# Homework 9

INTRO TO DATA SCIENCE FALL 2024

DUE MONDAY 11/25

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**How to submit:** You will submit your handwritten work as a picture posted to Brightspace, and you will submit both a .py file and a .r file to Brightspace.

**Problem 1** A nurse in charge of hospital beds uses a Markov chain to model the state of patients the next day assuming they have four possibilities:

$$X_n \in \{\text{Healthy H, Sick S, Very sick V, Dead D}\}$$

with transition matrix:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0 & 0.3 & 0.4 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For parts $a$) through $c$), do your work by hand (ie do the matrix multiplication).

✓ a) Find $\mathbf{P}[X_2 = H | X_1 = V]$

✓ b) Find $\mathbf{P}[X_3 = H | X_1 = V]$

✓ c) Find $\mathbf{P}[X_3 = D | X_1 = S]$

For part $d$) through $f$), use Python:

✓ d) Write code using Python similar to the example on the provided slides for the Markov chain starting with "Sick" for any given number of steps.

✓ e) Run your simulation for 3 days five times. On your submitted paper, write each path which occured and the given probability that the path would happen.

✓ f) Run your simulation for 10 days enough times until you get simulations which end in both "Healthy" and "Dead." For your first and last simulation, write the path which occured and the given probability that the path would happen. How many paths did it take until you get both desired outcomes?


with transition matrix

$$P = \begin{bmatrix} 0.7 & 0.3 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.1 & 0.1 & 0.2 & 0.3 & 0.1 & 0.2 \\ 0.2 & 0 & 0.4 & 0.1 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 0 & 0.6 & 0.4 \end{bmatrix}$$

a) If the mosquito bites Alice, what is the probability that the mosquito next bites Ben?

b) If the mosquito has just bitten Criss, what is the probability that the next person the mosquito bites traps the mosquito into a two-person cycle forever?

c) If the mosquito bites Dorothy first, find the probability that the third person the mosquito bites is Ben. (note: you do NOT need the whole 2-step transition matrix. Just the entry for D $\rightarrow$ B).

For parts *d*) through *f*), use R

d) Write code in R to create a function which will find the desired $n$-step transition matrix.

e) Use *d*) to find (and write down) the 3 step transition matrix, and use it to find

i) $\mathbf{P}[X_4 = B | X_1 = A]$

ii) $\mathbf{P}[X_4 = B | X_1 = D]$

iii) $\mathbf{P}[X_4 = C | X_1 = F]$

f) Write down the 5-step transition matrix, the 8-step transition matrix, and the 12-step transition matrix with entries rounded to 6-decimal places (if provided by R).

g) Using your answer for *f*), if the mosquito starts by biting Criss, estimate the probability that the 1000th bite is each person.

h) Using your answer for *f*), if the mosquito starts by biting Ben, estimate the probability that the 1000th bite is each person.

3-step : (e.)

5-step :

8-step : (f.)

12-step

(g.)

(h.)

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source("/Users/samstrickler/Downloads/School/Intro Data Science/Programming Assignments/Assignment 9/as9.r", encoding = "UTF-8")
> source("/Users/samstrickler/Downloads/School/Intro Data Science/Programming Assignments/Assignment 9/as9.r", encoding = "UTF-8")
3-step Transition Matrix:
      [,1]  [,2]  [,3]  [,4]  [,5]  [,6]
[1,] 0.628 0.372 0.000 0.000 0.000 0.000
[2,] 0.620 0.380 0.000 0.000 0.000 0.000
[3,] 0.224 0.126 0.068 0.057 0.375 0.150
[4,] 0.228 0.122 0.076 0.049 0.383 0.142
[5,] 0.000 0.000 0.000 0.000 0.752 0.248
[6,] 0.000 0.000 0.000 0.000 0.744 0.256
P[X4 = B | X1 = A]: 0.372
P[X4 = B | X1 = D]: 0.122
P[X4 = C | X1 = F]: 0

5-step Transition Matrix:
        [,1]    [,2]    [,3]    [,4]    [,5]    [,6]
[1,] 0.62512 0.37488 0.00000 0.00000 0.00000 0.00000
[2,] 0.62480 0.37520 0.00000 0.00000 0.00000 0.00000
[3,] 0.24396 0.14354 0.01772 0.01353 0.43175 0.14950
[4,] 0.24412 0.14338 0.01804 0.01321 0.43207 0.14918
[5,] 0.00000 0.00000 0.00000 0.00000 0.75008 0.24992
[6,] 0.00000 0.00000 0.00000 0.00000 0.74976 0.25024

8-step Transition Matrix:
         [,1]     [,2]     [,3]     [,4]     [,5]     [,6]
[1,] 0.625001 0.374999 0.000000 0.000000 0.000000 0.000000
[2,] 0.624998 0.375002 0.000000 0.000000 0.000000 0.000000
[3,] 0.249256 0.149182 0.002233 0.001673 0.447731 0.149925
[4,] 0.249256 0.149182 0.002231 0.001676 0.447731 0.149925
[5,] 0.000000 0.000000 0.000000 0.000000 0.750001 0.249999
[6,] 0.000000 0.000000 0.000000 0.000000 0.749998 0.250002

12-step Transition Matrix:
         [,1]     [,2]    [,3]     [,4]     [,5]     [,6]
[1,] 0.625000 0.375000 0.00000 0.000000 0.000000 0.000000
[2,] 0.625000 0.375000 0.00000 0.000000 0.000000 0.000000
[3,] 0.249953 0.149949 0.00014 0.000105 0.449858 0.149995
[4,] 0.249953 0.149949 0.00014 0.000105 0.449858 0.149995
[5,] 0.000000 0.000000 0.00000 0.000000 0.750000 0.250000
[6,] 0.000000 0.000000 0.00000 0.000000 0.750000 0.250000

Estimated probabilities for the 1000th bite starting from Criss:
[1]  2.500000e-01  1.500000e-01 5.332935e-302 3.999701e-302  4.500000e-01
[6]  1.500000e-01

Estimated probabilities for the 1000th bite starting from Ben:
[1] 0.625 0.375 0.000 0.000 0.000 0.000
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$$X_n \in \{\text{Healthy H, Sick S, Very sick V, Dead D}\}$$

with transition matrix:

$$P = \begin{array}{c} \\ H \\ S \\ V \\ D \end{array} \begin{array}{c} \begin{array}{cccc} H & S & V & D \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0 & 0.3 & 0.4 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

For parts $a$) through $c$), do your work by hand (ie do the matrix multiplication).

a) Find $\mathbf{P}[X_2 = H | X_1 = V]$

b) Find $\mathbf{P}[X_3 = H | X_1 = V]$

c) Find $\mathbf{P}[X_3 = D | X_1 = S]$

(a.) $P[X_2 = H \mid X_1 = V] \Rightarrow$ Use 1-step transition matrix

$\Rightarrow P = 0$

(b.) $P[X_3 = H \mid X_1 = V] \Rightarrow$ Use 2-step transition matrix

$$\Rightarrow P^2 = \underset{4\times4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0 & 0.3 & 0.4 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{4\times4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0 & 0.3 & 0.4 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$= \underset{4\times4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.6 & 0.1 & 0.12 & 0.18 \\ 0.15 & 0.18 & 0.22 & 0.45 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$\Rightarrow P = 0.15$

(c.) $P[X_3 = D \mid X_1 = S] = 0$

**Problem 2** A mosquito is in a room with 6 friends named Alice, Ben, Criss, Dorothy, Elsie, and Fiona. After biting a person, the mosquito flies off and then bites a person again. The $n$-th person to be bitten can be modeled by a Markov chain with state space

$$X_n \in \{A, B, C, D, E, F\}$$

with transition matrix

$$P = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0.7 & 0.3 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.1 & 0.1 & 0.2 & 0.3 & 0.1 & 0.2 \\ 0.2 & 0 & 0.4 & 0.1 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 0 & 0.6 & 0.4 \end{bmatrix} \end{array}$$

✓ a) If the mosquito bites Alice, what is the probability that the mosquito next bites Ben?

✓ b) If the mosquito has just bitten Criss, what is the probability that the next person the mosquito bites traps the mosquito into a two-person cycle forever?

✓ c) If the mosquito bites Dorothy first, find the probability that the third person the mosquito bites is Ben. (note: you do NOT need the whole 2-step transition matrix. Just the entry for D $\rightarrow$ B).

(a.) $P(A \rightarrow B) = 0.3$

(b.) $P(C \rightarrow \infty) = 0$

(c.) $P(X_3 = B \mid X_1 = D) = 0.1$

given $P^2 = \begin{bmatrix} .64 & .36 & 0 & 0 & 0 & 0 \\ .6 & .4 & 0 & 0 & 0 & 0 \\ .2 & .1 & .16 & .09 & .31 & .14 \\ .2 & .1 & .12 & .13 & .31 & .14 \\ 0 & 0 & 0 & 0 & .76 & .24 \\ 0 & 0 & 0 & 0 & .72 & .28 \end{bmatrix}$

& Code submitted as text files on Brightspace!