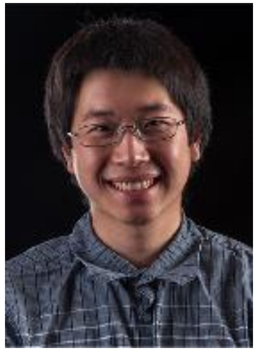
Tight Bounds for Adversarially Robust Streams and Sliding Windows via Difference Estimators



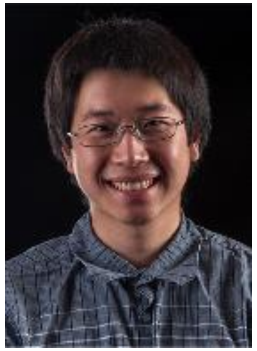
Samson Zhou

Carnegie Mellon University

(Based on work joint with David P. Woodruff)

Background

- Post-doc at Carnegie Mellon
- PhD in Computer Science from Purdue
- Bachelors in Math, Computer Science from MIT



- Research areas: Security and Privacy, Data Science, Sublinear Algorithms

Recent Work on Security and Privacy

- Private Data Stream Analysis for Universal Symmetric Norm Estimation (FORC 2022)
- On the Security of Proofs of Sequential Work in a Post-Quantum World (ITC 2021)
- Computationally Data-Independent Memory Hard Functions (ITCS 2020)
- Data-Independent Memory Hard Functions: New Attacks and Stronger Constructions (CRYPTO 2019)
- Bandwidth-Hard Functions: Reductions and Lower Bounds (CCS 2018)
- On the Economics of Offline Password Cracking (Security and Privacy 2018)

Recent Work on Data Science

- Learning-Augmented k-means Clustering (ICLR 2022)
- Fast Regression for Structured Inputs (ICLR 2022)
- New Coresets for Projective Clustering and Applications (AISTATS 2022)
- Dimensionality Reduction for Wasserstein Barycenter (NeurIPS 2021)
- Learning a Latent Simplex in Input Sparsity Time (ICLR 2021)
- Near Optimal Linear Algebra in the Online and Sliding Window Models (FOCS 2020)
- Data-Independent Neural Pruning via Coresets (ICLR 2020)
- Adversarially Robust Submodular Maximization under Knapsack Constraints (KDD 2019)

Recent Work on Sublinear Algorithms

- Memory Bounds for the Experts Problem (STOC 2022)
- The White-Box Adversarial Data Stream Model (PODS 2022)
- Truly Perfect Samplers for Data Streams and Sliding Windows (PODS 2022)
- Noisy Boolean Hidden Matching with Applications (ITCS 2022)
- Adversarial Robustness of Streaming Algorithms through Importance Sampling (NeurIPS 2021)
- Tight Bounds for Adversarially Robust Streams and Sliding Windows via Difference Estimators (FOCS 2021)
- Separations for Estimating Large Frequency Moments on Data Streams (ICALP 2021)
- Non-Adaptive Adaptive Sampling on Turnstile Streams (STOC 2020)

Model #1: Streaming Model

- Input: Elements of an underlying data set S, which arrives sequentially
- Output: Evaluation (or approximation) of a given function
- Goal: Use space *sublinear* in the size m of the input S

1 0 1 1 1 0 0 1

Heavy-Hitters

- Given a set S of m elements from $[n]$, let f_i be the frequency of element i. (How often it appears)

$$1\ 1\ 2\ 1\ 2\ 1\ 1\ 2\ 3 \rightarrow [5, 3, 1, 0] \coloneqq f$$

Heavy-Hitters

- Given a set S of m elements from $[n]$, let f_i be the frequency of element i. (How often it appears)
- Let L_2 be the norm of the frequency vector:

$$L_2 = \sqrt{f_1^2 + f_2^2 + \cdots + f_n^2}$$

- Goal: Given a set S of m elements from $[n]$ and a threshold ϵ, output the elements i such that $f_i > \epsilon \, L_2$...and no elements j such that $f_j < \frac{\epsilon}{16} \, L_2$
- Motivation: DDoS prevention, iceberg queries

Frequency Moments

- Given a set S of m elements from $[n]$, let f_i be the frequency of element i. (How often it appears)
- Let F_p be the frequency moment of the vector:

$$F_p = f_1^p + f_2^p + \cdots + f_n^p$$

- Goal: Given a set S of m elements from $[n]$ and an accuracy parameter ϵ, output a $(1+\epsilon)$-approximation to F_p
- Motivation: Entropy estimation, linear regression

Distinct Elements

- Given a set S of m elements from $[n]$, let f_i be the frequency of element i. (How often it appears)
- Let F_0 be the frequency moment of the vector:

$$F_0 = |\{i : f_i \neq 0\}|$$

- Goal: Given a set S of m elements from $[n]$ and an accuracy parameter ϵ, output a $(1+\epsilon)$-approximation to F_0
- Motivation: Traffic monitoring

$(1+\epsilon)$-Approximation Streaming Algorithms

- Space $O\left(\frac{1}{\epsilon^2} + \log n\right)$ algorithm for F_0 [KaneNelsonWoodruff10], [Blasiok20]
- Space $O\left(\frac{1}{\epsilon^2} \log n\right)$ algorithm for F_p with $p \in (0, 2]$ [BlasiokDingNelson17]
- Space $O\left(\frac{1}{\epsilon^2} n^{1-2/p} \log^2 n\right)$ algorithm for F_p with $p > 2$ [Ganguly11, GangulyWoodruff18]
- Space $O\left(\frac{1}{\epsilon^2} \log n\right)$ algorithm for L_2-heavy hitters [BravermanChestnutIvkinNelsonWangWoodruff17]

Model #2: Adversarially Robust Streaming

- Input: Elements of an underlying data set S, which arrives sequentially and *adversarially*
- Output: Evaluation (or approximation) of a given function
- Goal: Use space *sublinear* in the size m of the input S

1

1

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10

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101

2

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1010

3

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1010

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Model #2: Adversarially Robust Streaming

- Input: Elements of an underlying data set S, which arrives sequentially and *adversarially*
- Output: Evaluation (or approximation) of a given function
- Goal: Use space *sublinear* in the size m of the input S

- Adversarially Robust: "Future queries may depend on previous queries"
- Motivation: Database queries, adversarial ML

$(1+\epsilon)$-Robust Algorithms [Ben-EliezerJayaramWoodruffYogev20]

- Space $\tilde{O}\left(\frac{1}{\epsilon^3}\log n\right)$ algorithm for F_0
- Space $\tilde{O}\left(\frac{1}{\epsilon^3}\log n\right)$ algorithm for F_p with $p \in (0,2]$
- Space $\tilde{O}\left(\frac{1}{\epsilon^3}n^{1-2/p}\right)$ algorithm for F_p with $p > 2$
- Space $\tilde{O}\left(\frac{1}{\epsilon^3}\log n\right)$ algorithm for L_2-heavy hitters

"A general framework that loses* nothing in n and only $\frac{1}{\epsilon}$"

“What’s an epsilon between friends?

- Statista: $\sim 300B \sim 2^{38}$ e-mails sent per day
- Accuracy: $\epsilon = 0.01$
- Since $\frac{1}{\epsilon} > \log n$, we should care about $\frac{1}{\epsilon}$ factors!

$(1+\epsilon)$-Robust Algorithms [HassidimKaplanMansourMatiasStemmer20]

- Space $\tilde{O}\left(\frac{1}{\epsilon^{2.5}}\log^4 n\right)$ algorithm for F_0
- Space $\tilde{O}\left(\frac{1}{\epsilon^{2.5}}\log^4 n\right)$ algorithm for F_p with $p \in (0,2]$
- Space $\tilde{O}\left(\frac{1}{\epsilon^{2.5}}n^{1-2/p}\right)$ algorithm for F_p with $p > 2$
- Space $\tilde{O}\left(\frac{1}{\epsilon^{2.5}}\log^4 n\right)$ algorithm for L_2-heavy hitters

"$\frac{1}{\epsilon}$ losses are not necessary"

Our Results: $(1+\epsilon)$-Robust Algorithms

- Space $\tilde{O}\left(\frac{1}{\epsilon^2}\log n\right)$ algorithm for F_0
- Space $\tilde{O}\left(\frac{1}{\epsilon^2}\log n\right)$ algorithm for F_p with $p \in (0,2]$
- Space $\tilde{O}\left(\frac{1}{\epsilon^2}n^{1-2/p}\right)$ algorithm for F_p with integer $p > 2$
- Space $\tilde{O}\left(\frac{1}{\epsilon^2}\log n\right)$ algorithm for L_2-heavy hitters

"No losses* are necessary!"

Summary: $(1+\epsilon)$-Robust Algorithms

Problem	[BJWY20] Space	[HKM+20] Space	Our Result
Distinct Elements	$\tilde{\mathcal{O}}\left(\frac{\log n}{\varepsilon^3}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log^4 n}{\varepsilon^{2.5}}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log n}{\varepsilon^2}\right)$
F_p Estimation, $p \in (0,2]$	$\tilde{\mathcal{O}}\left(\frac{\log n}{\varepsilon^3}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log^4 n}{\varepsilon^{2.5}}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log n}{\varepsilon^2}\right)$
Shannon Entropy	$\tilde{\mathcal{O}}\left(\frac{\log^6 n}{\varepsilon^5}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log^4 n}{\varepsilon^{3.5}}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log^3 n}{\varepsilon^2}\right)$
L_2-Heavy Hitters	$\tilde{\mathcal{O}}\left(\frac{\log n}{\varepsilon^3}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log^4 n}{\varepsilon^{2.5}}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log n}{\varepsilon^2}\right)$
F_p Estimation, integer $p > 2$	$\tilde{\mathcal{O}}\left(\frac{n^{1-2/p}}{\varepsilon^3}\right)$	$\tilde{\mathcal{O}}\left(\frac{n^{1-2/p}}{\varepsilon^{2.5}}\right)$	$\tilde{\mathcal{O}}\left(\frac{n^{1-2/p}}{\varepsilon^2}\right)$
F_p Estimation, $p \in (0,2]$, flip number λ	$\tilde{\mathcal{O}}\left(\frac{\lambda \log^2 n}{\varepsilon^2}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log^3 n\sqrt{\lambda \log n}}{\varepsilon^2}\right)$	$\tilde{\mathcal{O}}\left(\frac{\lambda \log^2 n}{\varepsilon}\right)$

Model #3: Sliding Window Model

- Input: Elements of an underlying data set S, which arrives sequentially
- Output: Evaluation (or approximation) of a given function
- Goal: Use space *sublinear* in the size m of the input S

- Sliding Window: "Only the m most recent updates form the underlying data set S"

$$\boxed{1\ 0\ 1\ 1\ 1\ 0\ 0\ 1}$$

Model #3: Sliding Window Model

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Model #3: Sliding Window Model

- Input: Elements of an underlying data set S, which arrives sequentially
- Output: Evaluation (or approximation) of a given function
- Goal: Use space *sublinear* in the size m of the input S

- Sliding Window: "Only the m most recent updates form the underlying data set S"
  - Emphasizes recent interactions, appropriate for time sensitive settings

$$1\ 0\ 1\ \boxed{1\ 1\ 0\ 0\ 1\ 1\ 0\ 1}$$

$(1+\epsilon)$-Approximation Sliding Window Algorithms

- Space $\tilde{O}\left(\frac{1}{\epsilon^2}\log n\right)$ algorithm for F_0 [BravermanGrigorescuLangWoodruffZhou18]
- Space $\tilde{O}\left(\frac{1}{\epsilon^2}\log^3 n\right)$ algorithm for L_2-heavy hitters [BravermanGrigorescuLangWoodruffZhou18]

$(1+\epsilon)$-Approximation Sliding Window Algorithms

- Space $O\left(\frac{1}{\epsilon^3}\log^3 n\right)$ algorithm for F_p with $p \in (0,1)$ [BravermanOstrovsky07]
- Space $O\left(\frac{1}{\epsilon^{2+p}}\log^3 n\right)$ algorithm for F_p with $p \in (1,2]$ [BravermanOstrovsky07]
- Space $\tilde{O}\left(\frac{1}{\epsilon^{2+p}} n^{1-2/p}\right)$ algorithm for F_p with $p > 2$ [BravermanOstrovsky07]

"A general framework that loses* nothing in n and only $\frac{1}{\epsilon}$"

Our Results: $(1+\epsilon)$-Approximation Sliding Window Algorithms

- Space $\tilde{O}\left(\frac{1}{\epsilon^2}\log^3 n\right)$ algorithm for F_p with $p \in (0,2]$

Problem	[BO07] Space	Our Result
L_p Estimation, $p \in (0,1)$	$\tilde{\mathcal{O}}\left(\frac{\log^3 n}{\varepsilon^3}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log^3 n}{\varepsilon^2}\right)$
L_p Estimation, $p \in (1,2]$	$\tilde{\mathcal{O}}\left(\frac{\log^3 n}{\varepsilon^{2+p}}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log^3 n}{\varepsilon^2}\right)$
L_p Estimation, integer $p > 2$	$\tilde{\mathcal{O}}\left(\frac{n^{1-2/p}}{\varepsilon^{2+p}}\right)$	$\tilde{\mathcal{O}}\left(\frac{n^{1-2/p}}{\varepsilon^2}\right)$
Entropy Estimation	$\tilde{\mathcal{O}}\left(\frac{\log^5 n}{\varepsilon^4}\right)$	$\tilde{\mathcal{O}}\left(\frac{\log^5 n}{\varepsilon^2}\right)$

"$\frac{1}{\epsilon}$ losses are not necessary"

Format

- Part 1: Background
- Part 2: Frameworks
- Part 3: Difference Estimators

Questions?

AMS F_2 Algorithm

- Let $s \in \{-1, +1\}^n$ be a sign vector of length n
- Let $Z = \langle s, f \rangle = s_1 f_1 + \cdots + s_n f_n$ and consider Z^2

$$E[Z^2] = \sum_{i,j} E\left[s_i s_j f_i f_j\right] = f_1^2 + \cdots + f_n^2$$

$$Var[Z^2] \leq \sum_{i,j} E\left[s_i s_j s_k s_l f_i f_j f_k f_l\right] \leq 2F_2^2$$

- Take the mean of $O\left(\frac{1}{\epsilon^2}\right)$ inner products for $(1+\epsilon)$-approximation [AlonMatiasSzegedy99]

"Attack" on AMS

- Can learn whether $s_i = s_j$ from $\langle s, e_i + e_j \rangle$
- Let $f_i = 1$ if $s_i = s_1$ and $f_i = -1$ if $s_i \neq s_1$
- $Z = \langle s, f \rangle = s_1 f_1 + \cdots + s_n f_n = m$ and $Z^2 = m^2$ deterministically

- What happened? Randomness of algorithm not independent of input

Reconstruction Attack on Linear Sketches

- Linear sketches are “not robust” to adversarial attacks, must use $\Omega(n)$ space [HardtWoodruff13]
- Approximately learn sketch matrix U, query something in the kernel of U
- Iterative process, start with $V_1, \ldots, V_r$
- Correlation finding: Find vectors weakly correlated with U orthogonal to V_{i-1}
- Boosting: Use these vectors to find strongly correlated vector v
- Progress: Set $V_i = \text{span}(V_{i-1}, v)$

Insertion-Only Streams

- Key: Deletions are needed to perform this attack
- Similar lower bounds for the sliding window model [DatarGionisIndykMotwani02]

- Assume insertion-only updates
- How do the previous results work?

Robust Algorithms

- Suppose we are trying to approximate some given function
  1. Suppose we have a streaming algorithm for this function
  2. Suppose this function is monotonic and the stream is insertion-only
- Sketch switching framework [Ben-EliezerJayaramWoodruffYogev20] gives a robust for this function
- Start many instances of the streaming algorithm at the beginning
- Use an instance of the algorithm but "freeze" the output
- Each time the next instance has value $(1 + O(\epsilon))$ more than the "frozen" output, use the next instance and "freeze" its output

Robust Algorithms

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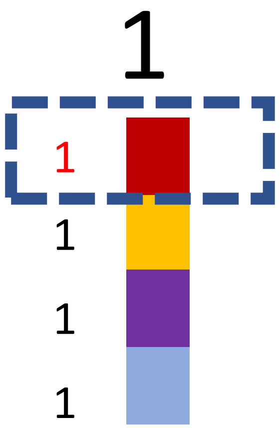

- Example: Number of ones in the stream (2-approximation)

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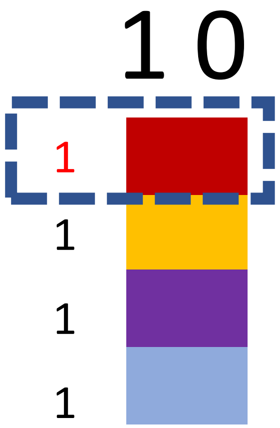

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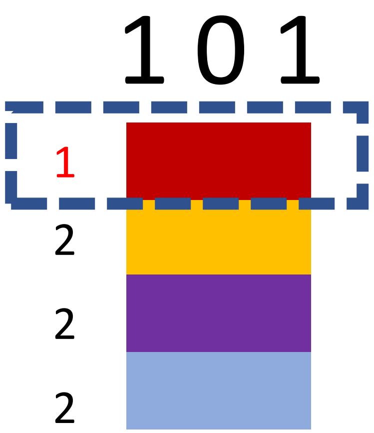

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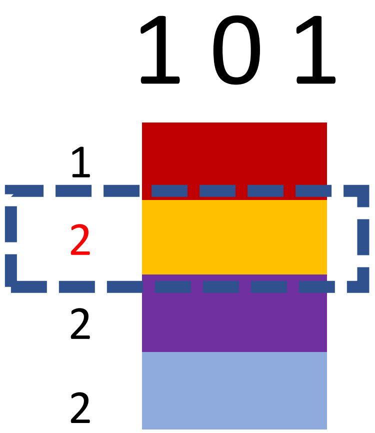

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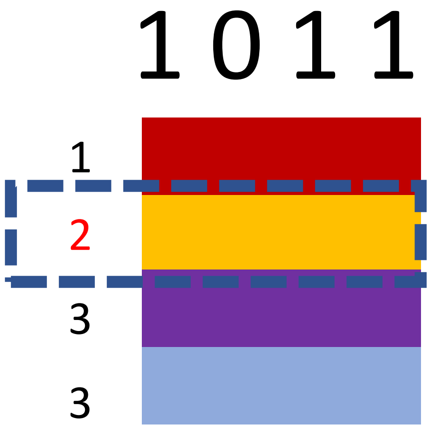

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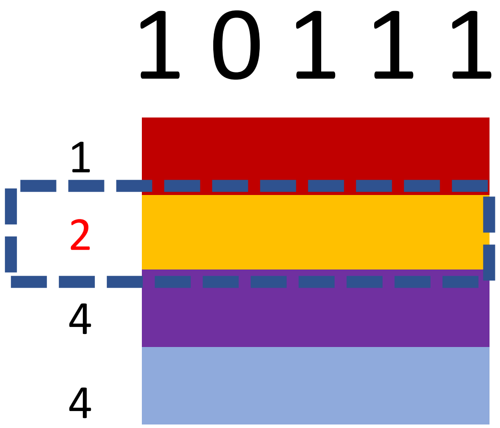

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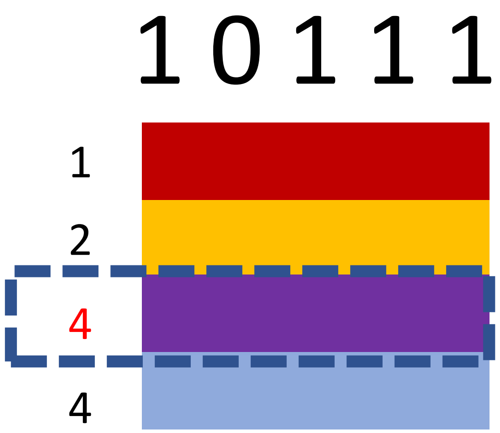

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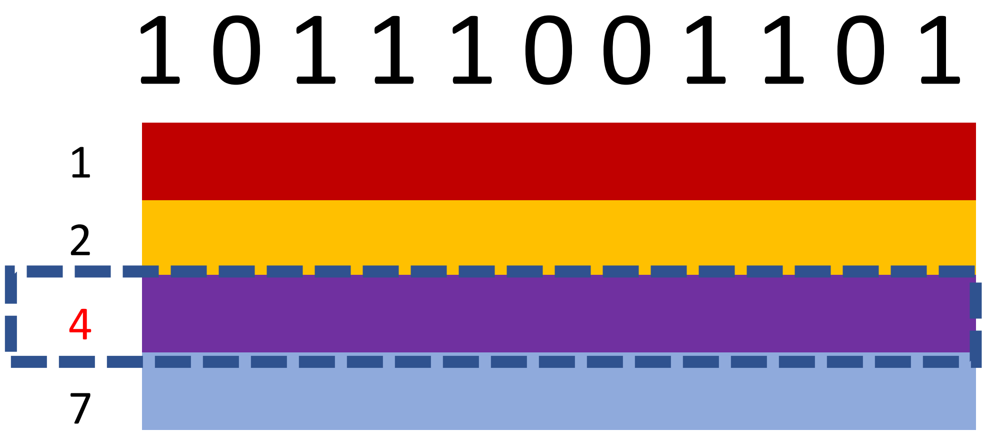

- Example: Number of ones in the stream (2-approximation)
- Number of ones stream is at least 4 and at most 8
- 4 is a good approximation

Summary

- ❖ Sketch switching for robust algorithms uses $\frac{1}{\epsilon^2}$ space each time F_p increases by $(1+\epsilon)$ and function increases $\frac{1}{\epsilon}$ times

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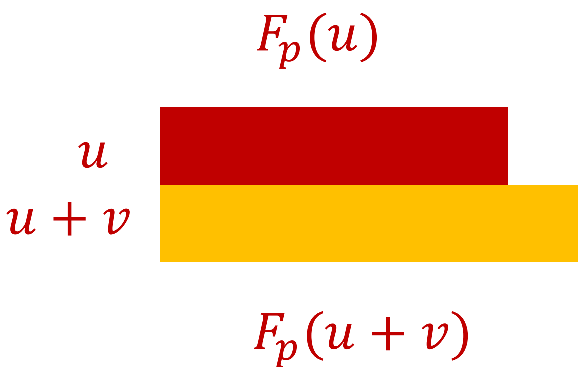
- ❖ Smooth histogram for sliding window algorithms uses $\frac{1}{\epsilon^2}$ space each time F_p increases by $(1+\epsilon)$ and function increases $\frac{1}{\epsilon}$ times for $p \in (0,1)$
- ❖ Smooth histogram for sliding window algorithms uses $\frac{1}{\epsilon^2}$ space each time F_p increases by $(1+\epsilon^p)$ and function increases $\frac{1}{\epsilon^p}$ times for $p \in (1,2)$

Intuition

❖ Do we really need to pay $\frac{1}{\epsilon^2}$ space each time F_p increases by $(1+\epsilon)$?



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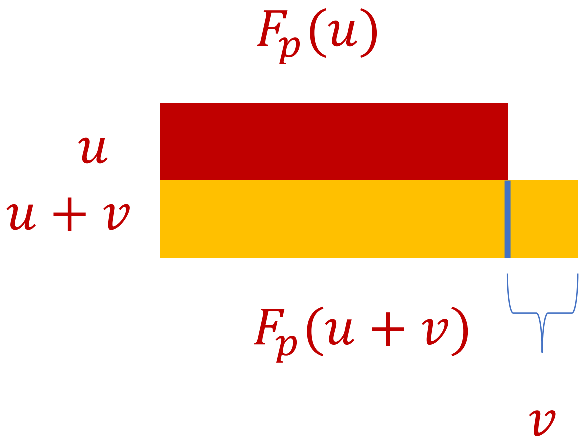

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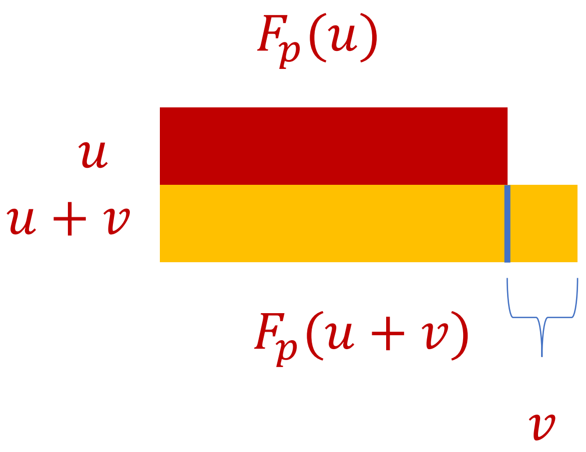
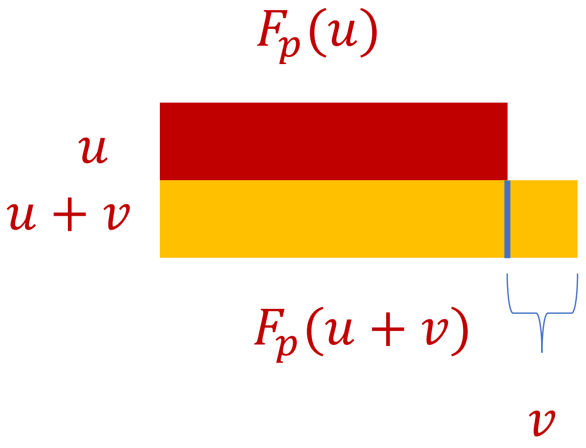


Suppose: $F_p(u+v) - F_p(u) = \epsilon F_p(u)$

Intuition

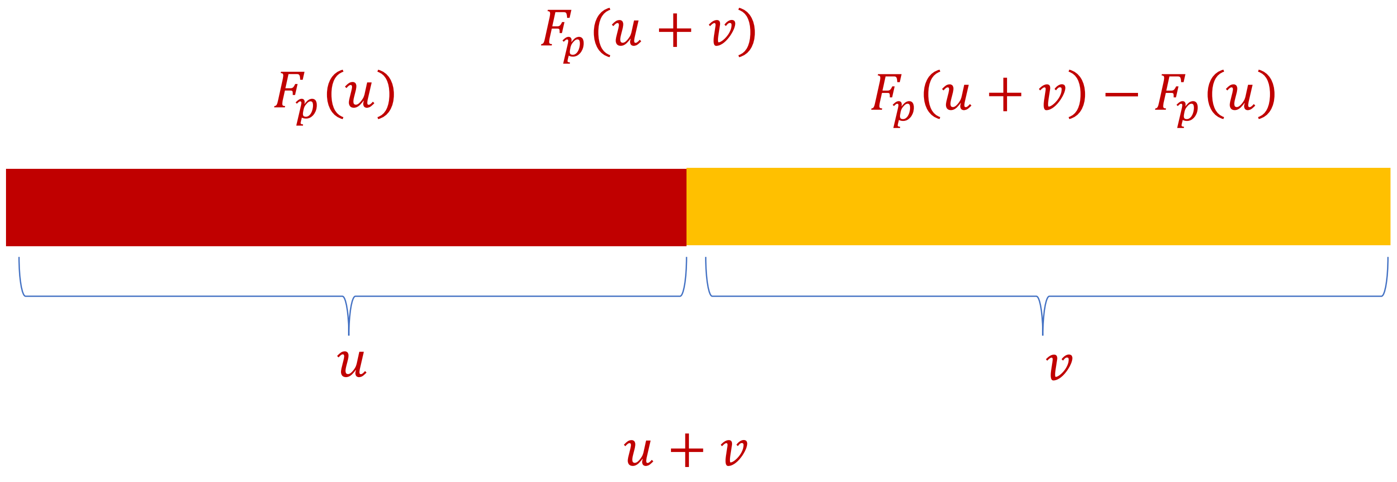
- Do we really need to pay $\frac{1}{\epsilon^2}$ space each time F_p increases by $(1+\epsilon)$?
- Only need constant factor approximation to $\epsilon F_p(u)$
- Only need constant factor approximation to $F_p(u+v) - F_p(u)$



Suppose: $F_p(u+v) - F_p(u) = \epsilon F_p(u)$

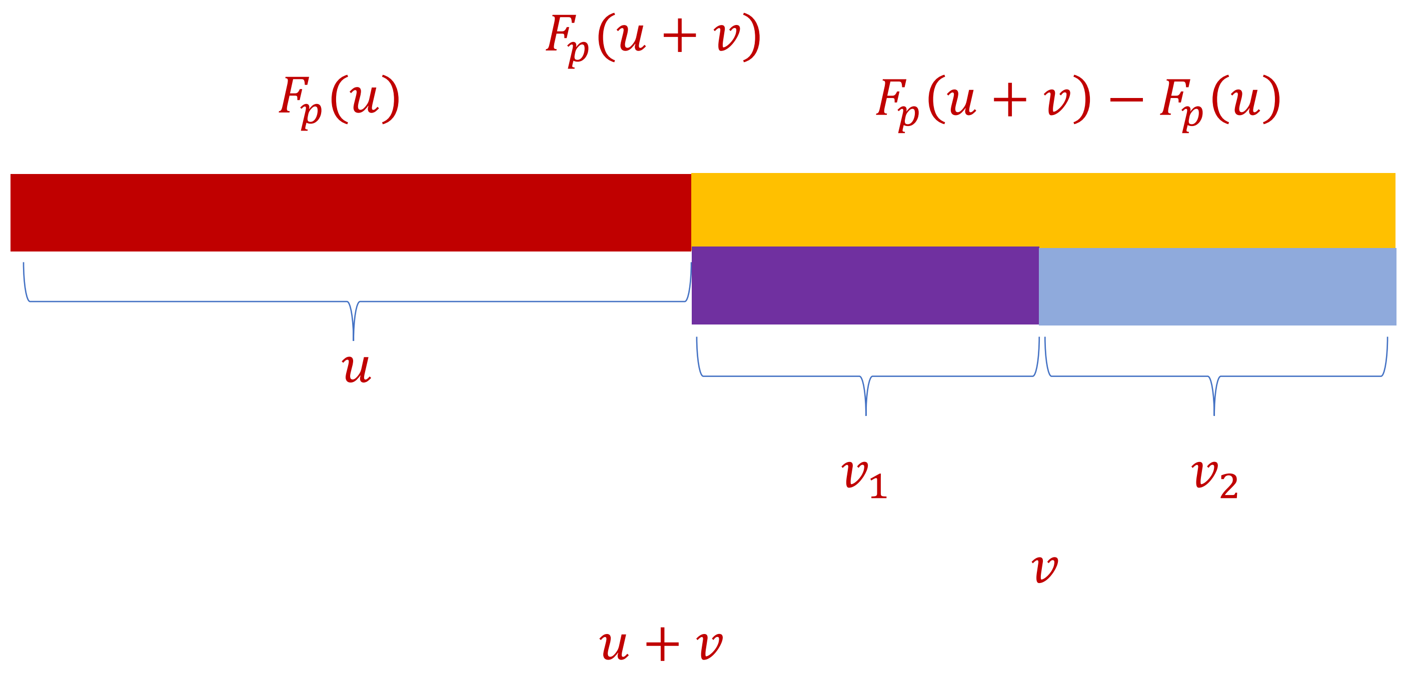
Sketch Stitching

- Suppose we want $F_p(u+v)$
- $F_p(u+v) = (F_p(u+v) - F_p(u)) + F_p(u)$



Sketch Stitching

- Suppose we want $F_p(u+v)$ and $v = v_1 + v_2 + \cdots + v_b$



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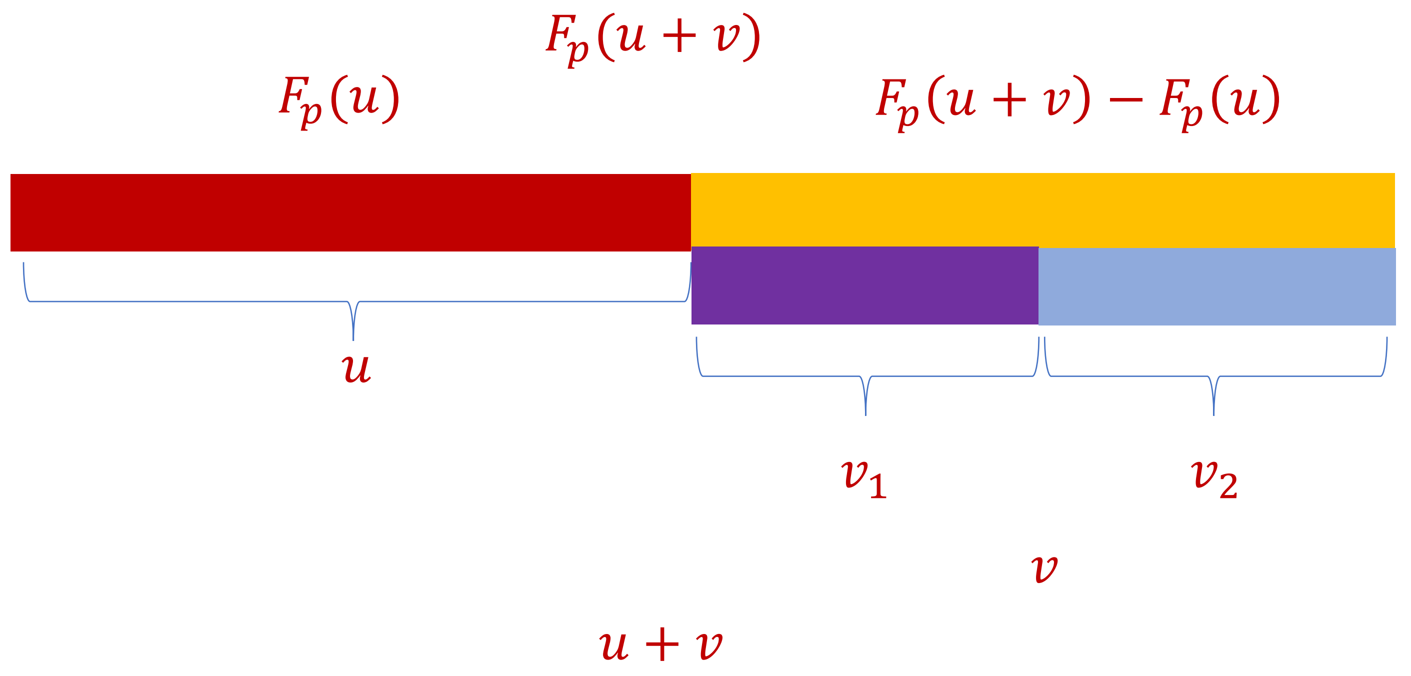


Sketch Stitching

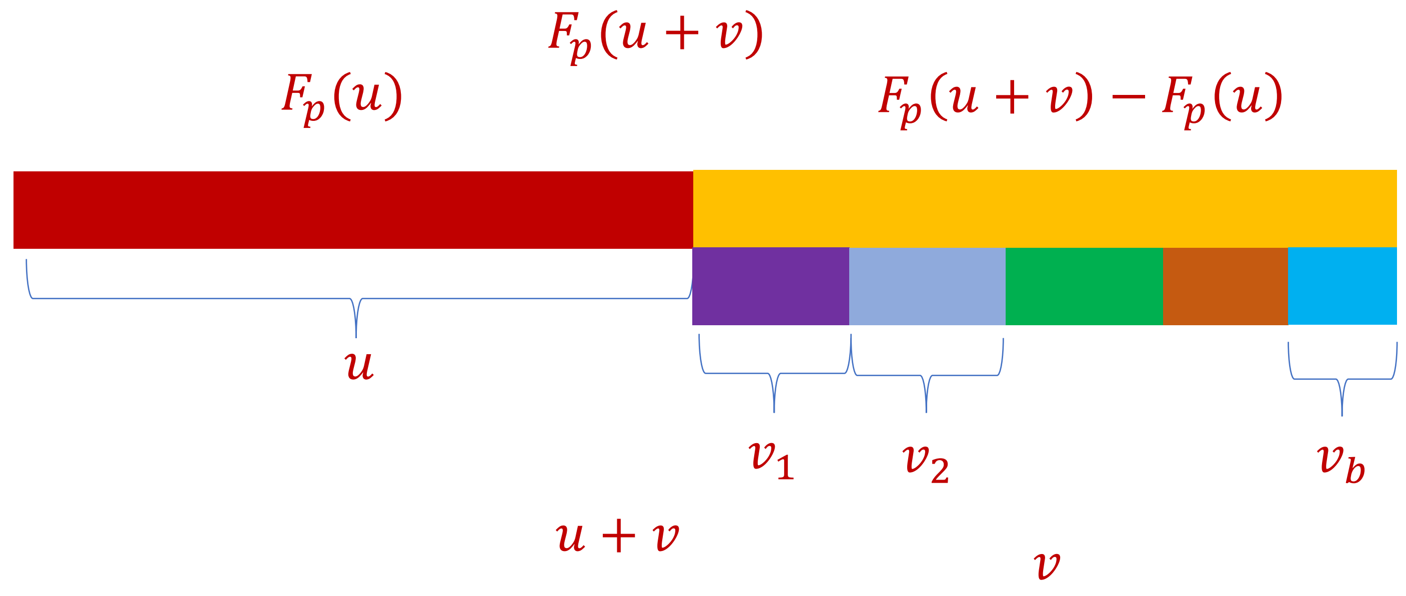
- Suppose we want $F_p(u+v)$ and $v = v_1 + v_2 + \cdots + v_b$
- $F_p(u+v) = (F_p(u+v) - F_p(u)) + F_p(u)$
- $F_p(u+v) = \left(F_p(u+v_1+\cdots+v_b) - \cancel{F_p(u+v_1+\cdots+v_{b-1})}\right) + \left(\cancel{F_p(u+v_1+\cdots+v_{b-1})} - \cancel{F_p(u+v_1+\cdots+v_{b-2})}\right) + \cdots + \left(\cancel{F_p(u+v_1)} - F_p(u)\right) + F_p(u)$

Sketch Stitching

- Suppose we want $F_p(u+v)$ and $v = v_1 + v_2 + \cdots + v_b$
- $F_p(u+v) = (F_p(u+v) - F_p(u)) + F_p(u)$
- $F_p(u+v) = \left(F_p(u + v_1 + \cdots + v_b) - F_p(u + v_1 + \cdots + v_{b-1})\right) + \left(F_p(u + v_1 + \cdots + v_{b-1}) - F_p(u + v_1 + \cdots + v_{b-2})\right) + \cdots + \left(F_p(u + v_1) - F_p(u)\right) + F_p(u)$

Sketch Stitching

- $F_p\,(u+v) = \left(F_p(u+v_1+\cdots+v_b) - F_p(u+v_1+\cdots+v_{b-1})\right) + \left(F_p(u+v_1+\cdots+v_{b-1}) - F_p(u+v_1+\cdots+v_{b-2})\right) + \cdots + \left(F_p(u+v_1) - F_p(u)\right) + F_p(u)$

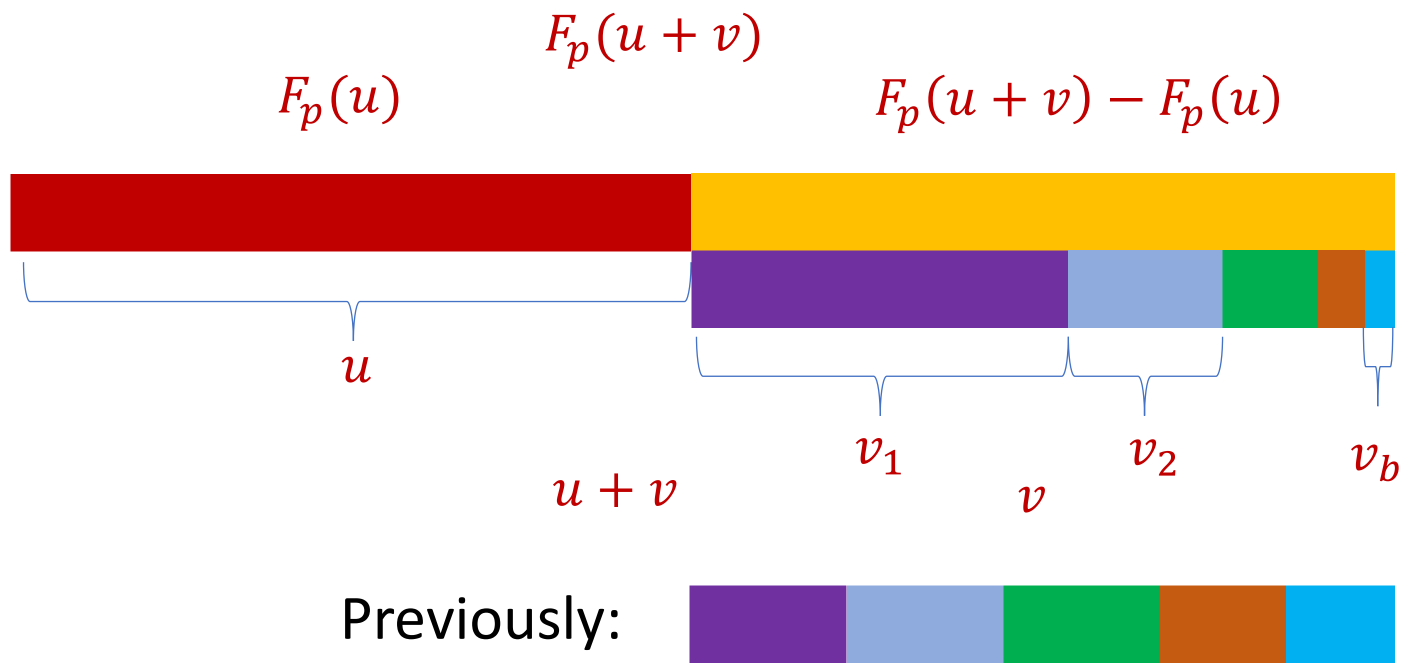


Granularity Change

- Set each difference to be exponentially decreasing
- $F_p\,(u+v) = \Big(F_p(u+v_1+\cdots+v_b) - F_p(u+v_1+\cdots+v_{b-1})\Big) + \Big(F_p(u+v_1+\cdots+v_{b-1}) - F_p(u+v_1+\cdots+v_{b-2})\Big) + \cdots + \Big(F_p(u+v_1) - F_p(u)\Big) + F_p(u)$
- $F_p(u+v_1+\cdots+v_b) - F_p(u+v_1+\cdots+v_{b-1}) = \frac{1}{2^b} F_p(u)$

Granularity Change

❖ $F_p(u+v_1+\cdots+v_b)-F_p(u+v_1+\cdots+v_{b-1})=\frac{1}{2^b}F_p(u)$



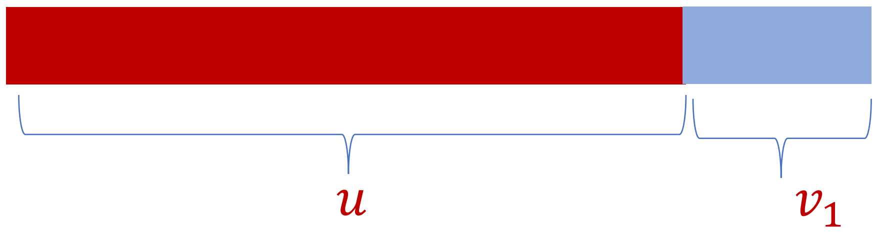
Granularity Change

- $F_p(u + v_1 + \cdots + v_b) - F_p(u + v_1 + \cdots + v_{b-1}) = \frac{1}{2^b} F_p(u)$
- Just need $2^b \epsilon$-approximation to $F_p(u + v_1 + \cdots + v_b) - F_p(u + v_1 + \cdots + v_{b-1})$

- Intuition: pay $\frac{1}{\epsilon^2}$ for $(1 + \epsilon)$-approximation
- Hope is to use space $\frac{1}{2^{2b}\epsilon^2}$ for $2^b \epsilon$-approximation

Framework

- Algorithms simultaneously running for each granularity
- Want space $\frac{1}{2^{2b}\epsilon^2}$ for granularity $\frac{1}{2^b}F_p(u)$
- Need 2^b instances for granularity $\frac{1}{2^b}F_p(u)$



- Total space $\sum \frac{1}{2^b\epsilon^2} = \frac{1}{\epsilon^2}$

Format

- Part 1: Background
- Part 2: Framework
- Part 3: Difference Estimators

Questions?

Difference Estimator

❖ If $F_p(u+v) - F_p(u) = 2^b \epsilon F_p(u)$, does there exist algorithm that approximates the difference with space $\frac{1}{2^{2b}\epsilon^2}$?

❖ Definition: $F_p(u+v) - F_p(u) = \gamma F_p(u)$, output an estimate to the difference with additive approximation $\epsilon F_p(u)$

Difference Estimator

- Definition: $F_p(u+v) - F_p(u) = \gamma F_p(u)$, output an estimate to the difference with additive approximation $\epsilon F_p(u)$
- F is generally non-linear
- Ex: $F_p(u+v) = \frac{1}{\epsilon^4}$, $F_p(u+v) - F_p(u) = 1$
- $(1+\epsilon)$ approximations to $F_p(u+v)$ and $F_p(u)$ give multiplicative approximation to the difference but use space $\frac{1}{\epsilon^2}$
- Constant factor approximations to $F_p(u+v)$ and $F_p(u)$ do not give additive approximation $\epsilon F_p(u)$ to the difference

Our Results: Difference Estimators

- Space $\tilde{O}\left(\frac{\gamma}{\epsilon^2}\log n\right)$ algorithm for F_0
- Space $\tilde{O}\left(\frac{\gamma^{2/p}}{\epsilon^2}\log n\right)$ algorithm for F_p with $p \in (0, 2]$
- Space $\tilde{O}\left(\frac{\gamma}{\epsilon^2} n^{1-2/p}\right)$ algorithm for F_p with integer $p > 2$

F_2 Difference Estimator

- Definition: $F_2(u+v) - F_2(u) = \gamma F_2(u)$, output an estimate to the difference with additive approximation $\epsilon F_2(u)$
- $F_2(u+v) - F_2(u) = \langle u+v, u+v\rangle - \langle u,u\rangle = 2\langle u,v\rangle + \langle v,v\rangle^2$
- Inner product property: $(1+\epsilon)$ -approximations to $\|u\|_2$ and $\|v\|_2$ gives an $\epsilon\|u\|_2\|v\|_2$ additive approximation to $\langle u,v\rangle$
- $\gamma F_2(u) \geq \langle v,v\rangle^2$ implies $2\langle u,v\rangle \leq 2\|u\|_2\|v\|_2 \leq 2\sqrt{\gamma}\, F_2(u)$
- Just need $\frac{\epsilon}{\sqrt{\gamma}}$ multiplicative approximation: $\tilde{O}\left(\frac{\gamma}{\epsilon^2}\log n\right)$ space!

F_2 Difference Estimator

- Difference estimator: Maintain $\left(1+\frac{\epsilon}{\sqrt{\gamma}}\right)$-approximations to $F_2(u+v)$ and $F_2(u)$ using AMS sketch
- For $F_2(u+v)-F_2(u)=\gamma F_2(u)$, difference of the outputs is an additive approximation $\epsilon F_2(u)$ to $F_2(u+v)-F_2(u)$

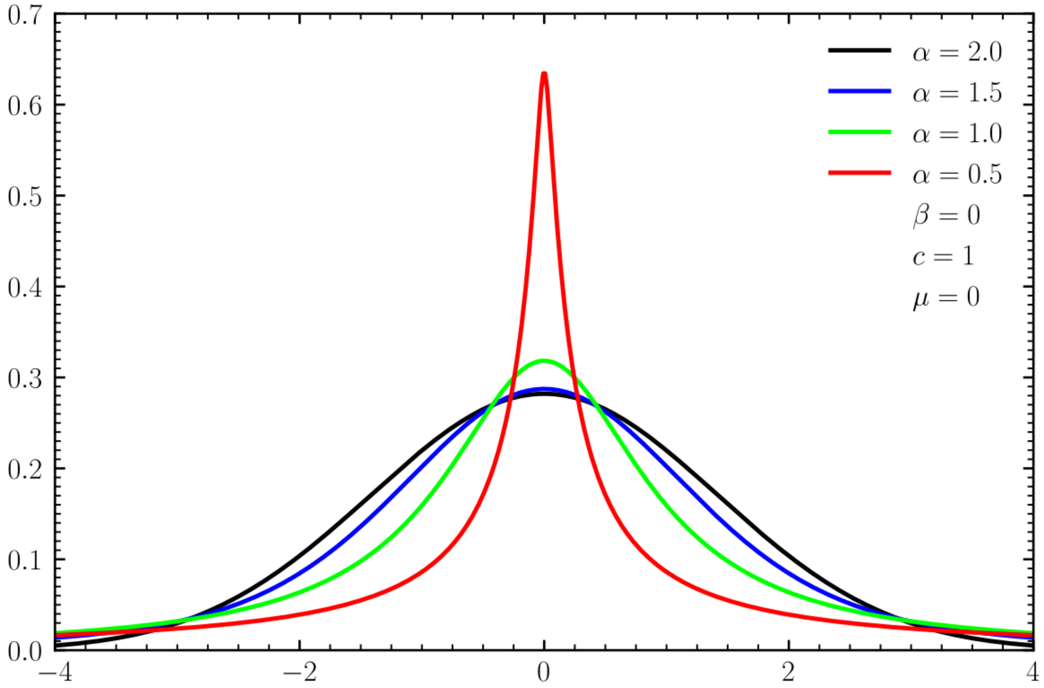
- Space $\tilde{O}\left(\frac{\gamma}{\epsilon^2}\log n\right)$ algorithm for difference estimator

Challenges for F_p Difference Estimators

❖ F_p difference estimator: Use p-stable random variables for $p \leq 2$?

❖ $\langle Z_p, f \rangle$ where Z_p has entries drawn from p-stable distribution

$$p(x) \sim \frac{1}{1 + |x|^{p+1}}$$



F_p Difference Estimators

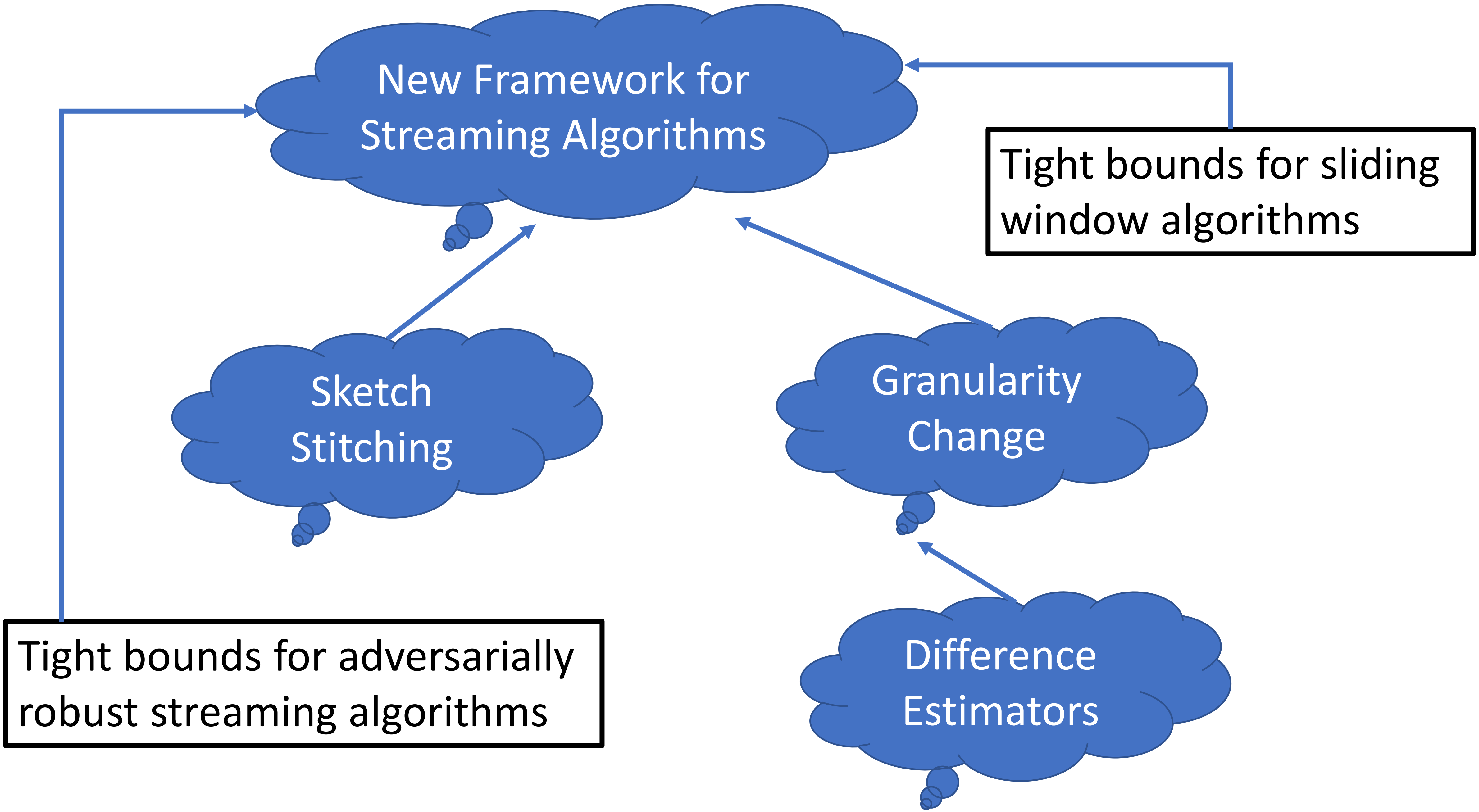
- $Z = median\langle Z_p, f\rangle$
- How to analyze median of each estimate of $F_p(u+v) - F_p(u)$?

- Use Li's geometric mean algorithm [Li08]
- Take the geometric mean of 3 inner products $\langle Z_p, f\rangle$
- Take the average of $O\left(\frac{1}{\epsilon^2}\right)$ geometric means

F_p Difference Estimators

- Difference estimator: Maintain $\left(1+\frac{\epsilon}{\gamma^{1/p}}\right)$-approximations to $F_p(u+v)$ and $F_p(u)$ using Li's geometric mean estimator
- $A(u+v)-A(u) \sim \langle p_1, v\rangle^{p/3}\langle p_2, v\rangle^{p/3}\,\langle p_3, v\rangle^{p/3} +$
$$\langle p_1, u\rangle^{p/3}\langle p_2, v\rangle^{p/3}\,\langle p_1, v\rangle^{p/3} +$$
$$\langle p_1, u\rangle^{p/3}\langle p_2, u\rangle^{p/3}\,\langle p_1, v\rangle^{p/3} + \cdots$$

- Each summand has $\langle p_1, v\rangle^{p/3}$ term, which has much smaller variance


New Framework for Streaming Algorithms
Tight bounds for sliding window algorithms
Sketch Stitching
Granularity Change
Difference Estimators
Tight bounds for adversarially robust streaming algorithms

Future Directions

- Additional applications of difference estimators, e..g, general $p > 2$?
- Uses of differential privacy for adaptive data analysis
- Algorithms robust to white-box adversaries?

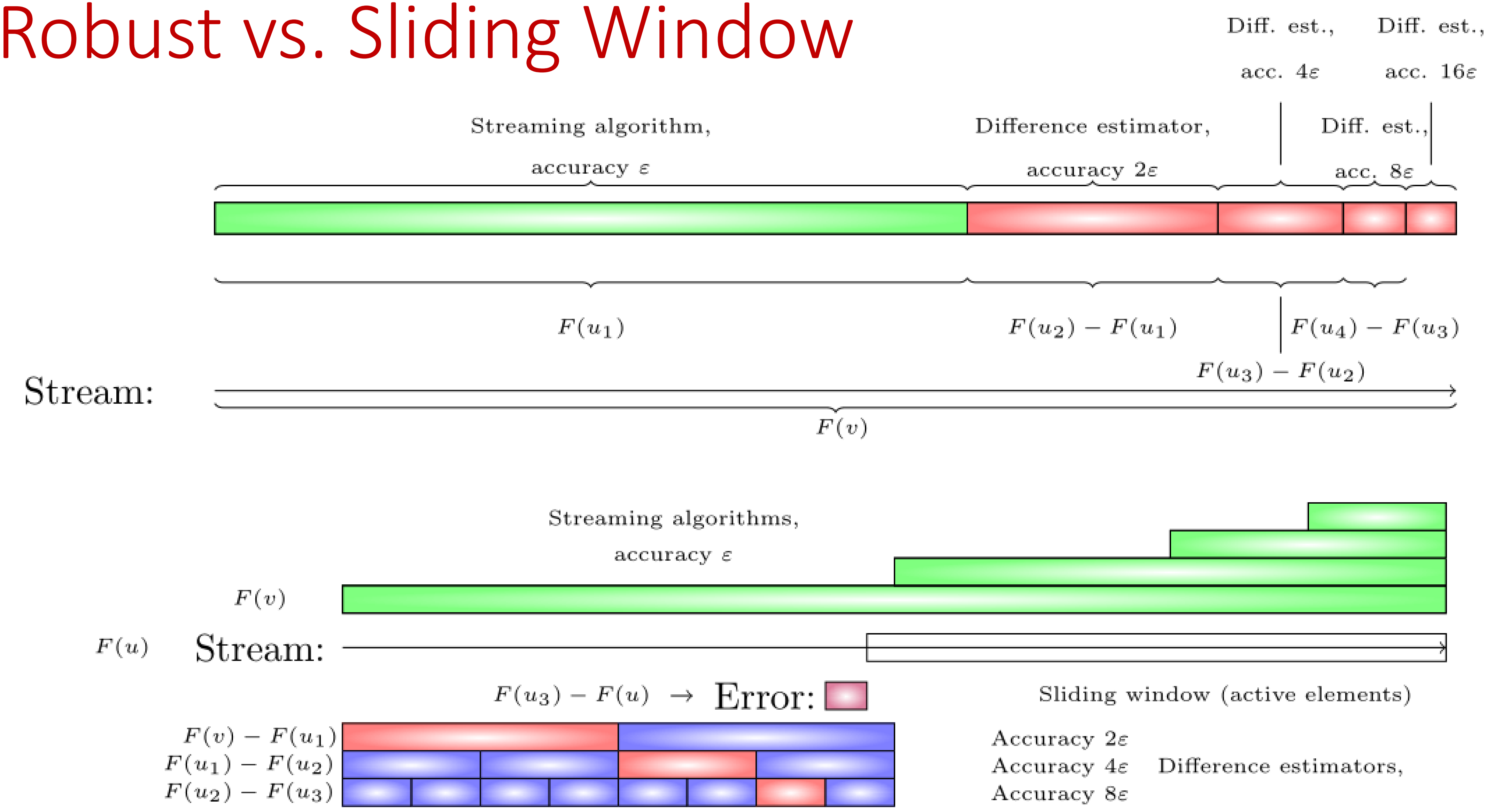
Challenges for F_p Difference Estimators

- F_p difference estimator: Generalization of inner products for $p > 2$?
- Variance can be much larger!
- Use heavy-hitter algorithm to explicitly track "heavy" elements
- Use L_2 sampling algorithm with $n^{1-2/p}$ buckets to sample "light" elements

General Challenges

- Use known structural results from chaining to remove $\log n$ factor in difference estimator
- Avoids typical Chernoff + union bound argument by considering the expected supremum of a process, “strong tracking”
- Use suffix argument to remove $\log n$ factor in framework
- Adaptation to sliding window model

Robust vs. Sliding Window

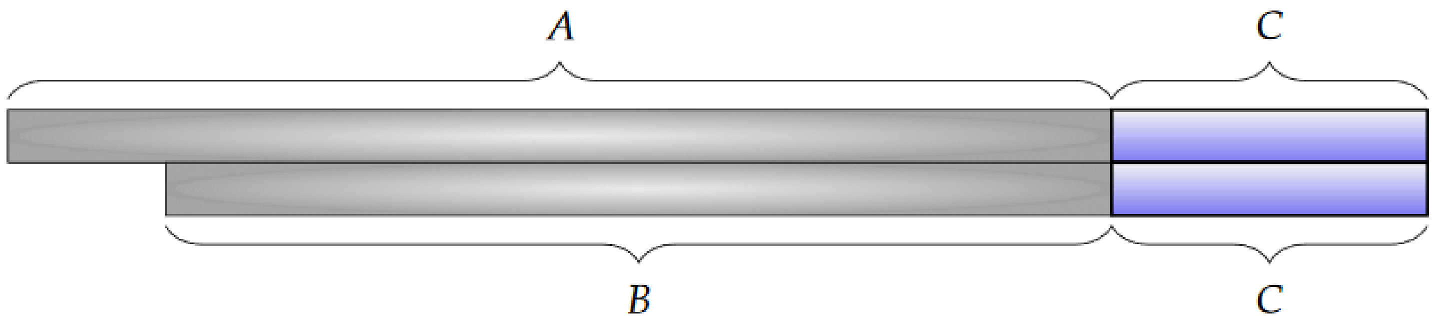


Literature

- Tight Bounds for Adversarially Robust Streams and Sliding Windows via Difference Estimators, David P. Woodruff, Samson Zhou (FOCS 2021)
- Adversarial Robustness of Streaming Algorithms through Importance Sampling, Vladimir Braverman, Avinatan Hassidim, Yossi Matias, Mariano Schain, Sandeep Silwal, Samson Zhou (NeurIPS 2021)
- The White-Box Adversarial Data Stream Model, Miklós Ajtai, Vladimir Braverman, T.S. Jayram, Sandeep Silwal, Alec Sun, David P. Woodruff, Samson Zhou (PODS 2022)

Sliding Window Algorithms

- Suppose we are trying to approximate some given function
  1. Suppose we have a streaming algorithm for this function
  2. Suppose this function is "smooth": If $f(B)$ is a "good" approximation to $f(A)$, then $f(B \cup C)$ will always be a "good" approximation to $f(A \cup C)$.



- Smooth histogram framework [BravermanOstrovsky07] gives a sliding window algorithm for this function

Smooth Histogram

- ❖ Suppose we are trying to approximate some given function
- ❖ Smooth histogram framework [BO07] gives a sliding window algorithm for this function
- ❖ Start a new instance of the streaming algorithm (along with existing instances) each time a new element arrives
- ❖ Each time there are three instances that report "close" values, delete the middle one
- ❖ Use different checkpoints to "sandwich" the sliding window

Smooth Histogram

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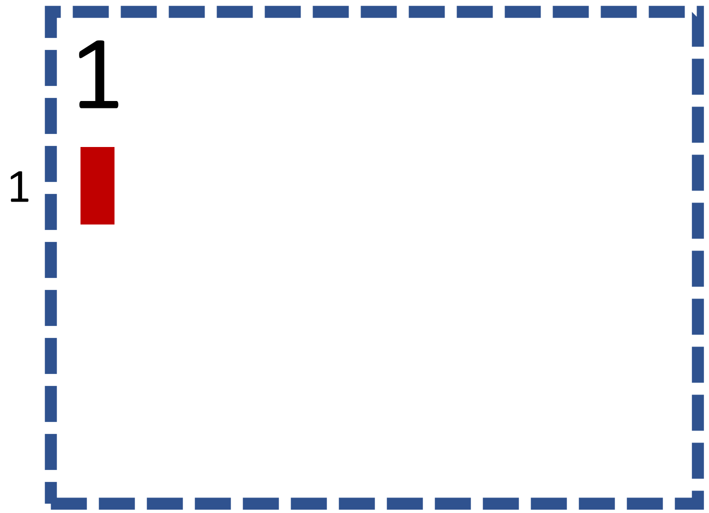

❖ Example: Number of ones in sliding window (2-approximation)

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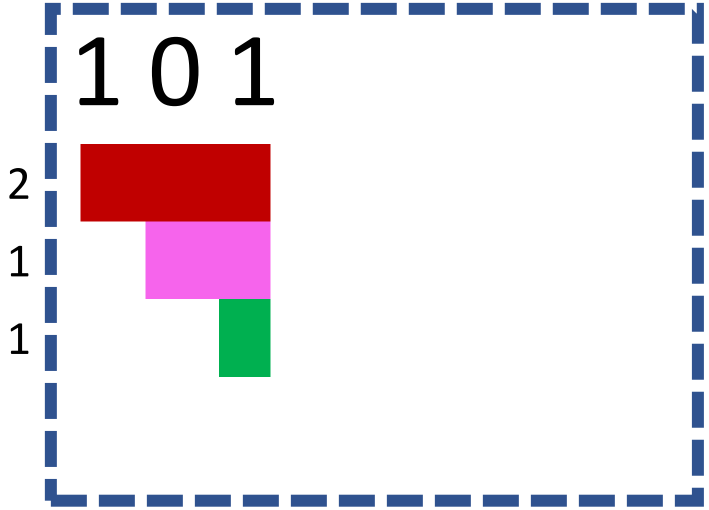

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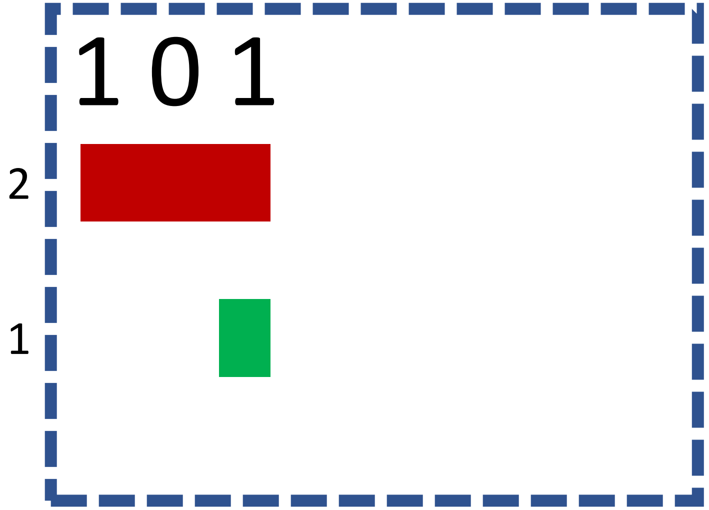

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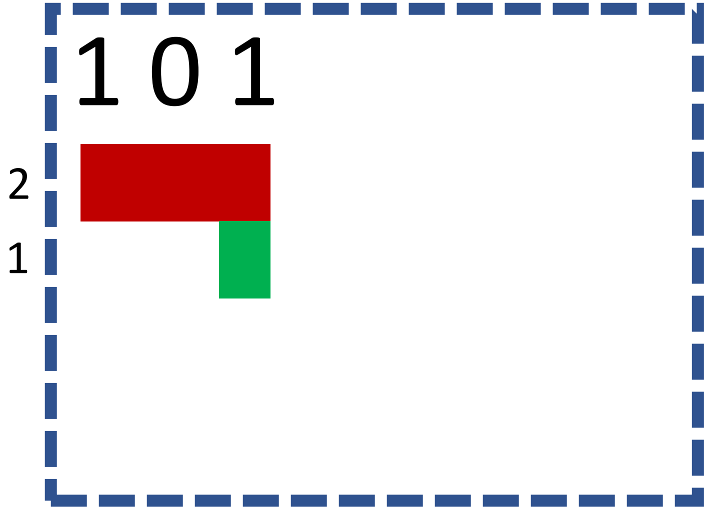

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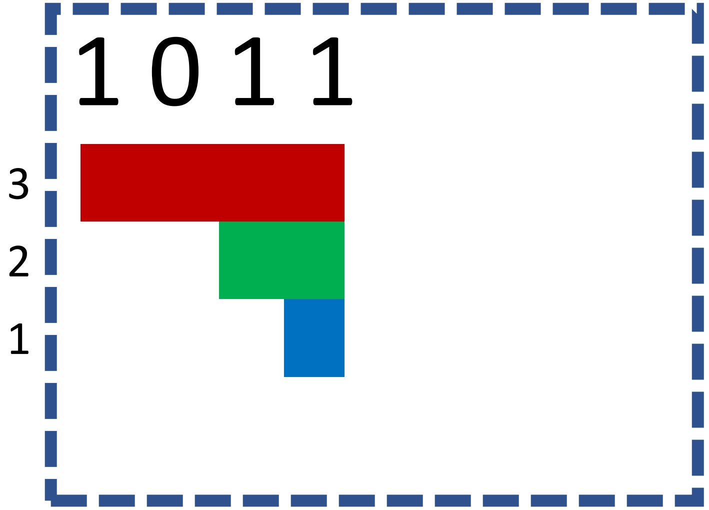

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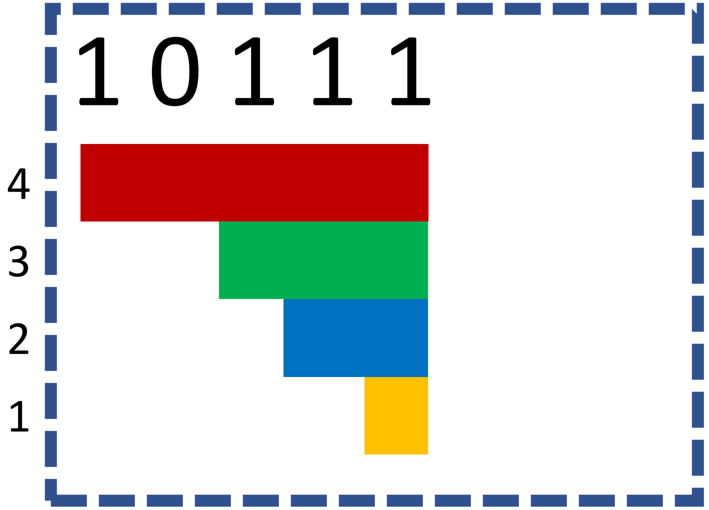

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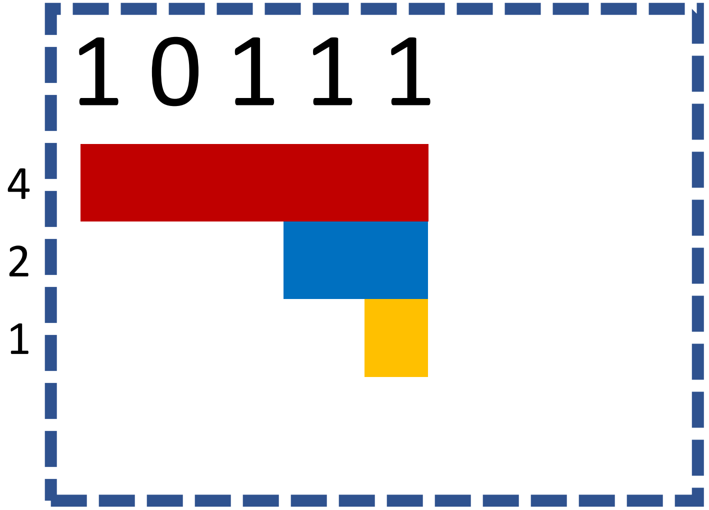

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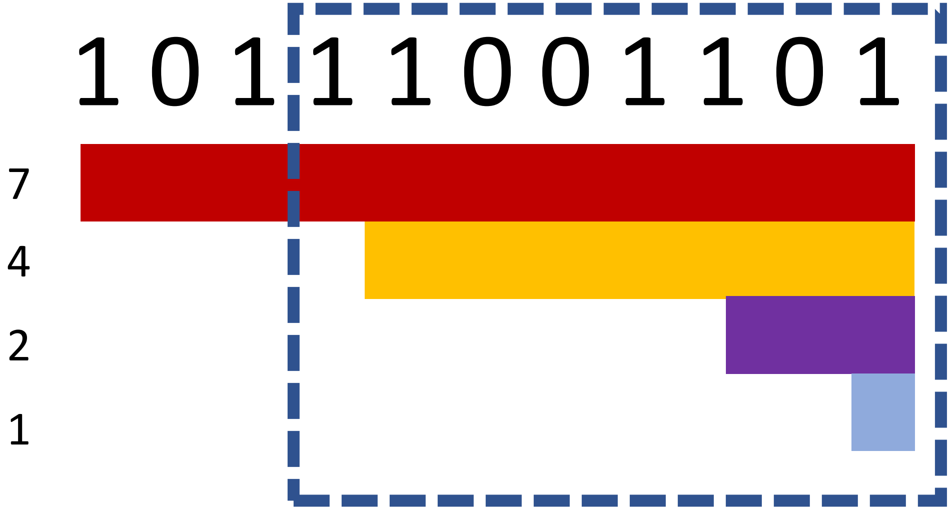

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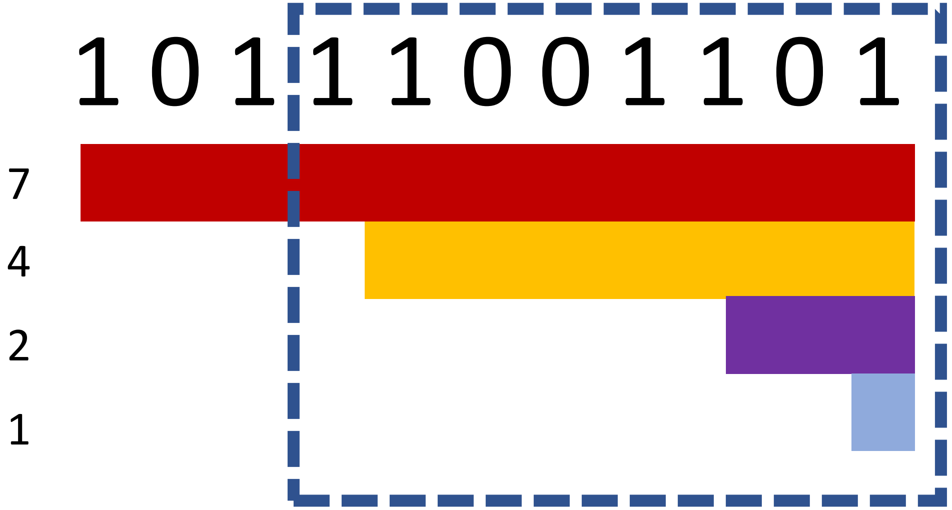

- ❖ Example: Number of ones in sliding window (2-approximation)
- ❖ Number of ones in sliding window is at least 4 and at most 7
- ❖ 4 is a good approximation

Granularity Change

- Set each difference to be exponentially decreasing
- $F_p\,(u+v) = \Big(F_p(u+v_1+\cdots+v_b) - F_p(u+v_1+\cdots+v_{b-1})\Big) + \Big(F_p(u+v_1+\cdots+v_{b-1}) - F_p(u+v_1+\cdots+v_{b-2})\Big) + \cdots + \Big(F_p(u+v_1) - F_p(u)\Big) + F_p(u)$
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