AN ANALYSIS OF BID PREFERENCE PROGRAMS

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BACKGROUND

- The California Department of Transportation (Caltrans) procurement auctions are used to allocate highway construction contracts.
- Descending auctions in which the bidder, a construction company, submits a bid that represents the cost it would charge the auctioneer, Caltrans, to complete the project.
- Caltrans utilizes a bid preference program to assist small businesses in winning procurement auctions.
- The bid preference comes in the form of a 5% bid discount applied to the bids of small businesses. This discount is only used in determining the winner of the auction and does not affect the payout.

Caltrans

QUESTIONS OF INTEREST

1	• Does the bid preference program increase or decrease the total procurement cost for Caltrans and by how much?
2	• How much do large businesses lose because of this program?
3	• Does the bid preference program increase competition between small and large businesses?

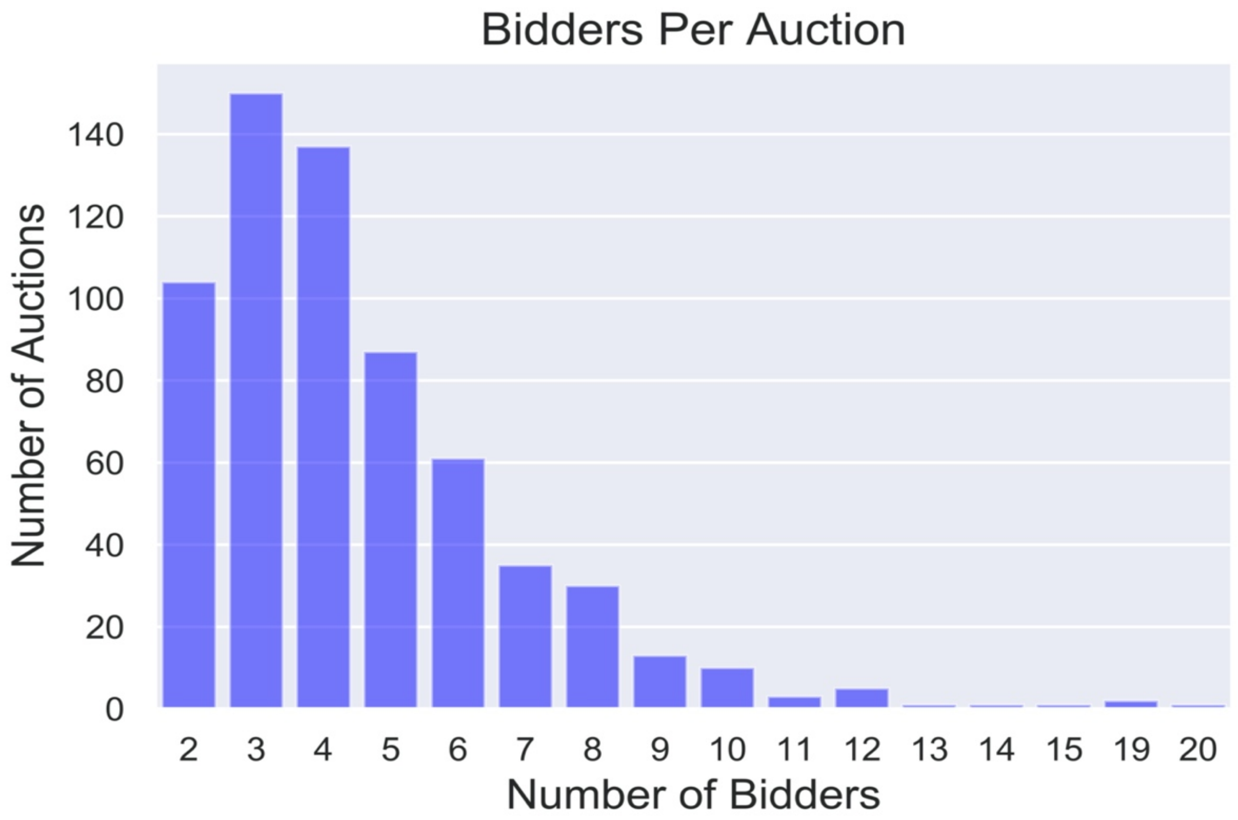
DEFINING A SMALL BUSINESS

1	• Must have no more than 100 employees.
2	• Be independently owned, operated, and based in the state of California.
3	• Have average annual gross receipts of no more than $10 million over the past 3 years.

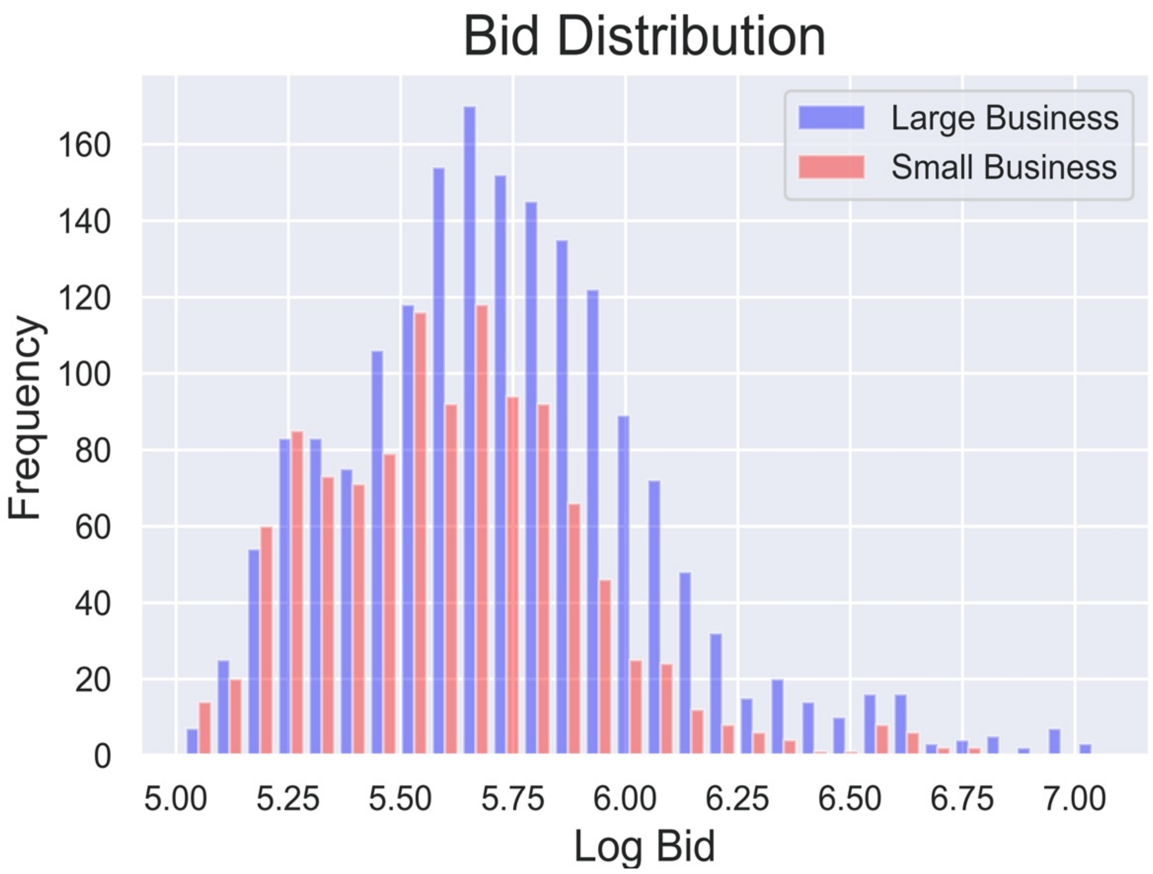
EXPLORATORY DATA ANALYSIS

- Their are a total of 641 procurement auctions (project IDs) within the dataset after the removal of outliers and erroneous data.
- Unique Bidders: 501
  - Small Business Bidders: 243
  - Large Business Bidders: 258

Number of Bidders					
Min	Quartile 1	Mean	Median	Quartile 3	Max
2	4	4.53	5	7	20



EXPLORATORY DATA ANALYSIS (CONTINUED)



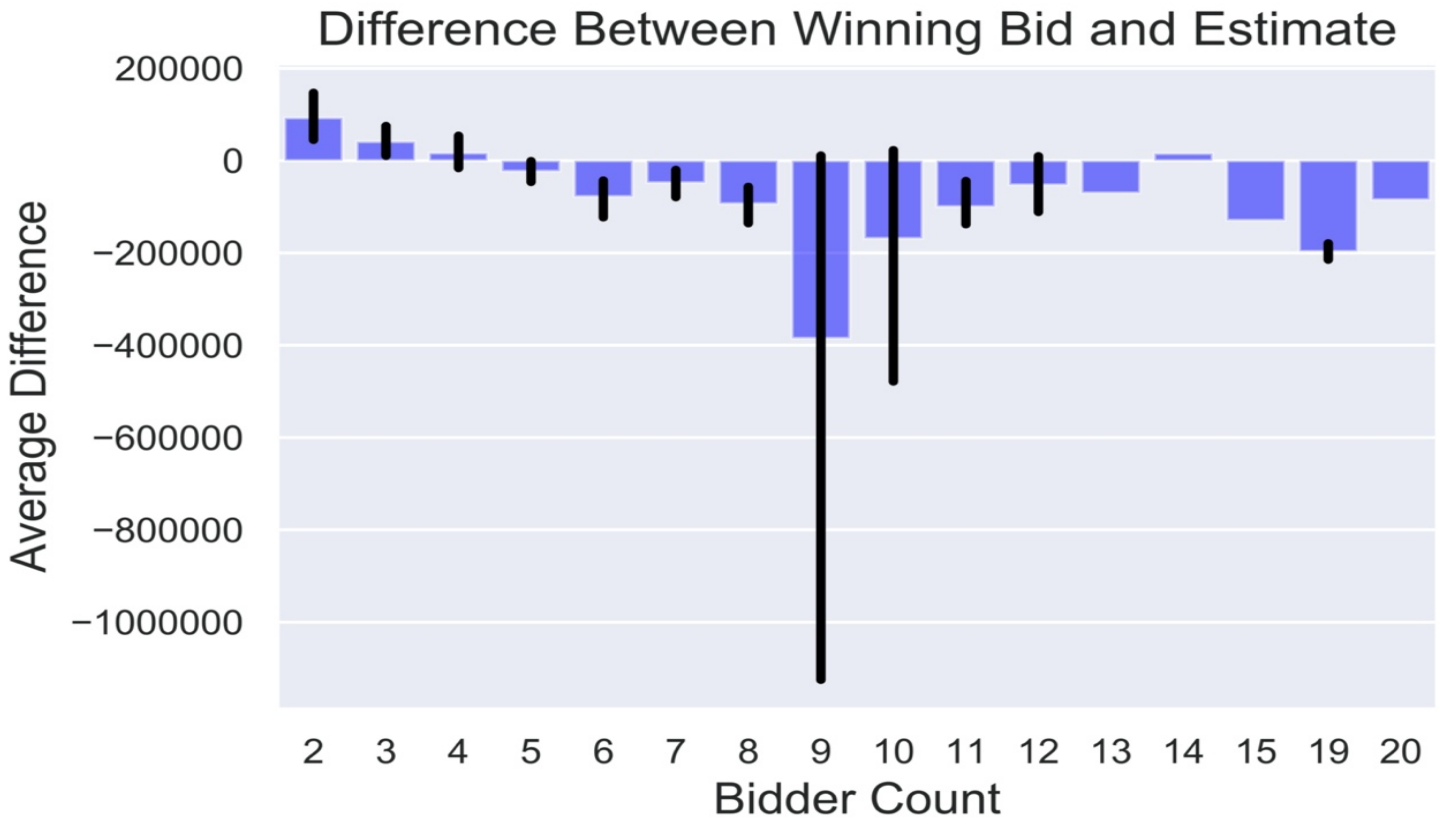
Distribution of All Bids				
Min	Quartile 1	Median	Quartile 3	Max
103,845.00	288,362.25	467,452.50	738,929.13	11,777,770.0

Distribution of Small Business Bids				
Min	Quartile 1	Median	Quartile 3	Max
103,845.00	245,786.00	395,526.00	601,299.00	5,965,853.00

Distribution of Large Business Bids				
Min	Quartile 1	Median	Quartile 3	Max
106,210.00	318,227.00	514,209.00	846,282.00	11,777,770.00

EXPLORATORY DATA ANALYSIS (CONTINUED)

- An increase in the number of auction participants decreases the procurement cost of Caltrans with respect to their engineers estimate.



EXPLORATORY DATA ANALYSIS (CONTINUED)

- Regression of the number of small business bidders, large business bidders, and cost estimate on the winning bids.
  - Statistically significant coefficients of **small business bidders (-18,730)** and **large business bidders (-42,680)**

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                            OLS Regression Results
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Dep. Variable:                    Bid   R-squared:                       0.892
Model:                            OLS   Adj. R-squared:                  0.892
Method:                 Least Squares   F-statistic:                     1755.
Date:                Mon, 10 Feb 2020   Prob (F-statistic):          2.08e-307
Time:                        22:49:20   Log-Likelihood:                -8902.0
No. Observations:                 641   AIC:                         1.781e+04
Df Residuals:                     637   BIC:                         1.783e+04
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept   2.003e+05   2.44e+04      8.204      0.000    1.52e+05    2.48e+05
SmallBiz   -1.873e+04   5570.464     -3.362      0.001   -2.97e+04   -7791.878
LargeBiz   -4.268e+04   6749.947     -6.323      0.000   -5.59e+04   -2.94e+04
Estimate       0.9199      0.013     70.151      0.000       0.894       0.946
==============================================================================
Omnibus:                      472.899   Durbin-Watson:                   1.506
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           150790.360
Skew:                          -2.128   Prob(JB):                         0.00
Kurtosis:                      78.018   Cond. No.                     2.51e+06
==============================================================================
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THEORETICAL MODEL – ENVIRONMENT

- For any auction with $n \geq 2$ bidders where $n = n_s + n_l$ we observe the following:
  - Small business bidders follow the cost distribution $C \sim F_s(\cdot)$
  - Large business bidders follow the cost distribution $C \sim F_l(\cdot)$
  - The distributions are asymmetric meaning $F_s(\cdot) \neq F_l(\cdot)$

THEORETICAL MODEL – LARGE BIDDERS

- For a **large business bidder** with bid b_i in an auction with n_s small and n_l large bidders the probability of winning is equivalent to:
  - Pr(i wins with bid b_i) = Pr(($n_l - 1$) bid $\geq b_i$) $*$ Pr(n_s bid $\geq 1.05 * b_i$) (1)

    $$= [1 - F_l\left(\beta_l^{-1}(b_i)\right)]^{(n_l-1)} * [1 - F_s\left(\beta_s^{-1}(1.05 * b_i)\right)]^{(n_s)} \quad (2)$$

- Because this bidder makes a profit equivalent to their bid b_i minus their individual cost C_i if they win, this bidder maximizes profit when:
  - $max_{b_i}\{ (b_i - C_i) * [1 - F_l\left(\beta_l^{-1}(b_i)\right)]^{(n_l-1)} * [1 - F_s\left(\beta_s^{-1}(1.05 * b_i)\right)]^{(n_s)} \}$ (3)

THEORETICAL MODEL – SMALL BIDDERS

- For a **small business bidder** with bid b_i in an auction with n_s small and n_l large bidders the probability of winning is equivalent to:
  - Pr(i wins with bid b_i) = Pr(($n_s - 1$) bid $\geq b_i$) $*$ Pr(n_l bid $\geq (b_i \,/\, 1.05)$) (1)

    $= [1 - F_l\left(\beta_l^{-1}((b_i \,/\, 1.05)\right)]^{(n_l)} * [1 - F_s(\beta_s^{-1}(b_i))]^{(n_s-1)}$ (2)

- Because this bidder makes a profit equivalent to their bid b_i minus their individual cost C_i if they win, this bidder maximizes profit when:
  - $max_{b_i}\{ (b_i - C_i) * [1 - F_l\left(\beta_l^{-1}(b_i \,/\, 1.05)\right)]^{(n_l)} * [1 - F_s(\beta_s^{-1}(b_i))]^{(n_s-1)} \}$ (3)

THEORETICAL MODEL – PROFIT MAXIMIZATION

- Taking the derivative of the profit maximization equations with respect to b_i and solving for the firms cost C_i we reach the following:

  - First order condition for **large business bidders:**

$$C_i = b_i - \left(\frac{(n_l - 1) * f_l\left(\beta_l^{-1}(b_i)\right)}{\left(1 - F_l\left(\beta_l^{-1}(b_i)\right)\right) * \beta_l'\left(\beta_l^{-1}(b_i)\right)} + \frac{1.05 * n_s * f_s\left(\beta_s^{-1}(1.05 * b_i)\right)}{\left(1 - F_s(\beta_s^{-1}(1.05 * b_i)\right) * \beta_s'\left(\beta_s^{-1}(1.05 * b_i)\right)} \right)^{-1}$$

  - First order condition for **small business bidders:**

$$C_i = b_i - \left(\frac{n_l * f_l\left(\beta_l^{-1}(b_i/1.05)\right)}{1.05 * \left(1 - F_l\left(\beta_l^{-1}(b_i/1.05)\right)\right) * \beta_l'\left(\beta_l^{-1}(b_i/1.05)\right)} + \frac{(n_s - 1) * f_s\left(\beta_s^{-1}(b_i)\right)}{\left(1 - F_s\left(\beta_s^{-1}(b_i)\right)\right) * \beta_s'\left(\beta_s^{-1}(b_i)\right)} \right)^{-1}$$

THE EMPIRICAL MODEL

- In order to implement our model in the empirical setting we will make several changes to the theoretical model.

  - In order to replace terms that involve F_l, f_l and β_l' we introduce the random variable G_l

  $$(1 - F_l\left(\beta_l^{-1}(b)\right)) = (1 - G_l(b|n_l, n_s))$$

  $$g_l(b|n_l, n_s) = f_l\left(\beta_l^{-1}(b)\right) \times \frac{1}{\beta_l'(\beta_l^{-1}(b))}$$

  - In order to replace terms that involve F_s, f_s and β_s' we introduce the random variable G_s

  $$(1 - F_s(\beta_s^{-1}(b))) = (1 - G_s(b|n_l, n_s))$$

  $$g_s(b|n_l, n_s) = f_s(\beta_s^{-1}(b)) \times \frac{1}{\beta_s'(\beta_s^{-1}(b))}$$

- We can estimate both $G_{l,s}(b|n_l, n_s)$ and $g_{l,s}(b|n_l, n_s)$ by using the distribution of observed bids and kernel density estimation.

THE EMPIRICAL MODEL (CONTINUED)

- Substituting in G and g into the theoretical model, we will now be applying equation (1) to large business bidders and equation (2) to small business to derive the cost distributions distributions $C \sim F_l(\cdot)$ and $C \sim F_s(\cdot)$.

$$C_i = b_i - \frac{1}{\left(\frac{(n_l - 1) * g_l(b_i|n_l, n_s)}{(1 - G_l(b_i|n_l, n_s))} + \frac{1.05(n_s) * g_s(1.05 * b_i|n_l, n_s)}{(1 - G_s(1.05 * b_i|n_l, n_s))}\right)} \quad (1)$$

$$C_i = b_i - \frac{1}{\left(\frac{(n_l) * g_l(b_i/1.05\,|n_l, n_s)}{1.05(1 - G_l(b_i/1.05\,|n_l, n_s))} + \frac{(n_s - 1) * g_s(b_i|n_l, n_s)}{(1 - G_s(b_i|n_l, n_s))}\right)} \quad (2)$$

- Note: from now on we will be conditioning G and g on the engineers estimate x in addition to the number of bidders.

ALGORITHM IMPLEMENTATION

Steps:

- Identify a combination of n_s & n_l bidders and subset the data to that combination.
- Remove auctions with bids that are considered outliers.
- Create the distributions $g(b|x)$ and $G(b|x)$ for both small and large bidders where x is the engineers estimate.
- Calculate the firm's cost C_i given bid b_i using the appropriate equation depending on the business classification (small or large).

Number of Bids per Bidder Combination:

	Number of Small Business Bidders	Number of Large Business Bidders	count
10	1	3	238
1	0	3	189
9	1	2	168
2	0	4	131
11	1	4	129
17	2	2	127
0	0	2	111
18	2	3	109
12	1	5	96
28	3	3	96

ESTIMATION RESULTS

- When carrying out estimation steps, I focused on auctions with 1 small bidder and 3 large bidders.
  - The auctions with this bidder combination had bid and cost distributions that were approximately normal after the removal of outliers allowing for easier estimation.
  - After the removal of outliers, there were only **26 auctions** with **104 bids**.

With the Bid Preference Program:

- The total procurement cost faced by Caltrans was **$10,470,375**

Without the Bid Preference Program:

- The total procurement cost faced by Caltrans was: **$13,519,275**

- The average procurement cost **decreased** by $117,265 in the presence of the bid preference program.
- Small businesses wins **increased** from 5 auctions without the discount to 9 auctions with the discount.

CONCLUSIONS AND NEXT STEPS

Conclusions:

- The bid preference program appears to increase competition and decrease total procurement cost.
- Due to the small sample size and single bidder combination I'd be hesitant to draw any concrete conclusions.

Next Steps:

- Try more bidder combinations to generalize the results.
- Rerun the simulations in log scale (or other transformation) to prevent errors in estimation such as negative values.

QUESTIONS?