3. The following transformation definitions are given:

$$ {}^{B}_{A}\mathbf{T} = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & 0.5 & 20 \\ 0 & -1 & 0 & 0 \\ 0.5 & 0 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{C}_{A}\mathbf{T} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{D}_{B}\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0.5 & 10 \\ 0 & -0.5 & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

(a) Determine the transformation ${}^{C}_{D}\mathbf{T}$.

(b) The coordinates of point number 1 are $[20, -30, 5]^{T}$ measured in the D coordinate system. Determine the coordinates of this point as measured in the A, B, and C coordinate systems.

4. Coordinate systems A and B are initially coincident. Coordinate system B is then rotated sixty degrees about a vector parallel to $[2, 4, 7]^{T}$, which passes through the point $[3, 4, -2]^{T}$. Determine the transformation ${}^{A}_{B}\mathbf{T}$.

8. Coordinate system B is initially aligned with coordinate system A. It is then rotated thirty degrees about an axis that is parallel to the X axis but that passes through the point $[10, 20, 10]^T$.

Coordinate system C is initially aligned with coordinate system A. It is then rotated sixty degrees about an axis $[2, 4, 6]^T$ that passes through the origin.

Determine the transformation that relates the C and B coordinate systems, that is, ${}^{B}_{C}\mathbf{T}$.