CSCE 411: Design and Analysis of Algorithms

Lecture 9: Intro to Graph Algorithms

Date: February 18 *Nate Veldt, updated by Samson Zhou*

1 Graph Notation and Terminology

An (undirected) graph $G = (V, E)$ is defined by

An edge between nodes i and j is denoted by ____________________

We can also denote an edge by ____________________

If $(i, j) \in E$, we say i and j are _______________. The neighborhood of node i is the set of nodes adjacent to it:

The *degree* of i is the number of neighbors it has: __________

1.1 Generalized graph classes

- Weighted:

- Directed:

1.2 Basic graphs and edge structures

- A **complete graph** is a graph in which

- A **bipartite graph** is a graph in which

- **Triangle**: set of three nodes that all share edges:

$$\{i, j, k\} \subseteq V \text{ such that } \{(i, j), (i, k), (j, k)\} \in E$$

- **Path**: is a sequence of edges joining a sequence of vertices:

$$\{i_1, i_2, \dots i_k\} \subseteq V \text{ where } (i_1, i_2) \in E, (i_2, i_3) \in E, \dots, (i_{k-1}, i_k) \in E.$$

- **Matching**: is a set of edges without common vertices

$$\mathcal{M} \subseteq E \text{ such that for all } e_i, e_j \in \mathcal{M} \text{ with } e_i \neq e_j, e_i \cap e_j = \emptyset.$$

- **Connected component**: a maximal subgraph in which there is a path between every pair of nodes in the subgraph.

1.3 Optimization Problems on Graphs

Many graph analysis problems amount to optimizing an objective function over a graph.

Example 1. Shortest path. Given a source node $s \in V$ and target node $t \in V$, find the shortest path of edges between s and t.

Example 2. Maximum bipartite matching. Let $G = (V, E)$ be a bipartite graph. Find a matching $\mathcal{M}$ with maximum sum of edge weights.

Example 3. Find connected components. Return the connected components of a graph:

1.4 Encoding a Graph

Consider a graph $G = (V, E)$ with a fixed node ordering $V = \{1, 2, \ldots, n\}$.

Adjacency Matrix The **adjacency matrix A** of G is defined so that

Adjacency List The **adjacency list Adj** of G is