EML6350 Nonlinear Control Systems
Project
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Nonlinear Control of Quadrotor for Point Tracking: Actual Implementation and Experimental Tests

Introduction-

In this paper, a nonlinear control scheme along with its simulation and experimental results for a quadrotor are presented. The problem illustrated in this paper is the complexity of controlling a quadrotor with 6 different states (x, y, z, φ, θ, ψ) with only four inputs (U1, U2, U3 and U4) in the form of current supplied to generate torque in each rotor. The previous methods of controllers were mostly PID-class controllers and linear quadratic regulators that excelled only under controlled environments. This paper proposes a novel nonlinear controller by using a backstepping-like feedback linearization method to control and stabilize the quadrotor. The dynamics of the quadrotor was obtained using the Euler-Lagrangian approach. The designed controller is divided into three sub controllers which are called attitude controller, altitude controller, and position controller. Stability of the designed controllers are verified by the Lyapunov stability theorem. The designed controllers are tested and validated under different simulated environments. The experimental results obtained show that the designed controller was able to carry out the tasks of taking off, hovering and positioning. The altitude controller will be analyzed and simulated because the attitude and position controllers are outside of the scope of this Nonlinear Controls course.

Analysis-

The dynamic model of the quadrotor is derived using the *Euler- Lagrangian* approach. The full dynamics of attitude and position of the quadrotor is basically those of a rotating rigid body with six degrees of freedom.

Let $q = (x, y, z, \varphi, \theta, \psi) \in R_6$ be the generalized coordinates where $\xi = (x, y, z) \in R_3$ denotes the absolute position of the mass of the quadrotor relative to a fixed inertial frame. Euler angles, which are the orientation of the quadrotor, are expressed by $\eta = (\varphi,\theta,\psi) \in R_3$, where φ is the roll angle around the x axis, θ is the pitch angle around the y axis, and ψ is the yaw angle around the z axis. It is assumed that the Euler angles are bounded as follows:

Roll: $-\pi/2 \leq \varphi \leq \pi/2$

Pitch : $-\pi/2 \leq \theta \leq \pi/2$

yaw : $-\pi \leq \psi < \pi$.

First, the dynamic model of the quadrotor in terms of translations (x, y, z) and rotations (ϕ,Θ,ψ) was derived using Euler-Lagrangian model as:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\phi s_\theta s_\psi - s_\phi c_\psi \\ c_\phi c_\theta \end{bmatrix} \frac{U_1}{m} + \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = f(\phi, \theta, \psi) + g(\phi, \theta, \psi)\tau_U$$

where c_ϕ is $\cos\phi$, s_ϕ is $\sin\phi$, and

$$f(\phi, \theta, \psi) = \begin{bmatrix} \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_r}{I_x}\dot{\theta}\Omega \\ \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) + \frac{J_r}{I_y}\dot{\phi}\Omega \\ \dot{\phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) \end{bmatrix}$$

$$g(\phi, \theta, \psi) = \begin{bmatrix} \frac{l}{I_x} & 0 & 0 \\ 0 & \frac{l}{I_y} & 0 \\ 0 & 0 & \frac{1}{I_z} \end{bmatrix}$$

The vertical force control input (U_1) controls the altitude subsystem and can be defined as the summation of the thrust moment of each motor. Therefore altitude error must be considered in this system as such:

$$e_7 = z - z_d$$

$$e_8 = \dot{z} - \dot{z}_d + k_7 e_7$$

Where we are told that k_7 is the control parameter which is always positive constant and the error e_8 converges to zero when e_7 is zero.

Once our error values are defined it is possible to choose a Lyapunov candidate (V_2) in order to stabilize the system. The Lyapunov equation used was:

$$V_2 = ½(e_7^2 + e_8^2)$$

Its derivative was found and values of error were plugged in:

$\dot{V_2} = e_7\dot{e}_7 + e_8\dot{e}_8$

$\dot{V_2} = e_7\dot{e}_7 + e_8(\ddot{z} - \ddot{z}_d + k_7\dot{e}_7)$

$\dot{V_2} = e_7(e_8 - k_7e_7) + e_8 \{(cos(\phi)cos(\Theta)\frac{1}{m} * U_1 - g - \ddot{z}_d + k_7\dot{e}_7\}$

$\dot{V_2} = e_7(e_8 - k_7e_7) + e_8 \{(cos(\phi)cos(\Theta)\frac{1}{m} * U_1 - g - \ddot{z}_d + k_7(e_8 - k_7e_7)\}$

In the paper it was assumed that the time derivative of the desired velocity of the altitude was zero ($\ddot{z}_d = 0$). Utilizing the definition of the Lyapunov Stability Theorem, we know that $\dot{V_2}$ should be positive semi-definite. U_1 can thus be defined as:

$$U_1 = \frac{m\{g + (k_7^2 - 1)e_7 - (k_7 + k_8)e_8\}}{\cos(\Phi)\cos(\theta)}$$

We are told that k_8 is the positive definite control parameter. Additionally, we know that when ϕ *or* Θ equals $\frac{\pi}{2}$ *or* $-\frac{\pi}{2}$, then the control input U_1 DNE since its denominator would be equal to zero. Boundary conditions for the Eurler angles were defined in the paper as:

roll : $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$

pitch : $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

We can then derive that the time derivative of the Lyapunov candidate is semi negative definite:

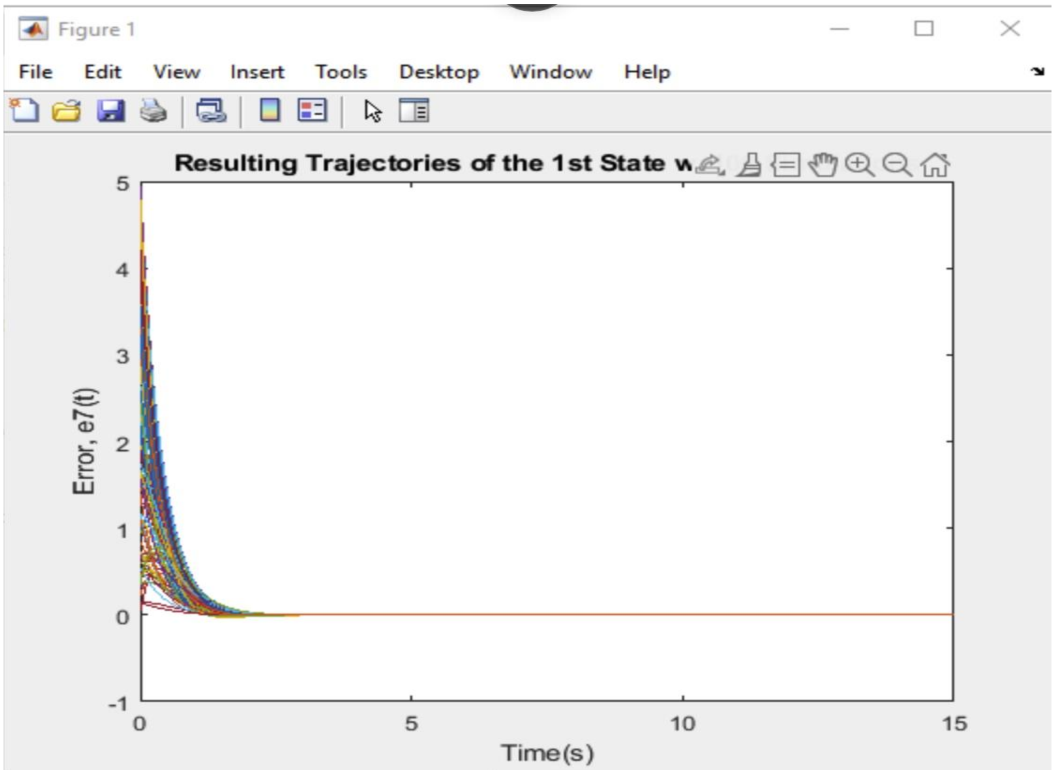
$\dot{V_2} = - k_7e_7^2 - k_8e_8^2 \leq 0$

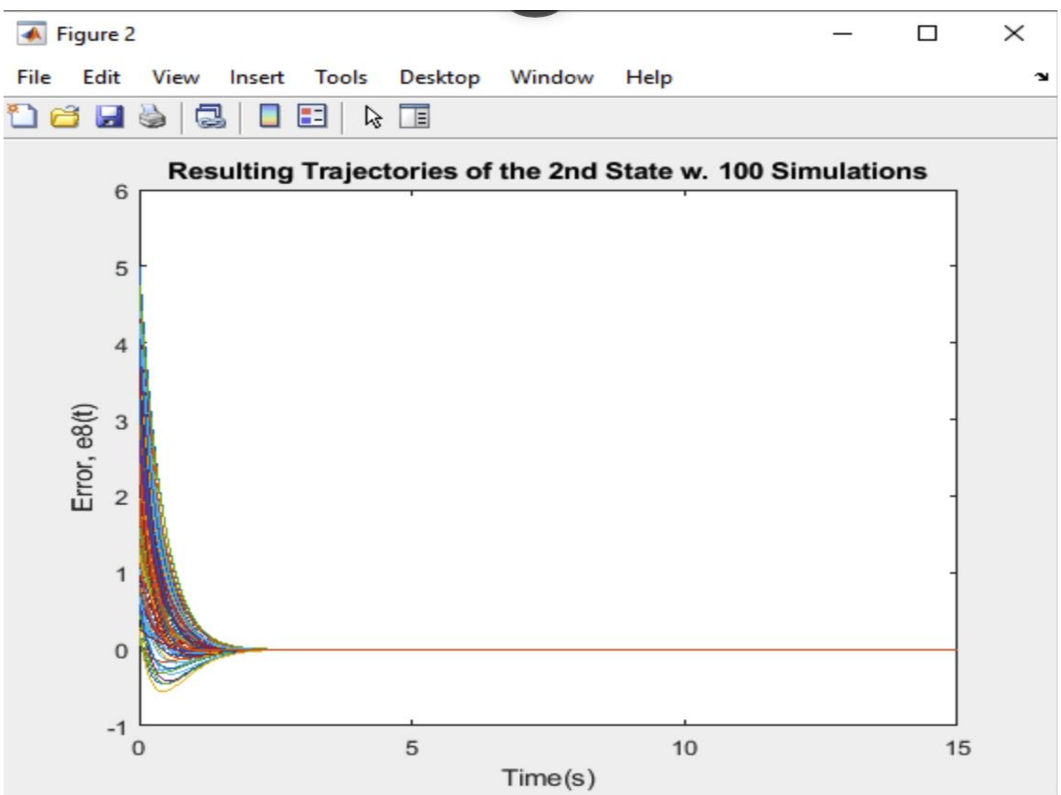
Therefore, the paper concluded that the altitude controller was asymptotically stable (AS).

Implementation- Please check the MATLAB code and graphs attached in the zip file.

Results-

(Figure 1 to Figure 4 show the simulated results using MATLAB)





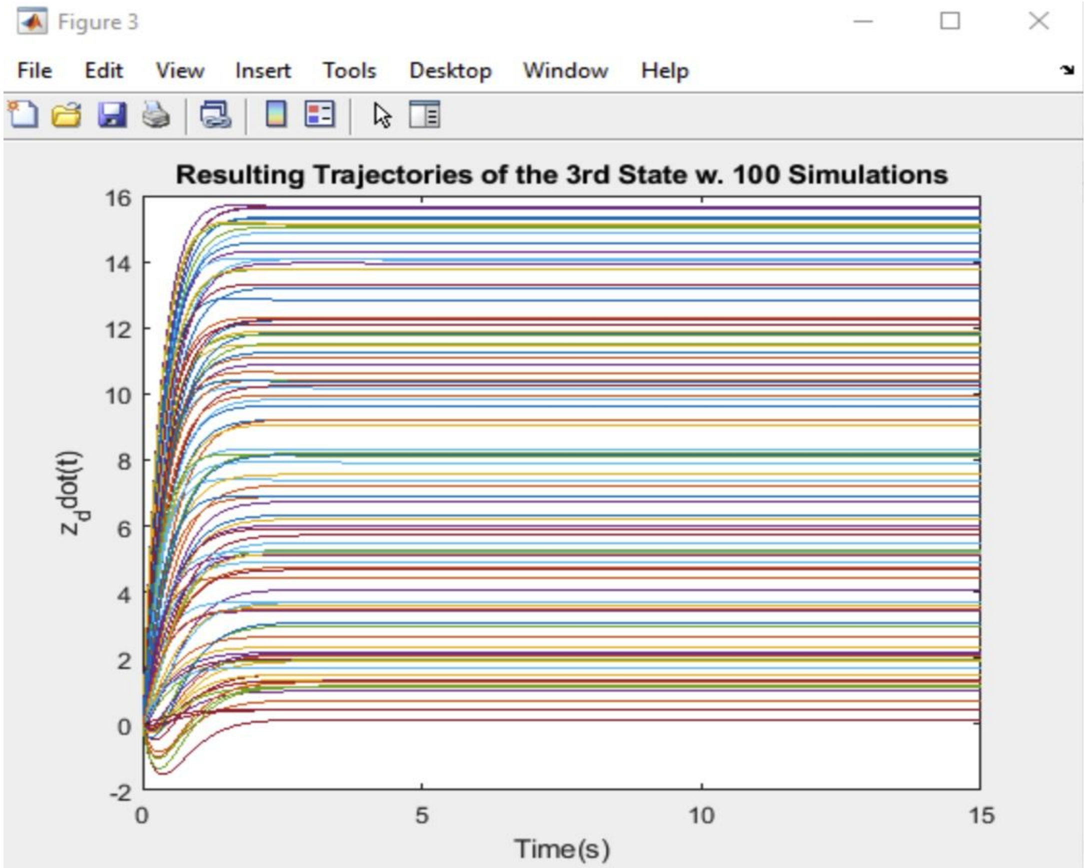


Figure 3
File Edit View Insert Tools Desktop Window Help
Resulting Trajectories of the 3rd State w. 100 Simulations
$z_d$dot(t)
Time(s)
16
14
12
10
8
6
4
2
0
-2
0
5
10
15

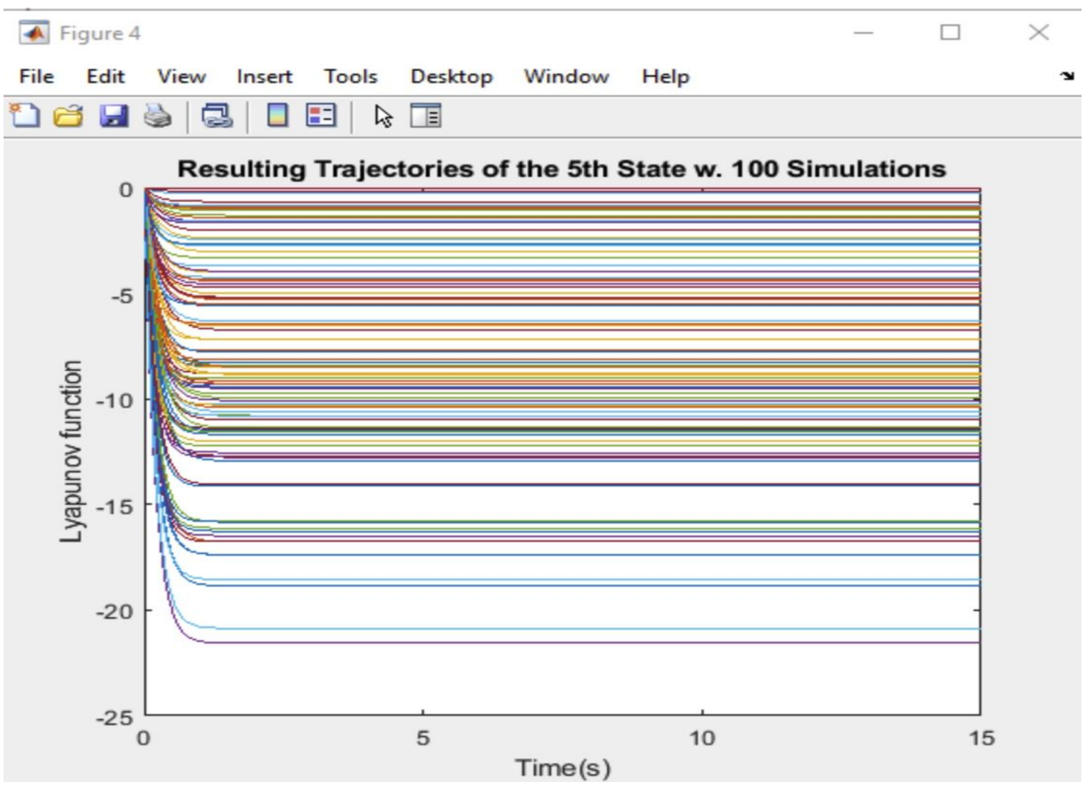


Figure 4
File Edit View Insert Tools Desktop Window Help
Resulting Trajectories of the 5th State w. 100 Simulations
Lyapunov function
Time(s)
0
-5
-10
-15
-20
-25
0
5
10
15

(The following figures 13 and 17 shows the experimental results found in the paper)

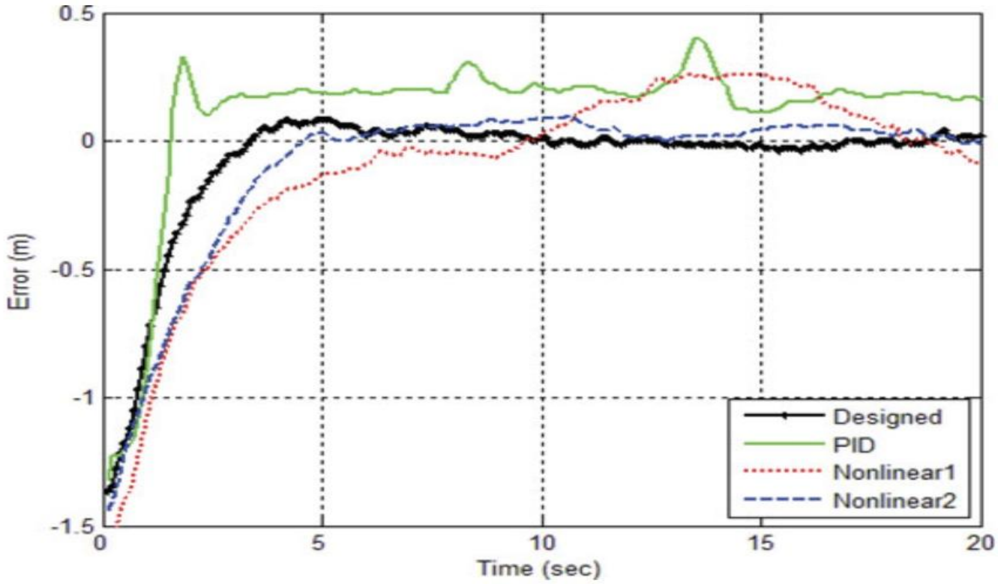


Fig. 13. Experimental result of the altitude error (Take-off).

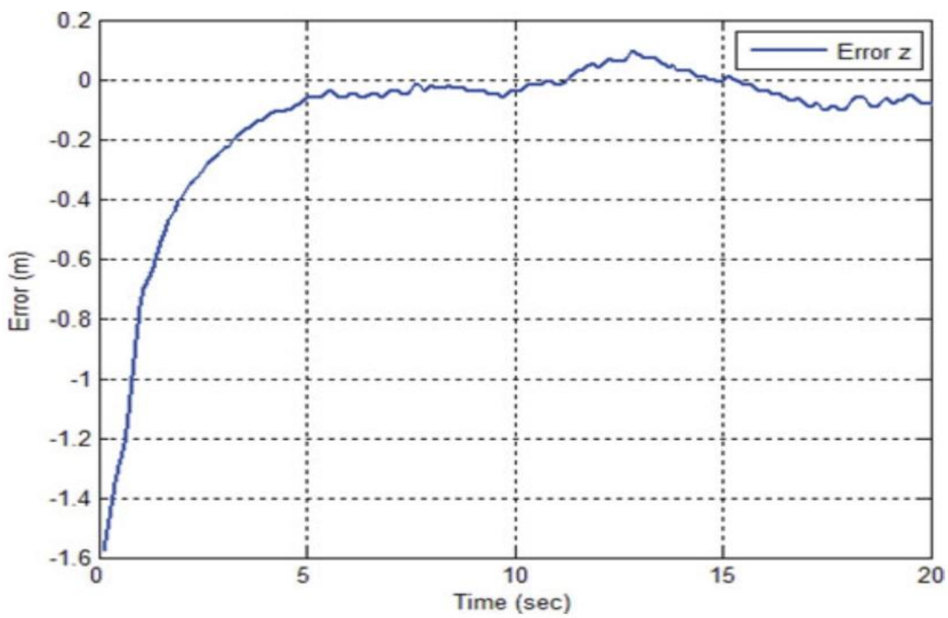


Fig. 17. Experimental result of the altitude error (Tracking).

From our implemented graphs we can conclude that the lyapunov function is following asymptotic stability as it is always decreasing until it reaches a stable state. Graph 1 suggests that altitude tracking error e7 goes to zero as time (t) goes to infinity. A similar observation can be made about altitude tracking error e8 from Graph 2. Graph 3 shows the measured altitude (z) of the quadrotor at every instance of time (t). This graph shows that the controller is stable as all the simulations settle after some time for random orientation and disturbances in the system. The results achieved in our simulation replicate the results observed in the paper.

Conclusion-

In conclusion, this paper presented the first stable controller successfully implemented in a real environment with unknown disturbances and noise. The paper was able to implement nonlinear control algorithms into an unidirectional quadrotor including all physical problems and outdoor experiments. As future work we can extend this design in several directions to increase the accuracy. One way of doing this would be to increase the speed of the motor and adding additional sensors for feedback control. The quadrotor can be designed to be more aerodynamic to improve the performance and maneuverability. It will also be interesting to see how this design will fare in extreme environments(ie. rescue situation during natural calamities, or combat, or for surveillance in hard to reach places).