Microeconomic Theory — ECON 323 503
Math Review

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Outline

1. Real numbers
2. Functions
3. Lines
4. Derivatives
5. Some properties of functions
6. Functions of more than one variable
7. Partial derivatives
8. Finding the maximum or minimum of a function

Real numbers

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Real numbers are important in economics because they represent:

- Amounts of resources
- Amounts of money
- Interest rates
- Probabilities
- And so many more things

Real line

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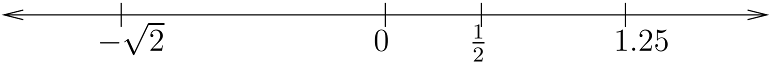
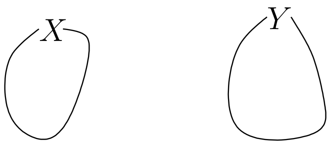


Functions of a single variable

Two sets: X and Y (might be the same)



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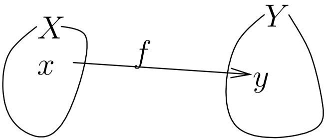

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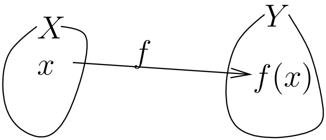

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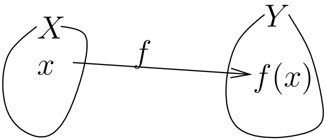

For our purposes, X and Y will usually be the real numbers ($\mathbb{R}$).

Examples of functions

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- Linear: for each $x \in X$, $f(x) = mx + c$ where m and c are fixed.

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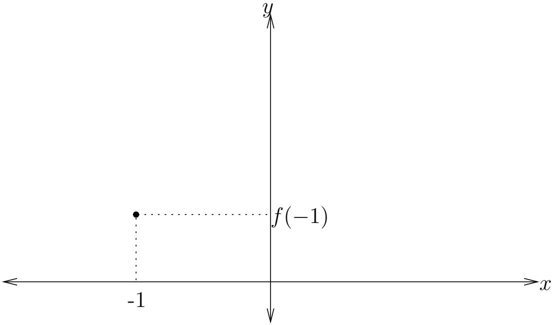

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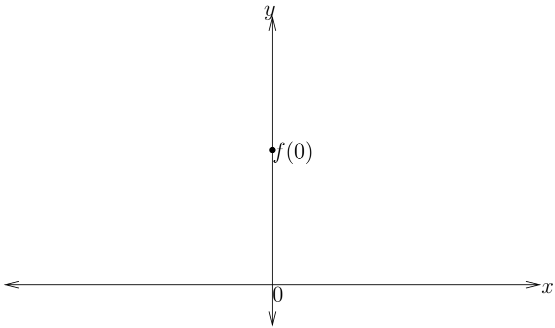

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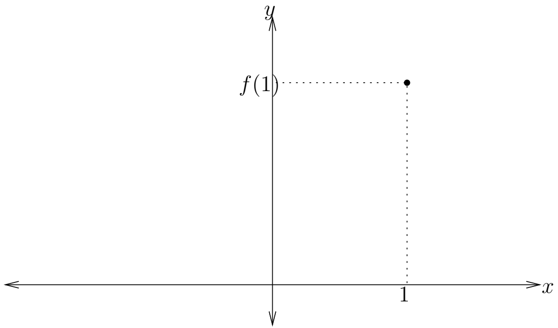

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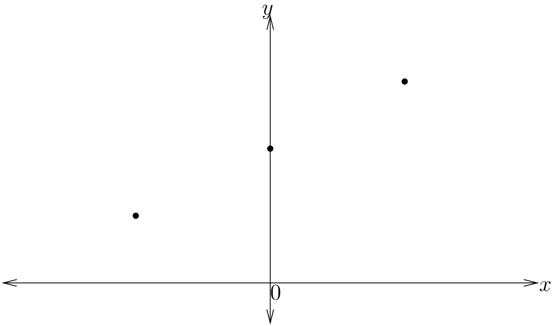
Suppose $m = 0.5$ and $c = 1$. Then $f(x) = 0.5x + 1$.

Connect the dots.



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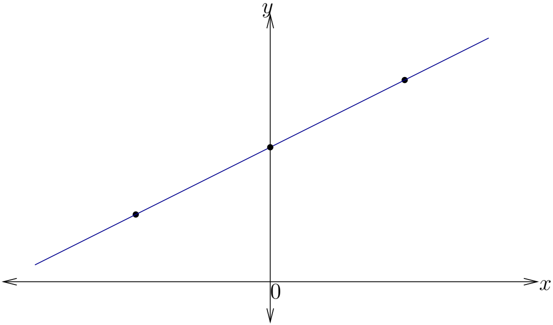
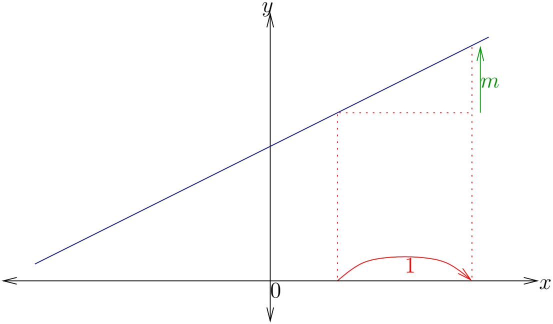
Now you have a line.



Slope of a line

We call m the "slope" of the line $mx + c$.
That's because for increment in x, the value of $mx + c$ increases m times.



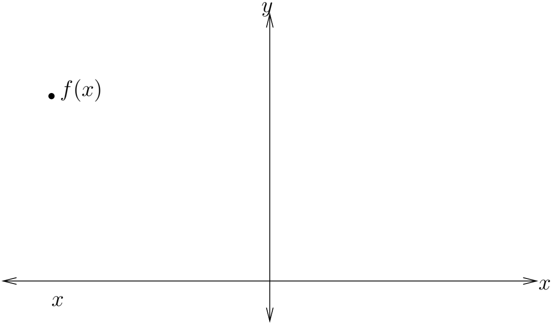
Slope of a line

So the slope of a line is the ratio of the change in $f(x)$ to the change in x. Suppose x changes from x_1 to x_2:

$$\text{slope } = m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

Graph of a function

We can draw the "graph" of any function in this way, by connecting the dots representing each x and corresponding $f(x)$.



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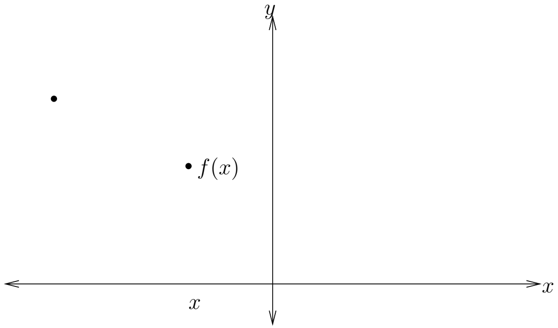

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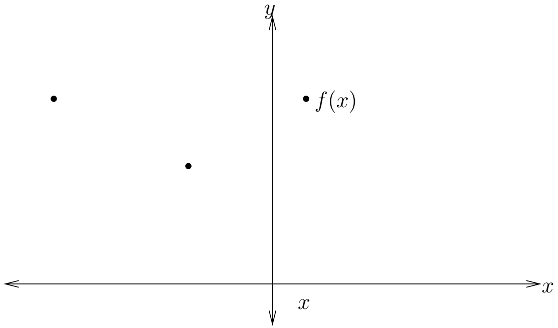

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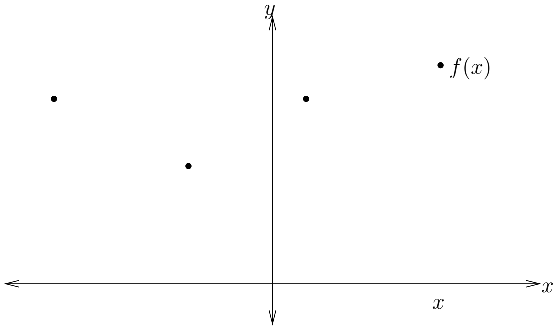

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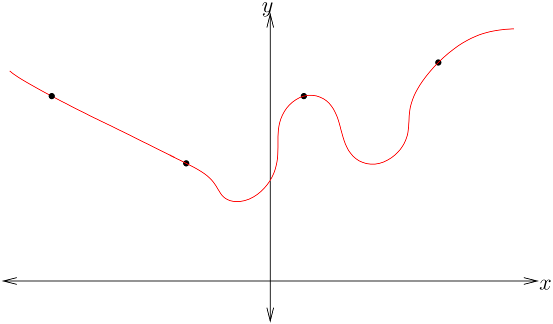


Derivative of a function

Recall the slope of a line is $\frac{f(x_2)-f(x_1)}{x_2-x_1}$. It doesn't depend on how far apart or where x_1 and x_2 are.

Derivative of a function

If f isn't linear, it will. So what is the slope of f? It's the same thing where the change in x is as small as possible.

Derivative of a function

So, if x changes from x to $x+h$, the ratio of the change in $f(x)$ to the change in x is just

$$\frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}.$$

Derivative of a function

To make h *as small as possible*, we take the limit as h goes to zero:

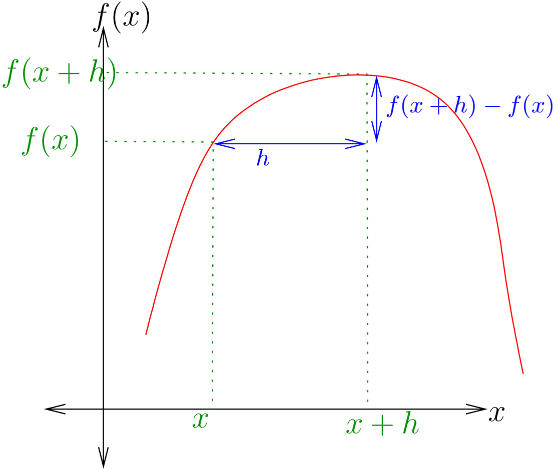
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

(Don't let the $\lim_{h \to 0}$ scare you...)

Derivative of a function

We'll represent this with the notation $\frac{df(x)}{dx}$.

Geometrically



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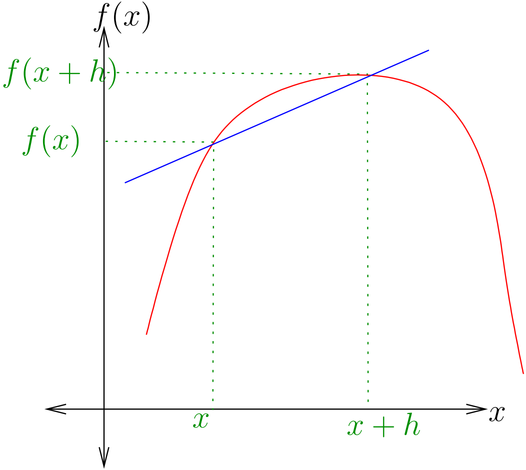

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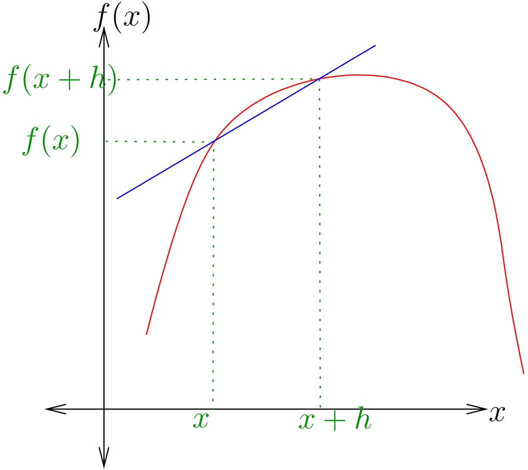

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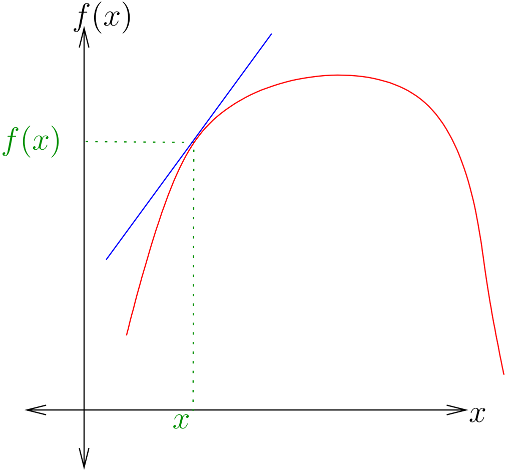

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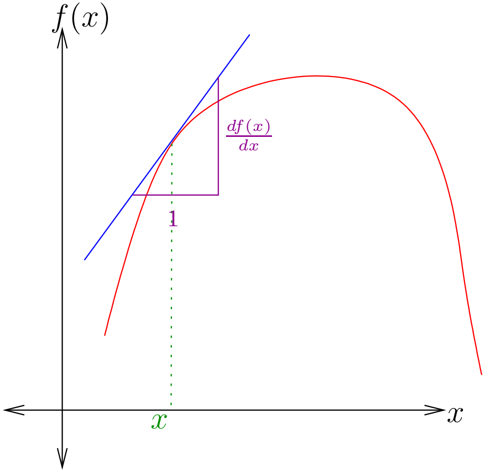

Some basic rules to calculate derivatives

1. Constant: If $f(x) = c$ then $\frac{df(x)}{dx} = 0$.

Some basic rules to calculate derivatives

2. Addition: If $f(x) = g(x) + h(x)$ then
$$\frac{df(x)}{dx} = \frac{dg(x)}{dx} + \frac{dh(x)}{dx}.$$

Some basic rules to calculate derivatives

3. Multiplication: If $f(x) = g(x)h(x)$ then
$$\frac{df(x)}{dx} = \frac{dg(x)}{dx}h(x) + g(x)\frac{dh(x)}{dx}.$$

Some basic rules to calculate derivatives

4. Power: If $f(x) = ax^b$ then

$$\frac{df(x)}{dx} = abx^{b-1}.$$

Some basic rules to calculate derivatives

5. Logarithm: If $f(x) = ln(x)$ then
$$\frac{df(x)}{dx} = \frac{1}{x}.$$

Some basic rules to calculate derivatives

6. Chain: If $f(x) = g(h(x))$ then

$$\frac{df(x)}{dx} = \frac{dg(h(x))}{dh(x)}\frac{dh(x)}{dx}$$

Derivatives of more complicated functions?

You can use the basic rules as building blocks to take derivatives of many functions (and all of the ones that we'll see in this class).

An example

$$f(x) = 5x^4 + xln(x)$$

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4. $\frac{dg'(x)}{dx} = 1$.
5. $\frac{dh'(x)}{dx} = \frac{1}{x}$.
6. Putting these two together, using the product rule,

$$\frac{dh(x)}{dx} = \frac{dg'(x)}{dx}h'(x)+g'(x)\frac{dh'(x)}{dx} = 1ln(x)+x\frac{1}{x} = ln(x)+1.$$

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7. Finally, using the addition rule
   $$\frac{df(x)}{dx} = \frac{dg(x)}{dx} + \frac{dh(x)}{dx} = 20x^3 + ln(x) + 1.$$

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$\frac{dg(y)}{dy} = y^2$.

$\frac{dh(x)}{dx} = 1$.

Using the chain rule,

$$\frac{df(x)}{dx} = \frac{dg(h(x))}{dh(x)}\frac{dh(x)}{dx} = (h(x))^2 \times 1 = \left(x - \frac{1}{2}\right)^2.$$

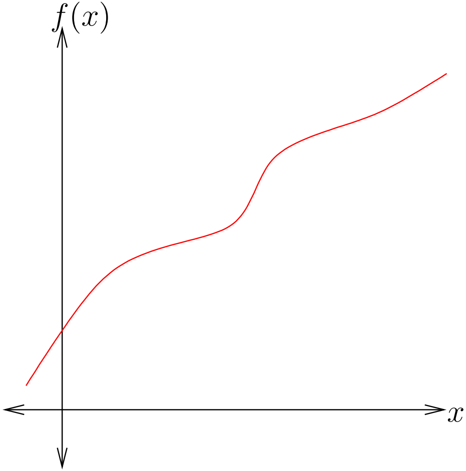
Some properties of functions

1. Monotonicity
2. Continuity
3. Concavity
4. Convexity

Monotonicity

The function is either always going up or always going down as you move x in the same direction.

This function is monotonic:



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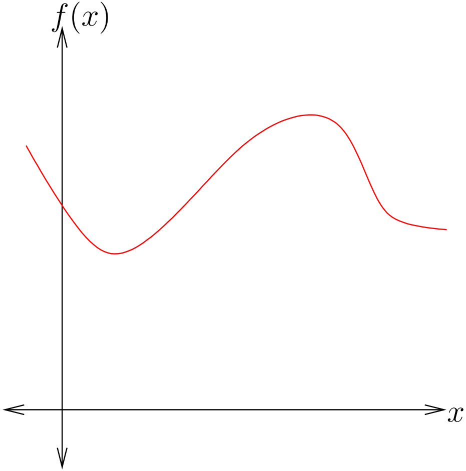
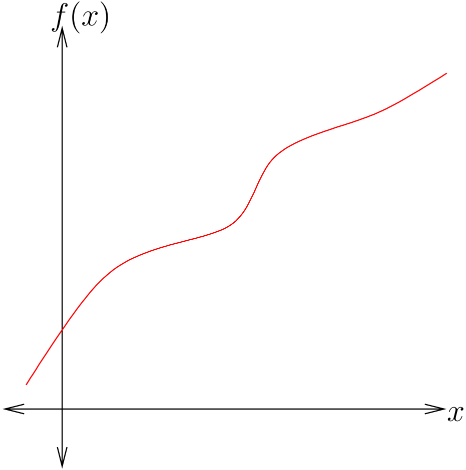


Continuity

The graph of the function doesn't "jump" or have any "breaks" in it.

This function is continuous:



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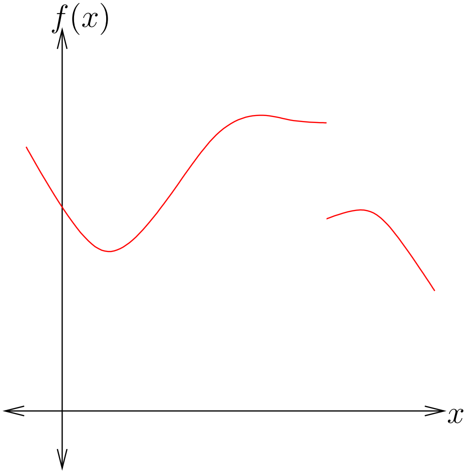
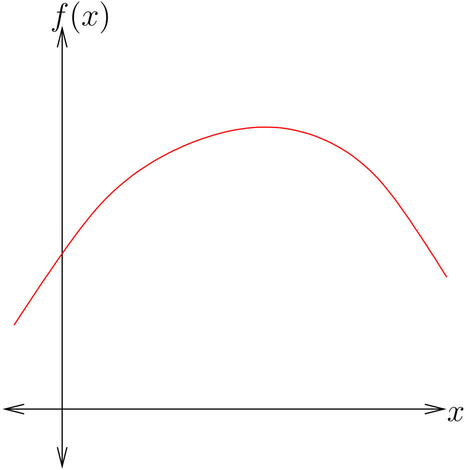


Concavity

What can we say about this derivative of this function as x moves to the right?



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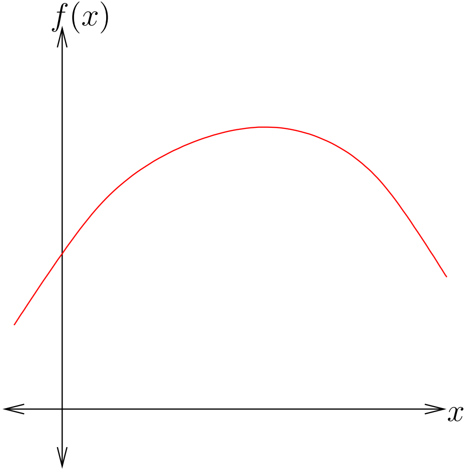

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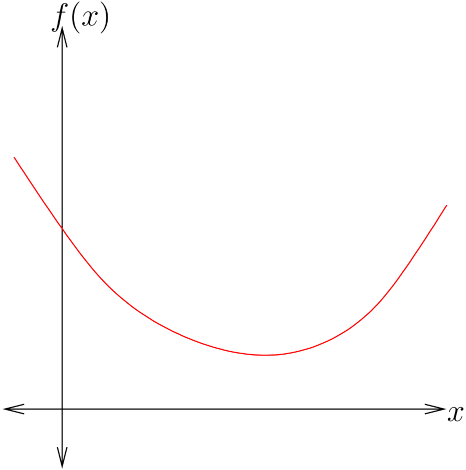

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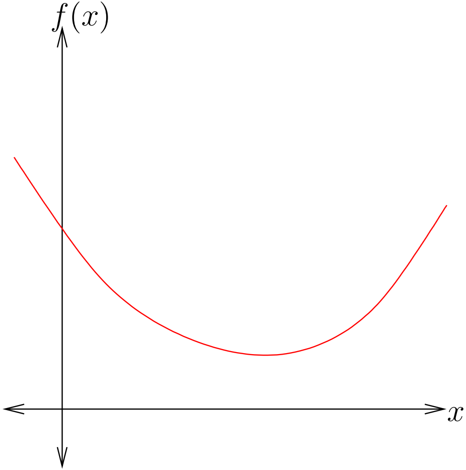
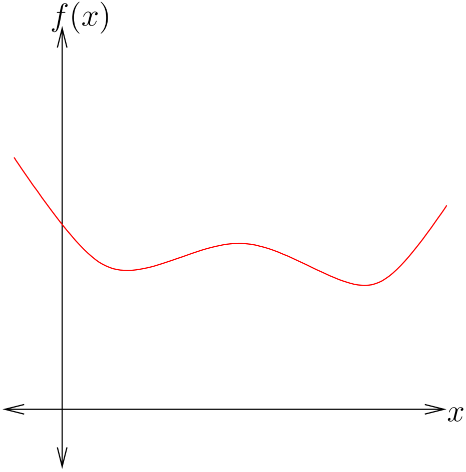


It's increasing.

Concavity and convexity

This function is neither concave nor convex.



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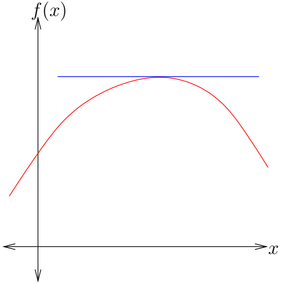

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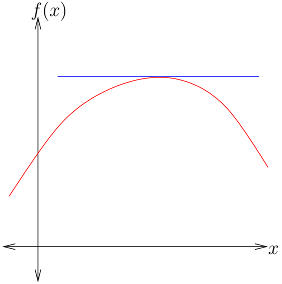

We just have to find the spot where the derivative is zero. For a concave function, it’s flat at the very top of the hill and for a convex function, it’s flat at the very bottom of the valley

Functions of more than one variable

1. f may map *pairs* of members of X with members of Y. Then $f : X^2 \to Y$. We write $f(x_1, x_2)$ to denote the value that the pair (x_1, x_2) is mapped to.

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3. ...
4. $\ldots f(x_1, x_2, \ldots, x_n)\ldots$

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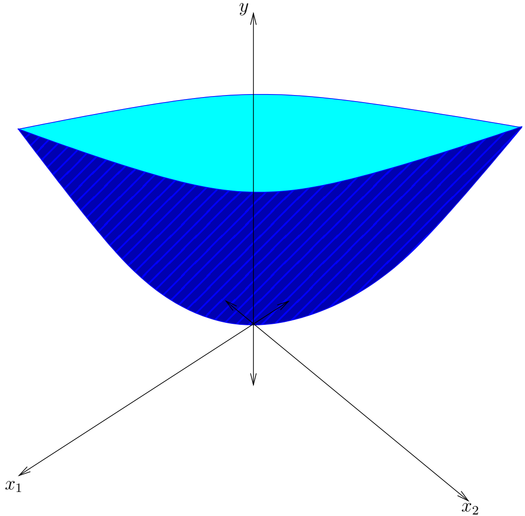


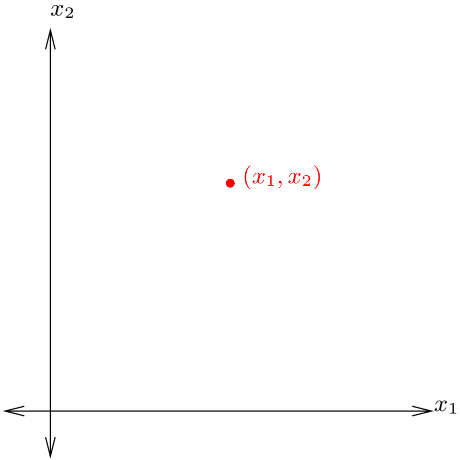
What about three variables?

Now you need four dimensions…

An easier way to visualize functions of two variables

Just like a topographic map, we just draw a curve through all the pairs (x_1, x_2) that are mapped to the same value.

Suppose $f(x_1, x_2) = y$.



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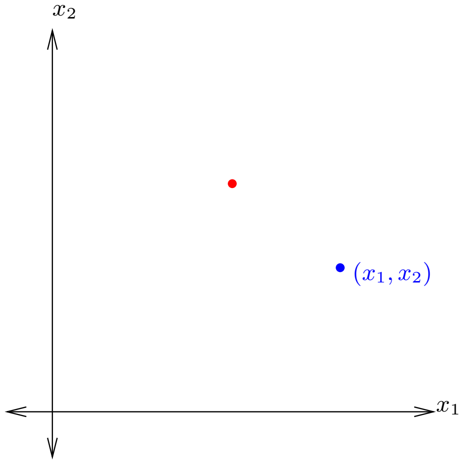
Just like a topographic map, we just draw a curve through all the pairs (x_1, x_2) that are mapped to the same value.

And $f(x_1', x_2') = y$ too.



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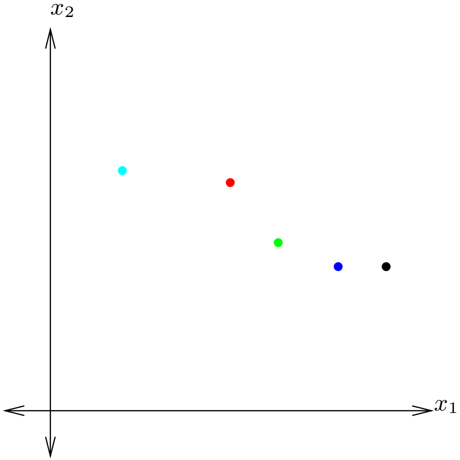

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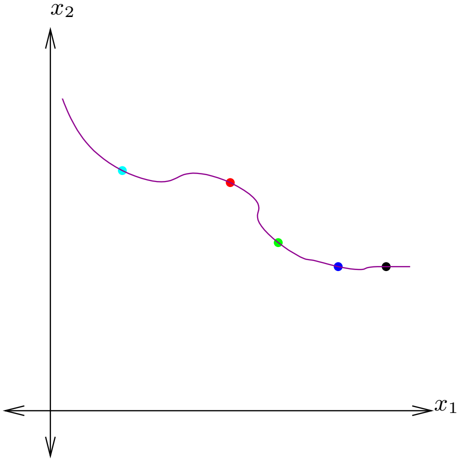

This is a “iso-level set” of f.

Partial derivatives

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We write $\frac{\delta f(x_1,x_2)}{\delta x_1}$ to represent the partial derivative of f with respect to x_1.

An example

$$f(x_1, x_2) = x_1^2 + x_2^2 + 2x_1x_2$$

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Just take the derivative, treating x_2 as a constant:

$$\frac{\delta f(x_1, x_2)}{\delta x_1} = 2x_1 + 0 + 2x_2 = 2x_1 + 2x_2.$$

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To find it, solve for x such that $\frac{df(x)}{dx} = 0$.

An example

$$\max_x \frac{1}{2}\left(x - \frac{1}{2}\right)^2$$

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These are the "first order conditions" for our maximization problem.

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$$\max_{x_1, x_2} \ \frac{1}{2}(x_1 - 4)^2 + \frac{1}{2}(x_2 - 9)^2$$

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Solve these two equations and you find that $x_1^* = 4$ and $x_2^* = 9$.

Constraints

You can't always just pick any old x_1 and x_2. In most economic problems, we have constraints: if you have \$10, apples cost \$1 each and bananas cost \$2 each, you can't buy 15 apples and 20 bananas.

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Suppose that the constraint is in the form of some function h:

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When we get to the point where we need a more sophisticated method (the Lagrange method), we'll go over that.

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Use this to find that $x_2^* = z - x_1^* = \frac{z}{2}$.