EML 6351 Fall 2022

Project 1

This is an open book and open notes project. Copying someone else's code or solution is considered a violation of the university honesty policy. **NOTE: READ ALL THE INSTRUCTIONS. If you need to make any assumptions, list them (e.g., if you need to restrict the domain, assume some trigonometric property, etc.). UPLOAD A SINGLE PDF OF YOUR SOLUTIONS AND YOUR CODE IN A SINGLE ZIP FILE ONLINE. PLOTS NOT FOLLOWING THE CORRECT FORMAT WILL NOT GET FULL CREDIT. IF SOMETHING IS NOT CLEAR PLEASE ASK.**

1 Dynamics

Consider the two-link dynamic system

$$M(\phi)\ddot{\phi} + C\left(\phi,\dot{\phi}\right) + G(\phi) = \tau$$

where $\phi \triangleq \begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix}, \dot{\phi} \triangleq \begin{bmatrix}\dot{\phi}_1\\ \dot{\phi}_2\end{bmatrix}, \ddot{\phi} \triangleq \begin{bmatrix}\ddot{\phi}_1\\ \ddot{\phi}_2\end{bmatrix}, \tau \triangleq \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} \in \mathbb{R}^2$ are the angle, angular velocity, angular acceleration, and input torque of the arm joints

$$\begin{aligned}
M(\phi) &\triangleq \begin{bmatrix}\left(m_1 l_1^2 + m_2\left(l_1^2 + 2l_1l_2c_2 + l_2^2\right)\right) & m_2\left(l_1l_2c_2 + l_2^2\right)\\ m_2\left(l_1l_2c_2 + l_2^2\right) & m_2l_2^2\end{bmatrix},\\
C\left(\phi,\dot{\phi}\right) &\triangleq \begin{bmatrix}-2m_2l_1l_2s_2\dot{\phi}_1\dot{\phi}_2\\ 0\end{bmatrix} + \begin{bmatrix}-m_2l_1l_2s_2\dot{\phi}_2^2\\ m_2l_1l_2s_2\dot{\phi}_1^2\end{bmatrix},\\
G(\phi) &\triangleq \begin{bmatrix}(m_1+m_2)\,gl_1c_1 + m_2gl_2c_{12}\\ m_2gl_2c_{12}\end{bmatrix},
\end{aligned}$$

$m_i, l_i \in \mathbb{R}_{>0}$ are the **unknown** constant mass and length of link i, $g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$ is gravity,

$$\begin{aligned}
c_i &= \cos(\phi_i),\\
s_i &= \sin(\phi_i),\\
c_{12} &= \cos(\phi_1+\phi_2),\\
s_{12} &= \sin(\phi_1+\phi_2).
\end{aligned}$$

2 Design and Simulation

Assume $\underline{m} \leq m_i \leq \overline{m}$, $\underline{l} \leq l_i \leq \bar{l}$ are **known** bounds on the **unknown** mass and length of link i, with $m_i = 2$kg, $l_i = 0.5$m, $\underline{m} = 1$kg, $\overline{m} = 3$kg, and $\underline{l} = 0.25$m, $\bar{l} = 0.75$m. Also, assume that $\phi, \dot{\phi}, \ddot{\phi}$ are all measurable. Let the desired trajectory have a magnitude of $\overline{\phi}_d \triangleq \begin{bmatrix}\overline{\phi}_{d1}\\ \overline{\phi}_{d2}\end{bmatrix} = \begin{bmatrix}\frac{\pi}{8}\\ \frac{\pi}{4}\end{bmatrix}$, at a frequency of $f_{\phi_d} \triangleq 0.2$Hz, a phase shift $a_{\phi_d} \triangleq \frac{\pi}{2}$, and bias of $b_{\phi_d} \triangleq \begin{bmatrix}b_{\phi_{d1}}\\ b_{\phi_{d2}}\end{bmatrix} = \begin{bmatrix}\frac{\pi}{2}\\ \frac{\pi}{4}\end{bmatrix}$, such that $\begin{bmatrix}\phi_{d1}(t)\\ \phi_{d2}(t)\end{bmatrix} \triangleq \begin{bmatrix}\overline{\phi}_{d1}\sin\left(2\pi f_{\phi_d}t - a_{\phi_d}\right) - b_{\phi_{d1}}\\ \overline{\phi}_{d2}\sin\left(2\pi f_{\phi_d}t - a_{\phi_d}\right) + b_{\phi_{d2}}\end{bmatrix}$ implying $\begin{bmatrix}\dot{\phi}_{d1}(t)\\ \dot{\phi}_{d2}(t)\end{bmatrix} \triangleq \begin{bmatrix}2\pi f_{\phi_d}\overline{\phi}_{d1}\cos\left(2\pi f_{\phi_d}t - a_{\phi_d}\right)\\ 2\pi f_{\phi_d}\overline{\phi}_{d2}\cos\left(2\pi f_{\phi_d}t - a_{\phi_d}\right)\end{bmatrix}$ and $\begin{bmatrix}\ddot{\phi}_{d1}(t)\\ \ddot{\phi}_{d2}(t)\end{bmatrix} \triangleq \begin{bmatrix}-\left(2\pi f_{\phi_d}\right)^2\overline{\phi}_{d1}\sin\left(2\pi f_{\phi_d}t - a_{\phi_d}\right)\\ -\left(2\pi f_{\phi_d}\right)^2\overline{\phi}_{d2}\sin\left(2\pi f_{\phi_d}t - a_{\phi_d}\right)\end{bmatrix}$.

1. (50 pts) Design a gradient adaptive controller that regulates the system to the desired trajectory and prove the designed controller is stable using Lyapunov-based methods. Show all your work and discuss the approach you took.

2. (50 pts) Simulate the system tracking the provided desired trajectory. Additionally, provide the following:

(a) Plot the norm of your tracking error and filtered tracking error over time.

   i. Based on these plots, how well did the designed controller track the desired trajectory?

(b) Plot the norm of your parametric error over time.

   i. Based on the plots how well did you estimator approximate the unknown parameters?

   ii. Plot the norm of the total input, the feedback portions of the input, and the feedforward portions of the input and discuss how the total input behaved over time (e.g., did the input rely on feedback the majority of the time or did the feedforward portion become the majority of the input over time).

(c) Based on the plots, use quantitative and qualitative descriptions of the plots and data to briefly discuss whether your simulation matches your stability result, and describe anything you think would improve the result.