**SINGLE AND TWO LAYER NEURAL NETWORK**

**DYNAMICS –**

Consider the two-link dynamic system

Where are the angle, angular velocity, angular acceleration, and input torque of the arm joints.

where are the unknow constant mass and length of the link and gravity is g = 9.8 m/s^2 and,

And is the unknown, unstructured state-dependent disturbance that is a function of the angles and angular velocity.

Assuming that are known bounds on the unknown mass and length of link , with Also, assuming that are all measurable.

Let the desired trajectory have a magnitude of , at a frequency of , a phase shift , and bias of , such that which implies and

Designing a controller under the assumption that the structure is completely unknown for

(1) Robust Controller

From the dynamics,

accounts to the structured uncertainties and represents the unstructured uncertainties. And as a result, we can design a linear in the parameter design for the structured uncertainties like the concurrent learning and integral concurrent learning design. And the unstructured uncertainties can be estimated using a single layer neural network.

For a single layer neural network can be estimated as

Where is the output weights, is the output basis functions, and the reconstruction error .

Error Dynamics-

Reference tracking error

Multiplying both sides by we get,

From the dynamics we know,

On substitution we get,

Add and subtract by

We can estimate since it is linear in the unknown parameters and we were able to develop the unknown parameters as follows,

And

Now,

Let the Lyapunov Candidate be,

We know,

On simplification we get,

Input design,

On substitution we get,

We can introduce an inequality since, if

Now,

Design Parametric error

We get,

Using Barbalat’s Lemma we can show the which implies Asymptotic tracking

**(2) Single Layer Neural Network Design**

From the dynamics,

accounts to the structured uncertainties and represents the unstructured uncertainties. And as a result, we can design a linear in the parameter design for the structured uncertainties like the concurrent learning and integral concurrent learning design. And the unstructured uncertainties can be estimated using a single layer neural network.

For a single layer neural network can be estimated as

Where is the output weights, is the output basis functions, and the reconstruction error .

Error Dynamics-

Reference tracking error

Multiplying both sides by we get,

From the dynamics we know,

On substitution we get,

Add and subtract by

We can estimate since it is linear in the unknown parameters and we were able to develop the unknown parameters as follows,

And

The unstructured state-dependent disturbance will be estimated using the neural network.

Now,

Let the estimated structured estimation error where

And unstructured estimation error where

Let the stacked errors

Where,

Let the Lyapunov candidate V be,

On differentiating we get,

Using

We get,

Designing Input

Where

Since we know,

The above equation can be simplified to

Choosing where represents the upper bound on , we can say

Therefore, the above equation can be lower bounded as follows

And form the property of trace

Designing structured estimate

And unstructured estimate

On substitution,

Since

Using Barbalat’s Lemma we can show the which implies Asymptotic tracking

**(3) Two-layer Neural Network**

From the dynamics,

accounts to the structured uncertainties and represents the unstructured uncertainties. And as a result, we can design a linear in the parameter design for the structured uncertainties like the concurrent learning and integral concurrent learning design. And the unstructured uncertainties can be estimated using a two-layer neural network.

For a two-layer neural network can be estimated as

Where is the output weights, is the activation function, is the inner features which has the input and inner weights where V is the inner weights and is the input and the reconstruction error is .

Error Dynamics-

Reference tracking error

Multiplying both sides by we get,

From the dynamics we know,

On substitution we get,

Add and subtract by

We can estimate since it is linear in the unknown parameters and we were able to develop the unknown parameters as follows,

And

The unstructured state-dependent disturbance will be estimated using a two-layer neural network.

Now,

Approximation for

Where,

Now on substitution we get,

Add and subtract

Let

Let the stacked errors

Where,

and

Lyapunov Candidate,

We know,

And

On substitution we get,

On simplification we get,

Design the input

Design for , we want to get,

So,

Design for , we want to get,

We know,

So,

Design for , we want to get,

We know,

So,

On substitution we get,

Using Barbalat’s Lemma we can show the which implies Asymptotic tracking

**SIMULATION AND RESULTS –**

(a) Selection of basis function -

For single-layer neural network, a polynomial basis was used to estimate the unstructured unknown since it was able to give a better estimate when compared to the Gaussian basis with uniformly distributed centers. And for two-layer neural network, Gaussian basis with uniformly distributed centers was used to estimate the unstructured unknown .

(b) Norm of tracking error and filtered tracking error

For Robust controller



For single-layer neural network

For two-layer neural network





Analysis –

The robust controller did not perform very well and had significantly large errors as indicated by the RMS filter tracking error norm of 1.32 and RMS error norm of 0.39. The single layer performed better that robust controller and brought the RMS filter tracking error to 0.7 and RMS error norm of 0.2. The two-layer performed a little better that single-layer controller in bringing down the RMS filter tracking error to 0.69 and RMS error norm remained the same at 0.2. The two-layer performed better overall.

(c) Function Approximation Error

For Robust Controller



For single-layer neural network



For two-layer neural network



Analysis –

Two-layer neural network performed the best in approximating the function with a RMS difference in function estimation of 4.97. The single-layer neural network was able did moderately well with a RMS difference in function estimation of 8.22, performing twice as better than the robust controller which has an RMS difference in function estimation of 21.9.

(d) Input Plots

For Robust Controller

Plot of total input



For Single-layer Neural Network

For Two-layer Neural Network



Analysis –

The robust controller relied mainly on the feedback terms of the input as seen in the plot above where the feedback curve in blue is fluctuating more compared to the neural networks. Whereas the single-layer and two-layer Neural Network relied mainly on the feedforward part of the input as indicated by the almost horizontal blue curve of feedback part of the input.

Two-layer neural network relied on the feedback part the least.