

Assignment 0

A.1 1. Pole: $s = -10$, $G_1(0) = 1$

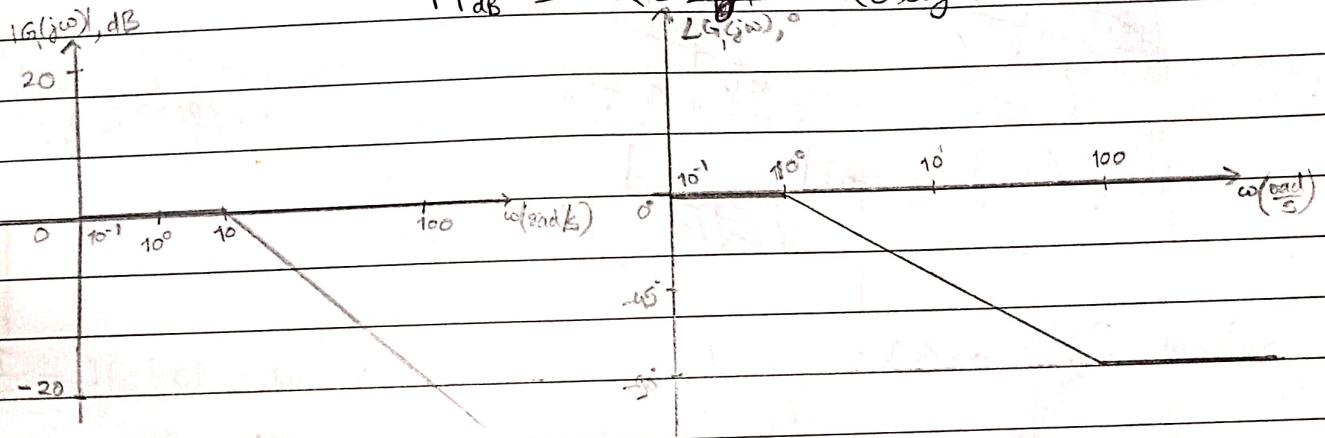
2. $G_1(s) = \frac{1}{1+\frac{s}{10}} \Rightarrow G_1(j\omega) = \frac{1}{1+\frac{j\omega}{10}}$

Corner freq. $\Rightarrow 10$

$$\omega < 10 \rightarrow \text{Magnitude, } M = \frac{1}{1} = 1 \rightarrow M_{dB} = 0$$

$$\omega > 10 \rightarrow M = \left(\frac{1}{j\omega/10}\right) = \left(\frac{10}{j\omega}\right) \Rightarrow \underline{\underline{M = \frac{10}{\omega}}}$$

$$M_{dB} = 20 \cancel{\log} \omega - 20 \log \omega$$



A.2 1. zero: $s = 2$, Pole: $s = -10$, $G_2(0) = -\frac{1}{5}$

2. $G_2(s) = \frac{(-2)(1+\frac{s}{-2})}{(10)(1+\frac{s}{10})} = \left(\frac{1}{5}\right)\left(1+\frac{s}{-2}\right) \frac{1}{(1+\frac{s}{10})}$

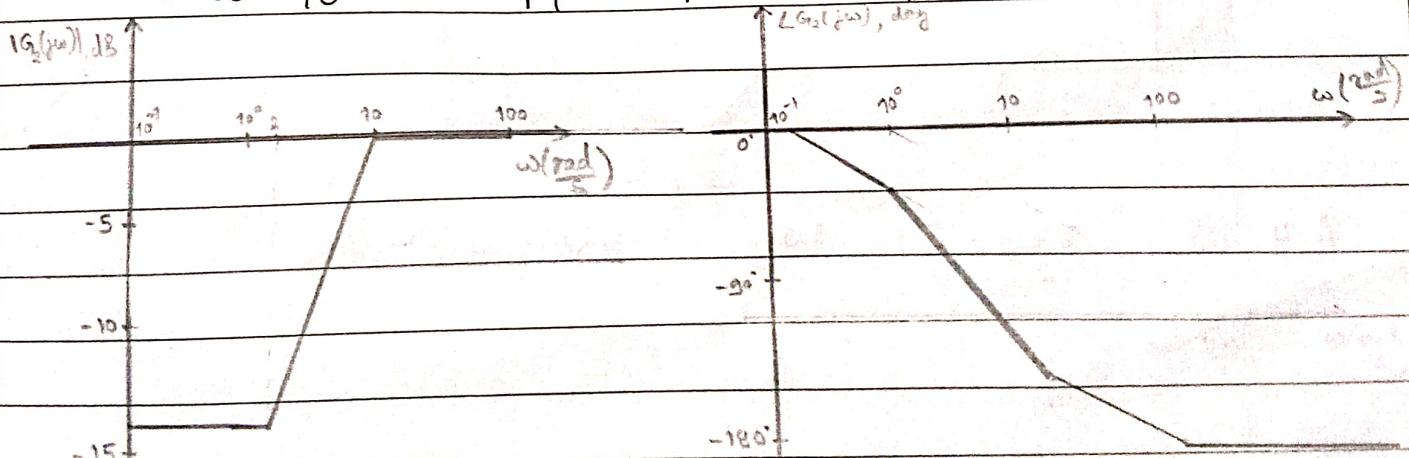
$$\Rightarrow G_2(j\omega) = \left(\frac{1}{5}\right) \left(1 - \frac{j\omega}{2}\right) \frac{1}{(1 + \frac{j\omega}{10})}$$

Corner frequencies $\Rightarrow 2, 10$

$$\omega < 2 \rightarrow M = +\frac{1}{5} \rightarrow M_{dB} = \cancel{-20 \log 5} - 20 \log \omega$$

$$2 < \omega < 10 \rightarrow M = \frac{j\omega}{10} \rightarrow M_{dB} = -20 + 20 \log \omega$$

$$\omega > 10 \rightarrow M = 1 \rightarrow M_{dB} = 0$$



4. Right-half plane zero causes phase decrease by 90° instead of increasing by 90° .

A.3 1. Pole: $s_{1,2} = -5 \pm j5\sqrt{3}$

~~2. $G_s(s) = \frac{1}{(s+5)^2 + (5\sqrt{3})^2}$~~

~~$\Rightarrow \omega_n = 10, \zeta = 0.5$~~

~~$(\frac{s}{10})^2 + 2 \cdot \frac{1}{2} \cdot \frac{5}{10} + 1$~~

~~$20 \log(2\zeta\sqrt{1-\zeta^2}) = 20 \log(\sqrt{3}/2)$~~

~~$20 \log(1/100) = -40$~~

~~2. $G_3(s) = \frac{1}{(\frac{s}{10})^2 + \frac{s}{10} + 1} \Rightarrow G_3(j\omega) = \frac{1 - (\frac{\omega}{10})^2 + j\frac{\omega}{10}}{1 - (\frac{\omega}{10})^2 + (\frac{\omega}{10})^2}$~~

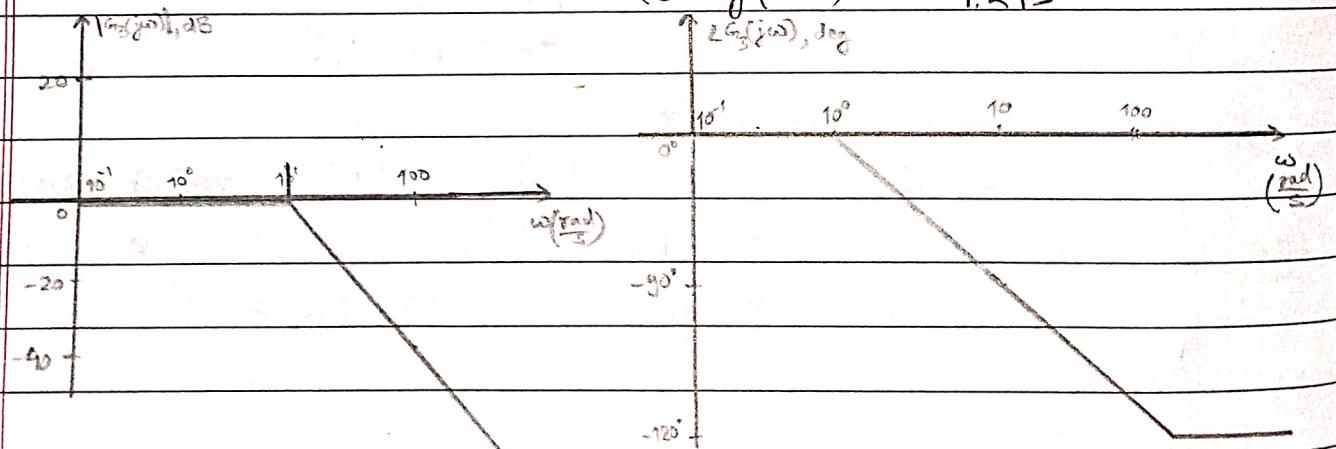
~~$|G_3(j\omega)| = \frac{1}{\sqrt{1 - (\frac{\omega}{10})^2 + (\frac{\omega}{10})^2}}$~~

2. $G_3(s) = \frac{1}{(\frac{s}{10})^2 + 2 \cdot \frac{1}{2} \cdot \frac{5}{10} + 1} \Rightarrow \omega_n = 10, \zeta = \frac{1}{2}$

$$\omega < \omega_n \rightarrow M = 1 \rightarrow M_{dB} = 0$$

$$\omega > \omega_n \rightarrow M = \frac{1}{\frac{\omega^2}{100}} \rightarrow M_{dB} = 40 - 40 \log \omega$$

$$\omega = \omega_n \rightarrow M_{dB} = -20 \log(2\zeta\sqrt{1-\zeta^2}) \\ = -20 \log(\sqrt{3}/2) \approx 1.249$$



A.4 1. Zero: $s = -10$, Pole: $s = -100$

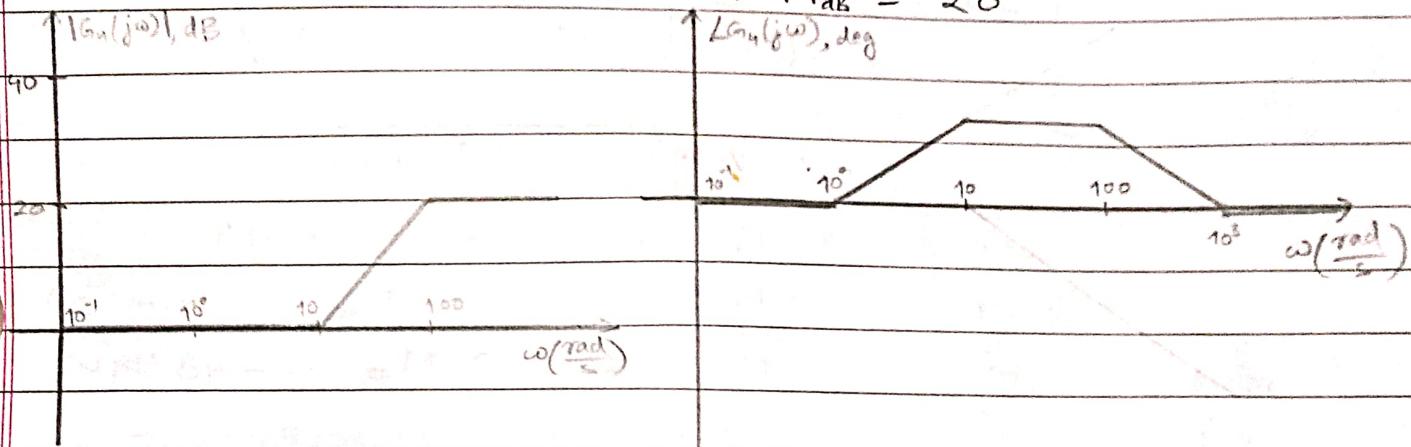
$$2. G_u(s) = \frac{1 + \frac{s}{10}}{1 + \frac{s}{100}} \Rightarrow G_u(j\omega) = \frac{1 + \frac{j\omega}{10}}{1 + \frac{j\omega}{100}}$$

Corner freq → 10, 100

$$\omega < 10 \rightarrow M = 1 \rightarrow M_{dB} = 0$$

$$10 < \omega < 100 \rightarrow M = \boxed{\frac{j\omega}{10}} \rightarrow M_{dB} = 20 \log \omega - 20$$

$$\omega > 100 \rightarrow M = 10 \rightarrow M_{dB} = 20$$



4. Between zero & pole, $G_u(s)$ tend to add neither +ve nor -ve phase.

B.1

$$1. \sum F = ma$$

$$\Rightarrow f(t) - kx(t) - c\dot{x}(t) = m\ddot{x}(t)$$

$$\therefore m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t)$$

2. Applying Laplace transform.

$$\therefore m s^2 X(s) + c s X(s) + k X(s) = F(s)$$

$$3. G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k}$$

B.2

$$1. G(s) = \frac{1}{s^2 + 16s + 4}$$

$$2. \text{Pole: } s_{1,2} = -8 \pm 2\sqrt{15}$$

$$\omega < \omega_0 \rightarrow M = \frac{1}{4} \rightarrow M_{dB} = -20 \log 4$$

$$\omega > \omega_0 \rightarrow M = \frac{1}{4} \cdot \frac{1}{\omega^2/4} \rightarrow M_{dB} = -40 \log \omega$$

$$\omega = \omega_0 \rightarrow M_{dB}$$

3. $G(s) = \frac{1}{(s+\omega_1)(s+\omega_2)}$, $\omega_1 = +8 + 2\sqrt{15}$
 $\omega_2 = +8 - 2\sqrt{15}$

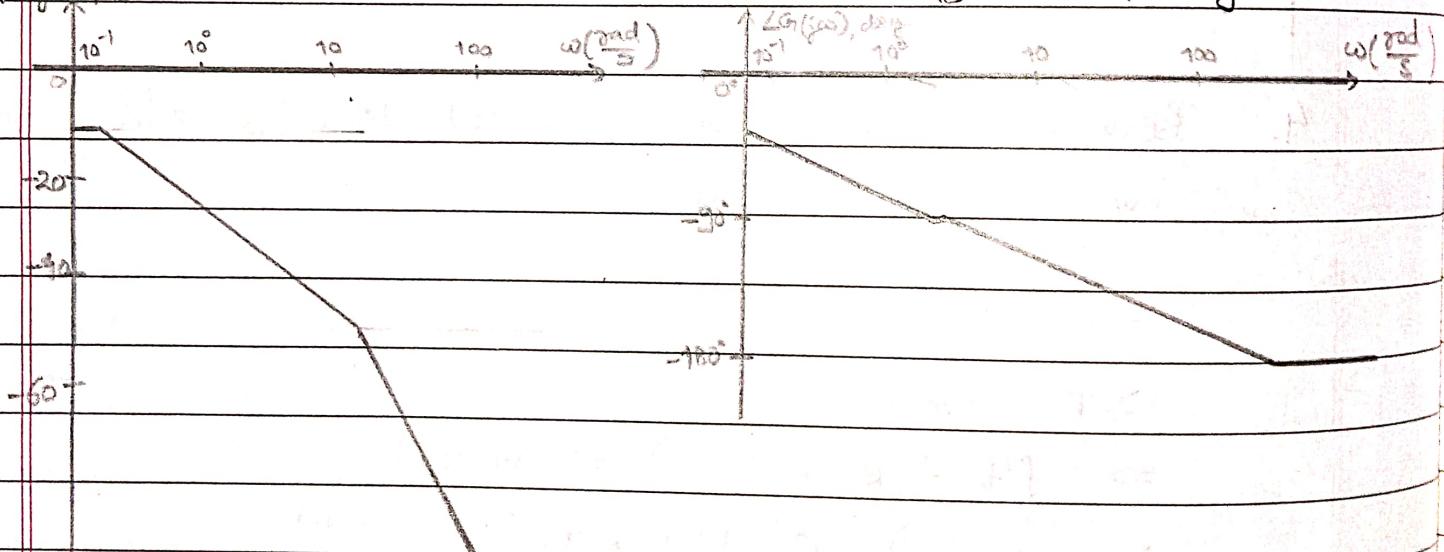
$$\Rightarrow G(s) = \frac{1}{4\sqrt{15}} \left(\frac{1}{s-\omega_2} - \frac{1}{s-\omega_1} \right)$$

$$\Rightarrow G(s) = \frac{1}{4 \left(\frac{s}{\omega_1} + 1 \right) \left(\frac{s}{\omega_2} + 1 \right)}$$

$$\omega < \omega_2 \rightarrow M = \frac{1}{4} \rightarrow M_{dB} = -20 \log 4$$

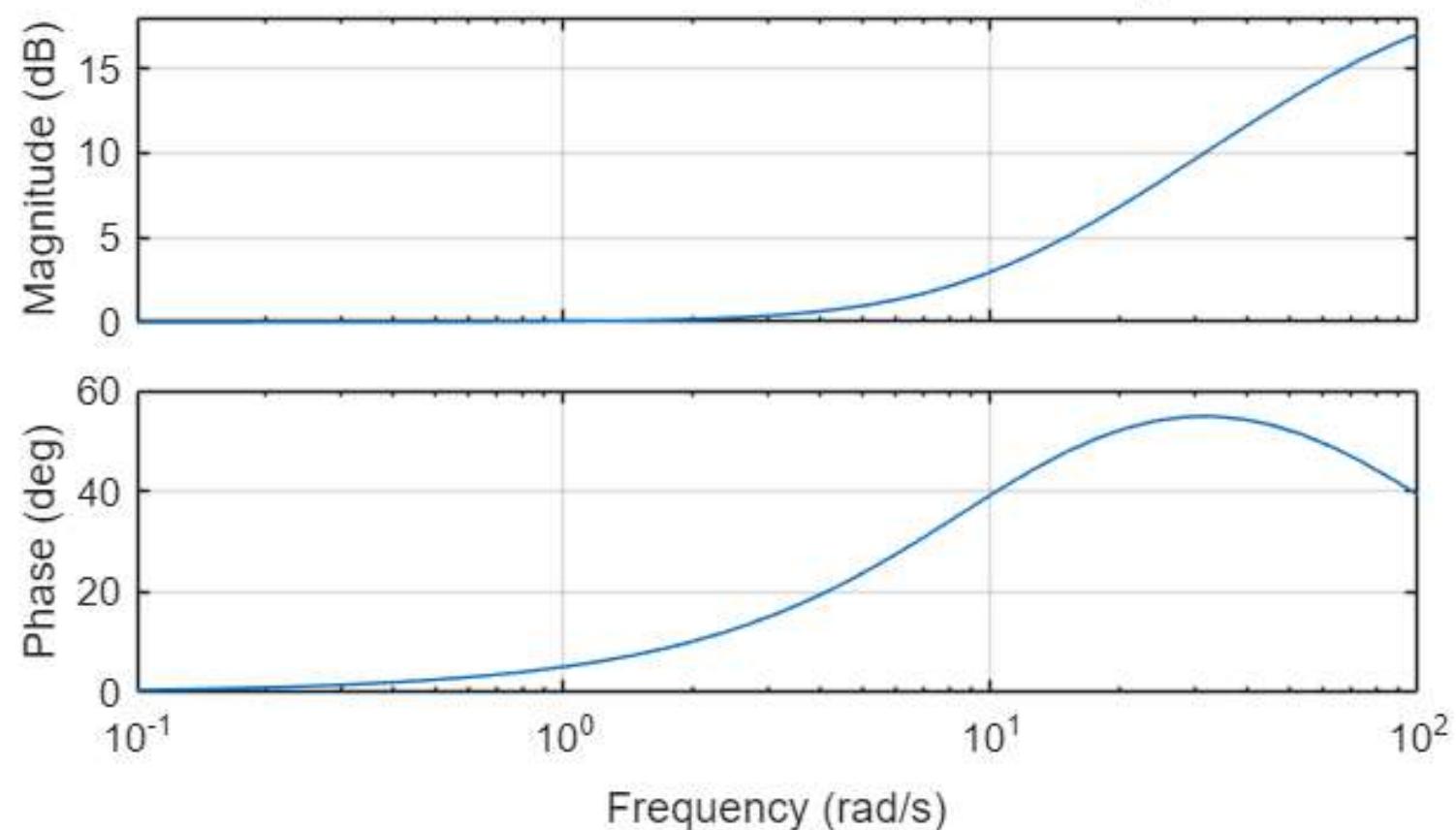
$$\omega_2 < \omega < \omega_1 \rightarrow M = \frac{\omega_2}{4j\omega} \rightarrow M_{dB} = 20 \log \left(\frac{\omega_2}{4} \right) - 20 \log \omega$$

$$10(\omega), dB \quad \omega > \omega_1 \rightarrow M = \frac{1}{\omega^2} \rightarrow M_{dB} = -40 \log \omega$$



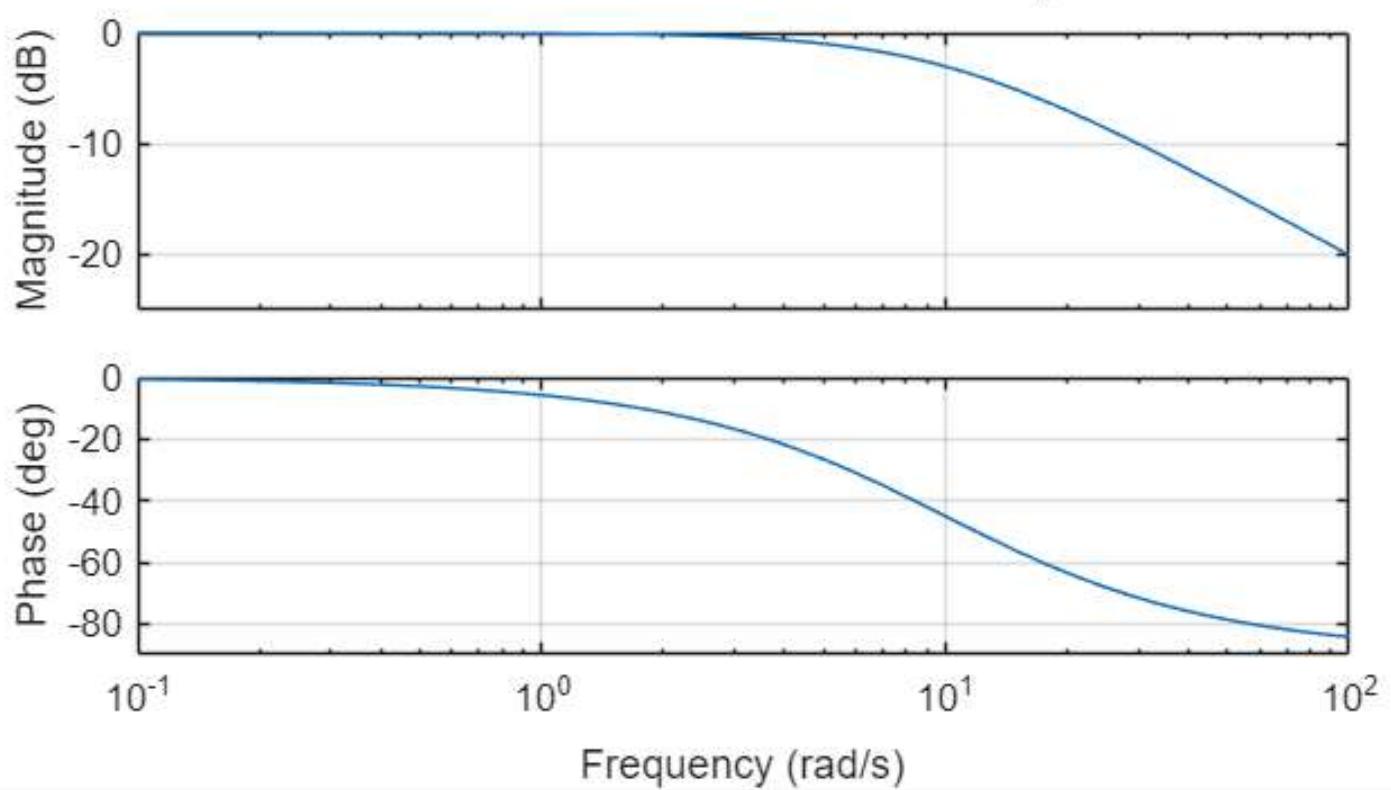
Bode Diagram

$G(s)$



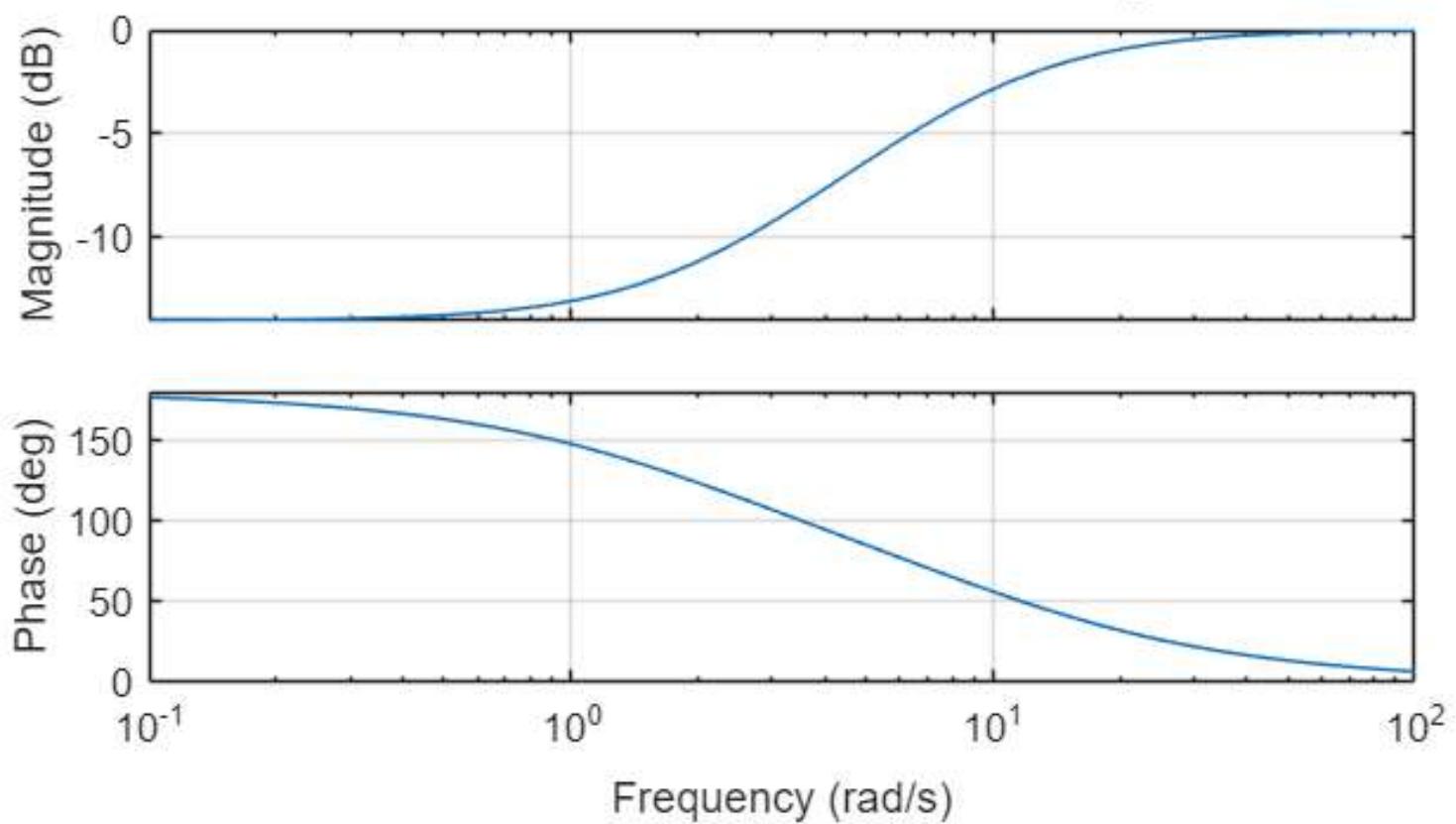
Bode Diagram

G_1

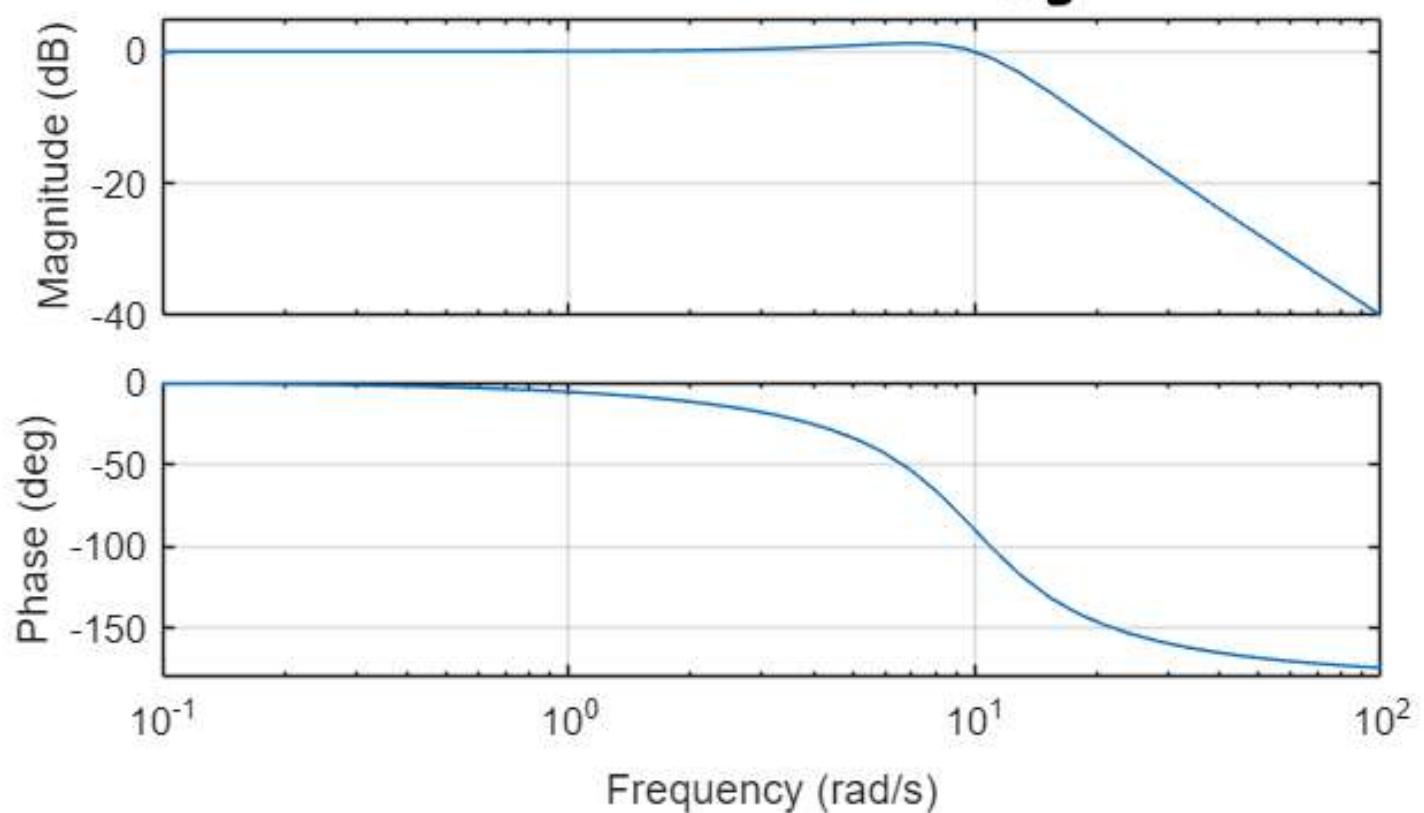


Bode Diagram

G_2



Bode Diagram G_3



Bode Diagram

G_4

