

Assignment 0

A.1 1. Pole: $s = -10$, $G_1(0) = 1$

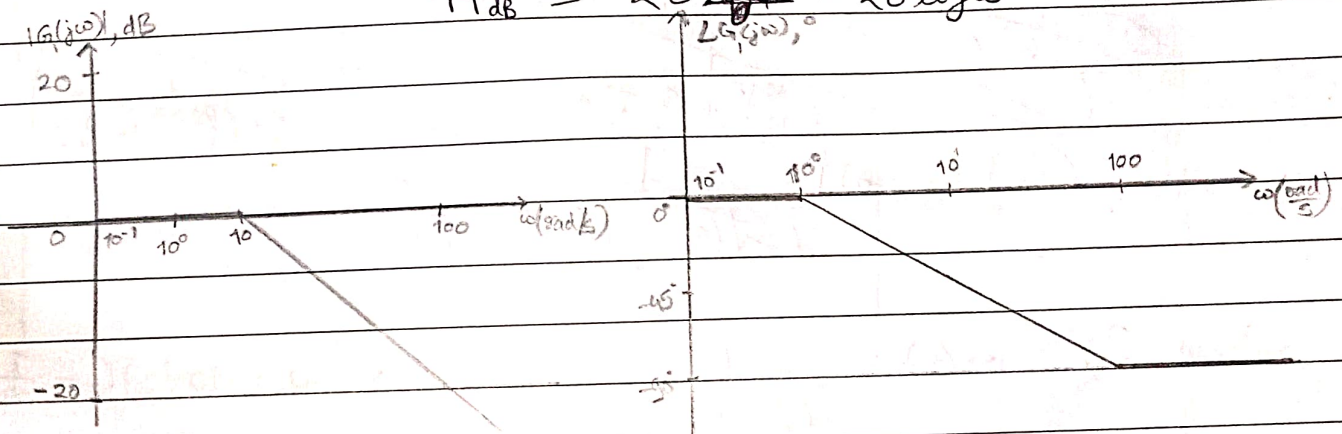
2. $G_1(s) = \frac{1}{1 + \frac{s}{10}} \Rightarrow G_1(j\omega) = \frac{1}{1 + \frac{j\omega}{10}}$

Corner freq. $\Rightarrow 10$

$\omega < 10 \Rightarrow$ Magnitude, $M = \frac{1}{1} = 1 \rightarrow M_{dB} = 0$

$\omega > 10 \Rightarrow M = \left(\frac{1}{j\omega/10} \right) = \left(\frac{10}{j\omega} \right) \Rightarrow \frac{10}{\omega}$

$M_{dB} = 20 \log \frac{10}{\omega} = 20 \log 10 - 20 \log \omega$



A.2 1. zero: $s = 2$, Pole: $s = -10$, $G_2(0) = -\frac{1}{5}$

2. $G_2(s) = \frac{(-2)(1 + \frac{s}{-2})}{(10)(1 + \frac{s}{10})} = \left(-\frac{1}{5} \right) \left(\frac{1 + \frac{s}{-2}}{1 + \frac{s}{10}} \right)$

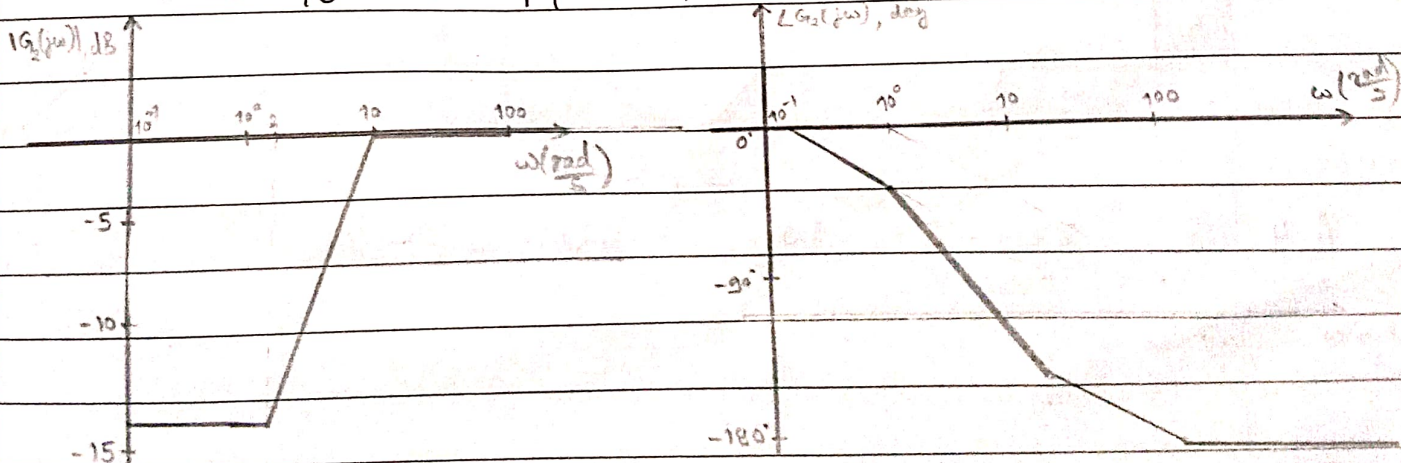
$\Rightarrow G_2(j\omega) = \left(-\frac{1}{5} \right) \left(\frac{1 - \frac{j\omega}{2}}{1 + \frac{j\omega}{10}} \right)$

Corner frequency $\Rightarrow 2, 10$

$\omega < 2 \rightarrow M = +\frac{1}{5} \rightarrow M_{dB} = -20 \log 5$

$2 < \omega < 10 \rightarrow M = \frac{j\omega}{10} \rightarrow M_{dB} = -20 + 20 \log \omega$

$\omega > 10 \rightarrow M = 1 \rightarrow M_{dB} = 0$



4. Right-half plane zero causes phase decrease by 90° instead of increasing by 90° .

A

A.3 1. Pole: $s_{1,2} = -5 \pm j5\sqrt{3}$

~~2. $G_2(s) = \frac{100}{(s/10)^2 + 2 \cdot \frac{1}{2} \cdot \frac{s}{10} + 1}$ $\Rightarrow \omega_n = 10, \zeta = 0.5$~~
 ~~$\frac{(s/10)^2 + 2 \cdot \frac{1}{2} \cdot \frac{s}{10} + 1}{100}$~~
 ~~$M = \frac{1}{\sqrt{1 - \zeta^2}} = \frac{1}{\sqrt{1 - 0.25}} = \frac{1}{\sqrt{0.75}} = \frac{2}{\sqrt{3}}$~~
 ~~$20 \log(2/\sqrt{3}) \approx 1.249$~~
 ~~$20 \log(100) = 40$~~

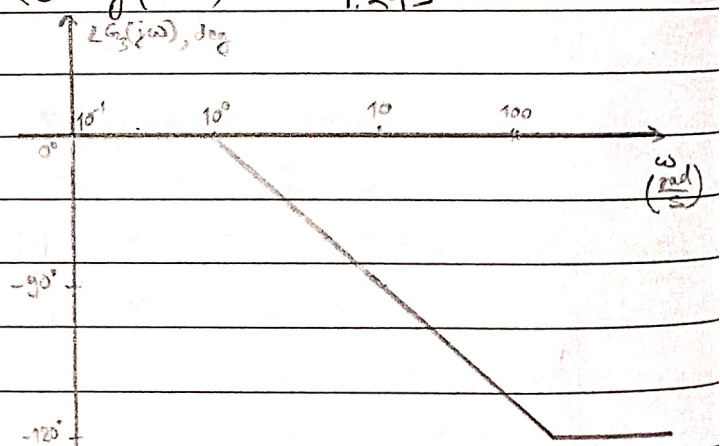
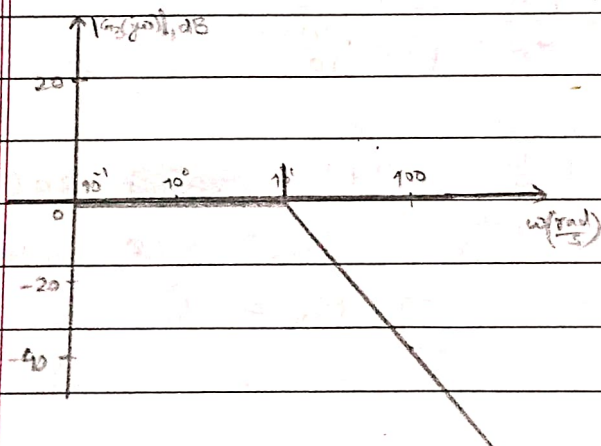
2. $G_3(s) = \frac{1}{(s/10)^2 + s/10 + 1} \Rightarrow G_3(j\omega) = \frac{1}{1 - (\frac{\omega}{10})^2 + j\frac{\omega}{10}}$
 $|G_3(j\omega)| = \frac{1}{\sqrt{(1 - (\frac{\omega}{10})^2)^2 + (\frac{\omega}{10})^2}}$

2. $G_3(s) = \frac{1}{(s/10)^2 + 2 \cdot \frac{1}{2} \cdot (s/10) + 1} \Rightarrow \omega_n = 10, \zeta = \frac{1}{2}$

$\omega < \omega_n \rightarrow M = 1 \rightarrow M_{dB} = 0$

$\omega > \omega_n \rightarrow M = \frac{1}{\frac{\omega^2}{100}} \rightarrow M_{dB} = 40 - 40 \log \omega$

$\omega = \omega_n \rightarrow M_{dB} = -20 \log(2\zeta\sqrt{1-\zeta^2})$
 $= 20 \log(\sqrt{3}/2) \approx 1.249$



A.4 1. Zero: $s = -10$, Pole: $s = -100$

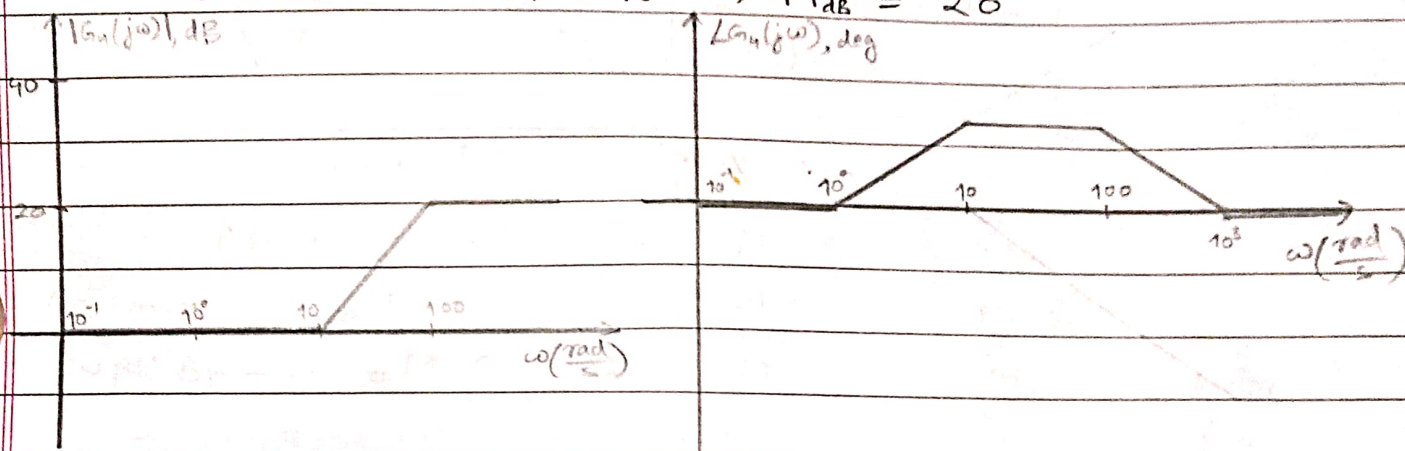
$$2. G_u(s) = \frac{1 + \frac{s}{10}}{1 + \frac{s}{100}} \Rightarrow G_u(j\omega) = \frac{1 + \frac{j\omega}{10}}{1 + \frac{j\omega}{100}}$$

Corner freq $\rightarrow 10, 100$

$$\omega < 10 \rightarrow M = 1 \rightarrow M_{dB} = 0$$

$$10 < \omega < 100 \rightarrow M = \frac{j\omega}{10} \rightarrow M_{dB} = 20 \log \omega - 20$$

$$\omega > 100 \rightarrow M = 10 \rightarrow M_{dB} = 20$$



4. Between zero & pole, $G_u(s)$ tend to add neither +ve nor -ve phase.

B.1 1. $\Sigma F = ma$

$$\Rightarrow f(t) - kx(t) - c\dot{x}(t) = m\ddot{x}(t)$$

$$\therefore m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$

2. Applying Laplace transform.

$$\therefore m.s^2.X(s) + c.s.X(s) + k.X(s) = F(s)$$

3. $G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$

B.2 1. $G(s) = \frac{1}{s^2 + 16s + 4}$

2. Pole: $s_{1,2} = -8 \pm 2\sqrt{15}$

$$\begin{aligned} \omega < \omega_0 &\rightarrow M = \frac{1}{4} \rightarrow M_{dB} = -20 \log 4 \\ \omega > \omega_0 &\rightarrow M = \frac{1}{4} \cdot \frac{1}{\omega^2/4} \rightarrow M_{dB} = -40 \log \omega \\ \omega = \omega_0 &\rightarrow M_{dB} = \end{aligned}$$

$$3. \quad G(s) = \frac{1}{(s+\omega_1)(s+\omega_2)}, \quad \begin{aligned} \omega_1 &= +8 + 2\sqrt{15} \\ \omega_2 &= +8 - 2\sqrt{15} \end{aligned}$$

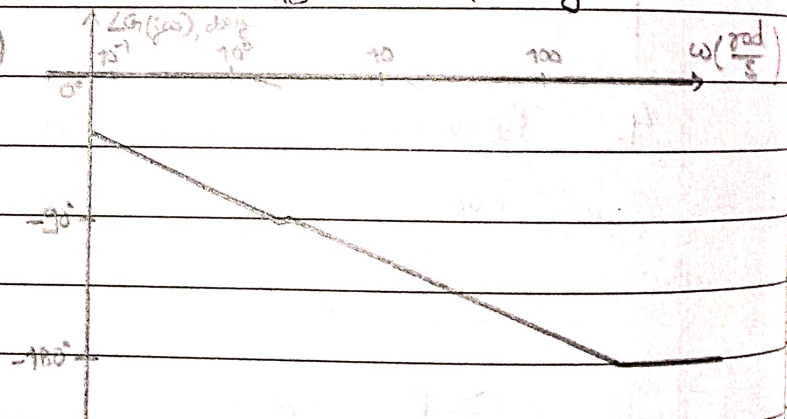
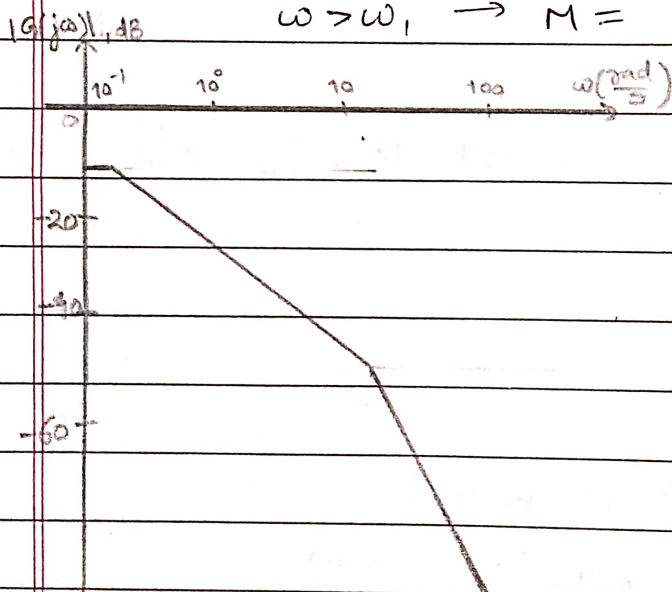
$$\Rightarrow G(s) = \frac{1}{(4\sqrt{15}) \left(\frac{s}{\omega_2} + 1 \right) \left(\frac{s}{\omega_1} + 1 \right)}$$

$$\Rightarrow G(s) = \frac{1}{4 \left(\frac{s}{\omega_1} + 1 \right) \left(\frac{s}{\omega_2} + 1 \right)}$$

$$\omega < \omega_2 \rightarrow M = \frac{1}{4} \rightarrow M_{dB} = -20 \log 4$$

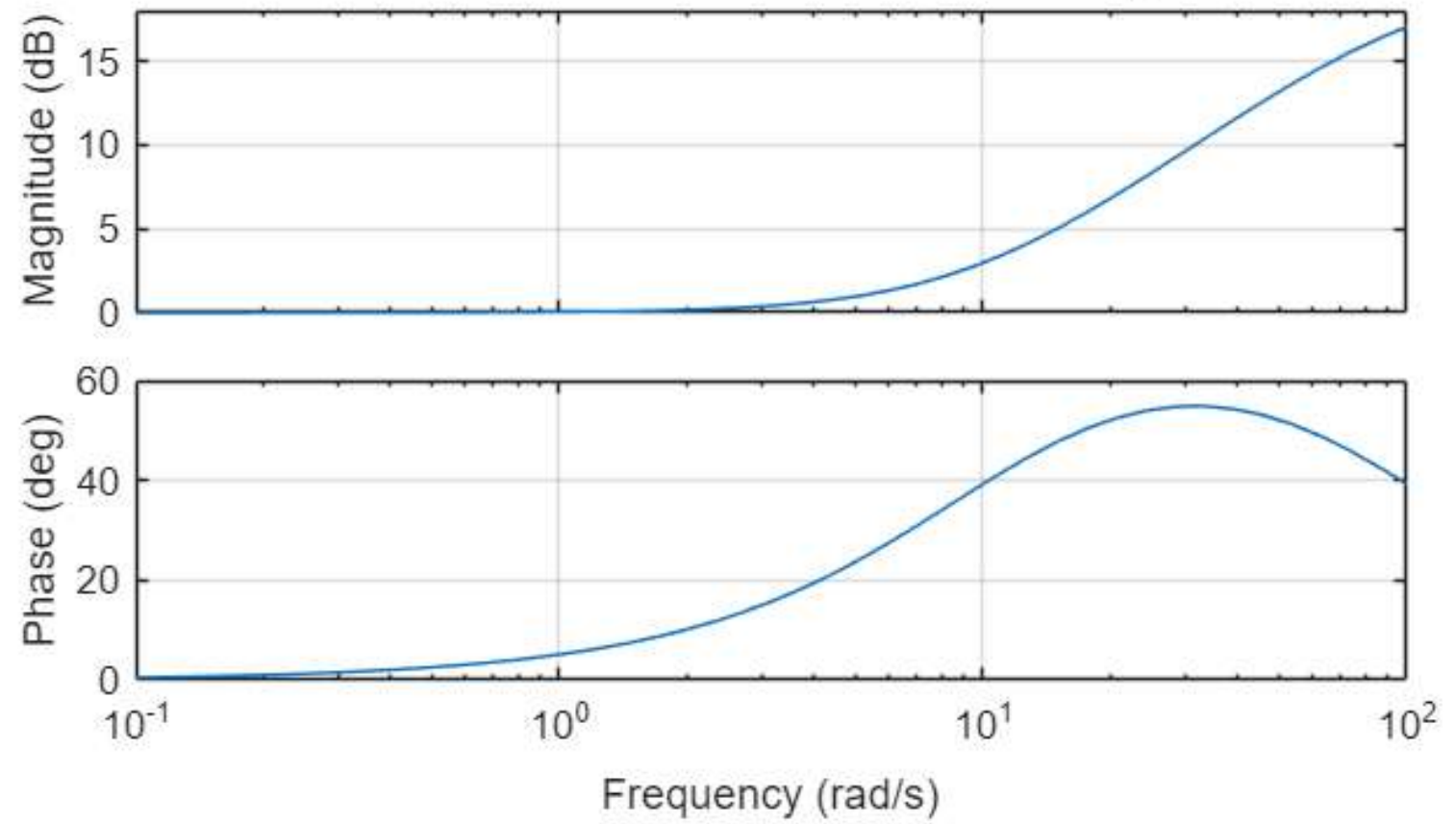
$$\omega_2 < \omega < \omega_1 \rightarrow M = \frac{\omega_2}{4j\omega} \rightarrow M_{dB} = 20 \log \left(\frac{\omega_2}{4} \right) - 20 \log \omega$$

$$\omega > \omega_1 \rightarrow M = \frac{1}{\omega^2} \rightarrow M_{dB} = -40 \log \omega$$



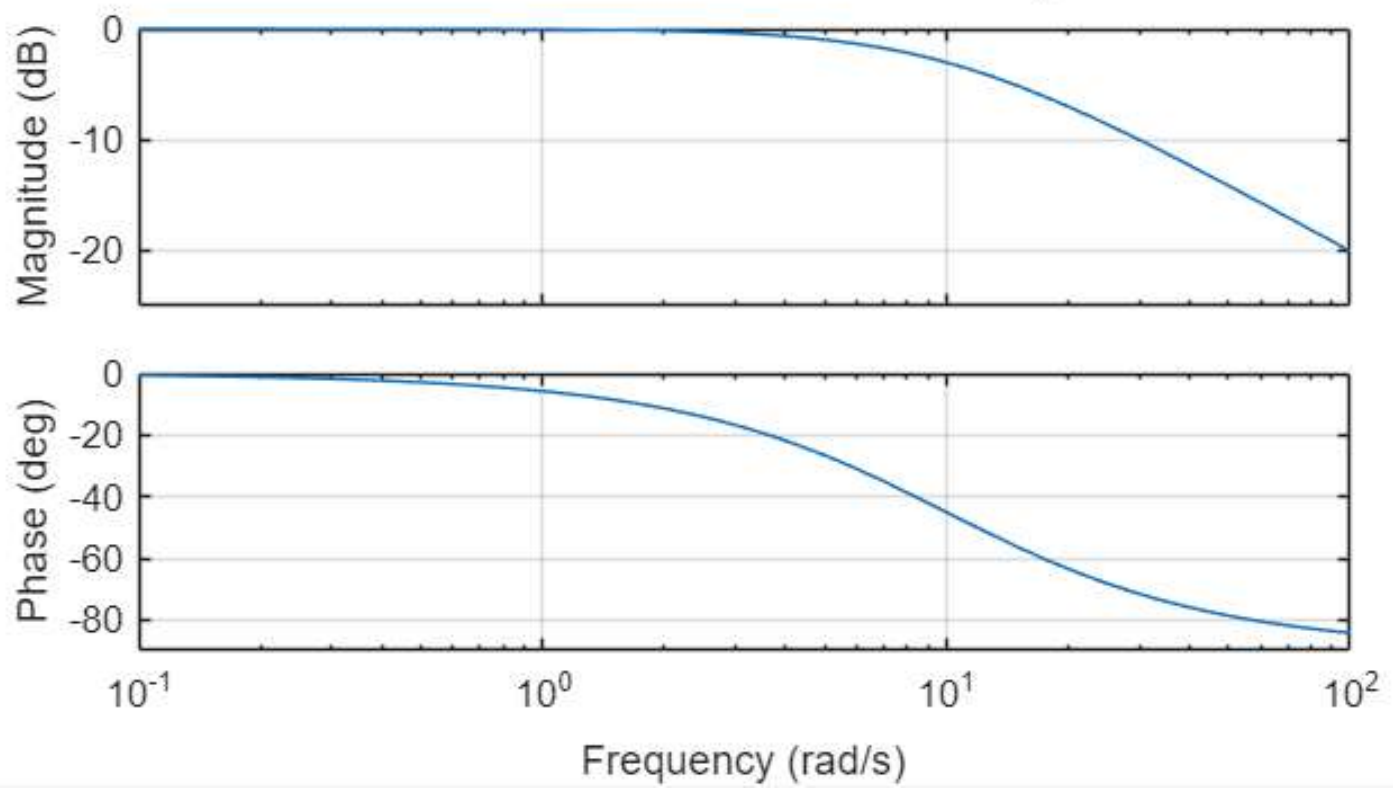
Bode Diagram

$G(s)$



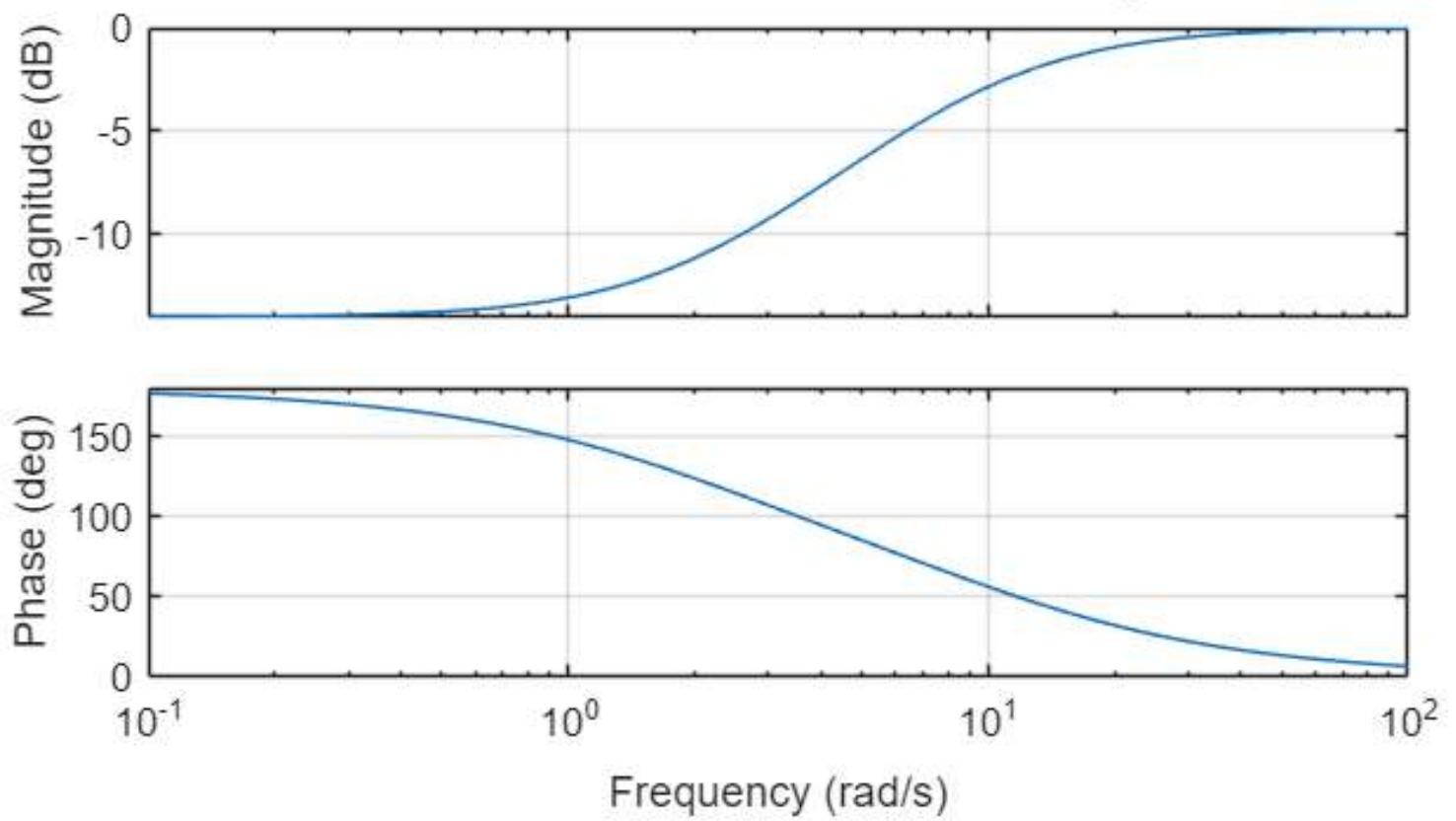
Bode Diagram

G_1

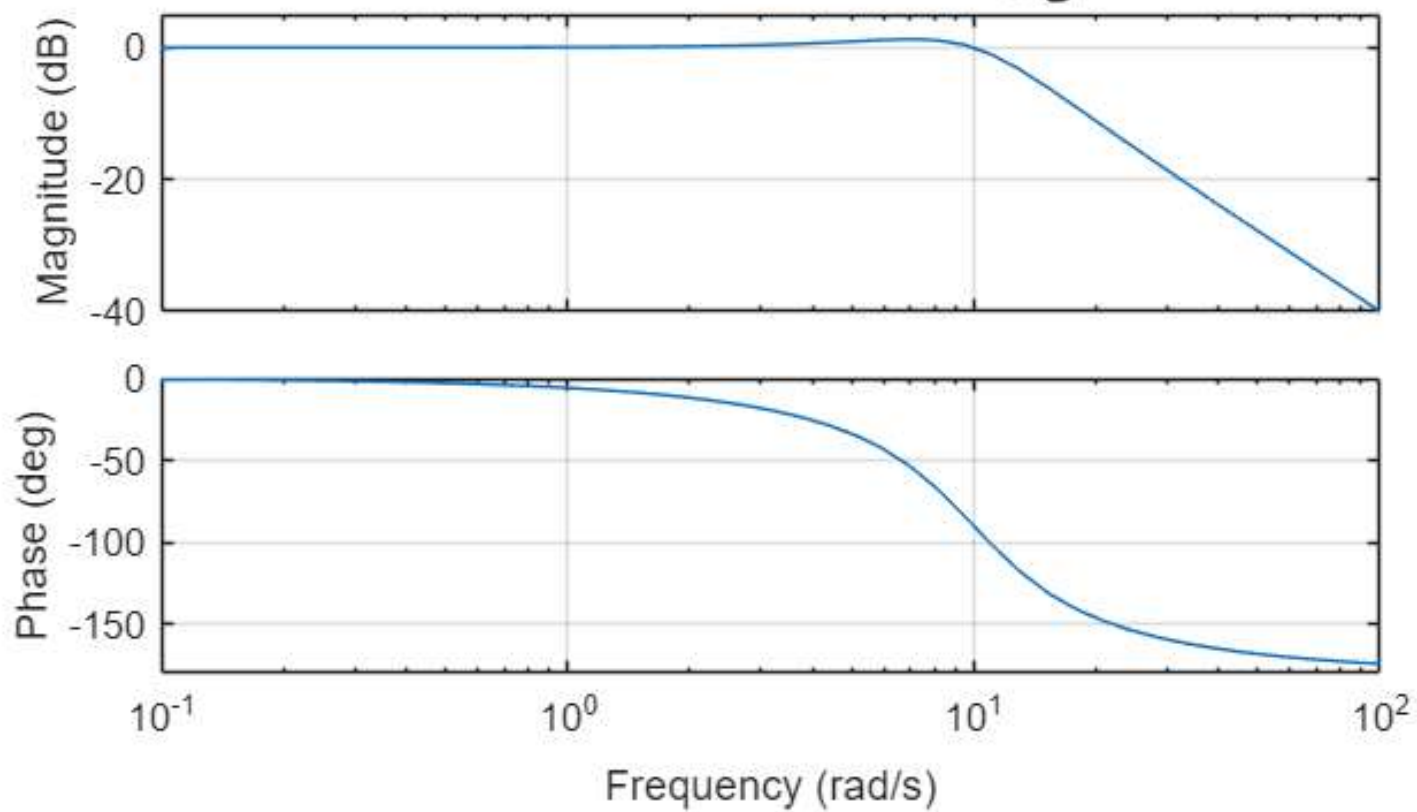


Bode Diagram

G_2



Bode Diagram G_3



Bode Diagram

G_u

