Microeconomic Theory — ECON 323 503
Chapter 4: Demand

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November 11, 2014

Outline

1. Deriving demand curves: Where do the demand curves that we saw in Chapter 2 come from?

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3. Effects of an increase in price: How does quantity demanded depend on the price of a good?

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1. Deriving demand curves: Where do the demand curves that we saw in Chapter 2 come from?
2. Effects of an increase in income: How does quantity demanded depend on income?
3. Effects of an increase in price: How does quantity demanded depend on the price of a good?
4. Revealed preference: How do we figure out preferences from the choices that we observe?

Deriving Demand Curves

We saw how a consumer chooses bundles (q_1, q_2) for given prices (p_1, p_2) and income (Y).
This describes a demand function for each good:

$$\begin{aligned} q_1 &= D_1(p_1, p_2, Y) \\ q_2 &= D_2(p_1, p_2, Y) \end{aligned}$$

Demand functions for particular utility functions

1. Perfect complements
2. Cobb-Douglas
3. Perfect substitutes
4. Quasilinear
5. Constant elasticity of substitution

Perfect complements

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If you spent all your money (Y), you'd buy $\frac{Y}{p_1+p_2}$ units. So

$$D_1(p_1, p_2, Y) = \frac{Y}{p_1+p_2}$$

$$D_2(p_1, p_2, Y) = \frac{Y}{p_1+p_2}$$

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To get the demand curve for good 1, leave the price of good 2 (p_2) and income (Y) fixed at say $p_2 = 3$ and $Y = 9$

$$D_1(p_1) = \frac{9}{p_1 + 3}$$

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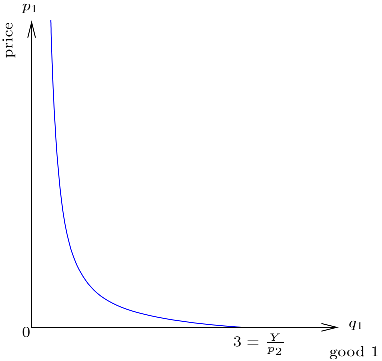

Cobb-Douglas

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$$\begin{aligned} D_1(p_1, p_2, Y) &= \frac{aY}{p_1} \\ D_2(p_1, p_2, Y) &= \frac{(1-a)Y}{p_2} \end{aligned}$$

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If $a = 0.25$ and $Y = 10$, then

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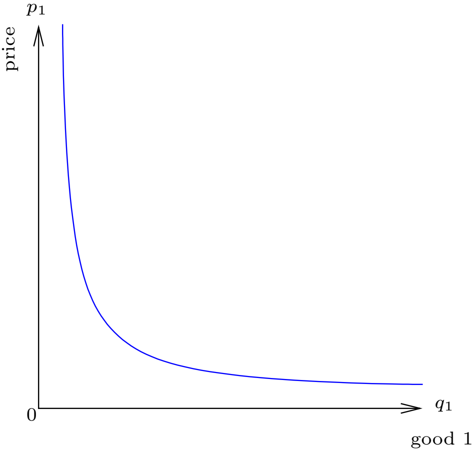

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$$D_1(p_1) = \begin{cases} \text{any } q_1 \text{ between } 0 \text{ and } \frac{12}{p_1} \text{ if } p_1 = 3 \\[2em] 0 \text{ if } p_1 > 3 \\[2em] \frac{12}{p_1} \text{ if } p_1 < 3 \end{cases}$$

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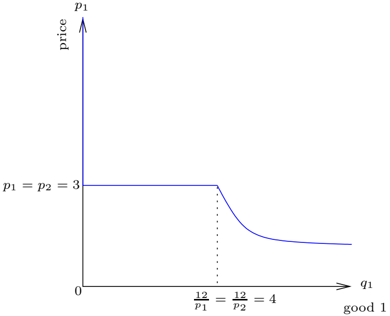

Quasilinear

This will depend on the exact utility function. Let's take an example:

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Interior solution if $Y > \frac{a^2 p_2}{4p_1}$

$$\begin{aligned} D_1(p_1, p_2, Y) &= \left(\frac{a p_2}{2p_1}\right)^2 \\ D_2(p_1, p_2, Y) &= \frac{Y}{p_2} - \frac{a^2 p_2}{4p_1} \end{aligned}$$

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Interior solution if $Y > \frac{a^2 p_2}{4p_1}$

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Corner solution if $Y \leq \frac{a^2 p_2}{4p_1}$

$$\begin{aligned} D_1(p_1, p_2, Y) &= \frac{Y}{p_2} \\ D_2(p_1, p_2, Y) &= 0 \end{aligned}$$

Constant elasticity of substitution (CES)

$$U(q_1, q_2) = (q_1^\rho + q_2^\rho)^{\frac{1}{\rho}}$$

Assuming $0 < \rho < 1$ and letting $\sigma = \frac{1}{\rho - 1}$,

$$\begin{aligned} D_1(p_1, p_2, Y) &= \frac{Y p_1^\sigma}{p_1^{\sigma+1} + p_2^{\sigma 1}} \\ D_2(p_1, p_2, Y) &= \frac{Y p_2^\sigma}{p_1^{\sigma+1} + p_2^{\sigma 1}} \end{aligned}$$

Deriving demand curves graphically

What happens as we hold everything fixed ($p_2 = 1$ and $Y = 20$) but vary p_1?

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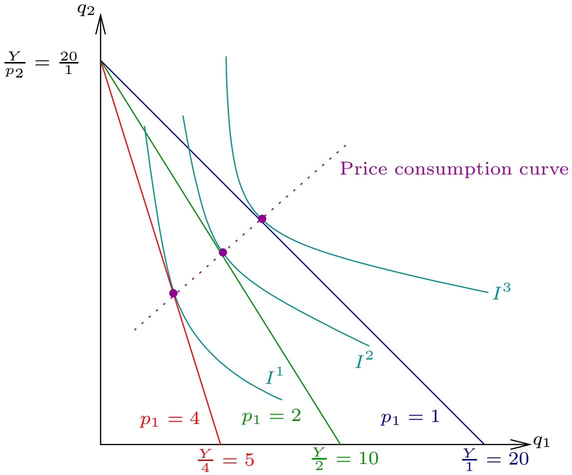

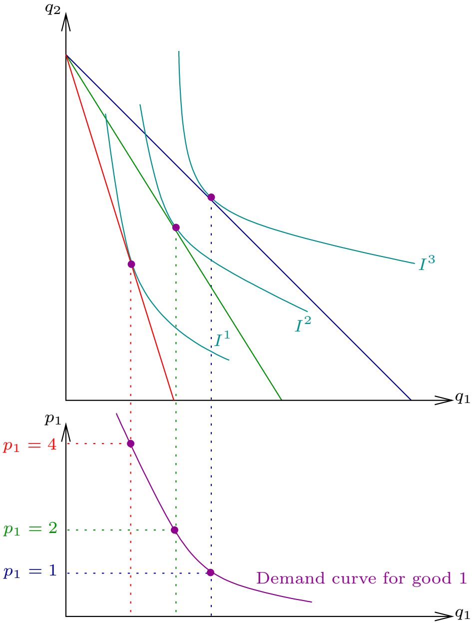

q_2
I^3
I^2
I^1
q_1
p_1
$p_1 = 4$
$p_1 = 2$
$p_1 = 1$
Demand curve for good 1
q_1

Textbook exercise 1.5

$$U(q_1, q_2) = 2\sqrt{q_1} + q_2$$

What are the demand functions for the two goods?

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What we want to find are $D_1(p_1, p_2, Y)$ and $D_2(p_1, p_2, Y)$.

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$$\max_{q_1 q_2} 2\sqrt{q_1} + q_2$$
$$\text{s.t. } p_1 q_1 + p_2 q_2 = Y.$$

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$$\max_{q_1 q_2} 2\sqrt{q_1} + q_2$$
$$\text{s.t. } p_1 q_1 + p_2 q_2 = Y.$$

Substituting $q_2 = \frac{Y - p_1 q_1}{p_2}$,

$$\max_{q_1} \; 2\sqrt{q_1} + \frac{Y - p_1 q_1}{p_2}.$$

Textbook exercise 1.5

First order condition:

$$2\frac{1}{2}\frac{1}{\sqrt{q_1}} - \frac{p_1}{p_2} = 0$$

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So, $D_1(p_1, p_2, Y) = \left(\frac{p_2}{p_1}\right)^2$

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Substituting back to solve for q_2:

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So, $D_2(p_1, p_2, Y) = \frac{Y}{p_2} - \frac{p_2}{p_1}$.

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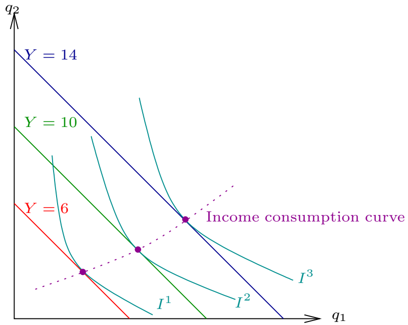
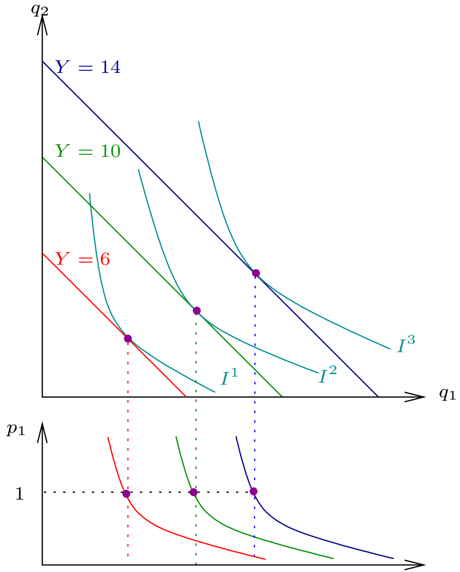
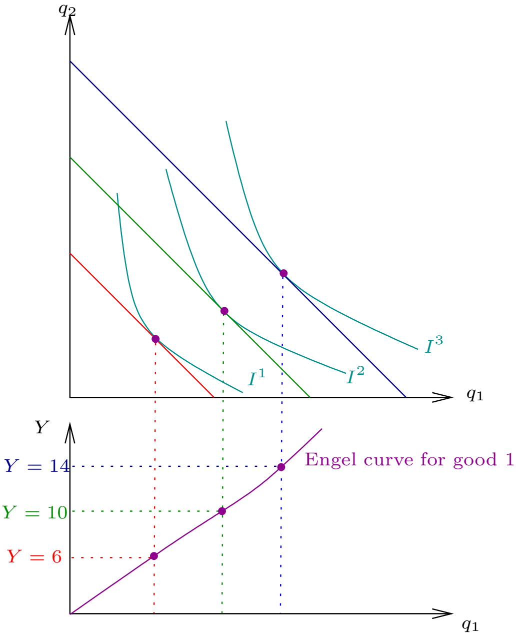
Suppose that $p_1 = p_2 = 1$.





q_2
$Y = 14$
$Y = 10$
$Y = 6$
I^1
I^2
I^3
q_1
p_1
1

The Engel curve



Income elasticity

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$$\xi = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in income}} = \frac{\delta Q}{\delta Y}\frac{Y}{Q}$$

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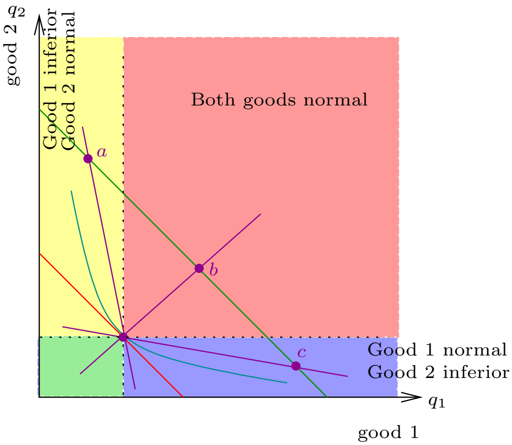
What if $\xi > 1$?

Your consumption of such a good grows *faster* than your income. *Luxury goods*.

If $0 \leq \xi \leq 1$ the good is called a *necessity*.

Income consumption curves and income elasticities

The shape of the ICC tells us if goods are normal or inferior.



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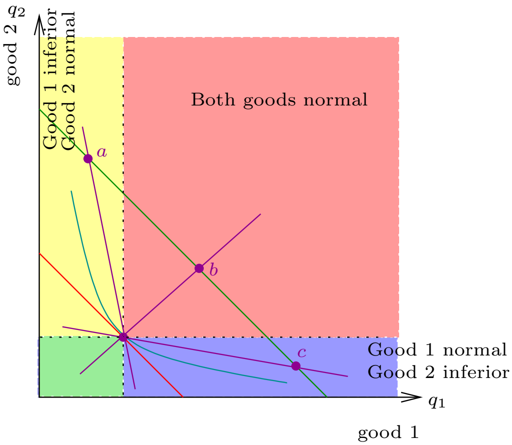

This picture shows us that both goods can't be inferior: there's always at least one normal good.

There's always a normal good

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Differentiating with respect to Y:

$$p_1 \frac{dq_1}{dY} + p_2 \frac{dq_2}{dY} + \cdots + p_n \frac{dq_n}{dY} = 1.$$

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Multiply and divide each term by $q_i Y$:

$$\frac{p_1 q_1}{Y} \frac{dq_1}{dY} \frac{Y}{q_1} + \frac{p_2 q_2}{Y} \frac{dq_2}{dY} \frac{Y}{q_2} + \cdots + \frac{p_n q_n}{Y} \frac{dq_n}{dY} \frac{Y}{q_n} = 1.$$

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Since $\xi_i = \frac{dq_1}{dY} \frac{Y}{q_1}$,

$$\theta_1 \xi_1 + \theta_2 \xi_2 + \cdots + \theta_n \xi_n = 1$$

where $\theta_i = \frac{p_i q_i}{Y}$.

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where $\theta_i = \frac{p_i q_i}{Y}$.

Since the θs sum to 1, there has to be at least one ξ that is positive.

Textbook exercise 2.4

$$U(q_1, q_2) = q_1^{0.6} q_2^{0.4}$$

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What we want is Y as a function of q_2.

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Since this is a Cobb-Douglas utility function, we know that

$$q_2 = \frac{(1-a)Y}{p_2} = \frac{0.4Y}{p_2}.$$

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So

$$Y = \frac{p_2 q_2}{0.4}$$

Textbook exercise 2.4

$$U(q_1, q_2) = q_1^{0.6} q_2^{0.4}$$

Find the equation of the Engel curve for good 2.

What we want is Y as a function of q_2.

Since this is a Cobb-Douglas utility function, we know that

$$q_2 = \frac{(1-a)Y}{p_2} = \frac{0.4Y}{p_2}.$$

So

$$Y = \frac{p_2 q_2}{0.4}$$

This should be easy for you to graph.

Effects of a price increase

The effect of a price increase has two parts:

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Knowing how the overall effect is split between these two allows us to better forecast the effects of policies.

Income and substitution effects with a normal good

Recall that an increase in p_1 causes the budget line to rotate inwards.

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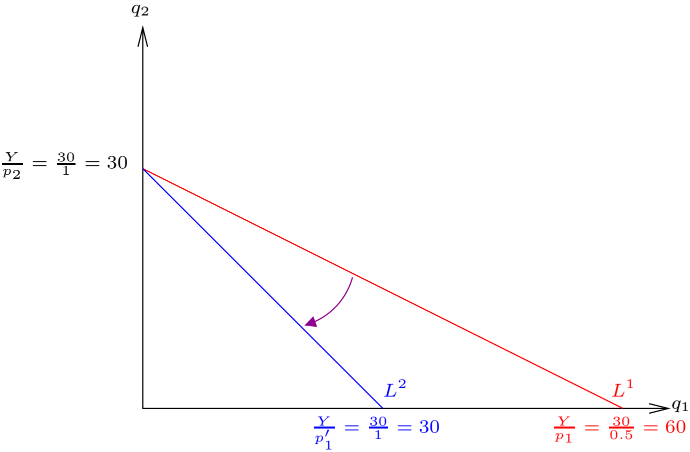
It is twice as steep:

$$\text{Slope of } L^1 = -\frac{p_1}{p_2} = -\frac{0.5}{1} = -0.5$$

$$\text{Slope of } L^2 = -\frac{p_1'}{p_2} = -\frac{1}{1} = -1$$

The budget line rotates

If $Y = 30$,



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$$\begin{aligned} q_1 &= 0.4\frac{Y}{p_1} = 0.4\frac{30}{0.5} = 24 \\ q_2 &= 0.6\frac{Y}{p_2} = 0.6\frac{30}{1} = 18 \end{aligned}$$

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What happens when the price of good 1 rises?

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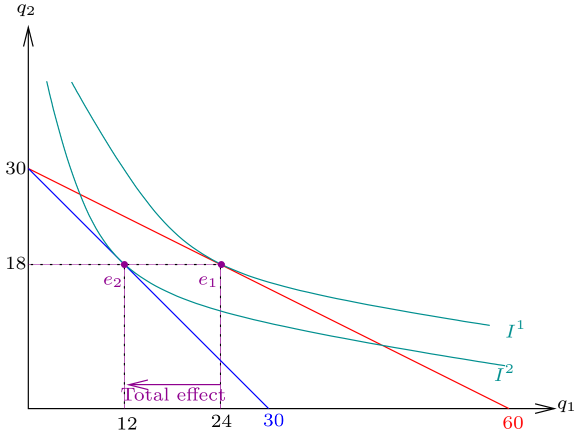
Price: $0.5 \rightarrow 1$

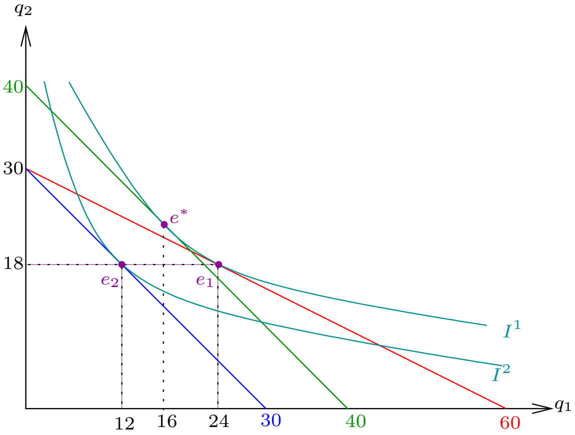
Quantity demanded: $24 \rightarrow 12$.

Income and substitution effects with a normal good



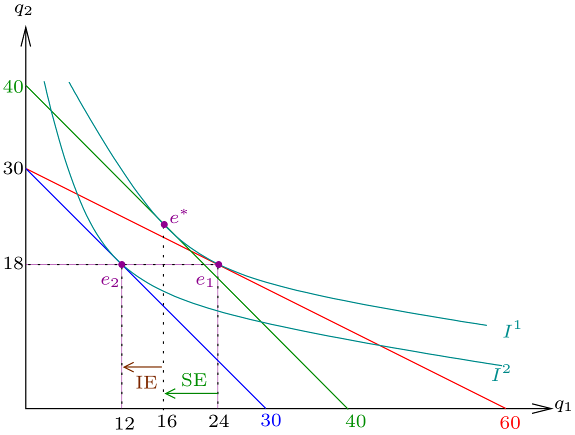
We can split this into the two parts.

Income and substitution effects with a normal good



Increase income to keep consumer on the same IC.

Income and substitution effects with a normal good



This tells us how to split the total effect.

How'd we figure out how much to increase income?

At original prices:

$$U_1(q_1, q_2) = 24^{0.4} \times 18^{0.6} = (0.8 \times 30)^{0.4} (0.6 \times 30)^{0.6} = 30 \times 0.8^{0.40} 6^{0.6}.$$

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Solving this, $Y \approx 40$.

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Substitution effect causes you to move along your indifference curve: remember that the slope changes as we move.

Income effect causes you to move down to a lower indifference curve. For *normal goods*, this means that you consume less as the price rises.

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Law of demand is an empirical observation. There are hardly any examples of real world Giffen goods.

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Typical (*Marshallian or uncompensated demand*) demand curve: how quantity demanded varies as price changes.

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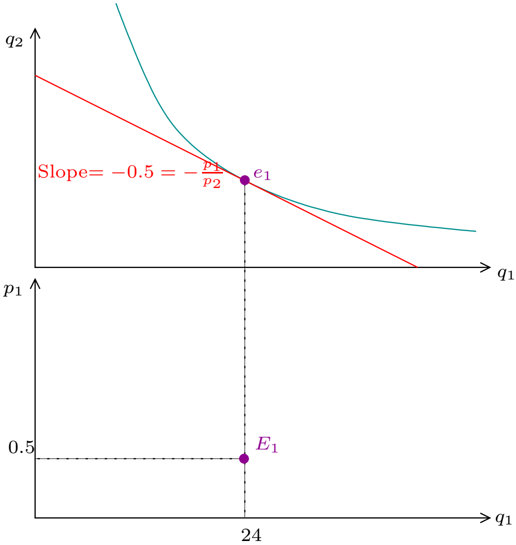
Compensated demand depends on utility level rather than income: $H_1(p_1, p_2, \overline{U})$.

Cannot observe compensated demand: we don't know people's utility functions.

Graphing the compensated demand curve



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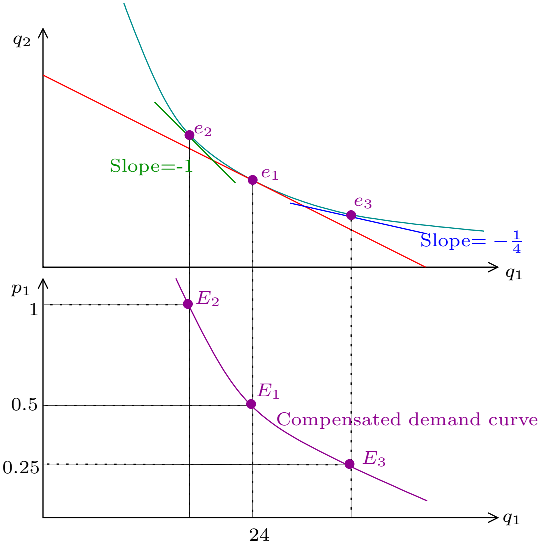

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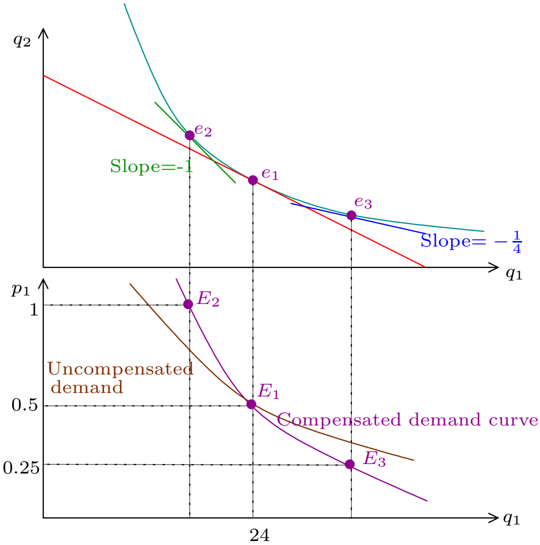

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Why is compensated demand curve steeper?

It's only reflects the substitution effect. The total change is bigger due to the income effect.

Cost minimization

Target utility: $\overline{U}$

Prices: p_1, p_2

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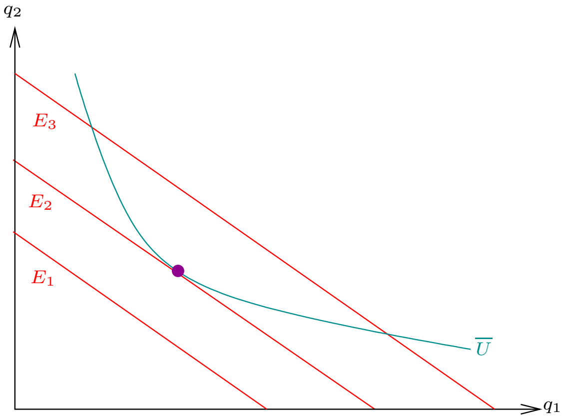
This minimum value is the lowest income level at which you can get to utility level $\overline{U}$: $E(p_1, p_2, \overline{U})$.

Cost minimization



Deriving compensated demand

Use *expenditure function* to derive compensated demand:

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This is *Shephard's Lemma*. We won't prove it.

Textbook exercise 3.11

$$U(q_1, q_2) = q_1 + 2q_2$$

What is the compensated demand function for good 1?

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So

$$E(p_1, p_2, \overline{U}) = \begin{cases} \frac{\overline{U}}{2} p_2 \text{ if } p_1 > \frac{p_2}{2} \\ \\ \overline{U} p_1 \text{ if } p_1 \leq \frac{p_2}{2} \end{cases}$$

Textbook exercise 3.11

Step 2: Differentiate the expenditure function

$$H(p_1, p_2, \overline{U}) = \frac{\delta E}{\delta p_1} = \begin{cases} 0 \text{ if } p_1 > \frac{p_2}{2} \\ \\ \overline{U} \text{ if } p_1 \leq \frac{p_2}{2} \\ \end{cases}$$

The Slutsky equation

Pure substitution elasticity of demand:

$$\varepsilon^* = \frac{\text{\% change in compensated demand}}{\text{\% change in price}} = \frac{\delta H}{\delta p_1}\frac{p_1}{H}$$

Describes the substitution effect.

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$$\underset{\text{total effect}}{\varepsilon} = \underset{\text{substitution effect}}{\varepsilon^*} + \underset{\text{income effect}}{(-\theta\xi)}$$

Revealed preference

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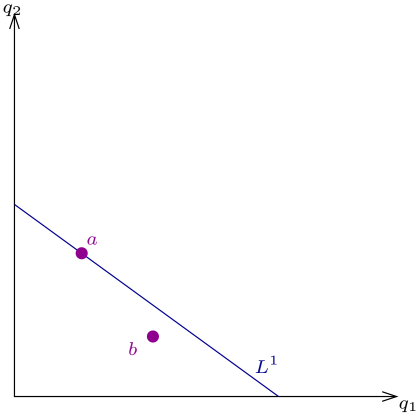
This is what revealed preference is.

Assumption: consumers maximize utility subject to a budget constraint.

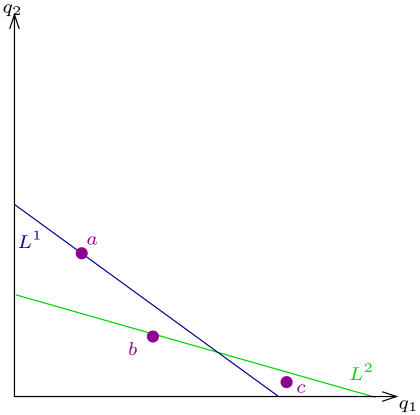
See what a consumer picks at various budgets and use this to back out his preferences.

Backing out preferences



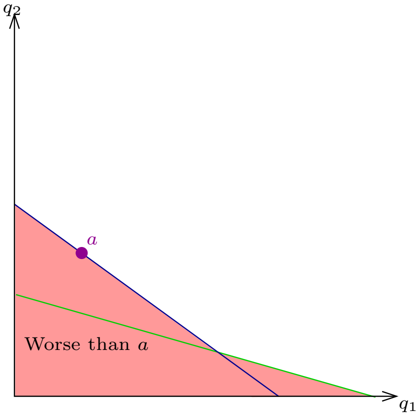
You could have had b but you picked a. So a is *revealed preferred to* b.

Backing out preferences



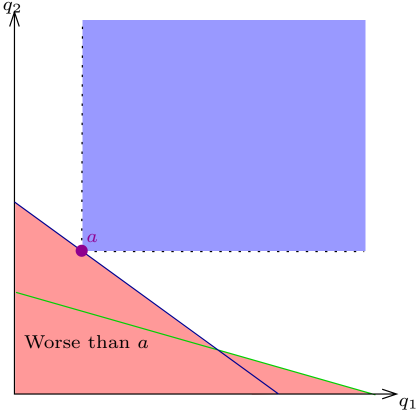
If you had already revealed that you prefer b to c, then you now have revealed that you prefer a to c as well.

Backing out preferences



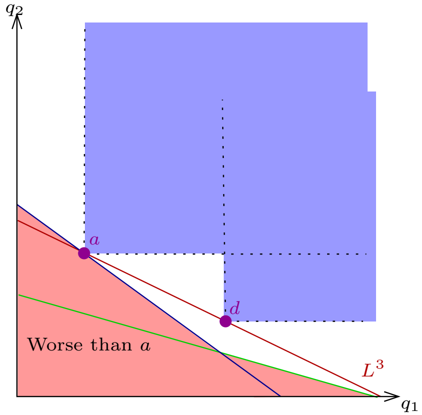
This reasoning would tell us that you prefer a to every bundle in the shaded area.

Backing out preferences



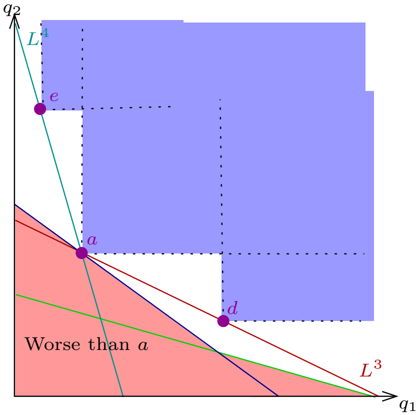
Since preferences are monotonic, any bundle that has more of each good is better than a.

Backing out preferences



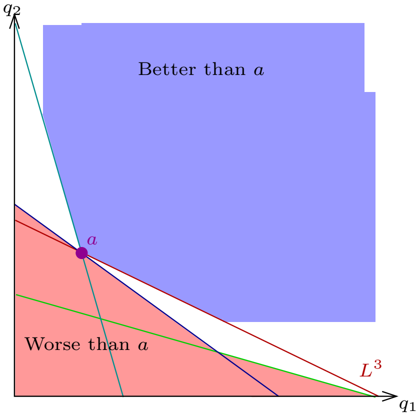
If you pick d when you could have a, then you reveal that d is preferred to a. Anything better than d is also better than a.

Backing out preferences



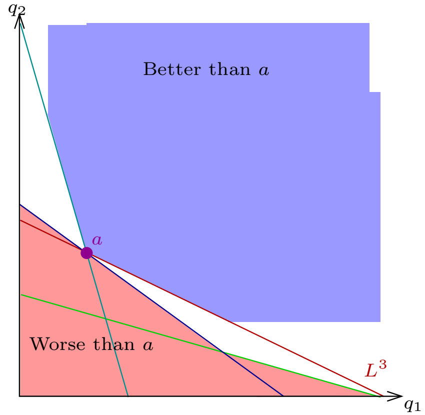
You could similarly reveal that you prefer e to a.
And so on.

Backing out preferences



By convexity of preferences, we know that you prefer every bundle in the blue shaded area to a.

Backing out preferences



The indifference curve through a would have to pass between the blue and the red shaded areas.

Backing out preferences

With enough observations we can find the IC.

Backing out preferences

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With fewer observations, we can approximate it.

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$$p_1 q_1 + p_2 q_2 \leq p_1 q_1' + p_2 q_2' \tag{1}$$

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At p_1' you pick (q_1', q_2'). So

$$p_1' q_1' + p_2 q_2' \leq p_1' q_1 + p_2 q_2 \tag{2}$$

Substitution effect

Suppose you're indifferent between bundles (q_1, q_2) and (q_1', q_2').

Fix p_2 and suppose that you are minimizing cost to stay on that indifference cost.

At p_1 you pick (q_1, q_2). So

$$p_1 q_1 + p_2 q_2 \leq p_1 q_1' + p_2 q_2' \tag{1}$$

At p_1' you pick (q_1', q_2'). So

$$p_1' q_1' + p_2 q_2' \leq p_1' q_1 + p_2 q_2 \tag{2}$$

If you subtract (2) from (1)

$$(p_1 - p_1')(q_1 - q_1') \leq 0$$

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If you subtract (2) from (1)

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This is the law of demand for compensated demand!
If $p_1 > p_1'$ then $q_1 < q_1'$ and vice versa.

Textbook exercise 5.3

When $Y = 1,000$, $p_1 = 100$, and $p_2 = 10$, consumer picks bundle $(q_1, q_2) = (2, 80)$.

When $Y' = 1,200$, $p_1' = 150$, and $p_2' = 10$, consumer picks bundle (q_1', q_2') where $q_1' = 1$.

Can we use revealed preference to determine which of the two bundles the consumer prefers?

Textbook exercise 5.3

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When $Y' = 1,200$, $p'_1 = 150$, and $p'_2 = 10$, consumer picks bundle (q'_1, q'_2) where $q'_1 = 1$.

Can we use revealed preference to determine which of the two bundles the consumer prefers?

Revealed preference: If you pick x when you could have y, then you prefer x to y.

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Revealed preference: If you pick x when you could have y, then you prefer x to y.

Since $p_1' q_1 + p_2' q_2 = 150 \times 2 + 10 \times 80 = 1,100 \leq Y'$, the consumer could afford (q_1, q_2) when he picked (q_1', q_2') he must prefer (q_1', q_2') to (q_1, q_2).