# Microeconomic Theory — ECON 323 503
# Chapter 3: A Consumer's Constrained Choice

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# Outline

1. Preferences: How does a consumer decide which bundle of goods he prefers?

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2. Utility: A summary of a consumer's preferences.
3. Budget constraint: Prices, income, regulation limit what a consumer can have.
4. Constrained choice: How does a consumer choose when faced with a budget constraint?

# Preferences

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Keep in mind that each consumer has his own preferences. Not everyone is necessarily the same.

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Think of $\succeq$ the way you would $\geq$.

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"Consumer is *indifferent* between $a$ and $b$."

Think of $\sim$ the way you would $=$.

# Properties of $\succeq$

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3. Monotonic.

# Transitivity

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Does it make sense for $\succeq$ not to be transitive?

Economist's definition of rationality: Preferences are transitive.

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Not critical for the kinds of analysis we will do in this course (unlike transitivity and completeness). But makes life easier.

# Describing preferences

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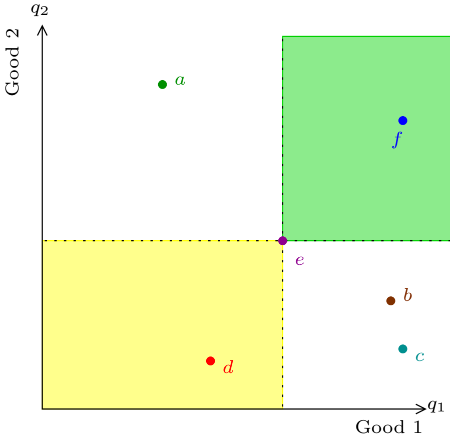
*Preference map*: draw a curve through all the bundles that the consumer is indifferent between.

Monotonicity restricts what these lines can look like.

# Monotonicity: graphically




$e$ has *less* of *every* good than $f$ so $f \succ e$.

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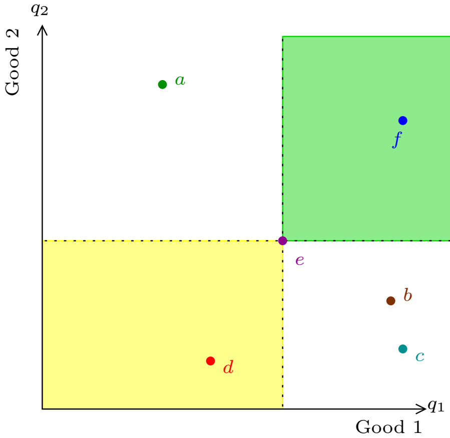


$e$ has *less* of *every* good than $f$ so $f \succ e$.

This is true for every bundle in the green region.

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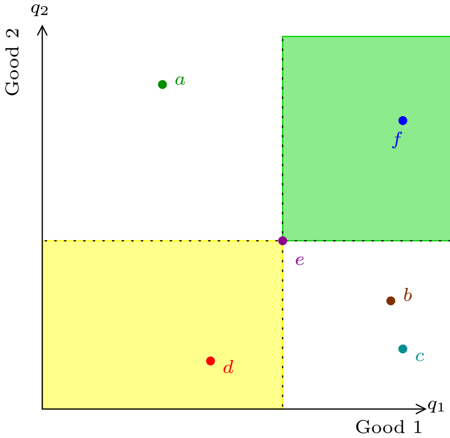


$e$ has *more* of *every* good than $d$ so $e \succ d$.

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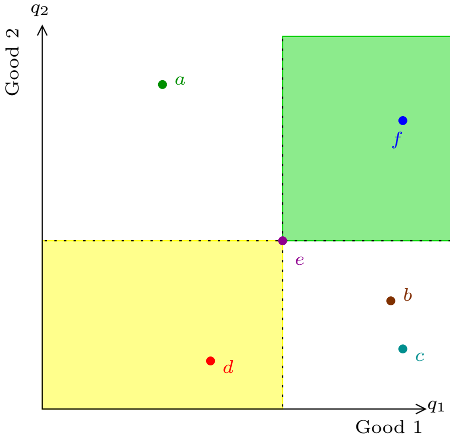


$e$ has *more* of *every* good than $d$ so $e \succ d$.

This is true for every bundle in the yellow region.

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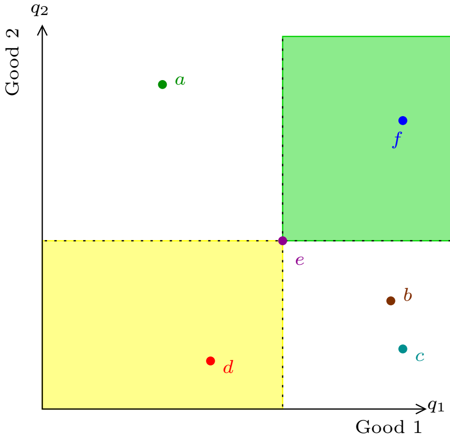


$e$ has more of good 1 and less of good 2 than $a$.

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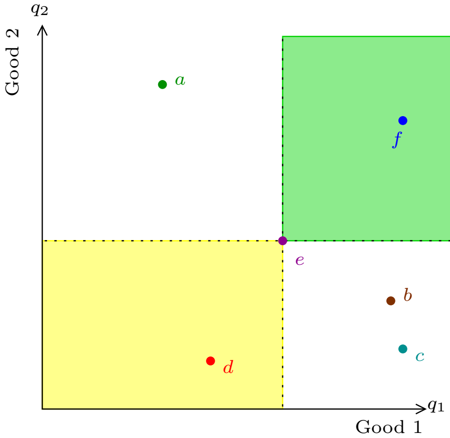
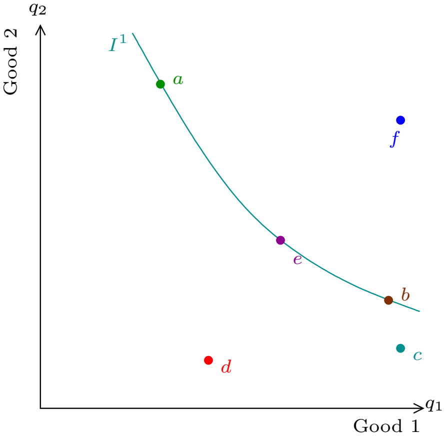



$e$ has more of good 1 and less of good 2 than $a$.

Bundles in unshaded area could be better, worse or indifferent.

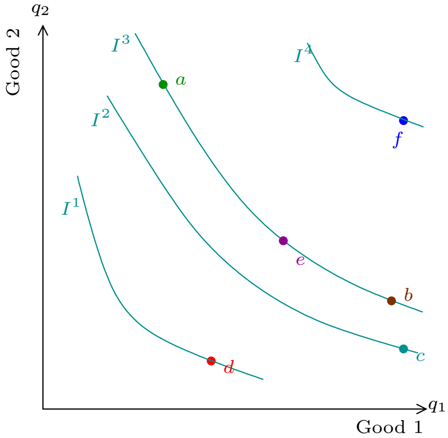
# Indifference curves




Curve through bundles the consumer is indifferent between.

$I^1$ is an "indifference curve."

# Preference map




Just do this for every possible bundle.

Collection of curves is a “preference map.”

# Properties of indifference curves

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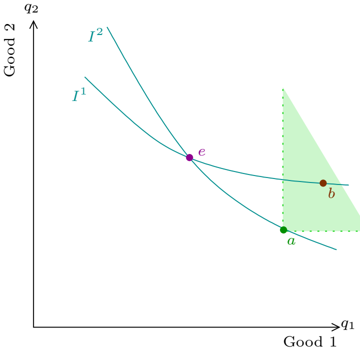
5. Can't be thick.

# Impossible preference maps

Crossing ICs:



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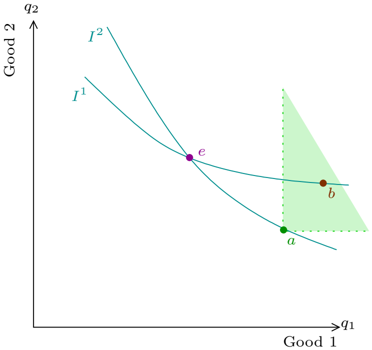


$e$ and $b$ are on $I^1$ so $e \sim b$.

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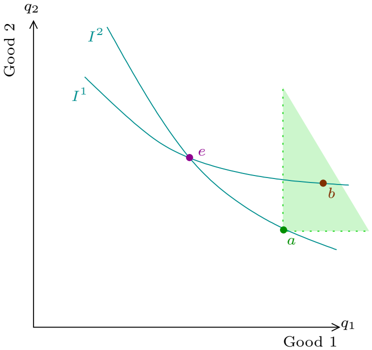


$e$ and $b$ are on $I^1$ so $e \sim b$.
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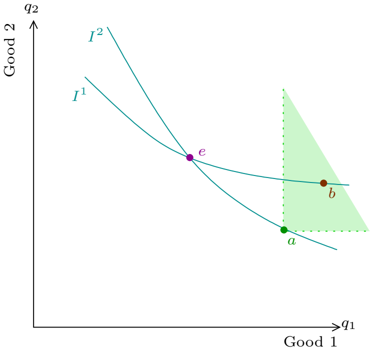


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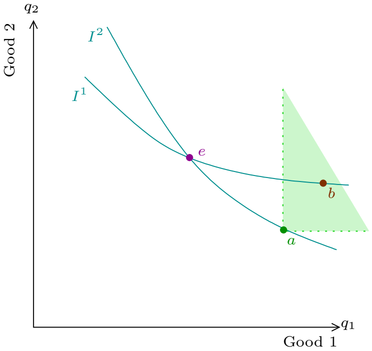


$e$ and $b$ are on $I^1$ so $e \sim b$.
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Transitivity says $a \sim b$.
But monotonicity says $b \succ a$.

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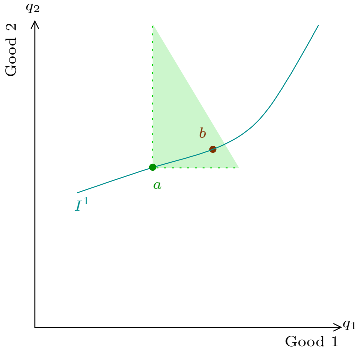
Upward sloping ICs:



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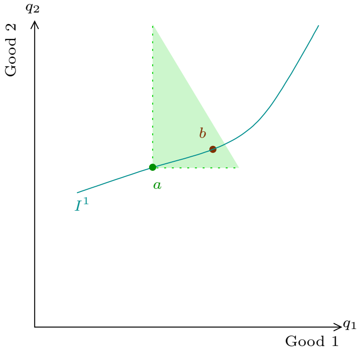


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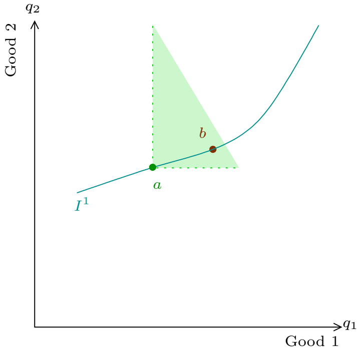


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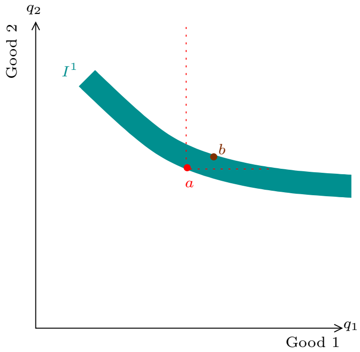

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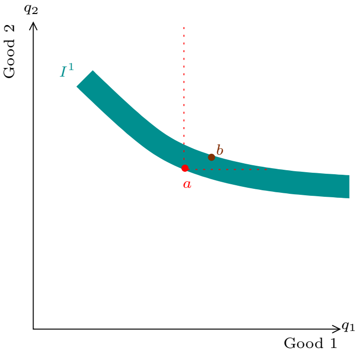


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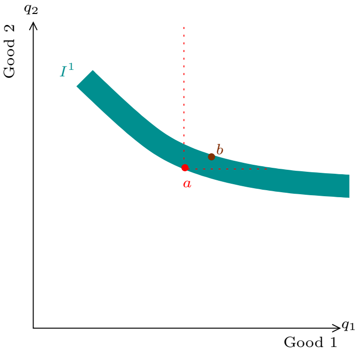


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# Utility

Associate a number with each IC. Higher numbers for ICs through better bundles.

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Example: Cobb-Douglas utility.

$$U(q_1, q_2) = q_1^a q_2^{1-a}.$$

where $a$ is a constant between 0 and 1.

# Utility

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Which bundle is better? (16,9) or (13,13)?

$U(16, 9) = \sqrt{16 \times 9} = 12$ and $U(13, 13) = \sqrt{13 \times 13} = 13$. So (13,13) is better than (16,9).

# Ordinal preferences

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Not really. A utility function only serves to help us *order* the different bundles.

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# Ordinal preferences

You prefer bundle $x$ to bundle $y$.

If $U$ is your utility function, we know that $U(x) > U(y)$. $U$ is a *description* of your preferences.

It could be that $U(x) = 5$ and $U(y) = 4$.

But we could just double all the numbers and it would still describe the *same* preferences

# Positive monotonic transformations

A function $F$ such that if $x > y$ then $F(x) > F(y)$.

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$$V(q_1, q_2) = F(U(q_1, q_2)).$$

$V$ defines the *same* preferences as $U$.

# An example

$$F(x) = a + bx$$

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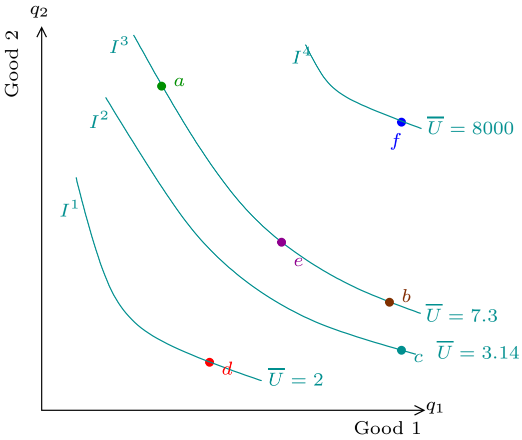
$$\begin{array}{rcl} U(q_1, q_2) & > & U(q_1', q_2') \\ & \Updownarrow & \\ a + bU(q_1, q_2) & > & a + bU(q_1', q_2') \\ & \Updownarrow & \\ V(q_1, q_2) & > & V(q_1', q_2'). \end{array}$$

# Utility functions and indifference curves

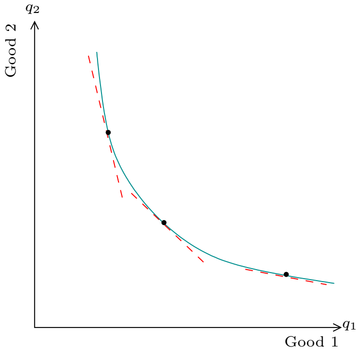
An indifference curve is described by, for each $\overline{U}$,

$$\overline{U} = U(q_1, q_2).$$



# Willingness to substitute between goods

*Marginal Rate of Substitution (MRS)*: Slope of a line tangent to IC. This is the ratio of changes in $q_2$ and $q_1$ that leave you indifferent. Since IC slopes downward, this is negative.



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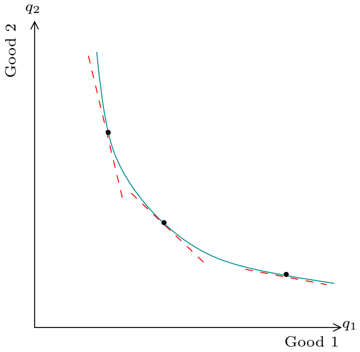


MRS tells us: How much of good 2 are you willing to give up for a tiny bit more of good 1?

# Calculating MRS from utility

*Marginal utility*: The additional utility from a tiny bit more of a good.
Just the partial derivative of $U$:

$$\begin{aligned}\text{Marginal utility from good 1 (MU}_1\text{)} &= \frac{\delta U}{\delta q_1} = U_1 \\ \text{Marginal utility from good 2 (MU}_2\text{)} &= \frac{\delta U}{\delta q_2} = U_2\end{aligned}$$

# Calculating MRS from utility

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Your utility loss from giving up some of good 2 is $MRS \times x \times U_2$.

Since you're indifferent, your total utility change is 0.

$$x \times U_1 + MRS \times x \times U_2 = 0.$$

In other words,

$$MRS = -\frac{U_1}{U_2} = -\frac{\frac{\delta U}{\delta q_1}}{\frac{\delta U}{\delta q_2}}.$$

# An example

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$$U(q_1, q_2) = q_1^a q_2^{1-a}.$$

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Step 1: Calculate marginal utility with respect to good 1:

$$U_1 = \frac{\delta U}{\delta q_1} = a q_1^{a-1} q_2^{1-a} = a \frac{U(q_1, q_2)}{q_1}.$$

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Step 2: Calculate marginal utility with respect to good 2:

$$U_2 = \frac{\delta U}{\delta q_2} = (1-a) q_1^a q_2^{-a} = (1-a) \frac{U(q_1, q_2)}{q_2}.$$

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Step 3: The negative of their ratio gives us the MRS:

$$MRS = -\frac{U_1}{U_2} = -\frac{a \frac{U(q_1, q_2)}{q_1}}{(1-a) \frac{U(q_1, q_2)}{q_2}} = -\frac{a}{1-a} \frac{q_2}{q_1}.$$

# Diminishing MRS

What happens to the shape of an IC as we move down and to the right?

# Diminishing MRS

What happens to the shape of an IC as we move down and to the right?
It gets flatter: as you get more and more of good 1, you're "less willing" to give up good 2 for more of good 1.

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$$U(q_1, q_2) = q_1^{0.5} q_2^{0.5}.$$

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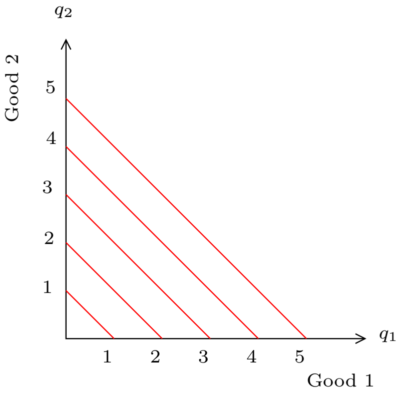
$$MRS = -\frac{q_2}{q_1}.$$

At (4,4), $MRS = -1$.

But at $(40, 4), MRS = -0.1$.

# Perfect substitutes

No matter how much of the goods you have, you'll give up one unit of good 1 for a unit of good 2 (and vice versa): you only care about the total number of units. E.g. Bananas from Ecuador vs Costa Rica.



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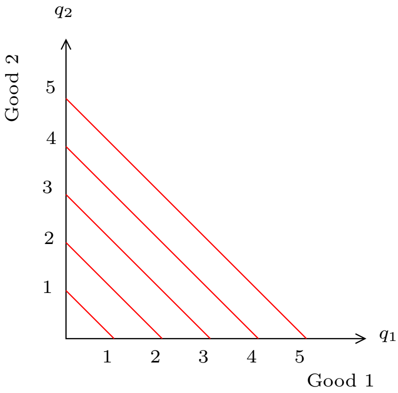


So your MRS is always -1.

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The MRS of -1 depends on the units of measurement. More generally, preferences are described by

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$$\begin{aligned} U_1 &= i \\ U_2 &= j \\ MRS &= -\tfrac{U_1}{U_2} = -\tfrac{i}{j}. \end{aligned}$$

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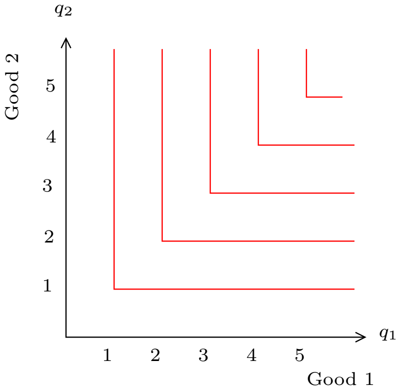
In general, preferences described by

$$U(q_1, q_2) = \min(q_1, q_2).$$

# Perfect complements



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where $0 < \rho \leq 1$.

$$\begin{aligned} U_1 &= (q_1^\rho + q_2^\rho)^{\frac{1-\rho}{\rho}} q_1^{\rho-1} \\ U_2 &= (q_1^\rho + q_2^\rho)^{\frac{1-\rho}{\rho}} q_2^{\rho-1} \\ MRS &= -\frac{U_1}{U_2} = -\left(\frac{q_1}{q_2}\right)^{\rho-1}. \end{aligned}$$

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Example: $u(q_1) = 4q_1^{0.5}$ so that $U(q_1, q_2) = 4q_1^{0.5} + q_2$.

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There are limits to what he can have: there are prices and he has a finite income.

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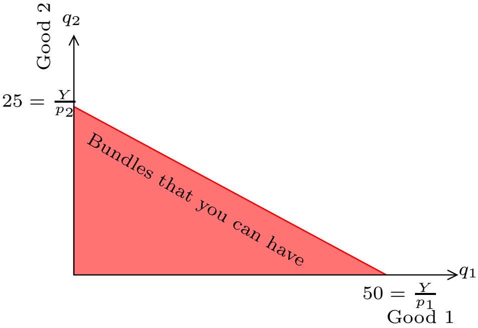
$$q_2 = \frac{Y}{p_2} - \frac{p_1}{p_2} q_1.$$

This tells us how to plot the budget constraint: intercept is $\frac{Y}{p_2}$ and slope is $-\frac{p_1}{p_2}$.

# Budget constraint

If $p_1 = \$1, p_2 = \$2$, and $Y = \$50$, budget line is

$$q_2 = \frac{50}{2} - \frac{1}{2}q_1 = 25 - \frac{1}{2}q_1.$$




Since you can have anything in the shaded area, we call it your *opportunity set.*

# Slope of budget line

$-\frac{p_1}{p_2}$ is the rate at which you trade off between the two goods.

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If $p_1 = \$1$ and $p_2 = \$2$, then $MRT = -\frac{1}{2}$: To get one more unit of good 1, you need to *give up* a half unit of good 2.

# Constrained consumer choice

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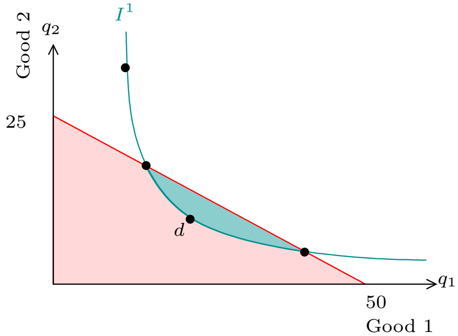
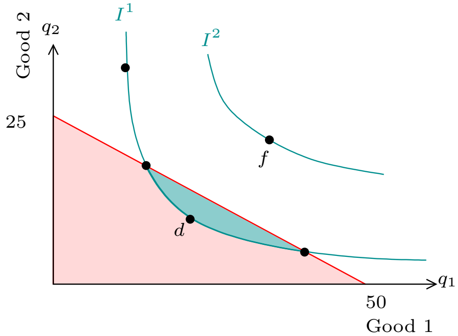
Now, we study how you choose from your budget set.

# Constrained consumer choice



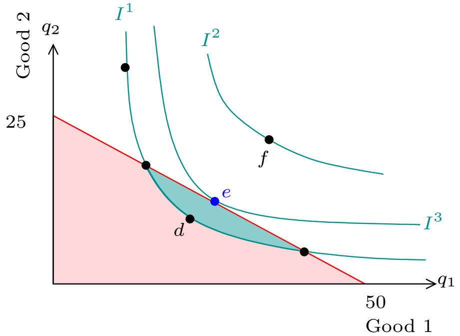

You can do better than $d$: all the bundles in the shaded area are better and you can afford them.

# Constrained consumer choice




Unfortunately, even though it's better than everything in your opportunity set, you can't afford $f$.

# Constrained consumer choice




The best you can do in your budget set is $e$. It's better than all the other bundles in the shaded area.

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This is one equation, the budget line is the other equation. now you can solve two equations for two unknowns ($q_1$ and $q_2$).

# Deriving the optimality condition

We just found the condition $\frac{U_1}{p_1} = \frac{U_2}{p_2}$ graphically. To make sure that it's right, we can also do it with calculus.

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We'll solve this using the *substitution method*.

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So the problem becomes

$$\max_{q_1} U\left(\frac{Y}{p_1} - \frac{p_2}{p_1} q_2, q_2\right)$$

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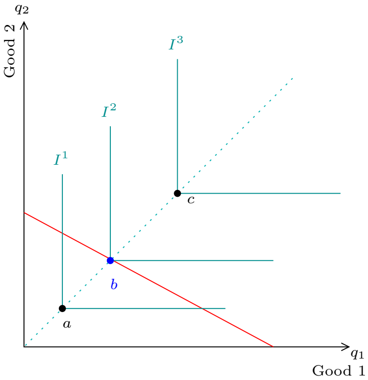
# Perfect complements

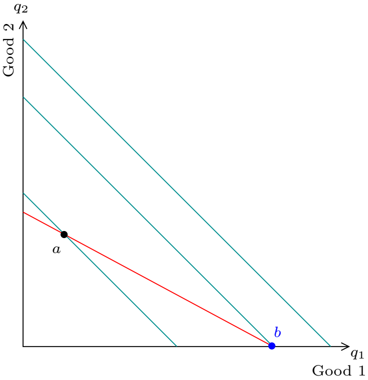
Perfect complements are a little different: The MRS is either $-\infty$,0, or undefined. So how do we maximize?




You would only buy bundles along the dotted line: they have the same amount of each good. The best bundle that you can afford is $b$.

# Corner solutions: perfect substitutes

The tangency argument only works on the "interior" not on the boundary.

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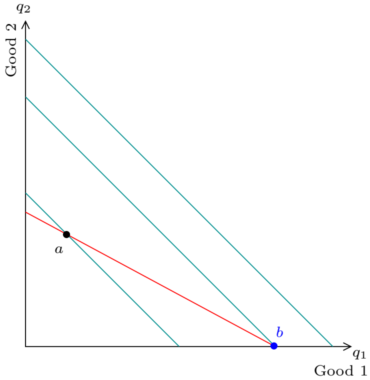



Since the slope of an IC is constant, it can’t be *tangent* to the budget line at any point. So $a$ can’t be the best you can do.

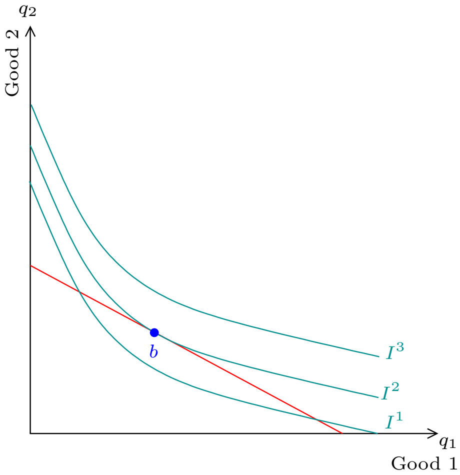
The best that you can do is $b$. But there’s no tangency there.

# Corner solutions: perfect substitutes

In this case, you only care about the total number of goods that you get ($U(q_1, q_2) = q_1 + q_2$).
It makes sense that you buy only the cheaper of the two goods.

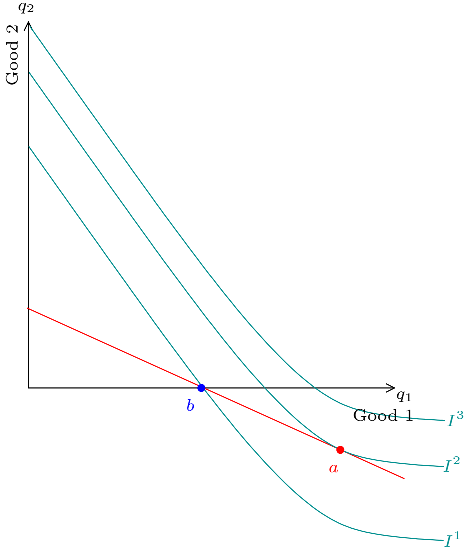
# Corner solutions: quasilinear preferences

While the ICs do cut the axis, you *may* have an interior solution with tangency.



# Corner solutions: quasilinear preferences

But often, this won't be the case. Tangency may only happen in an impossible area (negative amount of a good).



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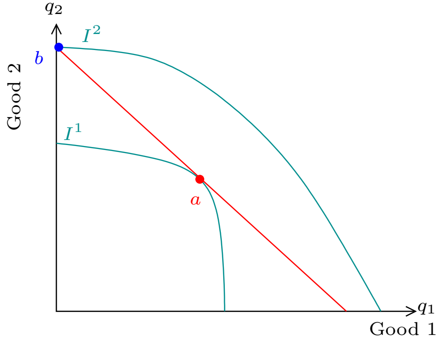
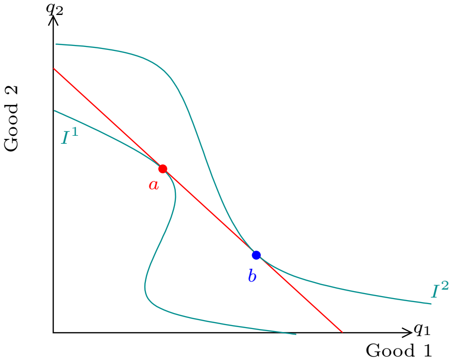



Despite tangency, $a$ is not the best. Usually "concave" preferences lead to corner solutions like $b$.

# What if indifference curves aren't convex?

For funky preferences, it gets even more complicated.




Tangency at both $a$ and $b$ but only $b$ is optimal.

# Solution of problem 4.11 from the textbook

$$U(q_1, q_2) = q_1^{0.75} q_2^{0.25}$$

What is the optimal bundle if $p_1 = \$1, p_2 = \$2$, and $Y = \$80$?

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Set up the maximization problem:

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That is,

$$\max_{q_1, q_2} q_1^{0.75} q_2^{0.25}$$
$$\text{s.t. } q_1 + 2q_2 = 80$$

# Solution of problem 4.11 from the textbook

Use the substitution method: $q_1 = 80 - 2q_2$, so the problem is actually

$$\max_{q_2} (80 - 2q_2)^{0.75} \, q_2^{0.25}$$

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$$\max_{q_2} \left(80 - 2q_2\right)^{0.75} q_2^{0.25}$$

First order condition:

$$-2 \times 0.75(80 - 2q_2)^{0.75-1} q^{0.25} + 0.25(80 - 2q_2)^{0.75} q^{0.25-1} = 0$$

# Solution of problem 4.11 from the textbook

$$-1.5\left(\frac{q_2}{80-2q_2}\right)^{0.25} + 0.25\left(\frac{80-2q_2}{q_2}\right)^{0.75} \quad = \quad 0$$

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Substituting $q_1 = 80 - 2q_2 = 80 - 2 \times 10 = 60.$

# Solution of problem 4.11 from the textbook

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$$\begin{aligned} 3\tfrac{q_2}{q_1} &= \tfrac{1}{2} \\ q_1 &= 6q_2 \end{aligned}$$

Second equation: $q_1 + 2q_2 = 80$.

Substitute first into second:

$$6q_2 + 2q_2 = 80.$$

So $q_2 = 10$ and $q_1 = 6q_2 = 60$.