HW5

Graded

Student

SAMUEL PHAM

Total Points

100 / 100 pts

Question 1

Q1

10 / 10 pts

- ✔ **− 0 pts** Part a correct
- ✔ **− 0 pts** Part b correct

Correct result, but we're not looking for implication by a knowledge base here. We're looking for exact truth table equivalence -- hence we do not need to worry about models and worlds.

Question 2

Q2

20 / 20 pts

- ✔ **+ 6 pts** Part a correct
- ✔ **+ 6 pts** Part b correct
- ✔ **+ 8 pts** Part c correct

Question 3

Q3

30 / 30 pts

- ✔ **− 0 pts** Correct

Question 4

Q4

20 / 20 pts

- ✔ **− 0 pts** Correct

Question 5

Q5

20 / 20 pts

- ✔ **− 0 pts** Correct

Question 6

Adjustments

0 / 0 pts

- ✔ **− 0 pts** N/A

Question assigned to the following page: 1

1. (10 pts) Use truth tables to show that the following pairs of sentences are equivalent:

- $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$
- $P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

- Δ : $P \Rightarrow \neg Q$ α : $Q \Rightarrow \neg P$

	P	Q	¬P	¬Q	Δ	α
w1	T	T	F	F	F	F
w2	T	F	F	T	T	T
w3	F	F	T	T	T	T
w4	F	T	T	F	T	T

$M(\Delta) = \{ w_2\ w_3\ w_4 \}$
$M(\alpha) = \{ w_2\ w_3\ w_4 \}$
$\rightarrow$ Equivalent

- Δ : $P \overset{\text{iff}}{\Leftrightarrow} \neg Q$ $\alpha : ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

	P	Q	¬P	¬Q	Δ	$P \wedge \neg Q$	$\neg P \wedge Q$	α
w1	T	T	F	F	F	F	F	F
w2	T	F	F	T	T	T	F	T
w3	F	F	T	T	F	F	F	F
w4	F	T	T	F	T	F	T	T

$M(\Delta) = \{ w_2\ w_4 \}$
$M(\alpha) = \{ w_2\ w_4 \}$
$\rightarrow$ Equivalent

Question assigned to the following page: 2

2. (20 pts) Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:

- (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
- (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
- ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Justify your answer using truth tables.

S = Smoke
F = Fire
H = Heat

- $\Delta : (S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$

	S	F	¬S	¬F	S ⇒ F	¬S ⇒ ¬F	Δ
w1	T	T	F	F	T	T	T
w2	T	F	F	T	F	T	T
w3	F	T	T	F	T	F	F
w4	F	F	T	T	T	T	T

$M(\Delta) = \{ w_1 \ w_2 \ w_4 \}$

→ Neither

- $\Delta : (S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$

	S	F	H	S ⇒ F	S ∨ H	(S ∨ H) ⇒ F	Δ
w1	T	T	T	T	T	T	T
w2	T	F	T	F	T	F	T
w3	T	T	F	T	T	T	T
w4	T	F	F	F	T	F	T
w5	F	F	F	T	F	T	T
w6	F	T	F	T	F	T	T
w7	F	F	T	T	T	F	F
w8	F	T	T	T	T	T	T

$M(\Delta) = \{ w_1 \cdots w_8 / w_7 \}$

→ Neither

- $((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

	S	F	H	S ∧ H	(S∧H) ⇒ F	S ⇒ F	H ⇒ F	(S ⇒ F) ∨ (H ⇒ F)	Δ
w1	T	T	T	T	T	T	T	T	T
w2	T	F	T	T	F	F	F	F	T
w3	T	T	F	F	T	T	F	T	T
w4	T	F	F	F	T	F	T	T	T
w5	F	F	F	F	T	T	T	T	T
w6	F	T	F	F	T	T	T	T	T
w7	F	F	T	F	T	T	F	T	T
w8	F	T	T	F	T	T	T	T	T

$M(\Delta) = W$ → valid

Question assigned to the following page: 3

3. (30 pts) Consider the following: ①

If the unicorn is mythical, then it is immortal, ② *but if it is not mythical, then it is a mortal mammal. If* ③ *the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.* ④

(a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).

(b) Convert the knowledge base into CNF.

(c) Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

Justify your answers using resolution by providing corresponding resolution derivations. Make sure to clearly define all propositional symbols (variables) first, then define your knowledge base, and finally give your derivations.

a, b)

A = Mythical

B = Mortal

M = Mammal

H = Horned

C = Magical

① $A \Rightarrow \neg B$

↳ $\neg A \vee \neg B$

② $\neg A \Rightarrow (B \wedge M)$

↳ $A \vee (B \wedge M)$

↳ $(A \vee B) \wedge (A \vee M)$

③ $(\neg B \vee M) \Rightarrow H$

↳ $\neg(\neg B \vee M) \vee H$

↳ $(B \wedge \neg M) \vee H$

↳ $(B \vee H) \wedge (\neg M \vee H)$

④ $H \Rightarrow C$

↳ $\neg H \vee C$

CNF : $(\neg A \vee \neg B) \wedge (A \vee B) \wedge (A \vee M) \wedge (B \vee H) \wedge (\neg M \vee H) \wedge (\neg H \vee C)$

c)

Mythical

Δ : CNF | $\Delta \models \alpha$?

α_1 : A

0.	$\neg A \vee \neg B$		Δ
1.	$A \vee B$		
2.	$A \vee M$		
3.	$B \vee H$		
4.	$\neg M \vee H$		
5.	$\neg H \vee C$		
6.	$\neg A$		$\neg\alpha$
7.	B	1, 6	$\frac{A \vee B,\ \neg A}{B}$
8.	$\neg A$	0, 7	$\frac{\neg A \vee \neg B,\ B}{\neg A}$
9.	M	2, 8	$\frac{A \vee M,\ \neg A}{M}$
10.	H	4, 9	$\frac{\neg M \vee H,\ M}{H}$
11.	C	5, 10	$\frac{\neg H \vee C,\ H}{C}$

→ can no longer apply rule

→ no contradiction

→ $\Delta \not\models \alpha_1$

→ $\Delta \wedge \neg\alpha_1$ is satisfiable

→ can't use KB to prove α_1

Magical

Δ : CNF | $\Delta \models \alpha$?

α_2 : C

0.	$\neg A \vee \neg B$		Δ
1.	$A \vee B$		
2.	$A \vee M$		
3.	$B \vee H$		
4.	$\neg M \vee H$		
5.	$\neg H \vee C$		
6.	$\neg C$		$\neg\alpha$
7.	$\neg H$	5, 6	$\frac{\neg H \vee C,\ \neg C}{\neg H}$
8.	$\neg M$	4, 7	$\frac{\neg M \vee H,\ \neg H}{\neg M}$
9.	A	2, 8	$\frac{A \vee M,\ \neg M}{A}$
10.	$\neg B$	0, 9	$\frac{\neg A \vee \neg B,\ A}{\neg B}$
11.	H	3, 10	$\frac{B \vee H,\ \neg B}{H}$

12. contradiction

→ H vs. $\neg H$

↳ $\Delta \models \alpha_2$

↳ $\Delta \wedge \neg\alpha_2$ is unsat

→ can use KB to prove α_2

Horn

Δ : CNF | $\Delta \models \alpha$

α_3 : H

0.	$\neg A \vee \neg B$		Δ
1.	$A \vee B$		
2.	$A \vee M$		
3.	$B \vee H$		
4.	$\neg M \vee H$		
5.	$\neg H \vee C$		
6.	$\neg H$		$\neg\alpha$
7.	$\neg M$	4, 6	$\frac{\neg M \vee H,\ \neg H}{\neg M}$
8.	A	2, 7	$\frac{A \vee M,\ \neg M}{A}$
9.	$\neg B$	0, 8	$\frac{\neg A \vee \neg B,\ A}{\neg B}$
10.	H	3, 9	$\frac{B \vee H,\ \neg B}{H}$

11. contradiction

→ H vs. $\neg H$

↳ $\Delta \models \alpha_3$

↳ $\Delta \wedge \alpha_3$ is unsat

↳ can use KB to prove α_3

Question assigned to the following page: 4

4. (20 pts) Consider the two NNF circuits in Figure 1 and Figure 2. Identify whether they are decomposable, deterministic, smooth and why.

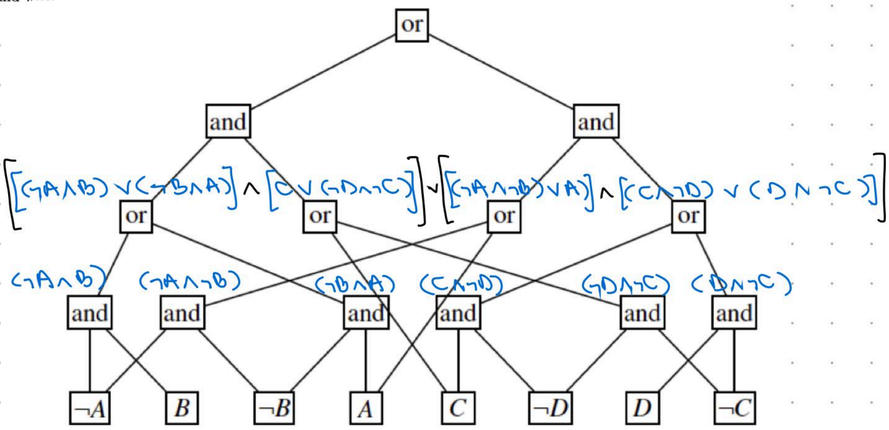

Figure 1

- decomposable ✓
  - → no share variables for every conjunctions
  - ↳ is decomposable
- deterministic
  - → is not deterministic b/c the top most OR have more than 1 True input
- smooth X
  - → Not smooth b/c $[C \vee (\neg D \wedge \neg C)]$

α only includes C whereas β includes C and D

→ $\alpha \neq \beta$ → Not smooth

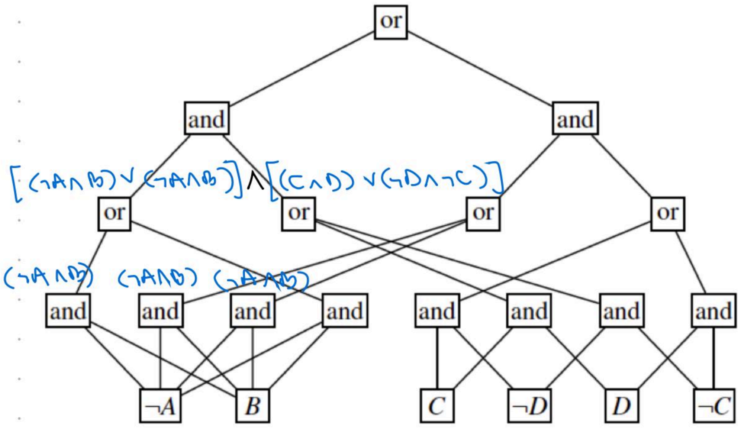

Figure 2

- Decomposable ✓
  - → no share variables for every conjunctions
- smooth ✓
  - → $var(\alpha) = var(\beta)$ $\forall$ $\alpha \vee \beta$
- deterministic X
  - ↳ is not deterministic b/c if A = F, B = T → both inputs will be T

Question assigned to the following page: 5

5. (20 pts) Given a propositional formula, where each literal has a weight ω in $[0, 1]$, the weight of a truth assignment is defined as the product of its literals weights. For example, $\omega(A, \neg B, C) = \omega(A)\omega(\neg B)\omega(C)$. The Weighted Model Count (WMC) of a propositional formula is defined as the added weight of its satisfying assignments (i.e., models).

Suppose we have the following literal weights: $\omega(A) = 0.1, \omega(\neg A) = 0.9, \omega(B) = 0.3, \omega(\neg B) = 0.7, \omega(C) = 0.5, \omega(\neg C) = 0.5, \omega(D) = 0.7, \omega(\neg D) = 0.3$.

(a) Compute the Weighted Model Count for formula $(\neg A \wedge B) \vee (\neg B \wedge A)$ by enumerating its models, computing their weights, then adding them up.

$$
\begin{aligned}
WMC &= w(\neg A, B) + w(\neg B, A) \\
&= w(\neg A)w(B) + w(\neg B)w(A) \\
&= 0.9(0.3) + 0.7(0.1) \\
&= 0.34
\end{aligned}
$$

(b) Consider the decomposable, deterministic and smooth NNF circuit in Figure 3. If we assign the weights of literals to all the leaf nodes, the count of each $\wedge$ node is computed as the product of the counts of its children, and the count of each $\vee$ node is computed as the sum of the counts of its children. What is the relation between the count on the root with the Weighted Model Count for the formula?

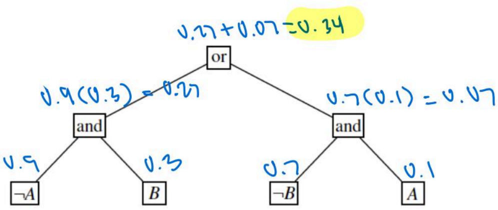

Figure 3

- root count : 0.34
- WMC of $(\neg A \wedge B) \vee (\neg B \wedge A)$ is the same as a) which is 0.34

$\boxed{\rightarrow \text{root and WMC has the same result for count}}$

(c) Compute the Weighted Model Count for the formula associated with the decomposable, deterministic and smooth NNF circuit in Figure 4.

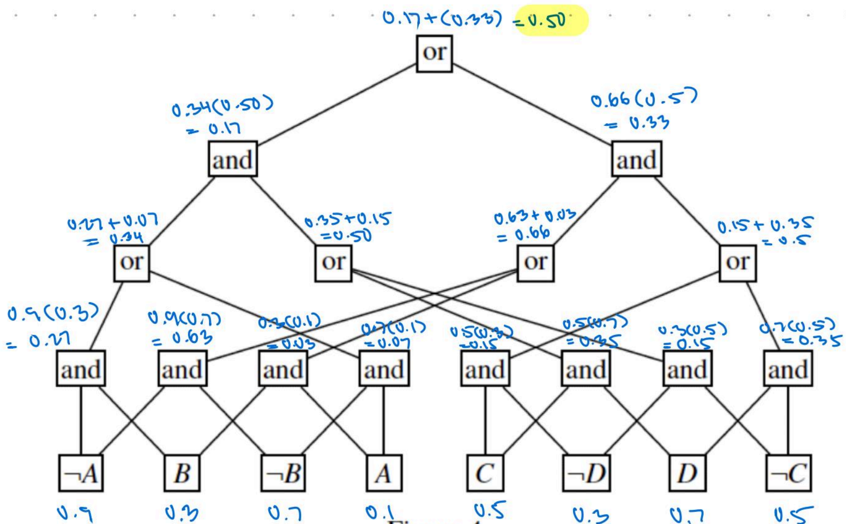

Figure 4

$\boxed{\text{Root count : 0.50}}$