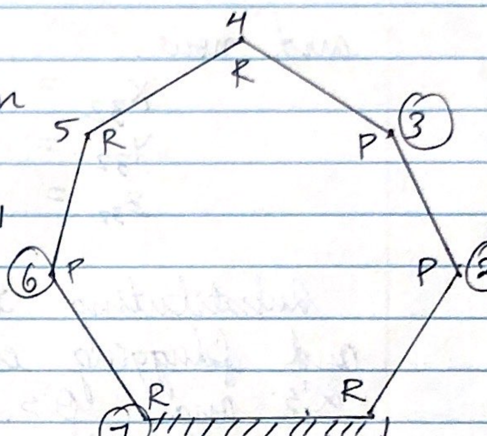
EML6281

Robot Geometry - 1

Homework #6

Problem 7.3



Given - Constant mechanism parameters.

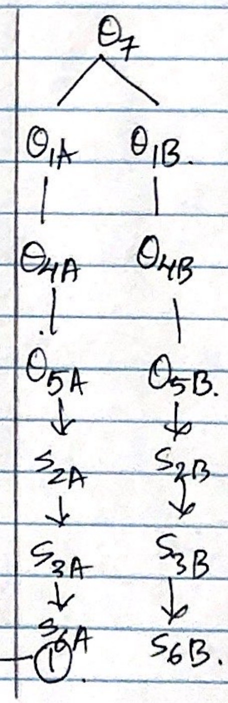
$\alpha_{12}, \alpha_{23}, \alpha_{34}, \alpha_{45}, \alpha_{56}, \alpha_{67}, \alpha_{71}$

$\theta_2, \theta_3, \theta_6$ - (prismatic joints).

θ_7 (input angle).

To find - θ_1, θ_4 and θ_5.

S_2, S_3 and S_6.



(a) Solving for θ_1 first - Looking at our cosine laws (the Z's) in our toolbox to find the equation with all the known θ angles and only θ_1 we find.

$$Z_{67123} = c_{45}$$

$$Z_{32176} = c_{45}.$$

Expanding the LHS of this equation we get.

$$s_{56}(X_{3217}\, s_6 + Y_{3217}\, c_6) + c_{56}\, Z_{3217} - c_{45} = 0. \quad (1)$$

We know that.

$$X_{3217} = X_{321}\, c_7 - Y_{321}\, s_7.$$

$$Y_{3217} = c_{67}(X_{321}\, s_7 + Y_{321}\, c_7) - s_{67} Z_{321}$$

$$Z_{3217} = s_{67}(X_{321}\, s_7 + Y_{321}\, c_7) + c_{67} Z_{321}$$

and

$$X_{321} = X_{32} C_1 - Y_{32} S_1$$
$$Y_{321} = C_{71}(X_{32} S_1 + Y_{32} C_1) - S_{71} Z_{32}$$
$$Z_{321} = S_{71}(X_{32} S_1 + Y_{32} C_1) + C_{71} Z_{32}.$$

and now.

$$X_{32} = \overline{X}_3 C_2 - \overline{Y}_3 S_2$$
$$Y_{32} = C_{12}(\overline{X}_3 S_2 + \overline{Y}_3 C_2) - S_{12} \overline{Z}_3$$
$$Z_{32} = S_{12}(\overline{X}_3 S_2 + \overline{Y}_3 C_2) + C_{12} \overline{Z}_3.$$

Substituting these equation back in equation ① and plugging in all the numerical values of known α's and θ's. we can rearrange the eqn ① in the form of

$$A C_1 + B S_1 + D = 0. \quad -②.$$

Expressing sine and cosine theta 1 (θ_1) in terms of tan half angles such as.

$x_1 = \tan \theta/2$. we can rewrite equation ② as.

$$A\left(\frac{1-x_1^2}{1+x_1^2}\right) + B\left(\frac{2x_1}{1+x_1^2}\right) + D = 0.$$
$$A(1-x_1^2) + B(2x_1) + D(1+x_1^2) = 0.$$
$$(D-A)\, x_1^2 + 2B x_1 + (D+A) = 0.$$

$$\Rightarrow \boxed{x_1 = \frac{-2B \pm \sqrt{(2B)^2 - 4(D-A)(D+A)}}{2(D-A)}}$$

this will ~~say~~ give us 2 solution to the equation ② ie θ_{1A} and θ_{1B} respectively.

(b) Looking at the Buddy equations.

$$\left.\begin{aligned} X_{67123} &= S_{45}\, S_4 \\ Y_{67123} &= S_{45}\, C_4 . \end{aligned}\right\}$$

By expanding the LHS and substituting all the known values for α's and θ's and θ'_{1A} (for A case) we can find a unique value for S_{4A} and C_{4A} from which we can determine a unique value for θ_{4A} (for A case).

Similarly by substituting all known values of α's and θ's in equation ~~an~~ and θ_{1B} (for B case) we can find a unique value for S_{4B} and C_{4B} from which we can determine a unique value for θ_{4B} (for B case).

To find θ_5

(c) Now ~~to~~ we look at the buddy equations.

$$X_{32176} = s_{45} s_5$$
$$Y_{32176} = s_{45} c_5 .$$

Expanding the LHS,

$$\left.\begin{array}{l} X_{3217} c_6 - Y_{3217} s_6 = s_{45} s_5 . \\ c_{56}(X_{3217} s_6 + Y_{3217} c_6) - s_{56} Z_{3217} = s_{45} c_5 \end{array}\right\} - (*)$$

we can find a numerical value to the LHS by plugging in the known values for α's and θ's and θ_{1A} to find the respective values for s_{5A} and c_{5A} from which we can determine a unique value for θ_{5A}.

Similarly by plugging in the known values for known α's and θ's and θ_{1B} we can find numerical values for s_{5B} and c_{5B} from which we can determine a unique value for the B case ie θ_{5B}.

(d) To find the joint offset S_2, S_3 and S_6 for the Prismatic joints.

Using the vector loop equation we know.

$$S_1 \underline{S_1} + a_{12} \underline{a_{12}} + S_2 \underline{S_2} + a_{23} \underline{a_{23}} + S_3 \underline{S_3} + a_{34} \underline{a_{34}} + S_4 \underline{S_4}$$
$$+ a_{45} \underline{a_{45}} + S_5 \underline{S_5} + a_{56} \underline{a_{56}} + S_6 \underline{S_6} + a_{67} \underline{a_{67}} + S_7 \underline{S_7}$$
$$+ a_{71} \underline{a_{71}} = 0$$

At this point, two solutions sets exists for the joint angles of the spatial mechanism. Corresponding values of S_2, S_3 and S_6 can be determined by projecting onto any three linearly independent directions. to yield 3 scalar equations in 3 unknowns. S_2, S_3 and S_6.

Projecting the vector loop equation onto these vectors will result in three scalar equation in three unknowns S_2, S_3 and S_6 which can be solved to get corresponding values by substituting all the known values and calculated θ values for A and B case, we can determine (S_{2A}, S_{3A}, S_{6A}) and $(S_{2B}, S_{3B}$ and $S_{6B})$

The $\underline{S}$ and $\underline{a}$ values are selected from one of the ~~to~~ sets in our toolbox.