**How does a two-asset portfolio compare to single-asset portfolios?**

We will examine this question from the point of view of an extremely risk-averse investor—one who wishes to minimize portfolio beta as much as possible, even at the expense of potential returns. To construct such a portfolio from two assets, we choose weights of an equity index (all of which have positive betas) and of a volatility index (all of which have negative betas) such that our portfolio beta (the weighted average of the asset betas) is zero.

We perform a bootstrap to estimate how such a portfolio performs. We buy $10,000 worth of assets, weighted such that portfolio beta is zero and bootstrap a month’s worth of returns (20 days). We replicate this process 500 times to arrive at 500 sample months of data. We repeat this process combining each of the three equity indices with each of the three volatility indices for a total of nine 500-month bootstrap samples. Tabular and graphical results of these bootstraps are presented in the appendix.

VXD has the highest mean for all equity indices, and RVX has the lowest. VXD is also the riskiest: It has the highest potential returns (highest 95th percentile) but also the highest potential losses (lowest 5th percentile).

Betas

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| VIX | SPY | RVX | RUT | DJI | VXD | VTI |
| -4.1713 | 0.9862 | -3.1513 | 1.1996 | 0.8856 | -3.8791 | 1.0000 |

Single-Asset Bootstrap Summary

|  |  |  |  |
| --- | --- | --- | --- |
|  | VIX | RVX | VXD |
| Min. | $ 3,301.00 | $ 5,051.00 | $ 4,138.00 |
| 1st Qu. | $ 8,185.00 | $ 8,343.00 | $ 8,170.00 |
| Median | $ 10,150.00 | $ 9,953.00 | $ 9,972.00 |
| Mean | $ 10,640.00 | $ 10,260.00 | $ 10,560.00 |
| 3rd Qu. | $ 12,550.00 | $ 11,650.00 | $ 12,570.00 |
| Max. | $ 24,210.00 | $ 20,120.00 | $ 31,550.00 |
| 5% | $ 6,285.50 | $ 6,758.78 | $ 6,164.85 |
| 95% | $ 17,125.63 | $ 14,952.02 | $ 16,727.71 |
|  |  |  |  |
|  | SPY | RUT | DJI |
| Min. | $ 8,201.00 | $ 8,190.00 | $ 8,486.00 |
| 1st Qu. | $ 9,656.00 | $ 9,596.00 | $ 9,740.00 |
| Median | $ 10,050.00 | $ 10,050.00 | $ 10,050.00 |
| Mean | $ 10,060.00 | $ 10,120.00 | $ 10,080.00 |
| 3rd Qu. | $ 10,420.00 | $ 10,590.00 | $ 10,430.00 |
| Max. | $ 13,260.00 | $ 12,870.00 | $ 11,890.00 |
| 5% | $ 9,100.07 | $ 8,892.36 | $ 9,216.73 |
| 95% | $ 11,058.40 | $ 11,422.66 | $ 10,907.71 |
|  |  |  |  |
|  | VTI |  |  |
| Min. | $ 7,719.00 |  |  |
| 1st Qu. | $ 9,760.00 |  |  |
| Median | $ 10,120.00 |  |  |
| Mean | $ 10,130.00 |  |  |
| 3rd Qu. | $ 10,500.00 |  |  |
| Max. | $ 12,500.00 |  |  |
| 5% | $ 9,124.79 |  |  |
| 95% | $ 11,226.74 |  |  |

Two-Asset Bootstrap Summary

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | VXD-DJI | VIX-DJI | RVX-DJI | ALL-DJI |
| Min. | $ 8,800.00 | $ 8,778.00 | $ 9,153.00 | $ 8,778.00 |
| 1stQu. | $ 9,866.00 | $ 9,855.00 | $ 9,845.00 | $ 9,855.00 |
| Median | $ 10,120.00 | $ 10,110.00 | $ 10,080.00 | $ 10,110.00 |
| Mean | $ 10,150.00 | $ 10,130.00 | $ 10,110.00 | $ 10,130.00 |
| 3rdQu. | $ 10,410.00 | $ 10,360.00 | $ 10,370.00 | $ 10,380.00 |
| Max. | $ 12,580.00 | $ 11,870.00 | $ 11,690.00 | $ 12,580.00 |
| 5% | $ 9,503.70 | $ 9,513.07 | $ 9,504.10 | $ 9,509.87 |
| 95% | $ 10,937.91 | $ 10,831.26 | $ 10,784.14 | $ 10,849.91 |
|  |  | | | |
|  | VXD-RUT | VIX-RUT | RVX-RUT | ALL-RUT |
| Min. | $ 8,546.00 | $ 8,978.00 | $ 8,873.00 | $ 8,546.00 |
| 1stQu. | $ 9,813.00 | $ 9,794.00 | $ 9,817.00 | $ 9,811.00 |
| Median | $ 10,180.00 | $ 10,140.00 | $ 10,110.00 | $ 10,140.00 |
| Mean | $ 10,190.00 | $ 10,160.00 | $ 10,130.00 | $ 10,160.00 |
| 3rdQu. | $ 10,540.00 | $ 10,450.00 | $ 10,430.00 | $ 10,470.00 |
| Max. | $ 13,610.00 | $ 12,600.00 | $ 12,050.00 | $ 13,610.00 |
| 5% | $ 9,308.62 | $ 9,372.04 | $ 9,410.18 | $ 9,368.12 |
| 95% | $ 11,175.21 | $ 11,024.07 | $ 10,891.22 | $ 11,043.91 |
|  |  | | | |
|  | VXD-SPY | VIX-SPY | RVX-SPY | ALL-SPY |
| Min. | $ 8,871.00 | $ 8,996.00 | $ 9,153.00 | $ 8,871.00 |
| 1stQu. | $ 9,856.00 | $ 9,869.00 | $ 9,845.00 | $ 9,853.00 |
| Median | $ 10,140.00 | $ 10,120.00 | $ 10,080.00 | $ 10,130.00 |
| Mean | $ 10,160.00 | $ 10,150.00 | $ 10,110.00 | $ 10,150.00 |
| 3rdQu. | $ 10,440.00 | $ 10,380.00 | $ 10,370.00 | $ 10,410.00 |
| Max. | $ 12,000.00 | $ 11,770.00 | $ 11,690.00 | $ 12,000.00 |
| 5% | $ 9,419.78 | $ 9,491.67 | $ 9,504.10 | $ 9,474.27 |
| 95% | $ 10,963.51 | $ 10,870.15 | $ 10,784.14 | $ 10,901.31 |
|  |  | | | |