# Microeconomic Theory — ECON 323 503 Chapter 14: Oligopoly and Monopolistic Competitions

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### Outline

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- 2. The Stackelberg oligopoly model: two firms pick quantities one after another.
- 3. The Bertrand oligopoly model: two (or more) firms simultaneously pick prices.

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- 2. There are n airlines, not just 2.

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Assume constant MC of \$147 and no fixed cost so that AC = MC.

# Monopoly for comparison

If A were a monopoly,  $MR_A = 339 - 2q_A$ .

## Monopoly for comparison

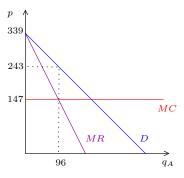
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$$R_A(q_A) = pq_A$$
  
=  $((339 - q_U) - q_A)q_A$   
=  $339q_A - q_Uq_A - q_A^2$ .

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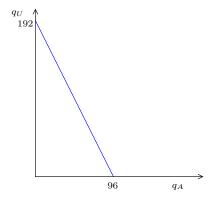
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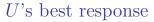
So A's best response to U's choice of  $q_U$  is

$$B_A(q_U) = 96 - \frac{1}{2}q_U.$$

### A's best response



If U doesn't produce anything, A picks 96. If U produces 192 or more, A's best response is to shut down.



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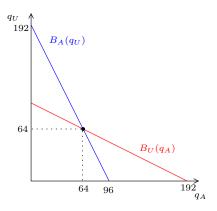
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 and  $q_B = 96 - \frac{1}{2}q_A$ .

Solving for  $q_A$  and  $q_U$ ,

$$q_A = q_U = 64$$
.

# Graphically



### With many firms

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Cost for firm i is  $C(q_i)$ .

## Many firms

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Firm 1's first order condition:

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Firm 1's best response to the other firms' choices is to pick  $q_1$  that solves this equation.

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$$\frac{p}{MC} = \frac{1}{\left[1 + \frac{1}{n\varepsilon}\right]}.$$

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So as the number of firms in the Bertrand game, the closer we get to the competitive outcome.

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If  $q_2 = q_3 = \cdots = q_n = q$ , then

$$q_1 = B_1(q_2, \dots, q_n) = \frac{a-m}{2b} - \frac{n-1}{2}q.$$

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Check that if n = 1 we get the monopoly price/quantity and if n = 2 we get the duopoly price/quantity.

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$$= 96q_A - \frac{1}{2}q_A^2.$$

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So, 
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Assume that MC is constant (MC = \$5)

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Unless a firm charges \$5 = MC, the other would under cut it.

Caveat: this reasoning relies on firms beign able to produce any quantity.

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If 
$$\varepsilon = -1$$
 the Cournot price would be \$10 (=  $\frac{MC}{1+\frac{1}{2\varepsilon}} = \frac{5}{1-\frac{1}{2}}$ )

Which is more realistic: Bertrand or Cournot?

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1. Observed oligopoly prices are typically, like the Cournot price, between the competitive price and the monopoly price.

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- 1. Observed oligopoly prices are typically, like the Cournot price, between the competitive price and the monopoly price.
- 2. The Bertrand price only depends on cost and is completely independent of demand. But observations are that prices vary with demand.