HW7

Graded

Student

SAMUEL PHAM

Total Points

100 / 100 pts

Question 1

Q1 **10** / 10 pts

✔ **– 0 pts** Correct

Question 2

Q2 **20** / 20 pts

✔ **– 0 pts** Correct: 0.833

Question 3

Q3 **20** / 20 pts

✔ **– 0 pts** Correct

Question 4

Q4 **30** / 30 pts

✔ **– 0 pts** Correct

Question 5

Q5 **20** / 20 pts

✔ **– 0 pts** Correct

Question 6

Points Adjustments **0** / 0 pts

✔ **– 0 pts** Correct

Question assigned to the following page: 1

1. (10 pts) Prove the following identity:

$$Pr(\alpha_1, \ldots, \alpha_n \mid \beta) = Pr(\alpha_1 \mid \alpha_2, \ldots, \alpha_n, \beta) Pr(\alpha_2 \mid \alpha_3, \ldots, \alpha_n, \beta) \ldots Pr(\alpha_n \mid \beta).$$

Proof by Induction:

- base case

$n = 1$ $\quad Pr(\alpha_1 \mid \beta) = Pr(\alpha_1 \mid \beta)$ ✓

- Inductive case

  - Assume T for $n = k$

$$Pr(\alpha_1, \ldots, \alpha_k \mid \beta) = \prod_{i=1}^{k} Pr(\alpha_i \mid \alpha_{i+1}, \ldots, \alpha_k, \beta)$$

  - Assume T for $n = k+1$

$$Pr(\alpha_1, \ldots, \alpha_{k+1} \mid \beta) = \prod_{i=1}^{k+1} Pr(\alpha_i \mid \alpha_{i+1}, \ldots, \alpha_{k+1}, \beta)$$

$$LHS = Pr(\alpha_1, \ldots, \alpha_k, \alpha_{k+1} \mid \beta) = Pr(\alpha_1, \ldots, \alpha_k \mid \alpha_{k+1}, \beta)\, Pr(\alpha_{k+1} \mid \beta)$$

$$Pr(\alpha_1, \ldots, \alpha_k \mid \alpha_{k+1}, \beta) = \prod_{i=1}^{k} Pr(\alpha_i \mid \alpha_{i+1}, \ldots, \alpha_k, \alpha_{k+1}, \beta)$$

$$Pr(\alpha_1, \ldots, \alpha_{k+1} \mid \beta) = \prod_{i=1}^{k+1} Pr(\alpha_i \mid \alpha_{i+1}, \ldots, \alpha_{k+1}, \beta) \quad \square$$

Question assigned to the following page: 2

2. (20 pts) A well is being drilled on a farm. Based on what has happened to similar farms, we judge the probability of oil being present to be 0.5, the probability of natural gas being present to be 0.2 and the probability of neither being present to be 0.3. If oil is present, a geological test will give a positive result with probability 0.9; if natural gas is present, it will give a positive result with probability 0.3; and if neither are present, the test will be positive with probability 0.1. Suppose the test comes back positive. What's the probability that oil is present?

let O = presence of oil
G = presence of gas
T = test w/ positive result

$$Pr(O|T) = \frac{Pr(T|O)\ Pr(O)}{Pr(T)}$$

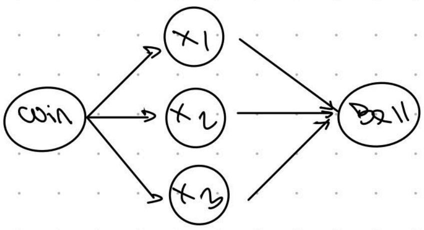
$$= \frac{Pr(T|O)\ Pr(O)}{Pr(T|O)\ Pr(O) + Pr(T|G)\ Pr(G) + Pr(T|\neg O, \neg G)\ P(\neg O, \neg G)}$$

$$= \frac{0.9(0.5)}{0.9(0.5) + 0.3(0.2) + 0.1(0.3)} = \boxed{0.833}$$

Question assigned to the following page: 3

3. (20 pts) We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 40%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2, and X_3. A bell will ring "on" if all coins flips come out the same. Draw the Bayesian network corresponding to this setup and define the necessary CPTs (Conditional Probability Tables).

Graph



CPTs

coin	Pr(C)
a	1/3
b	1/3
c	1/3

	X1	Pr(X1 | C)
a	H	0.2
	T	0.8
b	H	0.4
	T	0.6
c	H	0.8
	T	0.2

	X2	Pr(X2 | C)
a	H	0.2
	T	0.8
b	H	0.4
	T	0.6
c	H	0.8
	T	0.2

	X3	Pr(X3 | C)
a	H	0.2
	T	0.8
b	H	0.4
	T	0.6
c	H	0.8
	T	0.2

X1	X2	X3	Bell	P(Bell | X1, X2, X3)
H	H	H	on	1
H	H	H	off	0
H	H	T	on	0
H	H	T	off	1
H	T	H	on	0
H	T	H	off	1
H	T	T	on	0
H	T	T	off	1
T	H	H	on	0
T	H	H	off	1
T	H	T	on	0
T	H	T	off	1
T	T	H	on	0
T	T	H	off	1
T	T	T	on	1
T	T	T	off	0

Question assigned to the following page: 4

4. (30 pts) Consider the DAG in Figure 1:

(a) List the Markovian assumptions asserted by the DAG.

(b) True or false? Why?

- $d\_separated(A, F, E)$
- $d\_separated(G, B, E)$
- $d\_separated(AB, CDE, GH)$

(c) Express $Pr(a, b, c, d, e, f, g, h)$ in factored form using the chain rule for Bayesian networks.

(d) Compute $Pr(A = 1, B = 1)$ and $Pr(E = 0 \mid A = 0)$. Justify your answers.

a)

I(A, ϕ , BE)
I(B, ϕ, AC)
I(C, A, BDE)
I(D, AB, CE)
I(E, B, ACDFG)
I(F, CD, ABEH)
I(G, F, ABCDEH)
I(H, EF, ABCDG)

b)

d_separated(A, F, E)

- False because F doesn't block ADBE and all path between A and E

d_separated(G, B, E)

- True because B blocks all path between G and E

d_separated(AB, CDE, GH)

- True because CDE blocks all path between AB and GH

c)

Pr(a, b, c, d, e, f, g, h)
= Pr(a) * Pr(b) * Pr (c|a) * Pr(d|a, b) * Pr(e|b) * Pr(f | c,d) * Pr(g|f) * Pr(h| e, f)

d)

- A and B are independent from I(A, ϕ, BE)

Pr(A = 1, B = 1) = Pr(A = 1) * Pr(B = 1) = (0.2) * (0.7) = 0.14

- A and E are independent from I(A, ϕ, BE)

Pr(E = 0 | A = 0)
= Pr(E = 0)
= Pr(E = 0 | B = 1)Pr(B = 1) + Pr (E = 0 | B = 0)Pr(B = 0)
= (0.9)(0.7) + (0.1)(0.3) = 0.66

Question assigned to the following page: 5

5. (20 pts) Consider the joint probability distribution in Table 1 and the propositional sentence $\alpha : A \Rightarrow B$.

(a) List the models of α.

(b) Compute the probability $Pr(\alpha)$.

(c) Compute the conditional probability distribution $Pr(A, B \mid \alpha)$ as in Table 1.

(d) Compute the probability $Pr(A \Rightarrow \neg B \mid \alpha)$.

	A	B	$Pr(A, B)$
w_0	T	T	0.3
w_1	T	F	0.2
w_2	F	T	0.1
w_3	F	F	0.4

Table 1: A joint probability distribution.

a) $\mathcal{M}(\alpha) = \{ w_0, w_2, w_3 \}$

b) $Pr(\alpha) = Pr(w_0) + Pr(w_2) + Pr(w_3)$
$= 0.3 + 0.1 + 0.4$
$\boxed{= 0.8}$

c)

	$Pr(A, B \mid \alpha)$
w_0	$0.3 / 0.8 = 0.375$
w_1	0
w_2	$0.1 / 0.8 = 0.125$
w_3	$0.4 / 0.8 = 0.5$

d) let $\Delta = A \Rightarrow \neg B$
$\mathcal{M}(\Delta) = \{ w_1, w_2, w_3 \}$

$Pr(\Delta \mid \alpha) = Pr(w_1) + Pr(w_2) + Pr(w_3)$
$= 0 + 0.125 + 0.5$
$\boxed{= 0.625}$