EML 6281

<u>Robot Geometry – I</u>

<u>Homework #5</u>

→ <u>Problem 6.2</u>
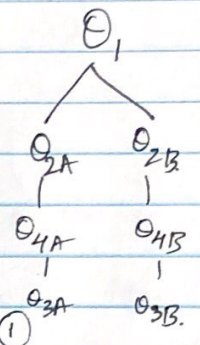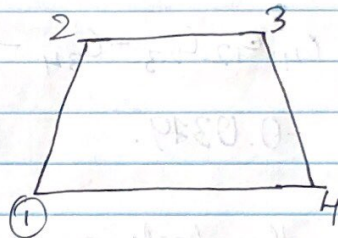
MATLAB Code.

→ <u>Problem 6.3</u>

Given – $\alpha_{12} = 75°$, $\alpha_{23} = 110°$, $\alpha_{34} = 60°$, $\alpha_{41} = 80°$.
$\theta_1 = 120°$.

we know,
$$Z_{12} = C_{34}$$
$$Z_{21} = C_{34}.$$

$$Z_{21} = S_{41}(\overline{X}_2 S_1 + \overline{Y}_2 C_1) + C_{41}\overline{Z}_2 - C_{34} = 0.$$

$$\overline{X}_2 = S_{23} S_2 \qquad \overline{Y}_2 = -(S_{12}C_{23} + C_{12}S_{23}C_2) \qquad \overline{Z}_2 = C_{12}C_{23} - S_{12}S_{23}C_2.$$

Plugging these values in equation ①.

$$S_{41}\left((S_{23}S_2)S_1 + (-(S_{12}C_{23} + C_{12}S_{23}C_2)C_1\right) + C_{41}(C_{12}C_{23} - S_{12}S_{23}C_2) - C_{34} = 0.$$

$$S_{41}S_{23}S_2 S_1 - S_{12}C_{23}C_1 S_{41} - C_{12}S_{23}C_2 C_1 S_{41} + C_{41}C_{12}C_{23} - S_{12}C_{41}S_{23}C_2 = 0.$$
$$\qquad\qquad - C_{34}$$

$$S_2\left(S_{41}S_{23}S_1\right) - C_2\left(C_{12}S_{23}C_1 S_{41} + S_{12}C_{41}S_{23}\right) - S_{12}C_{23}C_1 S_{41} + C_{41}C_{12}C_{23} - C_{34} = 0$$

$$A = S_{41}S_{23}S_1 \quad ; \quad B = C_{12}S_{23}C_1 S_{41} + S_{12}C_{41}S_{23}. \quad \text{and}$$

$$D = -S_{12}C_{23}C_1 S_{41} + C_{41}C_{12}C_{23} - C_{34} = 0.$$

<u>Aside</u>

From

$$X_2 = \frac{-2B \pm \sqrt{(2B)^2 - 4(D-A)(D+A)}}{2(D-A)}$$

$$-C_2\left(C_{12}S_{23}C_1S_{41} + S_{12}C_{41}S_{23}\right) + S_2\left(S_{41}S_{23}S_1\right) - S_{12}C_{23}C_1S_{41} + C_{41}C_{12}C_{23} - C_{34}^* = 0$$

$$AC_2 + BS_2 + D = 0.$$

from comparision we get.

$$A = -\left(C_{12}S_{23}C_1S_{41} + S_{12}C_{41}S_{23}\right), \quad B = \left(S_{41}S_{23}S_1\right)$$

$$D = C_{41}C_{12}C_{23} - C_{34} - S_{12}C_{23}C_1S_{41}.$$

$$A = -0.0379. \qquad B = 0.8014 \qquad D = -0.6780.$$
$$\text{(from MATLAB)}$$

from the Aside equation above.

$$X_2 = 0.5819 \text{ and } 1.9219.$$

$$X_2 = \tan\theta/2 \implies \theta_2 = 2 \times \tan^{-1}(X_2).$$

~~theta~~ $\theta_{2A} = 60.3869°$

and

$\theta_{2B} = 125.0222°$ } (From MATLAB).

Consider the buddy equation to find θ_4.

$$X_{21} = S_{234} S_4 \Rightarrow S_4 = \frac{X_{21}}{S_{34}} = \frac{\overline{X_2} C_1 - \overline{Y_2} S_1}{S_{34}}$$

$$\overline{X_2} = S_{23} S_2 \qquad \overline{Y_2} = \{-(S_{12} C_{23} + C_{12} S_{23} C_2)$$
$$= 0.8170 \qquad\qquad = 0.2102$$

$$S_4 = \frac{(0.8170) C_1 - (0.2102) S_1}{S_{34}}$$

$$S_4 = -0.6819.$$

theta 4a $= \theta_{4A} = -42.9886°$

11^{ly} theta 4b $= \theta_{4B} = -66.0978°$

To fend θ_3 consider. X_{12} and Y_{12}.

$$X_{12} = S_{34} S_3 \Rightarrow \boxed{S_3 = \frac{X_{12}}{S_{34}}}$$

$$X_{12} = X_1 C_2 - Y_1 S_2$$
$$= (S_{41} S_1) C_2 - (-(S_{12} C_{41} + C_{12} S_{41} C_1)) S_2.$$
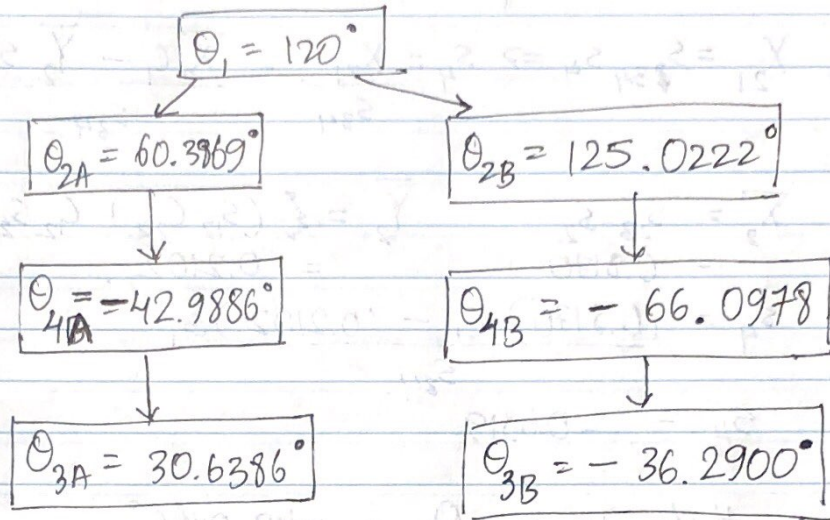
$$X_{12} = S_{41} S_1 C_2 + S_{12} C_{41} S_2 + C_{12} S_{41} C_1 S_2.$$
$$= 0.4413.$$

$$S_{3a} = 0.5096.$$
$$\Rightarrow \text{theta 3a} = \theta_{3A} = 30.6386°$$

11^{ly} theta 3b $= \theta_{3B} = -36.2900°$

Results of Problem 6.3.

$$\theta_1 = 120°$$

$$\theta_{2A} = 60.3869°$$

$$\theta_{2B} = 125.0222°$$

$$\theta_{4A} = -42.9886°$$

$$\theta_{4B} = -66.0978$$

$$\theta_{3A} = 30.6386°$$

$$\theta_{3B} = -36.2900°$$

→ Problem 6.5

MATLAB Code.

```
clear all
clc
alpha12=75;
alpha23=110;
alpha34=60;
alpha41=80;
theta1=120;
s12=sind(alpha12);
s23=sind(alpha23);
s34=sind(alpha34);
s41=sind(alpha41);
c12=cosd(alpha12);
c23=cosd(alpha23);
c34=cosd(alpha34);
c41=cosd(alpha41);
s1=sind(theta1);
c1=cosd(theta1);
B=s41*s23*s1
A=-(c12*s23*c1*s41+(s12*c41*s23))
D=-s12*c23*c1*s41+c41*c12*c23-c34
Q=(2*B)^2-4*(D-A)*(D+A);
X2a=(-2*B+sqrt(Q))/(2*(D-A))
X2b=(-2*B-sqrt(Q))/(2*(D-A))
theta2a=atand(X2a)*2
theta2b=atand(X2b)*2

s2=sind(theta2a);
c2=cosd(theta2a);
X_2bar=s23*s2
Y_2bar=-(s12*c23+c12*s23*c2)
s4=(X_2bar*c1-Y_2bar*s1)/s34
theta4a=asind(s4)

s2=sind(theta2b);
c2=cosd(theta2b);
X_2bar=s23*s2
Y_2bar=-(s12*c23+c12*s23*c2)
s4=(X_2bar*c1-Y_2bar*s1)/s34
theta4b=asind(s4)


%to find theta3
c2=cosd(theta2a);
s2=cosd(theta2a);
X_12=s41*s1*c2+s12*c41*s2+c12*s41*c1*s2
s3a=(X_12/s34)
theta3a=asind(s3a)

c2=cosd(theta2b);
s2=cosd(theta2b);
X_12=s41*s1*c2+s12*c41*s2+c12*s41*c1*s2
s3b=(X_12/s34)
theta3ab=asind(s3b)


B =

    0.8014
```

A =

   -0.0379

D =

   -0.6780

X2a =

   0.5819

X2b =

   1.9219

theta2a =

   60.3869

theta2b =

   125.0222

X_2bar =

   0.8170

Y_2bar =

   0.2102

s4 =

   -0.6819

theta4a =

   -42.9886

X_2bar =

   0.7695

Y_2bar =

   0.4699

```
s4 =

   -0.9142


theta4b =

  -66.0978


X_12 =

    0.4413


s3a =

    0.5096


theta3a =

   30.6386


X_12 =

   -0.5126


s3b =

   -0.5919


theta3ab =

  -36.2900
```