EML 6281

Robot Geometry - I

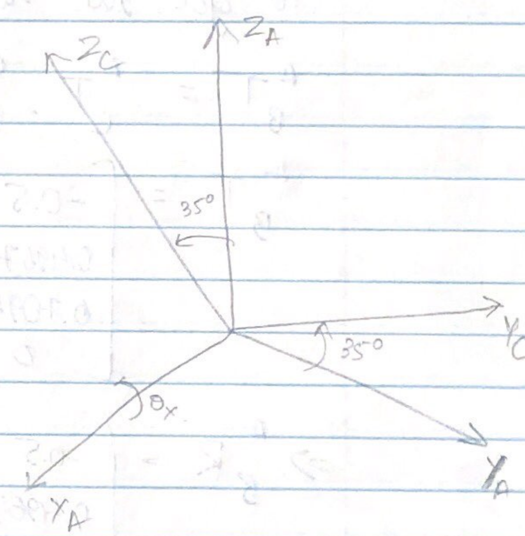
Homework # 2

Problem 2.9

Given – Coordinat system A and B are initially alligned and coincident. System B is then rotated by an angle $\theta_x = 35°$ and then rotated by $\theta_y = 120°$ about its then y-axis.

Consider a system C initially alligned with system A. which is then rotated by $\theta_x = 35°$ about x-axis.

$$^{A}_{C}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 35° & -\sin 35° \\ 0 & \sin 35° & \cos 35° \end{bmatrix}.$$

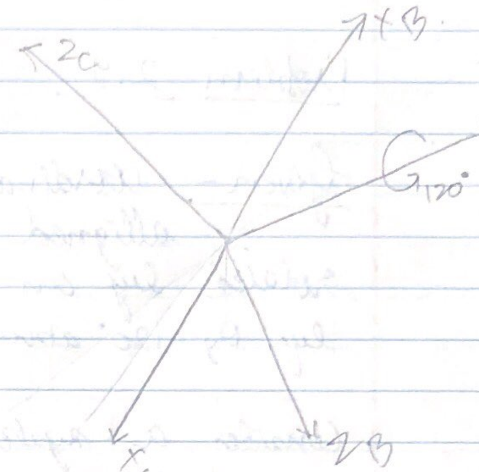


$$^{A}_{C}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8192 & -0.5736 \\ 0 & 0.5736 & 0.8192 \end{bmatrix}$$

$$^{A}_{C}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.8192 & -0.5736 & 0 \\ 0 & 0.5736 & 0.8192 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Consider coordinate system B initially alligned with C rotated about ~~120° ab~~ Y axis by $120° = \Theta_y$.

$$
{}^{C}_{B}R = \begin{bmatrix} \cos 120° & 0 & \sin 120° \\ 0 & 1 & 0 \\ -\sin 120° & 0 & \cos 120° \end{bmatrix}
$$



$$
{}^{C}_{B}T = \begin{bmatrix} -0.5 & 0 & 0.866 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

To get the relation from A to B (or B in A).

$$
{}^{A}_{B}T = {}^{A}_{C}T \cdot {}^{C}_{B}T.
$$

$$
{}^{A}_{B}T = \begin{bmatrix} -0.5 & 0 & 0.866 & 0 \\ 0.4967 & 0.8192 & 0.2868 & 0 \\ 0.7094 & 0.5736 & -0.4096 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\Rightarrow {}^{A}_{B}R = \begin{bmatrix} -0.5 & 0 & 0.866 \\ 0.4967 & 0.8192 & 0.2868 \\ 0.7094 & 0.5736 & -0.4096 \end{bmatrix}
$$

We know that,

$$
\cos\theta = \frac{r_{11} + r_{22} + r_{33} - 1}{2}
$$

$$
\cos\theta = \frac{-0.5 + 0.8192 - 0.4096}{2}
$$

$\cos\theta = -0.5452$

$\theta = \cos^{-1}(-0.5452).$

$\theta = \underline{\underline{\pm 123^\circ}}$

We know that,

$r_{32} - r_{23} = 2\, m_x \sin\theta.$

$\Rightarrow m_x = \dfrac{r_{32} - r_{23}}{2 \sin\theta.} = \dfrac{0.5736 - 0.2868}{2(\sin(123^\circ))}$

$m_x = \underline{\underline{0.171}}$

$r_{21} - r_{12} = 2 m_z \sin\theta.$

$\Rightarrow m_z = \dfrac{r_{21} - r_{12}}{2 \sin\theta.} = \dfrac{0.4967 - 0}{2 \sin(123^\circ)} = \underline{\underline{0.2961}}$

$\cancel{r_{32} - r_{23} = 2 m_x \sin\theta.}$

$\Rightarrow m_y = \dfrac{r_{13} - r_{31}}{2 \sin\theta} = \dfrac{0.866 - 0.7094}{2 \sin(123^\circ)} = \underline{\underline{0.9392}}$

$\boxed{m = [\,0.171,\ 0.9392\ \ 0.2961\,]^T_{//}.}$ when $\theta = 123^\circ.$

and when $\theta = -123^\circ$

$\boxed{m = [\,-0.171,\ -0.9392\ \ -0.2961\,]^T_{//}.}$

Problem 3.4

The variable parameters of this manipulator are.

- twist angles θ_1 and θ_2.
- Joint offset S_3

```
theta=35;
s=sind(theta);
c=cosd(theta);
X=[ 0 0 0 1];
Y=[ 0 0 0].';
R_AC=[ 1 0 0; 0 c -s; 0 s c];
T_AC=[R_AC,Y;X]
theta1=120;
s=sind(theta1);
c=cosd(theta1);
R_CB=[ c 0 s; 0 1 0; -s 0 c];
T_CB=[R_CB,Y;X]
T_AB=T_AC*T_CB
theta2=123;
mx=(0.5736-0.2868)/(2*sind(theta2))
mz=(0.4967-0)/(2*sind(theta2))
my=(0.866+0.7094)/(2*sind(theta2))
mag=sqrt(mx^2+my^2+mz^2)
```

```
T_AC =

    1.0000         0         0         0
         0    0.8192   -0.5736         0
         0    0.5736    0.8192         0
         0         0         0    1.0000


T_CB =

   -0.5000         0    0.8660         0
         0    1.0000         0         0
   -0.8660         0   -0.5000         0
         0         0         0    1.0000


T_AB =

   -0.5000         0    0.8660         0
    0.4967    0.8192    0.2868         0
   -0.7094    0.5736   -0.4096         0
         0         0         0    1.0000


mx =

    0.1710


mz =

    0.2961


my =

    0.9392
```

```
mag =

    0.9995
```

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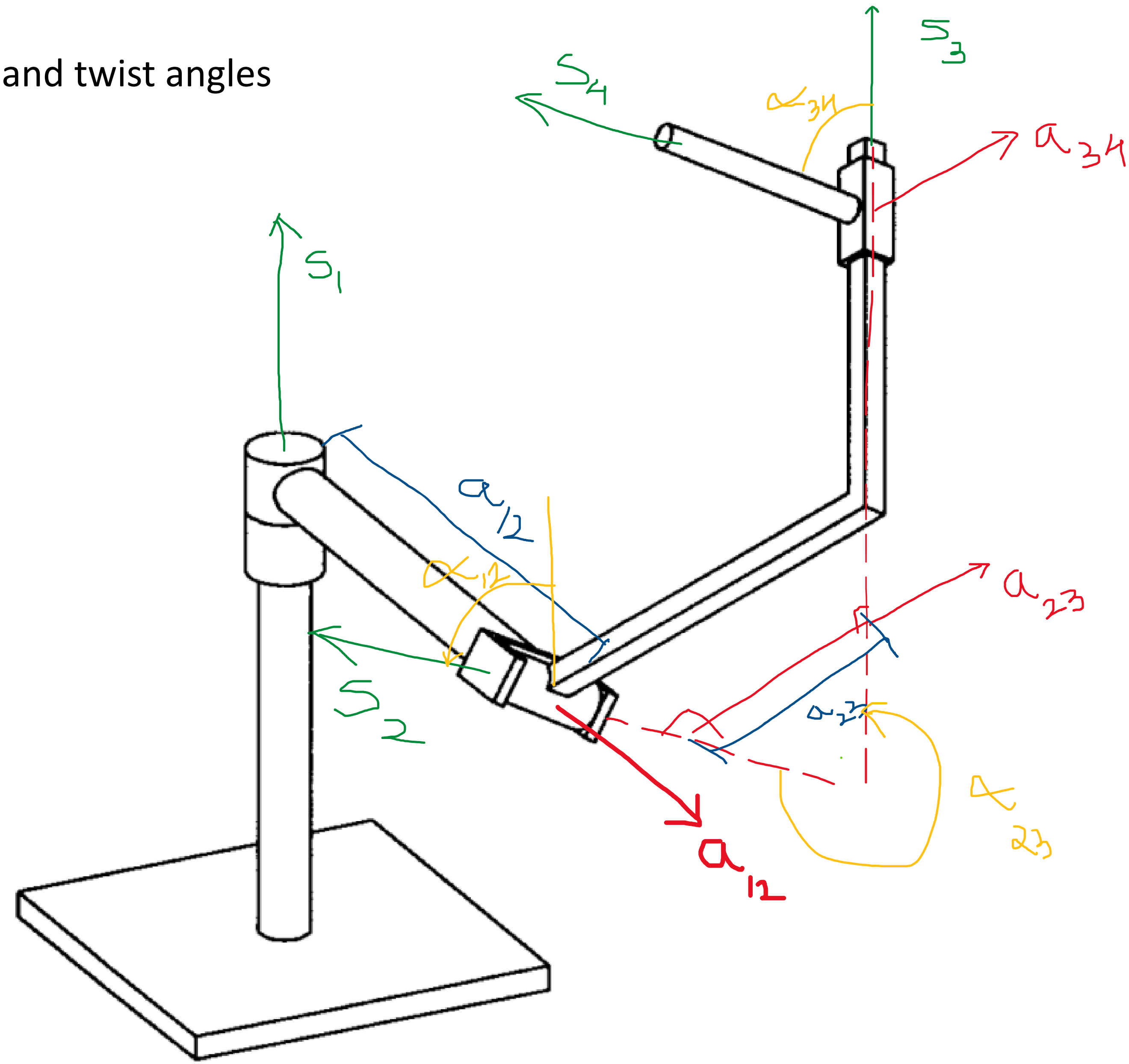
Link length and twist angles



Joint offset and twist angle