Microeconomic Theory — ECON 323 503
Chapter 7: Costs

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Outline

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5. Cost of producing multiple goods.

Measuring costs

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- Implicit costs: the opportunities foregone when using resources.

Opportunity cost

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The value of those other opportunities is your "opportunity cost" of being here today.

Opportunity cost: another example

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So your cost of employing yourself isn't the \$1,000 you pay yourself, but the \$11,000 you're forgoing.

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2. How do you deal with changes in the value of capital?

Capital costs: an example

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No. There's an opportunity cost: you could rent out the truck or even sell it rather than using it.

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That \$1,000 is part of the opportunity cost of the truck, bringing its total opportunity cost to \$4,000.

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You're forgoing that additional income from the resources that you have. That's an opportunity cost.

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An example: expenditure on specialized/custom equipment that you can't sell to anyone.

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Yes. By building the factory you end up with something worth \$40,000 $= \$240,000 - \$200,000$ more.

If you considered the original cost (which you've already paid), you'd come to the conclusion that if you build the factory, you'd have a loss of \$60,000.

Short-run costs

If you rent a space for your business, you have to pay that amount every month. You could decide not to do any business at all and avoid it (so it's not a sunk cost), but if you do any business at all you have to pay rent. This is your *fixed cost*, F. How much business you do doesn't affect it.

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Your *cost, C(q)* is the sum of these two:

$$C(q) = VC(q) + F.$$

Marginal cost

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Since F is a constant, $MC(q) = \frac{VC(q)}{dq}$.

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3. Average Cost: $AC(a) = \frac{C(q)}{q}$.

$$AC(q) = \frac{C(q)}{q} = \frac{VC(q)}{q} + \frac{F}{q} = AVC(q) + AFC(q).$$

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$AC(q)$ is what you need to know if you're making a profit or not.

Solved problem 7.2

$$C(q) = 100q - 4q^2 + 0.2q^3 + 450$$

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Marginal cost?

$$
\begin{aligned}
MC(q) &= \tfrac{dC(q)}{dq} \\
&= \tfrac{d(100q-4q^2+0.2q^3+450)}{dq} \\
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Average variable cost?

$$\begin{aligned} AVC(q) &= \tfrac{VC(q)}{q} \\ &= \tfrac{100q-4q^2+0.2q^3}{q} \\ &= 100 - 4q + 0.2q^2 \end{aligned}$$

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Average fixed cost?

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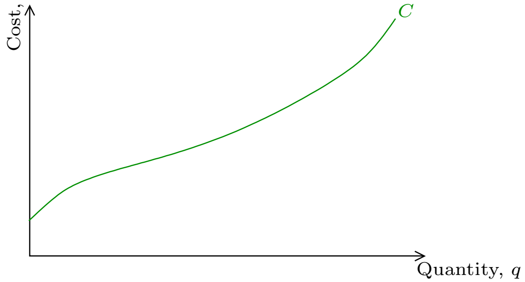
Average cost?

$$\begin{aligned} AC(q) &= \tfrac{C(q)}{q} \\ &= \tfrac{100q - 4q^2 + 0.2q^3 + 450}{q} \\ &= 100 - 4q + 0.2q^2 + \tfrac{450}{q} \\ &= AVC(q) + AFC(q). \end{aligned}$$

Short-run cost curves



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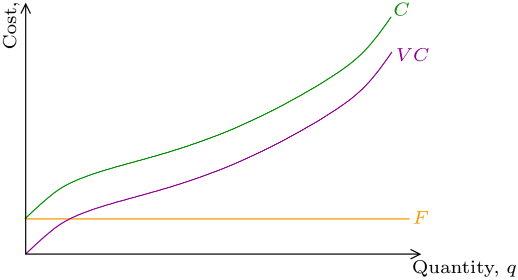
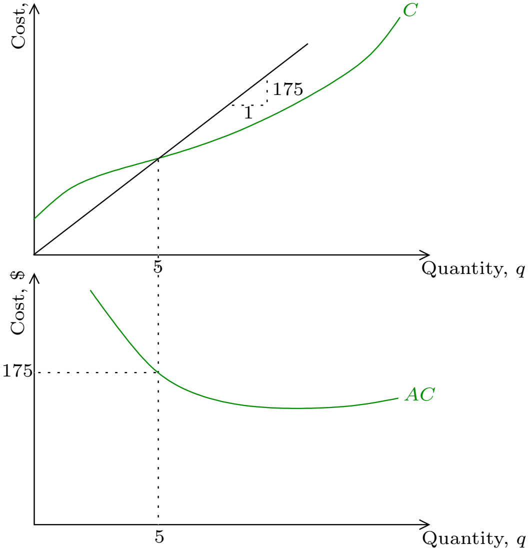


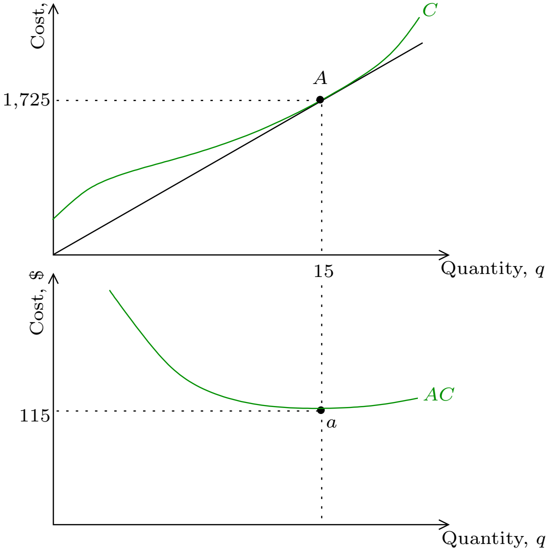
F doesn't vary with quantity while VC is a curve parallel to C (the vertical distance between points on VC and C) is F.

Short-run cost curves



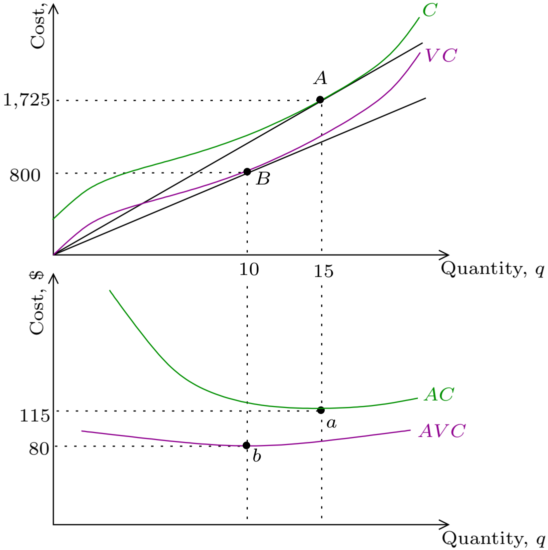
Average cost is the slope of a line from 0 to a point on C.

Short-run cost curves



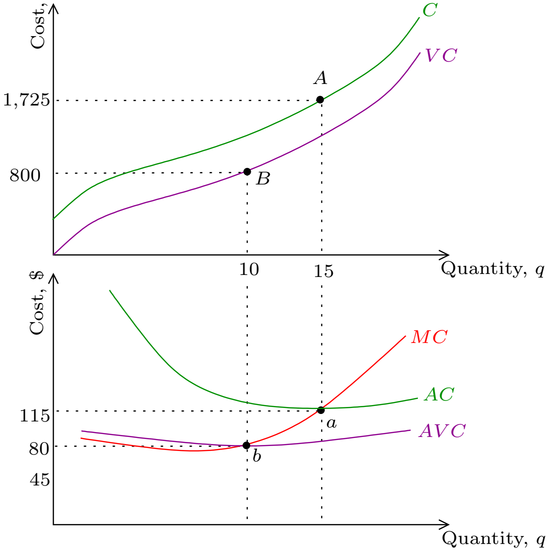
Average cost is minimized at $q = 15$.

Short-run cost curves



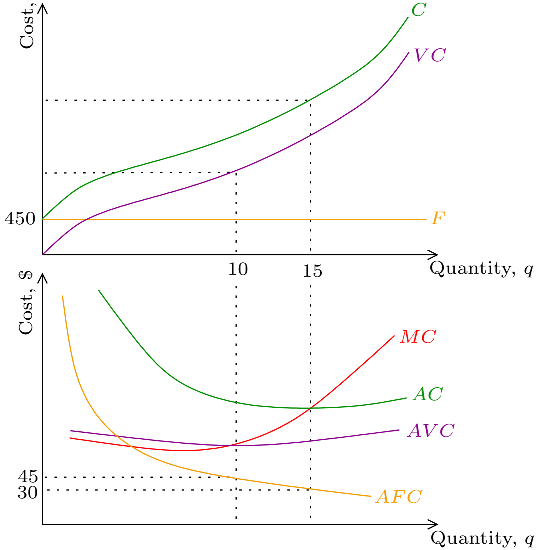
Similarly, AVC is minimized at $q = 10$.

Short-run cost curves



At each q, both C and VC have the same slope = MC.

Short-run cost curves



AFC is decreasing in q.

Production functions and the shape of cost curves

Production functions + prices of inputs $\longrightarrow$ cost curves

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Suppose that $q = f(L, \overline{K}) = g(L)$.

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Derive MC and AVC from g.

Shape of MC curve

$$MC(q) = \frac{dVC(q)}{dq} = \frac{dwg^{-1}(q)}{dq} = w\frac{1}{\frac{dg}{dq}} = w\frac{1}{MP_L}.$$

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$$\begin{aligned} \text{Diminishing } MP_L \quad &\Rightarrow \quad MP_L \text{ eventually decreases} \\ &\Rightarrow \quad MC \text{ eventually increases.} \end{aligned}$$

Shape of AC curve

$$AC(q) = \frac{VC(q)}{q} = \frac{wg^{-1}(q)}{q} = \frac{w}{\frac{q}{g^{-1}(q)}} = \frac{w}{\frac{q}{L}} = \frac{1}{AP_L}$$

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We saw that AP_L rises the falls so AC falls then rises.

Effects of taxes on costs

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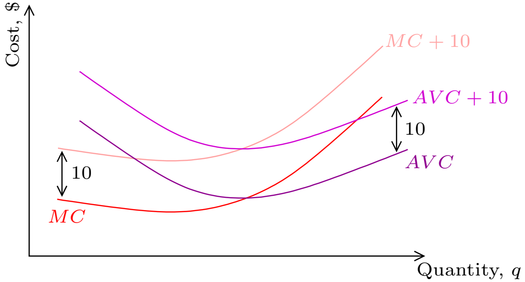

Long run costs

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Long run cost is never higher than short run costs. Why?

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Long run cost is never higher than short run costs. Why?

In the long run, you'd never be stuck with the wrong amount of inputs that are fixed in the short run. You've got more flexibility.

Input choice

Recall: An isoquant is a curve through all combinations of inputs that can be used to efficiently produce a particular amount of output.

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Somewhat like a budget line.

Isocost line

w — wage
r — rental rate

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$$\overline{C} = wL + rK$$

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To plot this, you can re-write it as

$$K = \frac{\overline{C}}{r} - \frac{w}{r}L.$$

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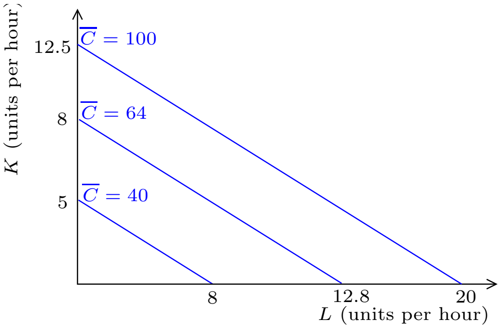
$$K = \frac{\overline{C}}{r} - \frac{w}{r}L.$$

Slope is $-\frac{w}{r}$ and vertical intercept is $\overline{C}r$ and horizontal intercept is $\overline{C}w$.

Isocost line

$w = \$5$ and $r = \$8$.



Higher isocost lines correspond to higher costs.

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3. Last-dollar rule: The last dollar spent one input should give you as much extra output as the last dollar spend on any other input.

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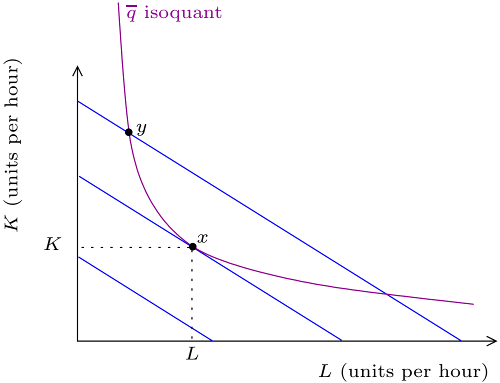
All equivalent.

Lowest-isocost rule



But this implies tangency of isocost line and isoquant.

Tangency rule

Recall: slope of isoquant is $MRTS$.

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Slope of isocost line is $-\frac{w}{r}$.

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So

$$MRTS = -\frac{w}{r}.$$

Tangency rule

Recall: slope of isoquant is $MRTS$.

Slope of isocost line is $-\frac{w}{r}$.

So

$$MRTS = -\frac{w}{r}.$$

Or

$$\frac{MP_L}{MP_K} = \frac{w}{r}.$$

Tangency rule

Recall: slope of isoquant is $MRTS$.

Slope of isocost line is $-\frac{w}{r}$.

So

$$MRTS = -\frac{w}{r}.$$

Or

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Rearranging this:

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Tangency rule

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That's the last-dollar rule.

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Need one more equation to solve for K and L!

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You started wanting to produce 100 units:

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Cost of producing 100 units is:

$$wL + rK = 50 \times \$24 + 100 \times \$8 = \$2,000.$$

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Notice that the firm substituted labor for capital as it became relatively cheaper.

Is this generally true?

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Differentiating both sides with respect to w,

$$\frac{d\left(\frac{K}{L}\right)}{dw} = \frac{1}{1.5r} > 0.$$

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So, holding r fixed, small increases in w lead to an increase of the capital-labor ratio $\left(\frac{K}{L}\right)$.

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Expansion path: curve through the cost minimizing inputs for different levels of output.

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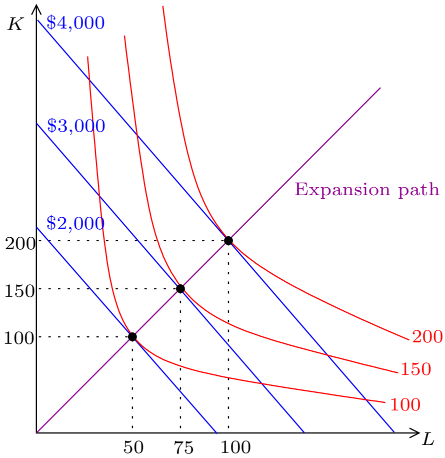
And so on.

Expansion path



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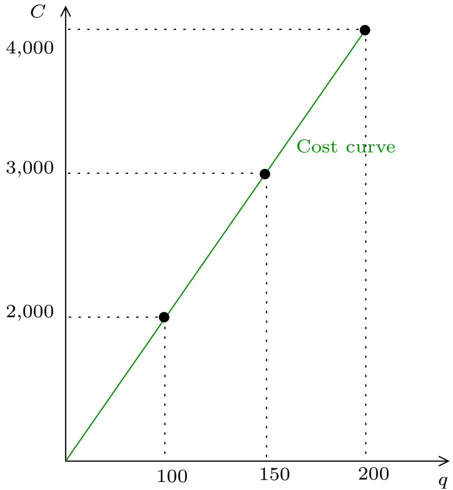
So

$$C(q) = wL + rK = w\frac{q}{2} + rq = \left(\frac{w}{2} + r\right) q = \left(\frac{24}{2} + 8\right) q = 20q.$$

Cost curve



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This means that it has the same “returns to scale” for all levels of output.

The shape of long-run cost curves

Just like short-run:

- MC and AC both depend on C.

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If returns to scale are increasing at first and then decreasing as quantity increases, AC is U-shaped.

Scale

For a U-shaped AC:

1. low levels of output where AC is decreasing: *economies of scale*.

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Competitive industries: firms have U-shaped AC curves.

Non-competitive industries: firms have either U or L-shaped AC curves.

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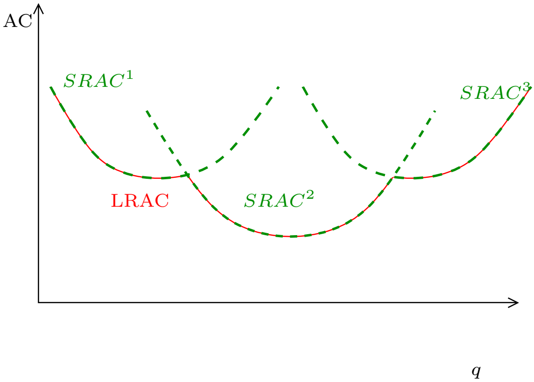
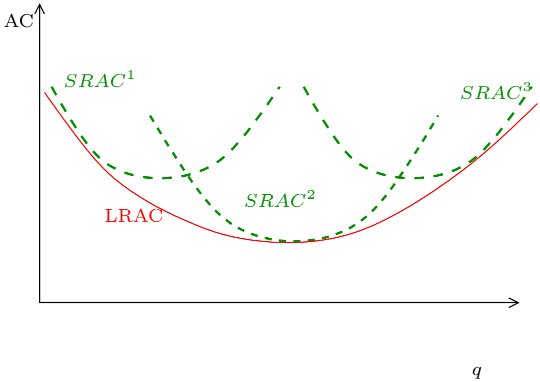
$$LRAC(q) = \min\{SRAC^s(q), SRAC^m(q), SRAC^l(q)\}.$$

Lower costs in the long run



Long-run AC is the least of the short-run ACs.

Lower costs in the long run



If the choice of K isn't only whole numbers, LRAC is smooth.

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Cost functions:

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AC^{\text{Laser}}(q) &= \frac{100}{q} + 0.04 \\
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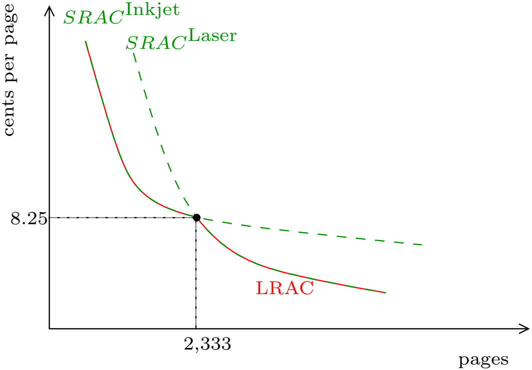
Which one should you buy?
It depends on q:

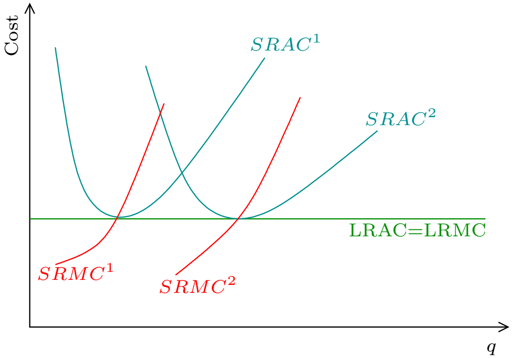
$$LRAC(q) = \begin{cases} \frac{100}{q} + 0.04 \text{ if } q > 2,333 \\ \frac{30}{q} + 0.07 \text{ if } q \leq 2,333 \end{cases}$$

Example



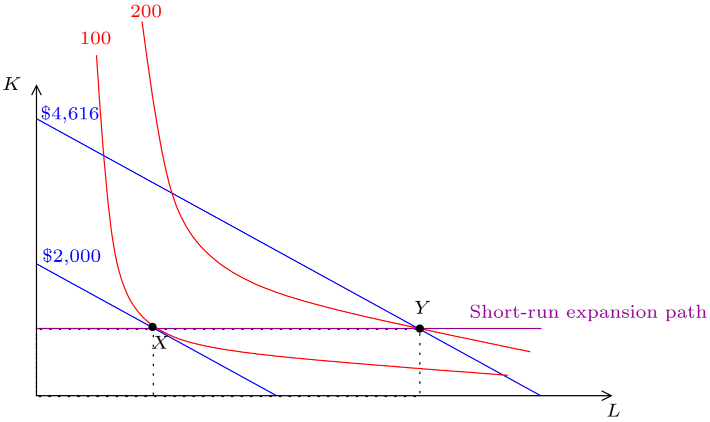
cents per page
$SRAC^{\text{Inkjet}}$
$SRAC^{\text{Laser}}$
8.25
LRAC
2,333
pages

Constant returns to scale and long-run AC/MC



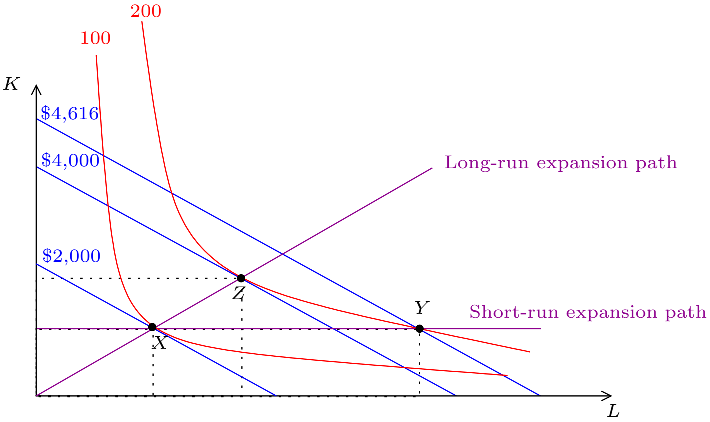
$CRS \Rightarrow MC = AC$ in the long run.

Short-run and long-run expansion paths



Short-run expansion path: horizontal line at fixed K.

Short-run and long-run expansion paths



Long-run expansion path: both inputs are varied.

Learning by doing

Reasons for long-run costs being lower than short-run costs:

1. More flexibility in choosing inputs.

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1. More flexibility in choosing inputs.
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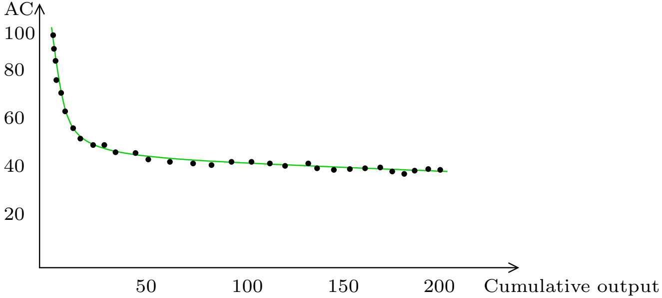
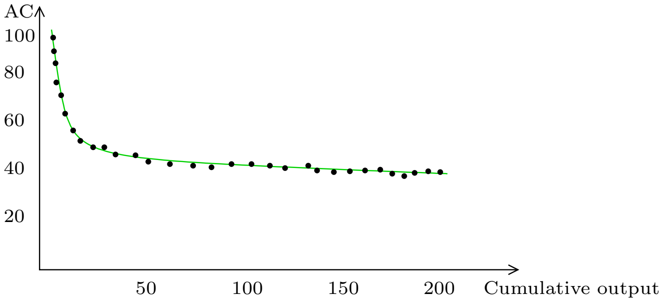
3. Learning by doing: the more your output (cumulatively), the more efficiently you produce it.

Learning by doing



Learning curve.

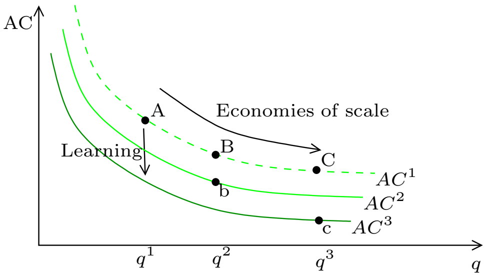
Learning by doing



Learning curve.

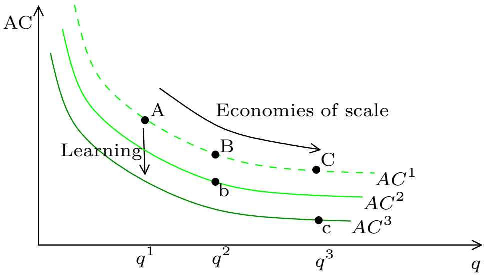
Despite common usage, a “steep learning curve” means you make quick progress!

Learning by doing: average cost over time



Larger output means:

Learning by doing: average cost over time



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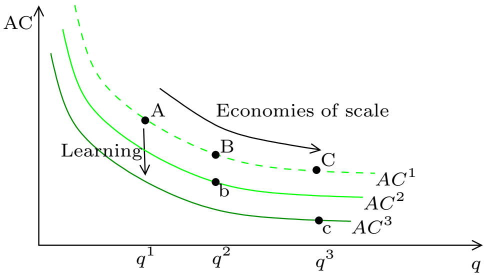

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1. lower average cost today because of economies of scale.
2. lower average cost tomorrow because of learning by doing.

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$C(q_1, 0) + C(0, q_2) - C(q_1, q_2)$ — Saving from joint production.

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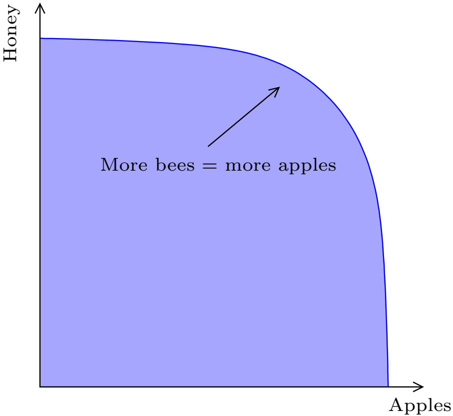
If $SC > 0$ then $C(q_1, 0) + C(0, q_2) > C(q_1, q_2)$: producing the goods separately costs *less* than producing them together. *Economies of scope.*

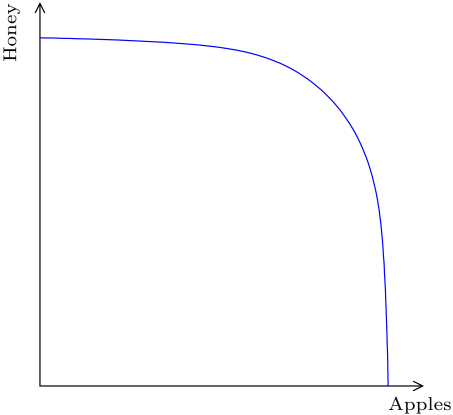
Production possibility frontier



Production possibility set is all of the pairs of output a fixed amount of input yields.

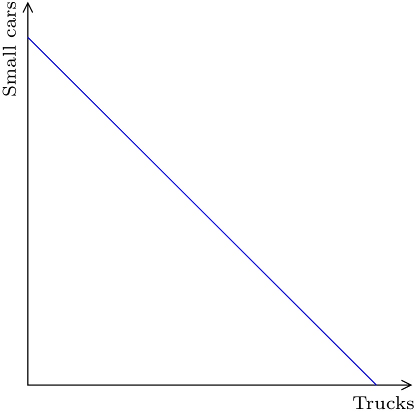
Production possibility *frontier* is the boundary of this set.

Production possibility frontier



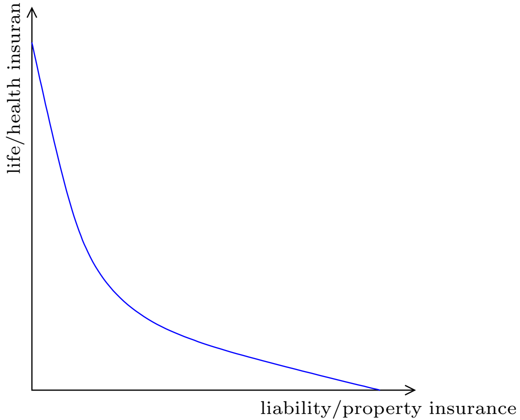
Economies of scope: PPF bowed outwards (from origin).

Production possibility frontier



No economies of scope: PPF straight line.

Production possibility frontier



Diseconomies of scope: PPF bowed inwards.