of the renormalization group equations. Such a calculation has been performed in models where the earlier discussed parameter μ is of order of $m_{3/2}$ and where $m_{3/2}$ is relatively small. In table 8.1 we show the comparison of a minimal model (MIM) with a cosmologically acceptable model (CAM) for comparable input parameters in this class $\mu \sim m_{3/2}$ [375]. There is no striking difference between the low energy spectra of these models. Apart from that it might be possible that the low energy spectrum in a given minimal model with certain input parameters A, $m_{3/2}$, m_0 and μ can be reproduced in a cosmologically acceptable model with somewhat different input parameters. I do not know whether this happens in general but it seems very likely. A similar behavior is of course expected in models with small μ and/or small m_0. In the models shown in table 8.1 it seems that a rather large $m_0/m_{3/2}$ is needed. It might very well be that such a large m_0 is needed in all cosmological acceptable models but this is not necessarily the case. In the models presented in the table one needs a large m_0 because one has used a small value $m_{3/2} \sim 15$ GeV and has to arrive at a mass $m \geq 20$ GeV for the partners of the right-handed leptons. It seems therefore that the constraints from cosmology on the spectrum of the low energy theory are not very restrictive – except for the mass of the lightest R-odd particle – although a complete analysis is only available in models with $\mu \sim m_{3/2}$. The lower bound on the mass of the lightest R-odd particle can usually be satisfied easily if this particle coincides with the photino. For a more detailed discussion see refs. [375] and [167].

9. Experimental signatures

We want to devote this chapter to a discussion of the potential experimental signatures that could either verify or falsify the discussed models. One relevant process is proton decay which we have discussed earlier in detail. The prediction of the supersymmetric models depend strongly on the chosen grand unified sector. There is however one mode $p \to K^+\nu$ which seems to be only predicted in the context of supersymmetry although there it needs not necessarily to be present. A detection of such a mode as the dominant mode of proton decay would therefore strongly support supersymmetric ideas. On the other hand such a mode is not easily accessible experimentally. All other proton decay modes can probably be accommodated both in conventional and supersymmetric models, and there remains the question in which models this can be done more easily. A dominant decay $p \to \pi^0 e^+$ would certainly favor conventional models. Absence of proton decay to a level of 10^{33} years on the other hand would favor supersymmetric models since we in general expect there a larger value of M_x. But whatever happens it will not be conclusive (with exception of $p \to K\nu$).

The other potential experimental predictions of the supersymmetric models are concerned with the low energy spectrum of the models, i.e. the region of order of 100 GeV. But before we discusss this we have to warn the reader. There are no real specific predictions of these classes of models. Some models predict the existence of certain particles in a certain mass range but other equally acceptable models (from a theoretical point of view) do predict completely different things. As always it would be easier to verify than to falsify the whole approach. Such a falsification would require complete measurements up to an energy of let us say 1000 GeV proving the nonexistence of supersymmetric (R-odd) particles and possibly a detection of conventional Higgses. If this is done one probably would think that the idea of a possible connection of supersymmetry and the breakdown scale of $SU(2) \times U(1)$ is wrong, although some people might argue the boundary of 1 TeV could be increased somewhat. One should compare the situation in the supersymmetric context to the prediction of the conventional standard model Higgs sector. It predicts a neutral Higgs particle somewhere in the range between 7 GeV and let us say 1 TeV. We can certainly construct supersymmetric models that look like this model and where all the new structure is pushed up to higher energies. In such models it is however hard to link supersymmetry to the breakdown of the weak interactions and these models look very contrived.

On the other hand supersymmetry might be relevant at much lower energies (as happens in the theoretically most promising models) and this has to be checked experimentally. We will concentrate mainly on the predictions of the supergravity models which usually contain less low lying states and therefore have more predictive power. This discussion then usually also covers the most relevant signatures of globally supersymmetric models with the difference that the lowest lying state in the global case is the goldstino. The supergravity models at low energies look like a special type of explicitly broken supersymmetric models and as such cover all relevant models. The low energy spectrum of these models is characterized by four parameters as we have seen in Chapter 7. These are the gravitino mass $m_{3/2}$, the trilinear coupling A, the gaugino masses m_0 and the parameter μ. The gravitino mass and m_0

(at M_x) are in general considered to be of the same order of magnitude although one might also imagine $m_0 \ll m_{3/2} \approx 20$–$400\,\mathrm{GeV}$. A is a number with $|A| < 3$. For theoretical reasons one might think that $\mu \ll m_{3/2}$ but such a restriction might just come from our ignorance about the high energy sector and should not be posed in this context where we want to discuss possible experimental signatures of all models. Specific models now correspond to specific choices of these parameters and as such a whole variety of experimental consequences might occur. In the cases in which $m_{3/2} \sim m_0$ (i.e., m_0 not large compared to $m_{3/2}$) one expects in general the photino to be the lightest newly introduced supersymmetric particle (R-odd particle) although other possibilities also might occur as we have pointed out earlier. Usually such a particle is considered to be stable unless one breaks R-parity and allows explicit lepton number nonconservation. If it is stable there is a cosmological bound of $m \gtrsim 2\,\mathrm{GeV}$. One can avoid this bound only through a breakdown of R-parity, which is usually hard to achieve, or through the construction of models in which the lowest R-odd particle is essentially massless (i.e. masses at most of the order of eV). In the globally supersymmetric models there is a candidate for such a particle: the goldstino. In models based on supergravity where $m_{3/2}$ is of the order of 100 GeV there is usually not such a candidate. The photino will in general receive a radiative mass much larger than eV even if $m_0 = 0$. To have other R-odd particles, like for example the scalar partners of neutrinos, massless some very exact adjustments are necessary since they in general receive mass contributions of order $m_{3/2}$ at the tree level. The only possible exception might be the partner of the invisible axion. In such a case the photino could decay and the mass bound on $\tilde{\gamma}$ could be lowered [361].

Let us now discuss the possible experimental signatures. These investigations have recently drawn very much attention. They include the search for gluinos [73, 74, 153, 183, 283, 304, 318, 348, 349, 355, 441], photinos [211, 164, 261, 355, 489] and for the scalar partners of quarks and leptons [259, 282, 284, 285, 290, 311, 315, 316, 388, 488, 502] including scalar neutrinos [50, 253, 328, 350]. Also gluino bound states have been considered [80, 194, 420]. The possible production mechanisms for such particles have been considered to be e^+e^-, e^-e^- reactions as well as deep inelastic scattering [12, 15, 120, 344, 356, 380] and $p\bar{p}$ collisions, where special emphasis has been given to W and Z decays [556, 36, 49, 51, 78, 95, 119, 162, 168, 215, 292, 411, 556]. Recently also some review articles on the subject are available [506, 158, 347]. We cannot discuss here all these possibilities in detail and will just describe the major directions of research. Let us start with :

9.1. *The scalar partners of quarks and leptons*

These are, with the exception of the scalar neutrinos, *charged* spin 0 particles. Bounds on their masses are already available from e^+e^- data and they imply these masses to be larger than something like 20 GeV, just given by the upper limit on the available center of mass energy of present e^+e^- machines [4, 32, 52, 53, 58, 529]. To avoid problems with flavor changing neutral currents particles of this type with same charge and same helicity have to be almost degenerate in mass (as we have discussed earlier in detail) which also is a presently available experimental limit. In the standard models in which the parameters are fixed at a large scale and then evolve through renormalization group equation flavor changing neutral currents can also give an absolute mass limit on the partners of the down quarks (d, s, b) of $m \geq 35\,\mathrm{GeV}$. This limit arose from gluino contributions to the real part of the K_L–K_S mass matrix as discussed in Chapter 7, and thus gives no restrictions on the partners of the up quarks (u, c, t). Such restrictions could come from D^0–$\bar{D}^0$ mixings but there the experimental results are less precise. There are also no such restrictions on the absolute mass value of partners of leptons since there the usually dominant gluino contribution is necessarily absent.

The masses of these particles are determined primarily by $m_{3/2}$ and m_0 and their splittings by A. Let us first discuss the case $m_0 = 0$. At the large scale M_x one starts with degenerate masses given by $m_{3/2}$. The two real scalars corresponding to the original left-handed fermions will be split if A does not vanish (compare (7.76))

$$m_{1,2}^2 = m_{3/2}^2 + m_Q^2 \pm A m_{3/2} m_Q\,, \tag{9.1}$$

where m_Q is the mass of the corresponding quark. With $m_0 = 0$ this will essentially be the situation at low energies. This might be different for the partners of the top quark due to the large Yukawa coupling where also the restrictions of flavor changing neutral currents are less stringent due to our ignorance of the top quark K–M angles. With $m_0 = 0$ all these particles are degenerate at the two mass values in (9.1) with the exception of the top scalars. Indeed in this case one expects one of the partners of the top quark to be the lightest partner of quarks and leptons. With $m_0 \neq 0$ this situation changes since they receive different radiative contributions to their masses at low energies. With m_0 comparable to $m_{3/2}$ one then expects the partners of quarks to be heaviest and the lightest of these particles will be the partners of the right-handed leptons since they only couple to hypercharge. In some models with large μ one could as well arrive at small $m \leq 20\,\mathrm{GeV}$ masses for the scalar neutrinos [330] but let us first concentrate on the charged scalars.

The experimentally cleanest way to detect these particles is a study of e^+e^- annihilation. Since they always appear in groups of at least three essentially degenerate in mass a first sign of their existence should show up in the total cross section where every particle gives a contribution of $\frac{1}{3}Q^2$ to R and Q is the electric charge. Once produced, these charged scalars will decay rapidly since for cosmological reasons they cannot be stable as we have discussed in the last chapter. The heavier ones could decay in cascades to the light ones and the light ones will then decay in $q\tilde{g}$ or $q\tilde{\gamma}$, where q denotes the corresponding quark and $\tilde{g}(\tilde{\gamma})$ the gluino (photino). The former decay mode of course occurs only in those models where the gluino mass is smaller than the one of the decaying particle. The gluino subsequently decays into $q\bar{q}\tilde{\gamma}$ where q, $\bar{q}$ denote light quarks. In globally supersymmetric models one would in general have $\tilde{g} \to gG$ where g is a gluon and G the goldstino. Independently of the special way these decays proceed we will finally have at least one of the lightest R-odd particle in the final state. Such particles are neutral stable particles which interact very weakly and will probably only be detectable as "missing energy" very similar to the neutrinos in ordinary decays of heavy leptons. A typical decay of these partners of quarks and leptons will thus proceed in this way; the final decay products will contain one or several of the lightest R-odd particles (to be specific let us assume that this is $\tilde{\gamma}$) which leave the detector unobserved. The signature is the cleanest if $\tilde{g}$ is heavier than the partners of quarks and leptons. In this case $\tilde{q}$ or $\tilde{\ell}$ will decay in $q(\ell) + \tilde{\gamma}$ and one will just observe $q(\ell)$ and missing energy out of which one could in general easily reconstruct the mass of the decaying particle. Independently of the fact whether $\tilde{\gamma}$ is the lightest R-odd particle or not one would expect this decay to occur, since one can hardly construct a model along the standard lines in which $\tilde{\gamma}$ is heavier than the charged partners of quarks and leptons. These particles, if they exist, are therefore easily detectable in e^+e^--annihilation experiments. This detection is of course not possible for the scalar partners of the neutrinos and also we have to wait some time for the e^+e^- machines of the next generation (SLC and LEP) to come into operation [506].

In the meantime one would therefore also like to investigate different ways to produce these particles. The highest center of mass energies today can be obtained at the $p\bar{p}$ colliders in which recently the weak gauge bosons W and Z have been detected. If allowed by phase space the scalar partners of quarks and leptons can be produced as decay products of W and Z^0. This also can now give information on the scalar partners of the neutrinos in the possible processes [36] $W \to \tilde{e}\tilde{\nu}$ or $Z \to \tilde{\nu}\tilde{\nu}$. Production and decay rates for these processes have been studied extensively in the literature and such processes were shown to be observable if they are kinematically allowed. Outside the W and Z resonances we can also expect these scalar partners of quarks and leptons $\tilde{q}(\tilde{\ell})$ to be produced provided their masses are small enough. This could either happen through the production $q\bar{q} \to \text{gluon} \to \tilde{q}\bar{\tilde{q}}$ or through $q\tilde{g} \to \tilde{q}$ where $\tilde{g}$ is a component in the "sea" of the proton [380]. Rates for these processes have been computed and in certain favorable cases could be even larger than $p\bar{p} \to W \to e\bar{\nu} + X$ [506]. We thus can expect that with an increase of the luminosity in the $p\bar{p}$ collider new bounds on the masses of $\tilde{q}$ and $\tilde{\ell}$ will become available although the signals are not as clean as the ones in e^+e^- machines at comparable energies. Similar conclusions can also be drawn for production of $\tilde{q}$ and $\tilde{\ell}$ in ep colliders (like projected in HERA) where they can either be produced directly in charged and neutral current reactions or by associated production. Rates for these processes have also been computed. The signatures are comparable to those in $p\bar{p}$ reactions.

9.2. Photinos and gluinos

In most models we expect the photino ($\tilde{\gamma}$) to be the lightest R-odd particle. It is then a stable neutral Majorana fermion with $m_{\tilde{\gamma}} \geq 2$ GeV. With other particles it can interact mainly through the exchange of scalar partners of quarks and leptons which are believed to be heavier than 20 GeV. It thus has only feeble interactions just like the neutrino in conventional models where its interactions are described by the exchange of W or Z bosons. We thus cannot expect its direct observation in presently available detectors. The photino will be a final decay product in any decay of heavier R-odd particles and will leave the detectors as missing energy. It could also be produced in $p\bar{p} \to \tilde{\gamma}\tilde{g} + X$ but a direct detection is not possible. In e^+e^- annihilation it could be produced [211, 164] via $e^+e^- \to \gamma\tilde{\gamma}\tilde{\gamma}$ (fig. 9.1) similar to the process $e + e^- \to \gamma\nu\bar{\nu}$. In fact all that has been said about neutrino detection can be carried over to the photino. This includes the influence on the decay width of the Z^0 the ψ, Υ or toponium [73] and also toponium to $\gamma\tilde{\gamma}\tilde{\gamma}$ decays. For a very heavy top quark, the decay $\text{top} \to \tilde{\text{top}} + \tilde{\gamma}$ might also be possible and proceed with a large branching ratio [183]. All these processes could give indirect information on the photino.

This is different for the gluino which also has strong interactions. The $\tilde{\gamma}$ and $\tilde{g}$ masses give information on the input parameter m_0 in the theory. We have seen in our earlier discussion that even if m_0 were small at the tree-level an effective m_0 will in general be generated through the heavy particles in the theory. The gluino–photino mass ratio is in general given by $m_{\tilde{g}}/m_{\tilde{\gamma}} \sim 3\alpha_3/8\alpha_{em}$ and it would therefore be surprising if $m_{\tilde{g}}$ would be smaller than let us say 5 GeV.

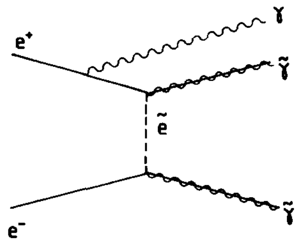

Fig. 9.1. Photino search in e^+e^- annihilation similar to neutrino counting. The cross section here depends crucially on the masses of the scalar partners of the electron. A careful measurement of this process at presently existing e^+e^- machines could give new information on $m_{\tilde{e}}$.

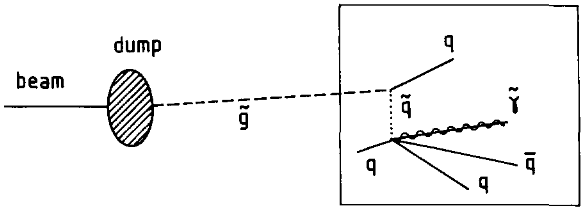

Fig. 9.2. Sketch of the gluino search in beam dump experiments.

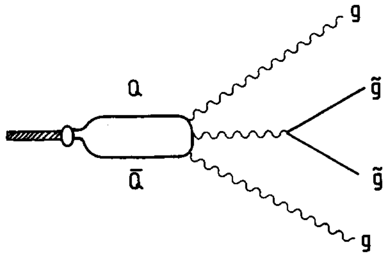

Fig. 9.3. Gluino search in quarkonium decay. This graph corresponds to a process that is independent of the mass of the Q partner.

There exist already some (model-dependent) experimental lower bounds on the mass of $\tilde{g}$ from beam dump experiments [27, 82] as sketched in fig. 9.2. To recover an information on $m_{\tilde{g}}$ from such a measurement depends on the parameters of the model under consideration. The crucial parameter is the mass of the partners of quarks ($\tilde{q}$). The absence of any event as the one shown in the figure gives $m_{\tilde{g}} \gtrsim 1$ GeV for $m_{\tilde{q}} = 100$ GeV or $m_{\tilde{g}} \gtrsim 4$ GeV if $m_{\tilde{q}} = 40$ GeV. Other information on gluino masses can come from quarkonium (QQ) decay. There is the possibility of $(Q\bar{Q}) \to gg\tilde{g}\tilde{g}$ as shown in fig. 9.3, a process that apart from the usual coupling constants solely depends on the gluino mass [73]. In such a process it seems however to be hard to isolate the produced gluinos. They will in general decay to light quarks and a photino $\tilde{g} \to q\bar{q}\tilde{\gamma}$. To isolate $\tilde{g}$ in the above process one thus has to look at four jet events in which two of them have missing energy. According to the decay $\tilde{g} \to q\bar{q}\tilde{\gamma}$ the gluino appears in its signatures as a heavy neutral lepton and all the methods to search for such particles can also be used for the gluino.

With respect to quarkonium decays one can have additional gluino decay modes like the ones displayed in fig. 9.4, $Q\bar{Q} \to \tilde{g}\tilde{g}$ or $g\tilde{g}\tilde{g}$. They lead to cleaner signatures (especially $\tilde{g}\tilde{g}$) than the decay previously discussed, but their rate depends on the mass of the scalar partner of Q. These rates have been computed for wide ranges of $m_{\tilde{g}}$ and $m_{\tilde{Q}}$ and it has been shown that the branching ratios could be substantial at least for the toponium case. One can therefore expect information on $m_{\tilde{g}}$ once toponium has been discovered [73, 74, 153, 183, 283, 304, 318, 348, 349, 355, 441].

Other than that gluino production could occur in the e^+e^- continuum in ep and $p\bar{p}$ scattering along the lines already discussed earlier. The gluino contribution could play a major role [380, 15] in the evolution of the nucleon structure functions at large Q^2. This would then also lead to the earlier discussed process $q + \tilde{g} \to \tilde{q}$ in $p\bar{p}$ scattering, where $\tilde{g}$ is in the "sea" of $\bar{p}$. We cannot discuss all these processes in detail and refer the reader to the original literature [164, 211, 261, 355, 489, 259, 282, 284, 285, 290, 311, 315, 316, 388, 488, 502].

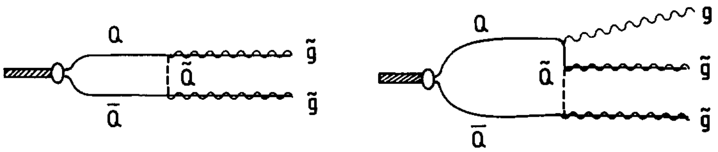

Fig. 9.4. Gluino production in quarkonium decay. The rate of these processes depends on the mass of the scalar partners of Q.

9.3. Wino, zino and Higgsino signals

A lot of work has been done to study production and decay of such particles since in a large class of models some of these particles might be lighter than the W and the Z [49, 51, 78, 95, 119, 162, 168, 215, 292, 411, 556].

Let us start with the charged gauginos and Higgsinos. In the breakdown of SU(2) × U(1) gauginos and Higgsinos will combine to form Dirac fermions in the supersymmetric Higgs effect. If we consider the minimal model we have to consider the left handed Weyl fermions $\tilde{\mathrm{H}}^-$, $\tilde{\mathrm{H}}^+$ and the partners $\lambda^\pm$ or $\mathrm{W}^\pm$ with mass matrix (compare (7.75))

$$\begin{array}{cc} & \begin{array}{cc} \lambda^+ & \tilde{\mathrm{H}}^+ \end{array} \\ \mathcal{M} = & \begin{pmatrix} \tilde{m}_2 & e_2 v_2 \\ e_2 v_1 & \mu \end{pmatrix} \end{array} \begin{array}{c} \lambda^- \\ \tilde{\mathrm{H}}^- \end{array} \tag{9.2}$$

in which v_1, v_2 denote the vacuum expectation values of the scalar Higgses ($M_{\mathrm{W}} = \frac{1}{2}e_2^2(v_1^2 + v_2^2)$, e_2 the SU(2) gauge coupling constant, $\tilde{m}_2$ the $\lambda^+\lambda^-$ mass term obtained from m_0 and μ the mass parameter in front of $\mathrm{H}\bar{\mathrm{H}}$ in the superpotential. With m_0 and μ large, these particles will of course all be heavy, but let us discuss here the cases where $\tilde{m}_2$ and/or μ are small compared to M_{W}.

Let us first consider $\tilde{m}_2 = 0$. We compute $\mathcal{M}\mathcal{M}^+$ and obtain the eigenvalues

$$\mathcal{M}^2 = \tfrac{1}{2}(e_2^2(v_1^2 + v_2^2) + \mu^2) \pm \tfrac{1}{2}[e_2^4(v_1^2 - v_2^2)^2 + 2\mu^2 e_2^2(v_1^2 + v_2^2) + \mu^4]^{1/2}\,, \tag{9.3}$$

which implies that one of the charged fermions is lighter than M_{W}. This light particle in the case $\tilde{m}_2 = 0$ has predominantly λ^+, λ^- components. Similarly one could consider the case $\mu \to 0$, $\tilde{m}_2 \neq 0$ and one again obtains a fermion with mass smaller than M_{W}, it now has predominantly $\tilde{\mathrm{H}}^+\tilde{\mathrm{H}}^-$ components. The limit in which both $\tilde{m}_2$ and μ are small (the Wiggsino limit) [292] implies the existence of a light charged fermion whose left-handed components are predominantly λ^+ and $\tilde{\mathrm{H}}^-$. In all of these three cases we have a charged fermion which could be found if W decays via $\mathrm{W} \to \lambda\tilde{\gamma}$ [556]. Let us here stress again that this must not be the case in general. It is just a possibility in a (although) wide class of models. μ and $\tilde{m}_2$ could both be larger than M_{W} and in such a case no such light charged fermion exists. Nonetheless this is an interesting possibility. The branching ratio for $\mathrm{W} \to \lambda\tilde{\gamma}$ has been computed and it was shown that it can well be comparable to the $\mathrm{W} \to \mathrm{e}\nu$ mode if λ and $\tilde{\gamma}$ are light enough. The signature, however, is not so clean as that of the $\mathrm{e}\nu$ mode because of the subsequent λ-decay which usually is expected to occur in several steps. The final decay products will be usual quarks and leptons as well as photinos (the lightest R-odd particle). They give rise to so-called Zen events [168] as shown in fig. 9.5, several hadronic jets recoiling against missing momentum. The decay chain of the λ starts

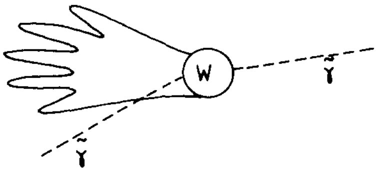

Fig. 9.5. Zen signature of W-decay in a cascade involving R-odd particles as given in ref. [168]. The one clapping hand represents the hadronic shower.

usually with $\lambda \to \lambda^0 + (e\nu \text{ or } q\bar{q})$ where λ^0 is a neutral gaugino which itself could decay into $\tilde{g} + q\bar{q}$. If the scalar partners of quarks and leptons are lighter than λ even more complicated decay chains could occur.

In this decay chain we expect the production of neutral gauginos and Higgsinos the masses of which we will discuss now. Apart from the photino which is a pure Majorana fermion that does not mix with the Higgsinos in the supersymmetric Higgs effect, there are three additional spin 1/2 neutral Weyl fermions in the minimal model. The mass matrix is given by (compare (7.68))

$$\begin{pmatrix} \tilde{m}_2 & 0 & -e_2 v_1/\sqrt{2} & e_2 v_2/\sqrt{2} \\ 0 & \frac{5}{3}(\alpha_1/\alpha_2)\tilde{m}_2 & e_1 v_1/\sqrt{2} & -e_1 v_2/\sqrt{2} \\ -e_2 v_1/\sqrt{2} & e_1 v_1/\sqrt{2} & 0 & \mu \\ e_2 v_2/\sqrt{2} & -e_1 v_2/\sqrt{2} & \mu & 0 \end{pmatrix} \begin{matrix} \lambda^0 \\ B^0 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{matrix} \tag{9.4}$$

where λ^0 and B^0 are the neutral wino of SU(2) and the bino of $U(1)_y$ respectively. The photino is given by the linear combination

$$\tilde{\gamma} = (e_1\lambda^0 + e_2 B^0)/(e_1^2 + e_2^2)^{1/2}, \tag{9.5}$$

with mass

$$m_{\tilde{\gamma}} \simeq \tfrac{8}{3}[e_1^2/(e_1^2 + e_2^2)]\tilde{m}_2, \tag{9.6}$$

and which usually is the lightest particle. In the case that either $\tilde{m}_2$ or μ or both are small, one additional light neutral fermion is present. As in our discussion of the charged fermions this additional light particle will predominantly be a gaugino if $\tilde{m}_2$ is small, or a Higgsino if μ is small, or a mixture if μ and $\tilde{m}_2$ are both small. A closer inspection of the mass matrix in (9.4) compared to that in (9.2) reveals that the light neutral fermion has a smaller mass than the light charged fermion such that a decay $\lambda \to \chi^0 + \text{all}$ is possible (χ^0 denotes the light neutral fermion in addition to the photino). Let us here point out again that a light χ^0 is a possibility in a certain class of models and is not a general prediction of supersymmetry.

The various branching ratios of the decay chain $W \to \lambda^{\pm} \to \lambda^0 \to \tilde{\gamma}$ have been computed and found to be nonnegligable. They depend on the parameters of the special models under consideration and for details we refer the reader to the original literature [49, 51, 78, 95, 119, 162, 168, 215, 292, 411, 556]. To detect these particles one has to be aware of the conventional backgrounds in W-decay. The most dangerous one is W-decay into heavy leptons (like τ) and neutrinos which has the same signature. The difference between these two processes is the Michel parameter which in the case of gauginos will in general not correspond to pure V − A or V + A interactions. These points have to be clarified to distinguish between new heavy leptons and gauginos. So much for gaugino production in W-decays. The neutral fermion χ^0 can of course also be produced in Z-decays. The production of these light fermions in e^+e^- annihilation have also been studied [489]. We, of course, know from there already that the charged gaugino–Higgsinos have to be heavier than 20 GeV and these particles will be easily detectable with e^+e^- machines at higher center of mass energies. The cross section for $e^+e^- \to \chi^0\tilde{\gamma}$ is of course smaller than that for $\lambda^+\lambda^-$, more comparable to $e^+e^- \to \nu\bar{\nu}$. Nonetheless for small masses of χ^0 and $\tilde{\gamma}$ such a reaction could be tested with high luminosity measurements at presently available e^+e^- machines and could give valuable information of m_{χ^0} and $m_{\tilde{\gamma}}$.

Let us close this chapter with some general remarks. We have not discussed everything that could be said about possible experimental signatures in supersymmetric models. This has its reason primarily in space limitations and also in the fact that many of the "predictions" are very model dependent and can only be presented including explanations of the very specific models under consideration. We have therefore focused on those predictions that appear in wide classes of models and we were of course also guided by theoretical prejudices. The investigations on the potential experimental signatures are still very much in progress at the time this paper is written and we can expect more information in the near future.

The most promising processes to find supersymmetric particles are in e^+e^- processes, W, Z and toponium decays and we have mainly concentrated on those reactions. At the present time we expect new information mainly from measurements at the $p\bar{p}$ collider with higher luminosity. The $e^+e^- \to \chi^0\tilde{\gamma}$ or $\gamma\tilde{\gamma}\tilde{\gamma}$ mode should be investigated with the presently available e^+e^- machines PETRA and PEP. The projected $p\bar{p}$, e^+e^- and ep machines at higher energy will certainly increase our knowledge about the (non) existence of these particles tremendously.

Let us close with a repetition of the warning given earlier. It might very well be that supersymmetry is related to the breakdown scale of the weak interactions and does not show traces of its existence in the mass region below 100 GeV. Once, however, the bounds on the masses of the R-odd particles are pushed up to higher and higher energies this possibility becomes more and more unlikely. In this sense we expect enough experimental information within the next 5 to 10 years to make up our mind.

10. Conclusion and outlook

Our aim was to construct phenomenologically acceptable models in which the breakdown scale M_W of the weak interactions was stabilized by supersymmetry. In addition we required the absence of small parameters in the theory that were artificially put in by hand. This then required a link between the supersymmetry breakdown and the breakdown of the weak interactions. Up to now there is no model which is theoretically completely satisfactory in this respect. We do not really know why supersymmetry is broken and hide our ignorance by constructing complicated additional sectors in the theory whose only role in life is to provide a breakdown of supersymmetry. It seems however that these questions can only be answered at energy scales much larger than $M_W \sim 100$ GeV where experimental information is very limited. Of more immediate interest at the moment is the physics at scales of order of M_W and let us recapitulate first the implications of supersymmetry there before we go to higher energies where things become much more unconstrained.

Supersymmetry requires the introduction of many new degrees of freedom. Every particle of the standard model receives a supersymmetric partner and in addition we have to introduce a second Higgs chiral superfield. This is the particle content of the minimal supersymmetric standard model, if supersymmetry would remain unbroken. To break supersymmetry one needs usually to introduce an additional sector, but let us discuss this later.

With this minimal set of superfields there exists the possibility of B and L violations in the low energy sector, contrary to the case in the nonsupersymmetric standard model where such violations have to be absent because of the symmetries and the requirement of renormalizability. To avoid these processes one usually introduces an R-symmetry or R-parity which explicitly forbids such terms and distinguishes between the Higgs H and the left-handed lepton doublet superfield. Once such an R-symmetry is present it also forbids a mass for, let us say the gluino. This in turn then implies a breakdown of this R-symmetry. Models with supersymmetry breakdown scale $M_S \lesssim 1$ TeV have usually problems to incorporate all these requirements (with the possible exception of supercolor where the R-symmetry could be broken by a supercolor anomaly) and this requires the investigation of models with breaking scale as high as $M_S \sim 10^{11}$ GeV. It is nontrivial that even with such a large M_S the mass scale M_W of some of the particles of the theory could remain stable. One has of course to be careful and hide the supersymmetry breaking sector sufficiently: a necessity partially already required from the constraints of the supertrace formula at the tree graph level. The particles in the hidden sector, that feel the breakdown of supersymmetry directly couple only very weakly (e.g. through the exchange of superheavy particles) to the low energy observable sector. The fact that M_S cannot be larger than 10^{11} GeV comes from the existence of gravity in which all particles (whether hidden or not) participate. In the framework of supersymmetry gravity is incorporated by the introduction of local supersymmetry. In such models M_W is then comparable to $m_{3/2}$ the gravitino mass and kept stable in a mass range of order of 100 GeV. The supersymmetric partners of the particles in the standard model also have masses of order $m_{3/2}$ and their existence provides a physical cutoff for the otherwise existing quadratic divergences of scalar particles. This existence of various new degrees of freedom in the 100–1000 GeV range seems to be a necessity in all attempts to understand M_W; compare the technicolor models [191] or models in which even quarks and leptons are bound states [475]. The question remains now whether the breakdown of $SU(2) \times U(1)$ can be related to the breakdown of supersymmetry. In the framework of supergravity, this can be achieved in a minimal model. Radiative corrections drive one of the m^2 of the Higgses to negative values and $SU(2) \times U(1)$ is broken as a result of the breakdown of supersymmetry. What is nice about this mechanism is the fact that it is one of the Higgs scalars that receives a negative m^2. We do not have to make tremendous efforts to avoid negative m^2 for the scalar partners of quarks and leptons, a potential danger in many other models. One can achieve this breakdown in various ways. Most straightforward it proceeds if the theory includes a large Yukawa coupling responsible for the mass of the top quark. If one allows for a $\mu H\bar{H}$ term in the superpotential with $\mu \sim m_{3/2}$ it can, however, work for arbitrary values of m_{top}.

This parameter μ actually poses a slight problem in the low energy theory. It has to be nonzero to avoid the existence of an unacceptable axion. Our whole motivation to study supersymmetry in this context on the other hand was to explain why μ is small compared to M_P. To do this in a satisfactory way one first looked for a situation in which $\mu = 0$. Having $\mu = 0$ at the tree level it is then not easy to generate $\mu \sim m_{3/2}$ in higher orders. This question, however, can only be answered if one also considers the models at high energies which allow for many possibilities and it remains open for the moment [365].

The minimal low energy model then depends on four new input parameters: the gravitino mass $m_{3/2}$, the scalar trilinear coupling A, gaugino masses m_0 and the parameter μ in $\mu H\bar{H}$. Nonminimal models of course have more new particles and input parameters and in general less predictive power. These four parameters should be explained through the theory at higher energies. There, however, things become arbitrary and unconstrained. We can construct at the moment various hidden sectors that serve as examples in which certain values of parameters $m_{3/2}$, A, m_0 and μ are obtained. Usually this is the only thing the considered hidden sector has to do and the whole situation remains arbitrary. Moreover in most cases certain parameters have to be chosen very carefully to arrive at $m_{3/2} \ll M_P$ and we do not really have a reason why M_S is small compared to M_P.

The inclusion of grand unification is straightforward. The minimal extension does not explain the absence of μ. Models with $\mu = 0$, however, tend to be very complicated [291]. With the experimental absence of proton decay one, however, might postpone the question of grand unification for the moment.

The construction of models beyond the TeV range is thus at the moment a place for relatively unconstrained theoretical speculation. The main question to study is an explanation of $M_S/M_P \sim 10^{-8}$. One might think that such a ratio is easier to understand than the corresponding ratio $M_W/M_P \sim 10^{-17}$ in the standard model but this need not necessarily be the case. The difference here of course is that M_S and consequently M_W are stable in perturbation theory.

Suggestions in this direction are already present in the supercolor models [123, 137] where one could understand M_S in the same way as one understands the masses of hadrons (via Λ_{QCD}) in the usual models. This could work for various values of M_S between 10^3 and 10^{11} GeV but our understanding of the dynamics of these models is very limited and still controversial. Another suggestion is that nonperturbative effects (like instantons) might not respect the nonrenormalization theorems and therefore induce a supersymmetry breakdown at a small scale [569] (i.e. small compared to an input scale like M_P or M_x). Attempts have been made in this direction [6, 7] but in four dimension such a breakdown of supersymmetry has not yet been established [104a].

Other models have been constructed in which the fundamental input scale is not M_P but rather $M_S \sim 10^{11}$ GeV. This scale plays a very special role. Apart from the fact that supersymmetry ought to be broken there it is also the scale of the SU(5) phase transition and the generation of the cosmological baryon asymmetry. Moreover in models where the strong CP problem is solved through the presence of a Peccei–Quinn symmetry one expects the breakdown scale of this symmetry somewhere between 10^9 and 10^{12} GeV. Inverted hierarchy models then start with such a scale and generate larger scales. For the model to be complete the generation of M_P is necessary. This could in principle work in the framework of some kind of induced gravity and remains an unsolved problem.

A satisfactory model should probably explain M_P, M_x, M_S and M_W in terms of each other. The above described models relate M_S and M_W even if $M_S/M_W \sim 10^8$, but the relation between M_P, M_x and M_S still remains a mystery. Possible descriptions have been given in various frameworks [444, 428, 264] but they can at most serve as toy models.

To shed some light on these questions one could also go beyond $N = 1$ supersymmetry. $N = 1$ supergravity is certainly not the final theory for gravitation; it is nonrenormalizable and there is the hope that extended $N > 1$ supergravities show an improved ultraviolet behavior. An application of extended supersymmetries to particle physics has however to face one central problem. The particle spectrum of these theories is always real with respect to the gauge symmetries under consideration, contrary to the spectrum of the particles in the standard $SU(3) \times SU(2) \times U(1)$ or SU(5) models. This is a serious problem since there is no way to remove some particles (give them a large mass) without breaking the gauge symmetries and arrive at chiral spectra at low energies, it seems to be a major stumbling block for generalizations of Kaluza–Klein ideas to extended supergravities [150, 188, 346, 572]. One could try to avoid this problem by assuming that the low energy spectrum does not contain the fundamental particles but dynamical boundstates [163, 110]. Even in this case, however, the problems seems to exist. Our knowledge of the structure of these models, however, is still limited at the moment and investigations of these models will go on. What would be of particular interest is the study of the breakdown chain $N \to N' \to 1 \to 0$ in supergravity and to find examples in which the found breakdown from $N = 1$ to $N = 0$ happens so late at $M_S \sim 10^{11}\,\text{GeV} \ll M_P$. Maybe progress in this direction would improve our situation concerning an understanding of M_S [109]. The corresponding question in the case of global N-extended supersymmetry is already answered: the breakdown of one supersymmetry implies the breakdown of all N of them. Interesting results could also be expected in extended supergravity with strong gauge interactions that could provide a supercolor mechanism. In all these cases there remains the problem of the reality of the representations and so at the moment one could use these more general models directly only for construction of the hidden sectors and try to understand the problem of understanding the value of M_S. Apart from that one could try to investigate the value of the cosmological constant in these models and perhaps find a connection to the breakdown of supergravity. At the moment all these questions have to remain open and we can only have the hope that the situation improves with a more complete understanding of these models.

From the experimental side it is the region of 100 GeV that is of immediate interest. Experiments can either falsify or confirm in the near future our theoretical prejudices whether supersymmetry is relevant in this energy range. If no traces of supersymmetry are found there one would rather believe that $m_{3/2}$ is as big as M_p and this question of M_S will become irrelevant for a long time. Supersymmetry then probably will be relevant only at this high scale (if at all). If, however, some of the supersymmetric particles are found in the 100 GeV range we would have of course a renewed motivation to investigate these questions on the origin of M_S.

The supersymmetric extension of the standard $SU(3)\times SU(2)\times U(1)$ model has to be checked experimentally and I am sure that this will be done in the near future. This extension, of course, is more complicated than the original standard model, but this seems to be a necessity in every model in which the breakdown of $SU(2)\times U(1)$ is explained by a more fundamental concept.

11. Appendix: notations and conventions

We follow closely the notation of ref. [296]. The transformation parameters θ_α $(\alpha = 1, 2)$ are two-component anticommuting objects. Indices can be raised by applying the two-dimensional ε tensor

$$\theta_\alpha = \varepsilon_{\alpha\beta}\theta^\beta\,, \tag{A.1}$$

where $\varepsilon_{\alpha\beta} = \varepsilon^{\alpha\beta}$ is antisymmetric in α and β $(\alpha, \beta = 1, 2)$ and $\varepsilon_{12} = +1$. The complex adjoint of θ is denoted by

$$\bar\theta^{\dot\alpha} = (\theta^\alpha)^+\,. \tag{A.2}$$

As before we have $\bar\theta_{\dot\alpha} = \varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\beta}$ with $\varepsilon_{\dot\alpha\dot\beta} = \varepsilon^{\dot\alpha\dot\beta}$ antisymmetric and $\varepsilon_{12} = +1$. The squares of θ and $\bar\theta$ are defined through

$$\theta^2 = \theta^\alpha\theta_\alpha = \varepsilon_{\alpha\beta}\theta^\alpha\theta^\beta\,, \qquad \bar\theta^2 = \bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha} = -\varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\alpha}\bar\theta^{\dot\beta}\,. \tag{A.3}$$

Observe that

$$\theta^\alpha\theta_\alpha = -\theta_\alpha\theta^\alpha\,. \tag{A.4}$$

For the ε symbol we have

$$\varepsilon^{\gamma\beta}\varepsilon_{\alpha\beta} = \delta^\gamma_\alpha\,, \tag{A.5}$$

where δ denotes the Kronecker symbol. The Pauli matrices carry a space–time index μ as well as a dotted and undotted index

$$\sigma_\mu^{\alpha\dot\beta} = \sigma^\mu_{\beta\dot\alpha} = (\mathbb{1}, \boldsymbol{\sigma})_{\mu;\alpha\beta}\,, \tag{A.6}$$

where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.7}$$

and we choose

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}. \tag{A.8}$$

It is sometimes worthwhile to know the following relations:

$$\sigma^\mu_{\alpha\dot\beta}\sigma_\nu^{\alpha\dot\beta} = 2g^\mu_{\ \nu}\,, \qquad \sigma^\mu_{\gamma\dot\delta}\sigma_\mu^{\alpha\dot\beta} = 2\delta^\alpha_\gamma\delta^{\dot\beta}_{\dot\delta}\,, \tag{A.9}$$

as well as

$$(\theta^\alpha\sigma_{\mu\alpha\dot\beta}\bar\theta^{\dot\beta})(\theta^\gamma\sigma_{\nu\gamma\dot\delta}\bar\theta^{\dot\delta}) = \tfrac{1}{2}g_{\mu\nu}\theta^2\bar\theta^2\,, \tag{A.10}$$

$$\sigma_\nu^{\alpha\dot\beta}\sigma_{\mu\beta\dot\beta}\sigma_{\rho\alpha\dot\gamma}\sigma_\sigma^{\beta\dot\gamma} = 2[g_{\nu\mu}g_{\rho\sigma} + g_{\nu\rho}g_{\mu\sigma} - g_{\nu\sigma}g_{\mu\rho} + \mathrm{i}\varepsilon_{\nu\mu\sigma\rho}] \tag{A.11}$$

with $\varepsilon_{\nu\mu\sigma\rho}$ totally antisymmetric and $\varepsilon_{0123} = +1$.

Differentiation with respect to θ is defined through

$$\partial_\alpha\theta^\beta = \frac{\partial\theta^\beta}{\partial\theta^\alpha} = \delta^\beta_\alpha\,, \qquad \bar\partial^{\dot\alpha}\bar\theta_{\dot\beta} = \frac{\partial\bar\theta_{\dot\beta}}{\partial\bar\theta_{\dot\alpha}} = \delta^{\dot\alpha}_{\dot\beta}\,. \tag{A.12}$$

With these rules one has, e.g.,

$$\partial_\alpha\theta^2 = 2\theta_\alpha\,, \qquad \bar\partial_{\dot\beta}\bar\partial^{\dot\beta}(\bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}) = -4\,. \tag{A.13}$$

Integration is defined through

$$\int \mathrm{d}\theta = 0\,, \qquad \int \theta\,\mathrm{d}\theta = 1\,. \tag{A.14}$$

The matrices $\bar\sigma$ are defined through

$$\bar\sigma_\mu^{\alpha\dot\beta} = (\mathbb{1}, -\boldsymbol{\sigma})_{\mu;\alpha\beta}\,. \tag{A.15}$$

A left- (right-) handed two-component Weyl-spinor transforms as $\theta(\bar\theta)$. A four-component Dirac spinor can be constructed with two Weyl spinors

$$\psi_a = \begin{pmatrix} \chi_\alpha \\ \bar{\xi}^{\dot\beta} \end{pmatrix} \tag{A.16}$$

where $a = 1, 2, 3, 4$, $\alpha = 1, 2$ and $\dot\beta = 3, 4$. For the Dirac matrices we have

$$\gamma^\mu_{ab} = \begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot\beta} \\ \sigma^{\mu\dot\beta\dot\alpha} & 0 \end{pmatrix} \tag{A.17}$$

i.e.,

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & \mathrm{i} \\ 0 & 0 & -\mathrm{i} & 0 \\ 0 & -\mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{A.18}$$

and

$$\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{A.19}$$

The symbol $\bar\psi$ for a Dirac spinor denotes

$$\bar\psi = \psi^+\gamma^0 , \tag{A.20}$$

which in the notation of (A.16) reads

$$\bar\psi^a = (\xi^\alpha, \bar\chi_{\dot\beta}) . \tag{A.21}$$

The mass term for a Dirac spinor is written as $m\bar\psi\psi$ which in terms of the two-component spinors is given by

$$m(\xi^\alpha\chi_\alpha + \bar\chi_{\dot\beta}\bar\xi^{\dot\beta}) . \tag{A.22}$$

A Majorana spinor can be written in terms of a Weyl spinor as

$$\eta_a = \begin{pmatrix} \chi_\alpha \\ \bar\chi^{\dot\alpha} \end{pmatrix} . \tag{A.23}$$

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