Database Management System

Lecture 4: Functional Dependency

Chapter: 7

Outline

- **Functional dependency (FD)**
- **Keys**
- **Closure set of attributes**
- **Canonical form**
- **Equivalence**
- **Finding Candidate key, Prime and Non Prime attributes**

Functional Dependency (FD)

- The functional dependency is a ***relationship*** that exists between two attributes. It typically exists between the primary key and non-key attribute within a table.
  - X → Y
- The left side of FD is known as a ***determinant***, the right side of the production is known as a ***dependent***.
- Functional dependency is used as a tool for normalization
- Data is dependent on FD, FD is not dependent on Data.

students

Id	Name	Roll	Phone	Address	gpa
1	Kabir	121	0178	Dhaka	3.5
2	Khan	122	019088	Barishal	3.45
3	Alice	123	0192320	Barishal	2.86
4	Bob	124	088889	Dhaka	4
5	Campher	125	09898089	New york	4
6	Alice	126	979009	Khulna	3.75

Functional Dependencies Definition

- Let R be a relation schema

$$\alpha \subseteq R \;\; and \;\; \beta \subseteq R$$

- The **functional dependency**

$$\alpha \rightarrow \beta$$

holds on R if and only if for any legal relations r(R), whenever any two tuples t_1 and t_2 of r agree on the attributes α, they also agree on the attributes β. That is,

$$t_1[\alpha] = t_2 [\alpha] \;\; \Rightarrow \;\; t_1[\beta] \; = t_2 [\beta]$$

- Example: Consider r(A,B) with the following instance of r.

1	4
1	5
3	7

- On this instance, $B \rightarrow A$ hold; $A \rightarrow B$ does **NOT** hold,

Examples

- $A \rightarrow BC$
- $DE \rightarrow C$
- $C \rightarrow DE$
- $BC \rightarrow D$

A	B	C	D	E
a	2	3	4	5
2	a	3	4	5
a	2	3	6	5
a	2	3	6	6

Types of FD

- Trivial
  - A $\rightarrow$ B has trivial functional dependency if B is a subset of A.
  - Examples: A $\rightarrow$ A, B $\rightarrow$ B, AB$\rightarrow$A
- Non Trivial
  - A $\rightarrow$ B has a non-trivial functional dependency if B is not a subset of A.
  - When A intersection B is NULL, then A $\rightarrow$ B is called as complete non-trivial.
  - Examples: A$\rightarrow$B, AB$\rightarrow$ABC

Closure of a Set of Functional Dependencies

- It is the ***complete set of all possible attributes*** that can be ***functionally derived from given functional dependency*** using the inference rules known as Armstrong's Rules.
- Given a set *F* set of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.
  - If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
- The set of **all** functional dependencies logically implied by *F* is the **closure** of *F*.
- We denote the *closure* of *F* by $\boldsymbol{F^+}$.

Armstrong's axioms or Inference Rules

- **Reflexivity Rule: If** $\beta \subseteq \alpha$ then: $\alpha \rightarrow \beta$
- **Augmentation Rule:** If $\alpha \rightarrow \beta$ then: $\alpha\gamma \rightarrow \beta\gamma$
- **Transitivity rule**. If $\alpha \rightarrow \beta$ holds and $\beta \rightarrow \gamma$ holds, then $\alpha \rightarrow \gamma$ holds

- ***Union rule****. If* $\alpha \rightarrow \beta$ *holds and* $\alpha \rightarrow \gamma$ *holds,*
  then $\alpha \rightarrow \beta\gamma$ *holds.*
- ***Decomposition rule****. If* $\alpha \rightarrow \beta\gamma$ *holds, then* $\alpha \rightarrow \beta$ *holds and* $\alpha \rightarrow \gamma$ *holds*

How to Find Closure set of Attributes

- R(ABC)
- *F:*
  - $A \rightarrow B$
  - $B \rightarrow C$
- $A^+ \rightarrow \{A,B,C\}$
- $B^+ \rightarrow \{B,C\}$
- $C^+ \rightarrow \{C\}$

Exercises

R (ABCDEFG)

FD:

- $A \rightarrow B$
- $BC \rightarrow DE$
- $AEG \rightarrow G$

$(AC)^{+} = ?$

R (ABCDE)

FD:

- $A \rightarrow BC$
- $CD \rightarrow E$
- $B \rightarrow D$
- $E \rightarrow A$

$(B)^{+} = ?$ And $(AB)^{+} = ?$

R (ABCDEFGH)

FD:

- $A \rightarrow BC$
- $CD \rightarrow E$
- $E \rightarrow C$, $D \rightarrow AEH$
- $ABH \rightarrow BD$, $DH \rightarrow BC$

$BCD \rightarrow H$?

Equivalence of FD

- R(ABCDEH)
- F_1:
  - $A \rightarrow C$
  - $AC \rightarrow D$
  - $E \rightarrow AD$
  - $E \rightarrow H$
- F_2:
  - $A \rightarrow CD$
  - $E \rightarrow AH$

Irreducible set of FD (Canonical Cover)

- A ***canonical cover*** is a simplified and reduced version of the given **set** of functional dependencies.

- **R(ABCD)**
- *FD:*
  - $B \rightarrow A$
  - $AD \rightarrow BC$
  - $C \rightarrow ABD$
- Canonical Cover (F^c):
  - $B \rightarrow A$
  - $AD \rightarrow C$
  - $C \rightarrow BD$

Keys→ SK, CK, PK

- R(ABCD)
  - FD1: A→BC
  - FD2: ABC→ D, AB→CD, A→BCD
  - FD3: B→ACD, ACD→B
- Super key is the key that can uniquely identify any record in a database
- Candidate Keys are super keys with the least number of attributes. *Any CK can not be proper sub set of any other super keys.*
  - *A →BCD*
  - *AB →CD [here A is the subset of the super key AB]*

Finding Candidate Keys

- R(ABCD)
  - A→B, B→C, C→A
- A(ABCD)
  - AB→CD, D→A
- R(ABCDEF)
  - AB→C, C→D, B→AE
- R(ABCDE)
  - AB→CD, D→A, BC→DE

Prime and Non-Prime Attribute

- Prime Attribute: Member of candidate key
- Non-Prime Attribute: Attributes those are not member of candidate key

Chapter 7: Functional Dependency

END OF LECTURE