0.01

EML 6350

Non-linear Control System

1. Given -

Switching dynamic system moving though a field with intermittent feedback

$$m\ddot{\eta}(t) = -c\dot{\eta}(t) + \tau(t) + \partial(t).$$

(a) To find maximum dwell-time.

When outside feedback region.

$\sigma = q$ and ~~$\hat{e} = e + \tilde{x}$~~

Let our lyapunov function be

$$V^q = \frac{1}{2}\tilde{\eta}^T\tilde{\eta}$$

taking derivative on both side, we get.

$$\dot{V}^q = \tilde{\eta}^T\dot{\tilde{\eta}}$$

we know.

$$\dot{\tilde{\eta}} = \dot{\eta} - \dot{\hat{\eta}} = \dot{\eta} - \dot{\eta}_m$$

$$\Rightarrow \quad \dot{V}^q = \tilde{\eta}^T(\dot{\eta} - \dot{\eta}_m).$$

$$\dot{V}_{q_j} = \tilde{\eta}^T(\cancel{\dot{\eta}} - (\cancel{\dot{\eta}} + w_{\dot{\eta}}(t))).$$

$$\dot{V}^q = \tilde{\eta}^T(-w_{\dot{\eta}}(t)).$$

$$\dot{V}^q = -\tilde{\eta}^T w_{\dot{\eta}}(t).$$

we know.

$$\tilde{\eta}^T w_{\dot{\eta}} \leq \|\tilde{\eta}^T\| \|w_{\dot{\eta}}\| \leq \|\tilde{\eta}^T\| \overline{w_{\dot{\eta}}}$$

$$\dot{V}^q \leq -\|\tilde{\eta}^T\| \, \overline{w}_{\eta}$$

Using Young's Inequality

$$\dot{V}^q \leq -\frac{1}{2}\|\tilde{\eta}\|^2 + \left(\frac{1}{2}\|\overline{w}_{\eta_i}\|\right)$$

$$\dot{V}^q \leq \frac{1}{2}\|\tilde{\eta}\|^2 + \left(-\frac{1}{2}\overline{w}_{\eta_i}\right)$$

From Ray layritz theorem we know.

$$\frac{1}{2}\|\tilde{\eta}\|^2 \leq V \leq \frac{1}{2}\|\tilde{\eta}\|^2$$

$$\|\tilde{\eta}\|^2 \leq 2V.$$

$$\Rightarrow \quad \dot{V}^q \leq -\frac{1}{\cancel{2}}(\cancel{2}V) + \frac{1}{2}\overline{w}_{\eta_i}$$

$$\dot{V}^q \leq -V + \underbrace{\frac{1}{2}\overline{w}_{\eta_i}}_{\epsilon_q}$$

$$\Rightarrow \quad \dot{V}^q \leq V + \epsilon_q.$$

Therefore,

$$V^q(\tilde{\eta}(t), t) \leq \left[V^q(\tilde{\eta}(t_j^q), t_j^q) + \epsilon_q\right] \exp(t - t_j^q) - \epsilon_q.$$

we know that the maximum dwell time should follow the condition:

$$V^q(\tilde{\eta}(t_{j+1}^s), t_{j+1}^s) \leq \overline{V}$$

We Know,

$$V^q(\tilde{\eta}(t_j^q), t_j^q) = \tfrac{1}{2}\tilde{\eta}^T\tilde{\eta}$$

As $\hat{\eta} \longrightarrow \eta$

$$\tilde{\eta} = -w_{\cancel{q}\eta} \Rightarrow \tilde{\eta} \leq \|w_q\| \leq \overline{w}_{\cancel{q}\eta}$$

$$\Rightarrow \quad V^q(\tilde{\eta}(t_j^q), t_j^q) \leq \frac{1}{2}\overline{w}_\eta^2$$

Now

$$\left(\frac{1}{2}\overline{w}_\eta^2 + \varepsilon_q\right)\exp\big(\underbrace{t_{j+1}^s - t_j^q}_{\Delta t_j^q}\big) - \epsilon_q \leq \overline{V}$$

and $V_j^q = \frac{1}{2}\tilde{\eta}^T\tilde{\eta} \Rightarrow \overline{V} = \frac{1}{2}\|\tilde{\eta}\|^2$

$$\Rightarrow \quad \exp(\Delta t_j^q) \leq \frac{\overline{V} + \epsilon_q}{\left(\frac{1}{2}\overline{w}_\eta^2 + \epsilon_q\right)}.$$

$$\Delta t_j^q \leq \ln\left[\frac{\overline{V} + \frac{1}{2}\overline{w}_{\dot{\eta}}^2}{\frac{1}{2}\overline{w}_\eta^2 + \frac{1}{2}\overline{w}_{\dot{\eta}}^2}\right]$$

To find max dwell time (numerical).

$$\overline{V} = \tfrac{1}{2}\|\tilde{\eta}\|^2 = \tfrac{1}{2}(10)^2 = 50\ m^2$$

$$\overline{w}_{\dot{\eta}} = 0.005\ m/sec \quad \overline{w}_\eta = 0.1\,m.$$

$$\Delta t_j^q \leq \ln\left(\frac{(50) + \frac{1}{2}(0.005)^2}{\frac{1}{2}(0.1)^2 + \frac{1}{2}(0.005)^2}\right).$$

$$\boxed{\Delta t_j^q \leq 9.21\ sec}$$

(b). Estimated tracking error

$$\hat{e} = \eta_d - \hat{\eta}$$

$$\dot{\hat{e}} = \dot{\eta}_d - \dot{\hat{\eta}} = \dot{\eta}_d - \dot{\eta}_m .$$

$$\dot{\hat{e}} = \dot{\eta}_d - \dot{\eta}_m .$$

$$\ddot{\hat{e}} = \ddot{\eta}_d - \ddot{\eta}_m = \ddot{\eta}_d - \left(\ddot{\eta}(t) + w_{\ddot{\eta}}(t)\right).$$

$$\ddot{\hat{e}} = \ddot{\eta}_d - \ddot{\eta}(t) - w_{\ddot{\eta}}(t)$$

$$m\ddot{\hat{e}} = m\ddot{\eta}_d - m\ddot{\eta} - m w_{\ddot{\eta}} \qquad -①$$

Filtered tracking error

$$\hat{r} = \dot{\hat{e}} + \alpha \hat{e}$$

$$\dot{\hat{r}} = \ddot{\hat{e}} + \alpha \dot{\hat{e}}$$

$$m\dot{\hat{r}} = m\ddot{\hat{e}} + m\alpha\dot{\hat{e}}$$

Sub equ ①

$$m\dot{\hat{r}} = \left(m\ddot{\eta}_d - m\ddot{\eta} - m w_{\ddot{\eta}}\right) + m\alpha\dot{\hat{e}}$$

$$m\dot{\hat{r}} = m\ddot{\eta}_d - \left(-c\dot{\eta} + \tau + d\right) - m w_{\ddot{\eta}} + m\alpha\dot{\hat{e}}$$

$$m\dot{\hat{r}} = m\left(\ddot{\eta}_d - w_{\ddot{\eta}} + \alpha\dot{\hat{e}}\right) + c\dot{\eta} - \tau - d$$

$$m\dot{\hat{r}} = \underbrace{\left[\ddot{\eta}_d - w_{\ddot{\eta}} + \alpha\dot{\hat{e}} \quad \dot{\eta}\right]}_{Y} \underbrace{\begin{bmatrix} m \\ c \end{bmatrix}}_{\Theta} - \tau - d$$

$$m\dot{\hat{r}} = Y\Theta - \tau - d$$

Let $\tilde{\theta} = \theta - \hat{\theta}$

$$Y\theta = m\dot{\hat{r}} - \tau - \partial$$

for concurrent learning.

$$y = Y \quad \text{and} \quad u = m\dot{\hat{r}} - \tau$$

$$\Rightarrow \quad y\theta = u - \partial$$

$$y^T y\theta = y^T u - y^T \partial$$

$$\sum_{i=1}^{N} y^T(t_i) y(t_i)\theta = \sum_{i=1}^{N} y^T(t_i) u(t_i) + \sum_{i=1}^{N} y^T(t_i)\partial(t_i).$$

$$\sum_{i=1}^{N} y^T(t_i) u(t_i) = \sum_{i=1}^{N} y^T(t_i) y(t_i)\theta - \sum_{i=1}^{N} y^T(t_i)\partial(t_i).$$

Let $\xi = [\hat{e} \quad \hat{r} \quad \tilde{\theta}]^T$

Lyapunov candidate.

$$V = \frac{1}{2}\hat{e}\hat{e}^T + \frac{1}{2}\hat{r}m\hat{r}^T + \frac{1}{2}\tilde{\theta}\tilde{\theta}^T$$

$$\dot{V} = \hat{e}^T\dot{\hat{e}} + \hat{r}^T m\dot{\hat{r}} + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}}$$

we know,

$$\hat{r} = \dot{\hat{e}} + \alpha\hat{e} \Rightarrow \dot{\hat{e}} = \hat{r} - \alpha\hat{e}$$

$$m\dot{\hat{r}} = Y\theta - \tau - \partial$$

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}} \quad (\because \theta \text{ is constant}).$$

$$\dot{V} = \hat{e}^T(\hat{r} - \alpha\hat{e}) + \hat{r}^T(Y\theta - \tau - \partial) + \tilde{\theta}^T\Gamma^{-1}(-\dot{\hat{\theta}}).$$

Design input τ as.

$$\tau = Y\hat{\theta} + \beta_\partial \operatorname{sgn}(\hat{r}) + \beta_r\hat{r} + \hat{e}$$

$Y\tilde{\theta}$

$$\dot{V} = \cancel{\hat{e}^T\hat{r}} - \hat{e}^T\alpha\hat{e} + \hat{r}^T(Y\theta - \cancel{\hat{e}} - Y\hat{\theta} - \beta_r\hat{r} - \beta_\partial sgn(\hat{r}) - \partial) - \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}}$$

$$\dot{V} = -\hat{e}^T\alpha\hat{e} + \hat{r}^TY\tilde{\theta} - \hat{r}^T\beta_r\hat{r} - \hat{r}^T\beta_\partial sgn(\hat{r}) - \hat{r}^T\partial - \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}}$$

we know,

$$-\hat{r}^T\beta_\partial sgn(\hat{r}) \leq -\underline{\beta_\partial}\,\|\hat{r}\|$$

$$-\hat{r}^T\partial \leq +\bar{\partial}\,\|\hat{r}\|.$$

$$\Rightarrow \quad -(\beta_\partial - \bar{\partial})\|\hat{r}\| \leq 0 \quad \text{if } \beta_\partial > \bar{\partial}$$

(so drop)

$$\dot{V} = -\hat{e}^T\alpha\hat{e} - \hat{r}^T\beta_r\hat{r} + \hat{r}^TY\tilde{\theta} - \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}}$$

Design $\dot{\hat{\theta}}$

$$\dot{\hat{\theta}} = \Gamma Y^T\hat{r} \quad \text{(Before collecting enough data)}$$

$$\dot{V} = -\hat{e}^T\alpha\hat{e} - \hat{r}^T\beta_r\hat{r} + \hat{r}^TY\tilde{\theta} - \tilde{\theta}^T\cancel{\Gamma^{-1}}(\cancel{\Gamma}Y^T\hat{r}).$$

$$= -\hat{e}^T\alpha\hat{e} - \hat{r}^T\beta_r\hat{r} + \underbrace{\cancel{\hat{r}^TY\tilde{\theta}}}_{=\tilde{\theta}^TY^T\hat{r}} - \cancel{\tilde{\theta}^TY^T\hat{r}}.$$

$$\dot{V} = -\hat{e}^T\alpha\hat{e} - \hat{r}^T\beta_r\hat{r}$$

$$\dot{V} \leq -\underline{\alpha}\|\hat{e}\|^2 - \underline{\beta_r}\,\|\hat{r}\|$$

$\dot{V}$ is NSD

Let

$$\dot{\hat{\theta}} = \Gamma Y^T\hat{r} + \Gamma K_\theta y^Ty\tilde{\theta} + \Gamma K_{CL}\left(\sum_{i=1}^{N} y^Ty\tilde{\theta} - \sum_{i=1}^{N} y^T\partial\right)$$

(when enough data is collected after $t \geq T$)

$$\dot{V} = -\hat{e}^T \alpha \hat{e} - \hat{\gamma}^T \beta_\gamma \hat{\gamma} + \hat{\gamma}^T Y \tilde{\theta} - \tilde{\theta}^T \Gamma^{-1} \left(\Gamma Y^T \hat{\gamma} + \Gamma K_\theta y^T y \tilde{\theta} + \Gamma K_{cl} \left(\sum_{i=1}^{N} y^T y \tilde{\theta} - \sum_{i=1}^{N} y^T \partial \right) \right)$$

$$\dot{V} = -\hat{e}^T \alpha \hat{e} - \hat{\gamma}^T \beta_\gamma \hat{\gamma} - \tilde{\theta}^T \Gamma^{-1} \Gamma K_\theta y^T y \tilde{\theta} - \tilde{\theta}^T \Gamma^{-1} \Gamma K_{cl} \left(\sum_{i=1}^{N} y^T y \tilde{\theta} - \sum_{i=1}^{N} y^T \partial \right)$$

$$\dot{V} = -\hat{e}^T \alpha \hat{e} - \hat{\gamma}^T \beta_\gamma \hat{\gamma} - K_\theta \tilde{\theta}^T y^T y \tilde{\theta} - K_{cl} \tilde{\theta}^T \tilde{\theta} \sum_{i=1}^{N} y^T y + K_{cl} \tilde{\theta}^T \sum_{i=1}^{N} y^T \partial$$

we know

1. $-\hat{e}^T \alpha \hat{e} \le -\alpha \|\hat{e}\|^2$
2. $-\hat{\gamma}^T \beta_\gamma \hat{\gamma} \le -\beta \|\hat{\gamma}\|^2$
3. ~~$-K_\theta \tilde{\theta}^T y^T y \tilde{\theta} \le -K_\theta \|\tilde{\theta}\|^2 \cdot \lambda$~~
4. ~~$K_{cl} \tilde{\theta}^T$~~

3. $-K_\theta \tilde{\theta}^T y^T y \tilde{\theta} \le -K_\theta \|\tilde{\theta}\|^2 \cdot \|y\|^2$
4. $-K_{cl} \tilde{\theta}^T \tilde{\theta} \sum_{i=1}^{N} y^T y \le -K_{cl} \|\tilde{\theta}\|^2 \lambda$ (λ is min eig value of $\sum_{i=1}^{N} y^T y$)
5. $K_{cl} \tilde{\theta}^T \sum_{i=1}^{N} y^T \partial \le -K_{cl} \|\tilde{\theta}\| N \bar{y} \partial$.

Therefore

$$\dot{V} \le -\alpha \|\hat{e}\|^2 - \beta \|\hat{\gamma}\|^2 - K_\theta \|\tilde{\theta}\|^2 \|y\|^2 - K_{cl} \|\tilde{\theta}\|^2 \lambda - K_{cl} \|\tilde{\theta}\| N \bar{y} \partial.$$

Split ① to use ~~current~~ ~~Nonlinear~~ Nonlinear damping

$$-\tfrac{1}{2} K_{cl} \|\tilde{\theta}\|^2 \lambda + K_{cl} \|\tilde{\theta}\| N \cdot \bar{y} \partial \le \frac{(K_{cl} \bar{y} \partial N)^2}{2 K_{cl} \lambda}.$$

So,

$$\dot{V} \leq -\underline{\alpha}\|\hat{e}\|^2 - \underline{\beta}\|\delta\|^2 - \underline{K_\theta}\|\tilde{\theta}\|^2\|y\|^2 - \tfrac{1}{2}\underline{K_{cl}}\|\tilde{\theta}\|^2\underline{\lambda} + \underbrace{\frac{(\underline{K_{cl}}\,\overline{y}\,\partial N)^2}{2\,\underline{K_{cl}}\,\underline{\Delta}}}$$

$$\dot{V} \leq -\underline{\alpha}\|\hat{e}\|^2 - \underline{\beta}\|\delta\|^2 - K_\theta\underbrace{\|\tilde{\theta}\|^2\|y\|^2}_{\leq 0} - \tfrac{1}{2}\underline{K_{cl}}\|\tilde{\theta}\|^2\underline{\lambda} + \epsilon$$

$$\dot{V} \leq -\min\{\underline{\alpha}, \underline{\beta}, \tfrac{1}{2}\underline{K_{cl}}\underline{\Delta}\}\|\xi\|^2 + \epsilon$$

and

$$V \leq \tfrac{1}{2}\max\{1, m, \Gamma^{-1}\}\|\xi\|^2$$

$$\frac{-2}{\max\{1, m, \Gamma^{-1}\}} \geq -\|\xi\|^2$$

We can write.

$$\dot{V} \leq \frac{+\min\{\underline{\alpha}, \underline{\beta}, \tfrac{1}{2}\underline{K_{cl}}\underline{\lambda}\}(2)}{\max\{1, m, \Gamma^{-1}\}} V + \epsilon.$$

This implies exponential bounds.

Therefore the system is GUUB.

(C) (i) Since our dynamic includes unknown disturbance and added noise during measurement I used backstepping with Euler-Legrange method to simplify the dynamics and then I used adaptive controller with concurrent learning to design our inputs to our design τ and $\dot{\tilde{\theta}}$. Using this approach I was able to prove GUUB using Lyapunov analysis.

(ii) A. Yes, the simulated position error remained less than the upper limit 10 m.

B. Yes, the position error got close ~~the~~ to the max duell time ~~when~~ during time close to max duell time in the unstable region (or no feedback region) initially.

C. Yes, I think the duell time was conservative.

(iii) ~~Fig. 1-7~~ Fig 1 to Fig 7 in MATLAB code.

(iv) Fig 8 and Fig 9.

(v) MATLAB file.