EML 6281

Robot Geometry II

Homework - 1

Problem 2.3

Given - $${}^{B}_{A}T = \begin{bmatrix} \sqrt{3}/2 & 0 & 0.5 & 20 \\ 0 & -1 & 0 & 0 \\ 0.5 & 0 & -\sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{C}_{A}T = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ \sqrt{2}/2 & -\sqrt{3}/2 & 0 & 0 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{D}_{B}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 0.5 & 10 \\ 0 & -0.5 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) find - ${}^{C}_{D}T$ -

$${}^{C}_{D}T = {}^{C}_{A}T \cdot \left({}^{B}_{A}T\right)^{T} \left({}^{D}_{B}T\right)^{T}$$

$$= \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 0 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 0 & 0.5 & 0 \\ 0 & -1 & 0 & 0 \\ 0.5 & 0 & -\sqrt{3}/2 & 0 \\ 20 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -0.5 & 0 \\ 0 & 0.5 & \sqrt{3}/2 & 0 \\ 0 & 10 & 0 & 1 \end{bmatrix}$$

$${}^{C}_{D}T = \begin{bmatrix} 0.6124 & 0.4356 & 0.6597 & 0 \\ 0.6124 & 0.7891 & -0.0474 & 0 \\ 199.5 & 100.433 & 0.75 & 10 \\ 20 & 10 & 0 & 1 \end{bmatrix}$$ (Using MATLAB)

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(b) <u>Given</u> - ${}^{D}P_1 = [20, -30, 5]^T$

find - ${}^{A}P_1$, ${}^{B}P_1$ and ${}^{C}P_1$.

we know that,

$${}^{B}P_1 = {}^{B}_{D}T \, {}^{D}P_1$$

$$\Rightarrow {}^{B}P_1 = \left({}^{D}_{B}T\right)^T \, {}^{D}P_1$$

$$= \begin{bmatrix} 20 \\ -13.4808 \\ 19.3301 \\ 1 \end{bmatrix}$$

$${}^{A}P_1 = {}^{A}_{B}T \, {}^{B}P_1 = \begin{bmatrix} 46.9856 \\ 13.4808 \\ -6.7404 \\ 1 \end{bmatrix}$$

$${}^{C}P_1 = {}^{C}_{A}T \, {}^{T}P_1 = \begin{bmatrix} 42.7562 \\ 23.6915 \\ 16.7404 \\ 1 \end{bmatrix}$$

Problem 2.4

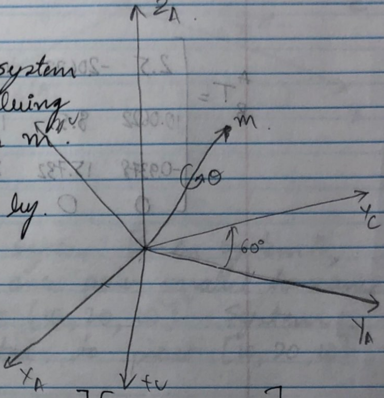
Given – Coordinate system A and B are initially alligned. B is then rotated 60° about m

$m = [2, 4, 7]^T$.

$\theta = 60°$.

Let us consider coordinate system C initially alligned with A being rotated by 60° about vector $\hat{m}$.

the relationship ${}^A_C R$ is given by.



$${}^A_C R = \begin{bmatrix} a_x & b_x & m_x \\ a_y & b_y & m_y \\ a_z & b_z & m_z \end{bmatrix} \begin{bmatrix} \cos 60° & -\sin 60° & 0 \\ \sin 60° & \cos 60° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ m_x & m_y & m_z \end{bmatrix}$$

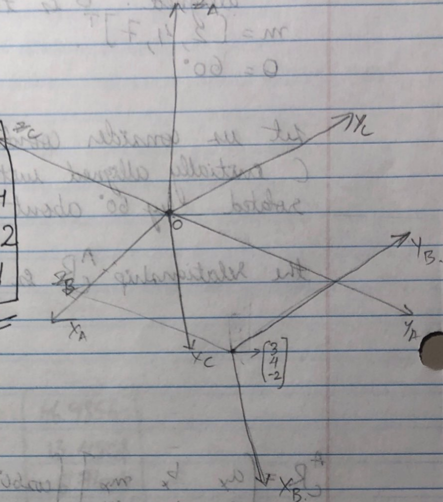
$$= \begin{bmatrix} m_x^2(1-\cos 60°) + \cos 60° & m_x m_y(1-\cos 60°) - m_z \sin 60° & m_x m_z(1-\cos 60°) + m_y \sin 60° \\ m_x m_y(1-\cos 60°) + m_z \sin 60° & m_y^2(1-\cos 60°) + \cos 60° & m_y m_z(1-\cos 60°) - m_x \sin 60° \\ m_x m_z(1-\cos 60°) - m_z \sin 60° & m_y m_z(1-\cos 60°) + m_x \sin 60° & m_z^2(1-\cos 60°) + \cos 60° \end{bmatrix}$$

$$= \begin{bmatrix} 4(\tfrac{1}{2}) + \tfrac{1}{2} & 8(\tfrac{1}{2}) - 7(\tfrac{\sqrt{3}}{2}) & 7 + 4(\tfrac{\sqrt{3}}{2}) \\ 4 + 7(\tfrac{\sqrt{3}}{2}) & 8 + \tfrac{1}{2} & 14 - \sqrt{3} \\ 7 - 7(\tfrac{\sqrt{3}}{2}) & 14 + 2(\tfrac{\sqrt{3}}{2}) & 49(\tfrac{1}{2}) + \tfrac{1}{2} \end{bmatrix}$$

$${}^A_C R = \begin{bmatrix} 2.5 & -2.0622 & 10.4641 \\ 10.0622 & 8.5 & 12.268 \\ -0.9378 & 15.732 & 25 \end{bmatrix}$$

Hence the vector passes through the point $[3,4,-2]^T$ system C has to be translated to the point $[3,4,-2]^T$ to determine ~~the~~ ${}^A_BT$.

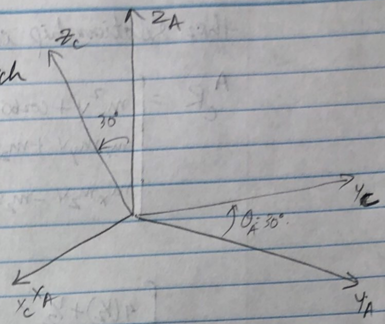
$$ {}^A_BT = \begin{bmatrix} 2.5 & -2.0622 & 10.4641 & 3 \\ 10.0622 & 8.5 & 12.268 & 4 \\ -0.9378 & 15.732 & 25 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



Problem 2.8

Given - Coordinate system B is initially alligned with system A. B is then rotated by $\theta_B = 30^\circ$ about ~~x axis~~ axis parallel to x axis passing through point $[10, 20, 10]^T$.

Consider a system C' initially alligned with system A which is rotated about X axis.



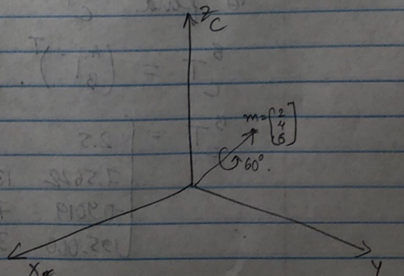
$$^{A}_{C'}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix}$$

$$^{A}_{C'}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

To find relation between System B in system A, which is rotated about an axis ~~pathr~~ parallel to x axis and passes through point $[10, 20, 10]^T$. System C' has to be translated to point $[10, 20, 10]^T$.

$$^{A}_{B}T = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & \sqrt{3}/2 & -1/2 & 20 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>given</u>.- System C is rotated 60° about $m = [2, 4, 6]^T$



$m = [2\ 4\ 6]$

ther relationship is given by.

$${}^{A}_{C}R = \begin{bmatrix} m_x^2 V + \cos 60° & m_x m_y V - m_z \sin 60° & m_x m_z V + m_y \sin 60° \\ m_x m_y V + m_z \sin 60° & m_y^2 V + \cos 60° & m_y m_z (1-\cos 60°) - m_x \sin 60° \\ m_x m_z V - m_z \sin 60° & m_y m_z V + m_x \sin 60° & m_z^2 V + \cos 60° \end{bmatrix}$$

where $V = 1 - \cos\theta$
$= 1 - \cos 60°$.

$$= \begin{bmatrix} 4(\tfrac{1}{2}) + \tfrac{1}{2} & 4-3 & 6+2 \\ 4+3 & 8+\tfrac{1}{2} & 12-1 \\ 6-3 & 12+1 & 36(\tfrac{1}{2}) + \tfrac{1}{2} \end{bmatrix}$$

$${}^{A}_{C}R = \begin{bmatrix} 2.5 & 1 & 8 \\ 7 & 8.5 & 11 \\ 3 & 13 & 18.5 \end{bmatrix}$$

$${}^{A}_{C}T = \begin{bmatrix} 2.5 & 1 & 8 & 0 \\ 7 & 8.5 & 11 & 0 \\ 3 & 13 & 18.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To find ${}^{B}_{C}T$

$${}^{B}_{C}T = ({}^{A}_{B}T)^{T} \cdot {}^{A}_{C}T$$

$${}^{B}_{C}T = \begin{bmatrix} 2.5 & 1 & 8 & 0 \\ 7.5622 & 13.8612 & 18.7763 & 0 \\ -0.9019 & 7.0083 & 10.5215 & 0 \\ 195.000 & 310 & 485 & 1 \end{bmatrix}$$ (using MATLAB)

```
T_CA=[sqrt(2)/2 sqrt(2)/2 0 0; sqrt(2)/2 -sqrt(2)/2 0 0; 0 0 -1 10; 0 0 0 1];
T_BA=[sqrt(3)/2 0 0.5 0; 0 -1 0 0; 0.5 0 -sqrt(3)/2 0; 20 0 0 1];
T_DB=[1 0 0 0; 0 sqrt(3)/2 -0.5 0; 0 0.5 sqrt(3)/2 0; 0 10 0 1];
D=T_CA*T_BA*T_DB
P_1inD=[20 -30 5 1]';
P_1inB=T_DB' * P_1inD
P_1inA=T_BA'*P_1inB
P_1inC=T_CA*P_1inA
```

```
D =

    0.6124   -0.4356    0.6597         0
    0.6124    0.7891   -0.0474         0
  199.5000  100.4330    0.7500   10.0000
   20.0000   10.0000         0    1.0000


P_1inB =

   20.0000
  -13.4808
   19.3301
    1.0000


P_1inA =

   46.9856
   13.4808
   -6.7404
    1.0000


P_1inC =

   42.7562
   23.6915
   16.7404
    1.0000
```

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```
T_AB=[1 0 0 10; 0 sqrt(3)/2 -1/2 20; 0 1/2 sqrt(3)/2 10; 0 0 0 1];
T_AC=[2.5 1 8 0; 7 8.5 11 0; 3 13 18.5 0; 0 0 0 1];
T_BC=T_AB'*T_AC
```

```
T_BC =

    2.5000    1.0000    8.0000         0
    7.5622   13.8612   18.7763         0
   -0.9019    7.0083   10.5215         0
  195.0000  310.0000  485.0000    1.0000
```

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