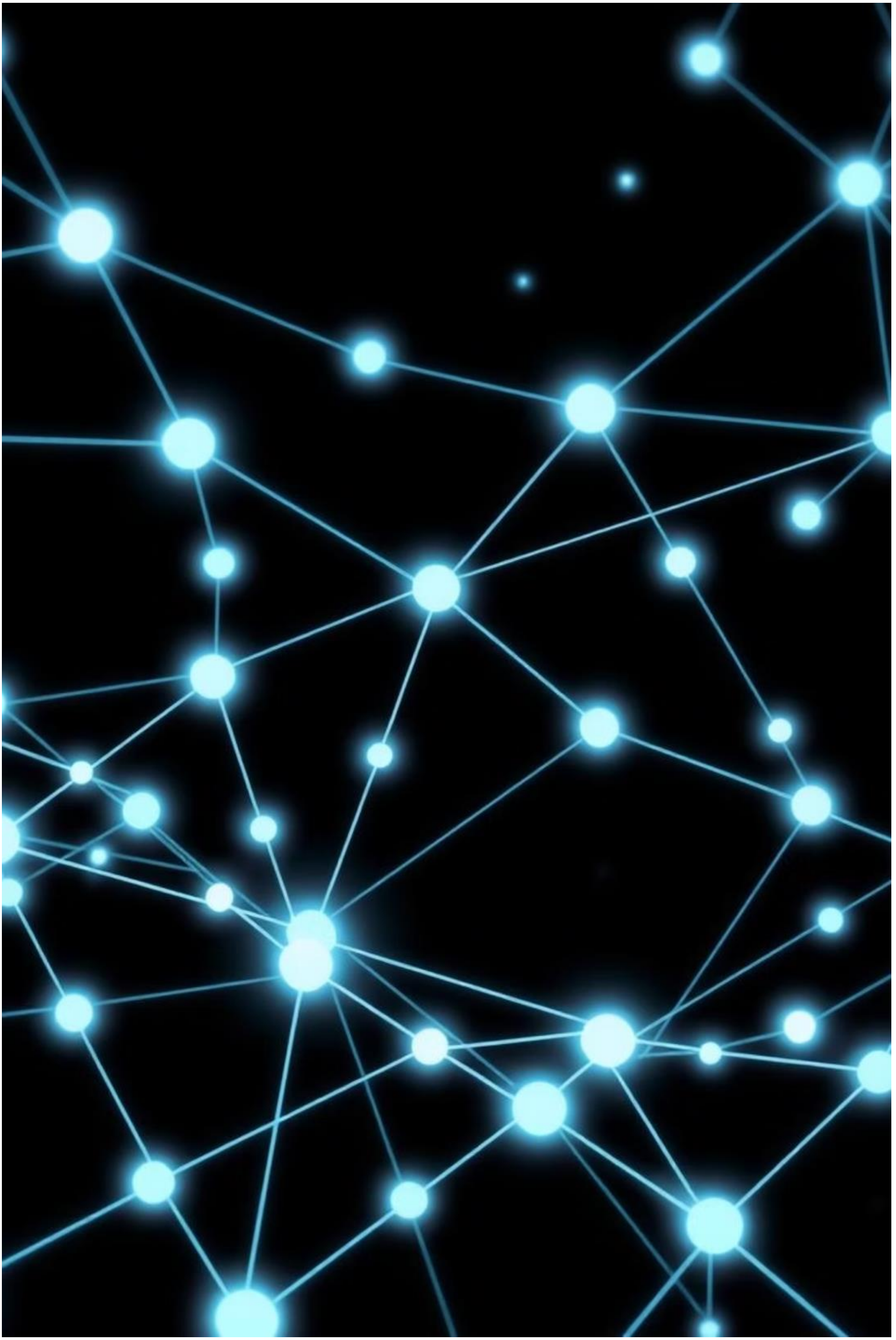

Data Structures

Lecture 8: Sorting

Instructor:
Md Samsuddoha
Assistant Professor
Dept of CSE, BU

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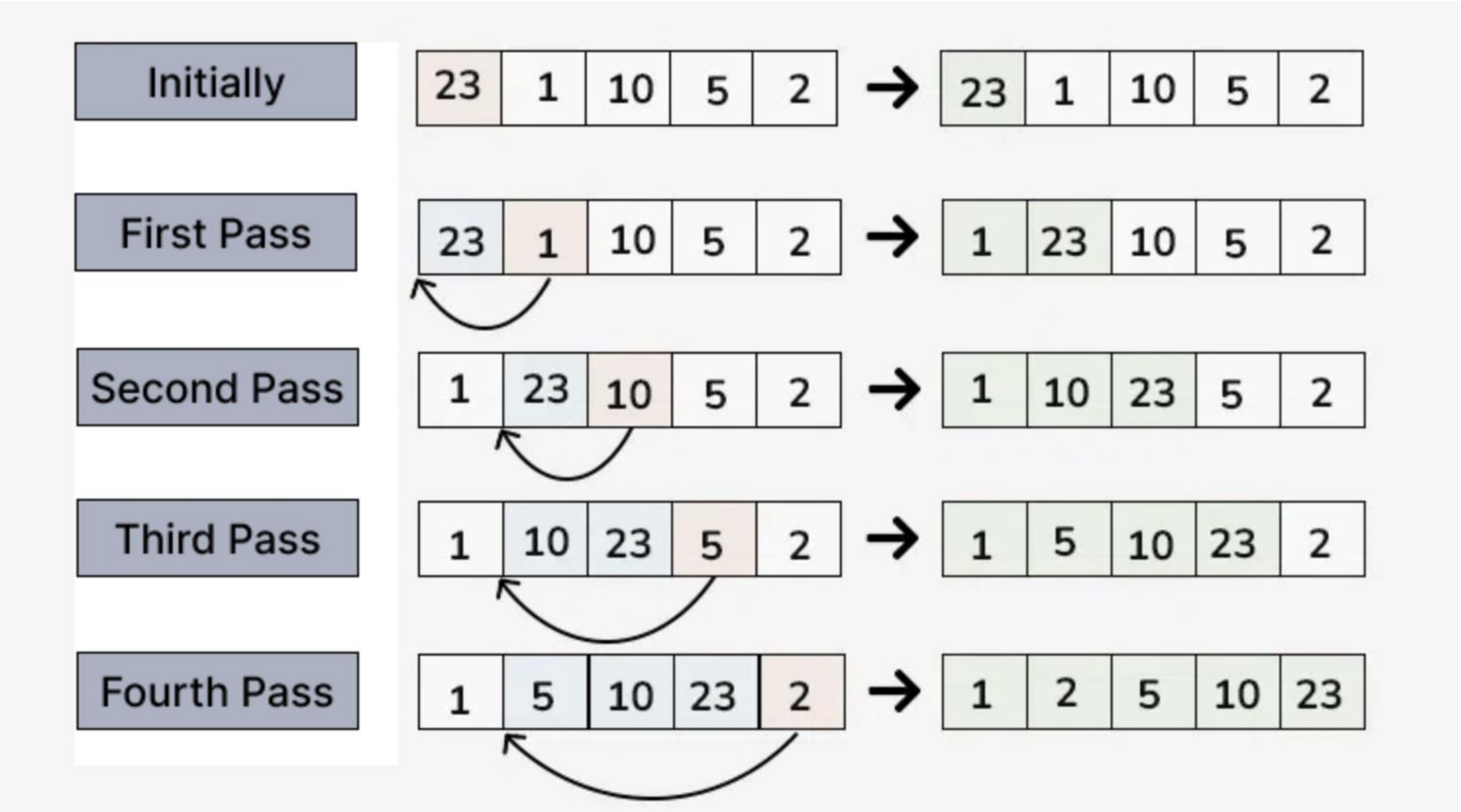
- Insertion Sort
- Counting Sort
- Merge Sort

Insertion Sort

- **Insertion sort** is a simple sorting algorithm that works by iteratively inserting each element of an unsorted list into its correct position in a sorted portion of the list.
- Process of Sorting:
  - We start with the ***second element*** of the array as the first element is assumed to be sorted.
  - Compare the second element with the first element if the second element is smaller then swap them.
  - Move to the third element, compare it with the first two elements, and put it in its correct position
  - Repeat until the entire array is sorted.


Initially
23 1 10 5 2
23 1 10 5 2
First Pass
23 1 10 5 2
1 23 10 5 2
Second Pass
1 23 10 5 2
1 10 23 5 2
Third Pass
1 10 23 5 2
1 5 10 23 2
Fourth Pass
1 5 10 23 2
1 2 5 10 23

Complexity

- Worst case: array is in **reverse order**.
- Number of Iterations: N-1
- Number of comparisons in each pass:
  - 1st pass → 1 comparison
  - 2nd pass → 2 comparisons
  - …
  - Last pass → n-1 comparisons
- Total comparisons = 1+2+3 …..+(n-1) = n(n-1)/2
- **Worst-case time complexity:** $O(n^2)$

Pseudocode

```
InsertionSort(A, n):
    for i from 1 to n-1:
        key = A[i]
        j = i - 1

        while j >= 0 and A[j] > key:
            A[j + 1] = A[j]
            j = j - 1

        A[j + 1] = key
```

Coding

- Write a C program for Insertion sort.

Counting Sort

- Counting Sort is a ***non-comparison-based*** sorting algorithm.
- Instead of comparing elements (like ***bubble, merge, quick***), Counting Sort counts how many times each value appears.
- The basic idea behind Counting Sort is to count the **frequency** of each distinct element in the input array and use that information to place the elements in their correct sorted positions.
- It is super fast when:
  - The range of values is small
  - Data contains integers or integer-like values (grades, ages, IDs, frequencies)

Counting Sort Algorithm

- Declare a count array **cntArr[]** of size **max(arr[])+1** and initialize it with **0**.
- Traverse input array **arr[]** and map each element of **arr[]** as an index of **cntArr[]** array, i.e., execute **cntArr[arr[i]]++** for **0 <= i < N**.
- Calculate the prefix sum at every index of **cntArr[]**.
- Create an array **ans[]** of size **N**.
- Traverse array **arr[]** from end and update **ans[cntArr[arr[i]] - 1] = arr[i]**. Also, update cntArr**[arr[i]] = cntArr[arr[i]]- -** .

01 Step | Find out the maximum element from the given array.

arr[]=

0	1	2	3	4	5	6	7
2	5	3	0	2	3	0	3

Max
5

02 Step

Initialize a cntArr[] of length max+1 with all elements as 0. This array will be used for storing the occurrences of the elements of the input array.

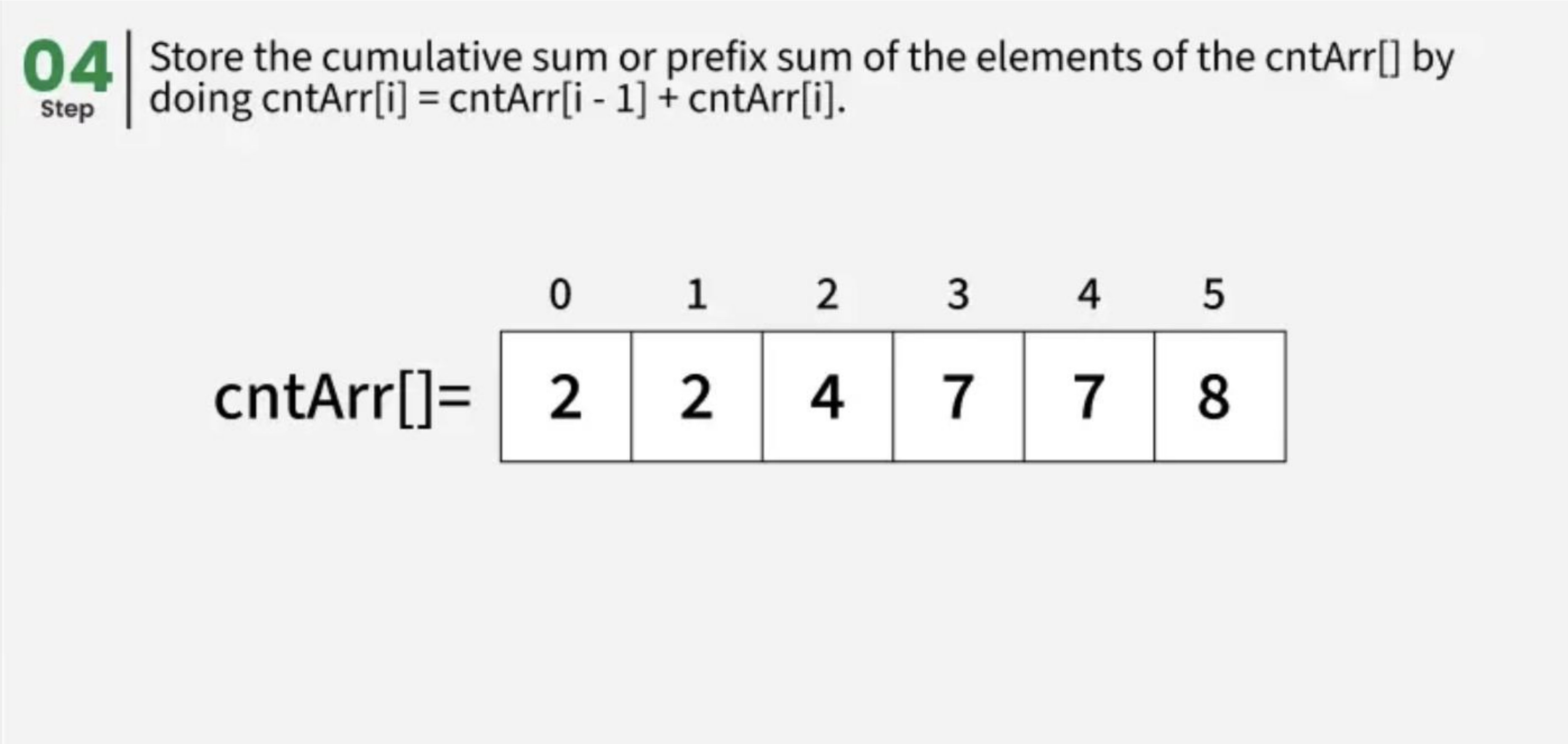
cntArr[]=

0	1	2	3	4	5
0	0	0	0	0	0

03 Step

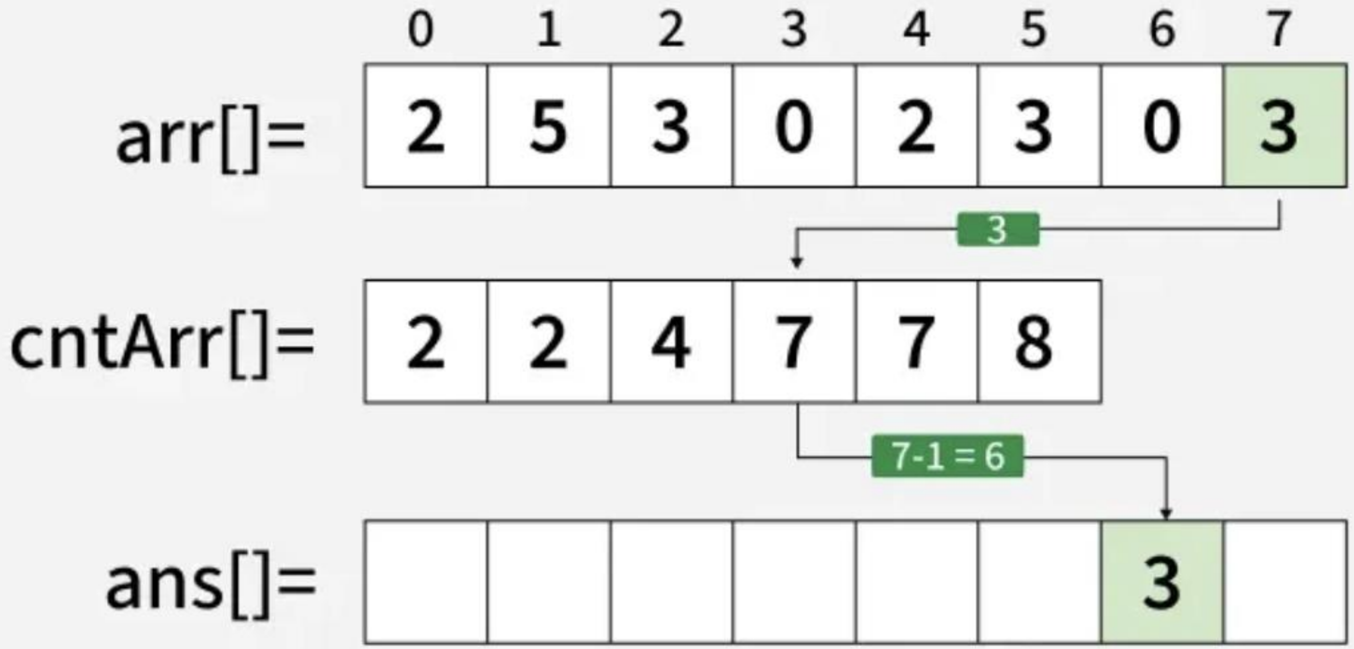
In the cntArr[], store the count of each unique element of the input array at their respective indices.

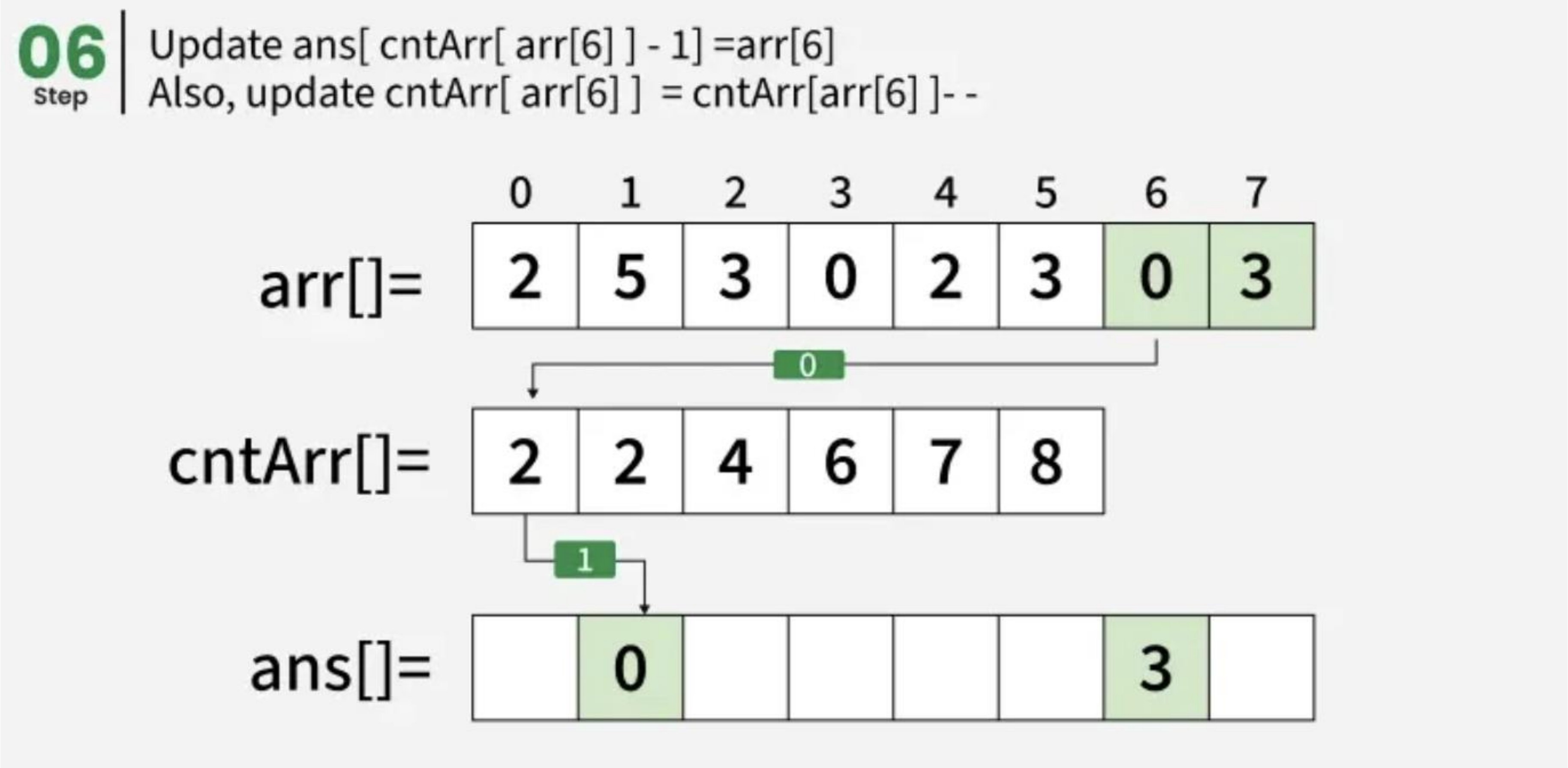
	0	1	2	3	4	5
cntArr[]=	2	0	2	3	0	1

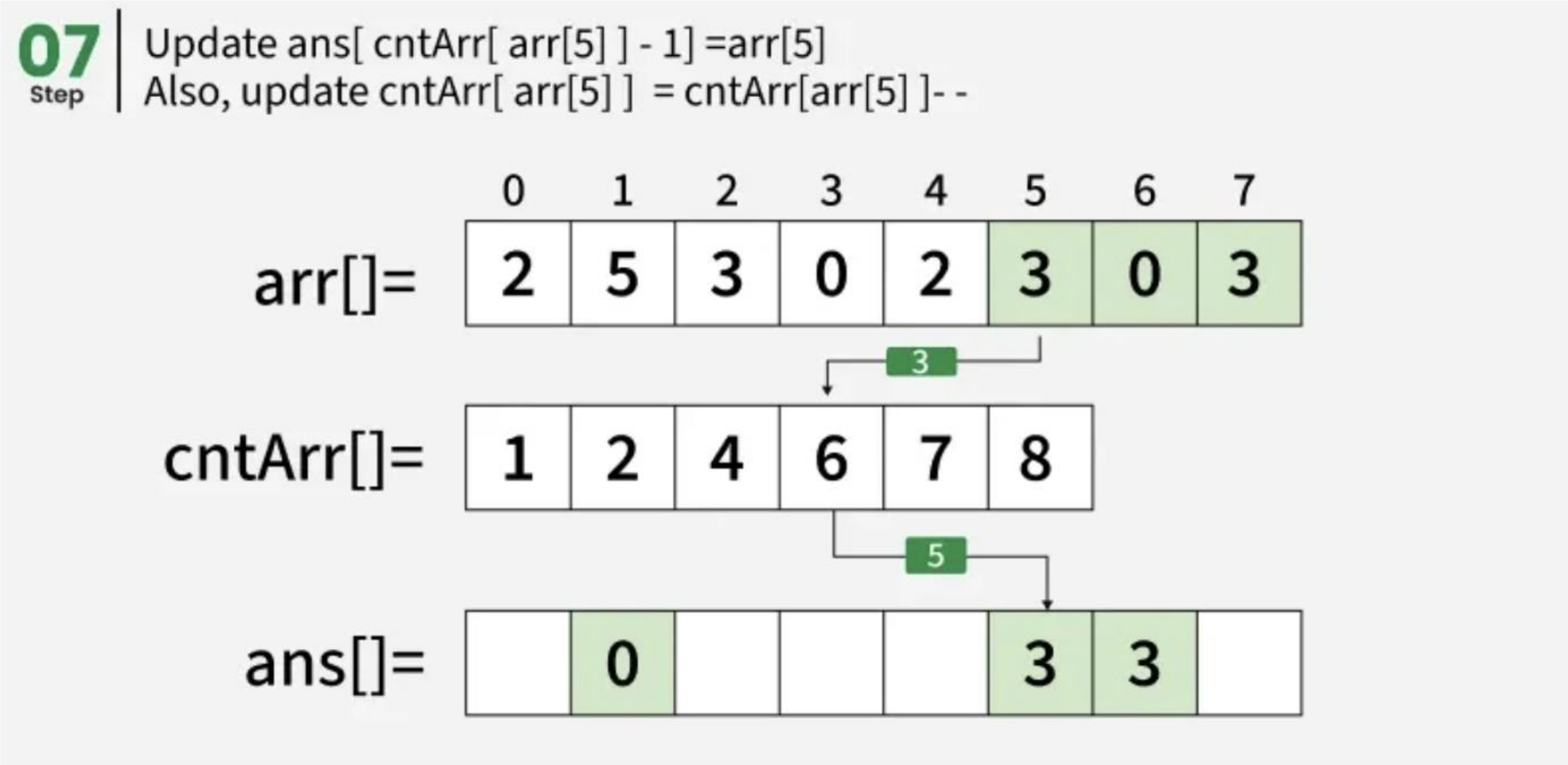

04
Step
Store the cumulative sum or prefix sum of the elements of the cntArr[] by doing cntArr[i] = cntArr[i - 1] + cntArr[i].
0 1 2 3 4 5
cntArr[]= 2 2 4 7 7 8

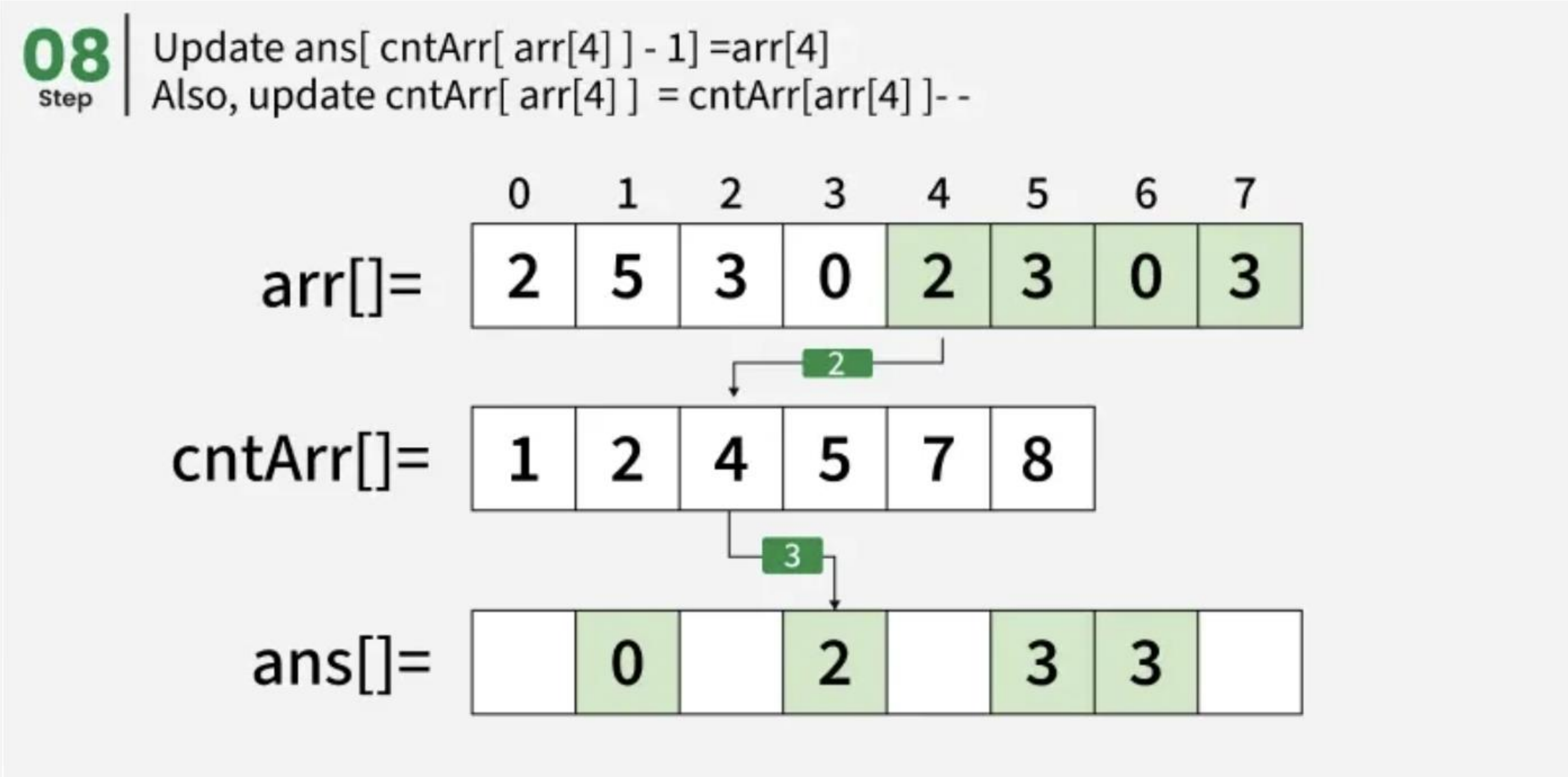
05 Step

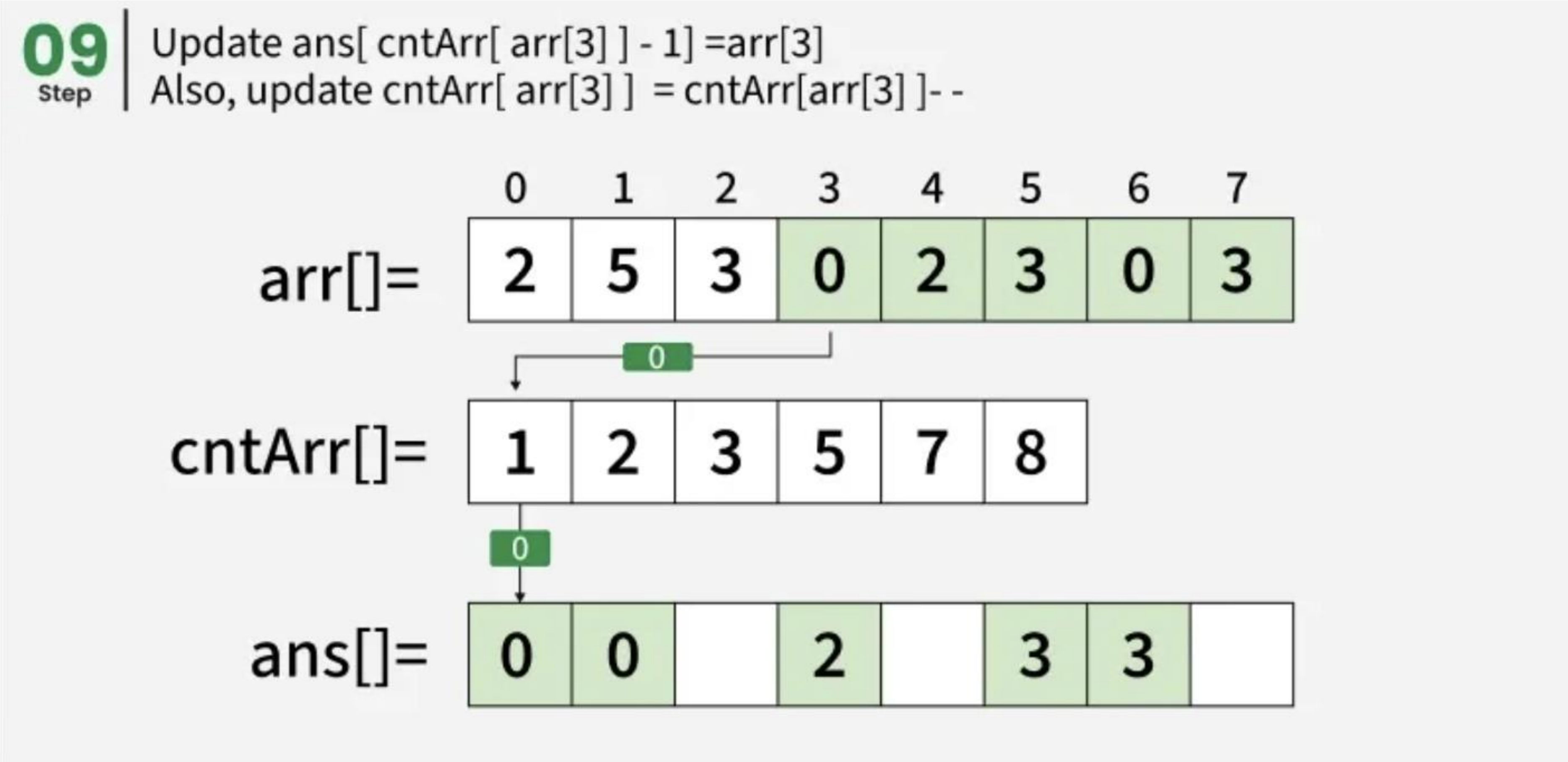
Update ans[cntArr[arr[i]] - 1] = arr[i] and decrement cntArr[arr[i]]. Traverse the input array in reverse to maintain the order of equal elements, ensuring the sort remains stable.

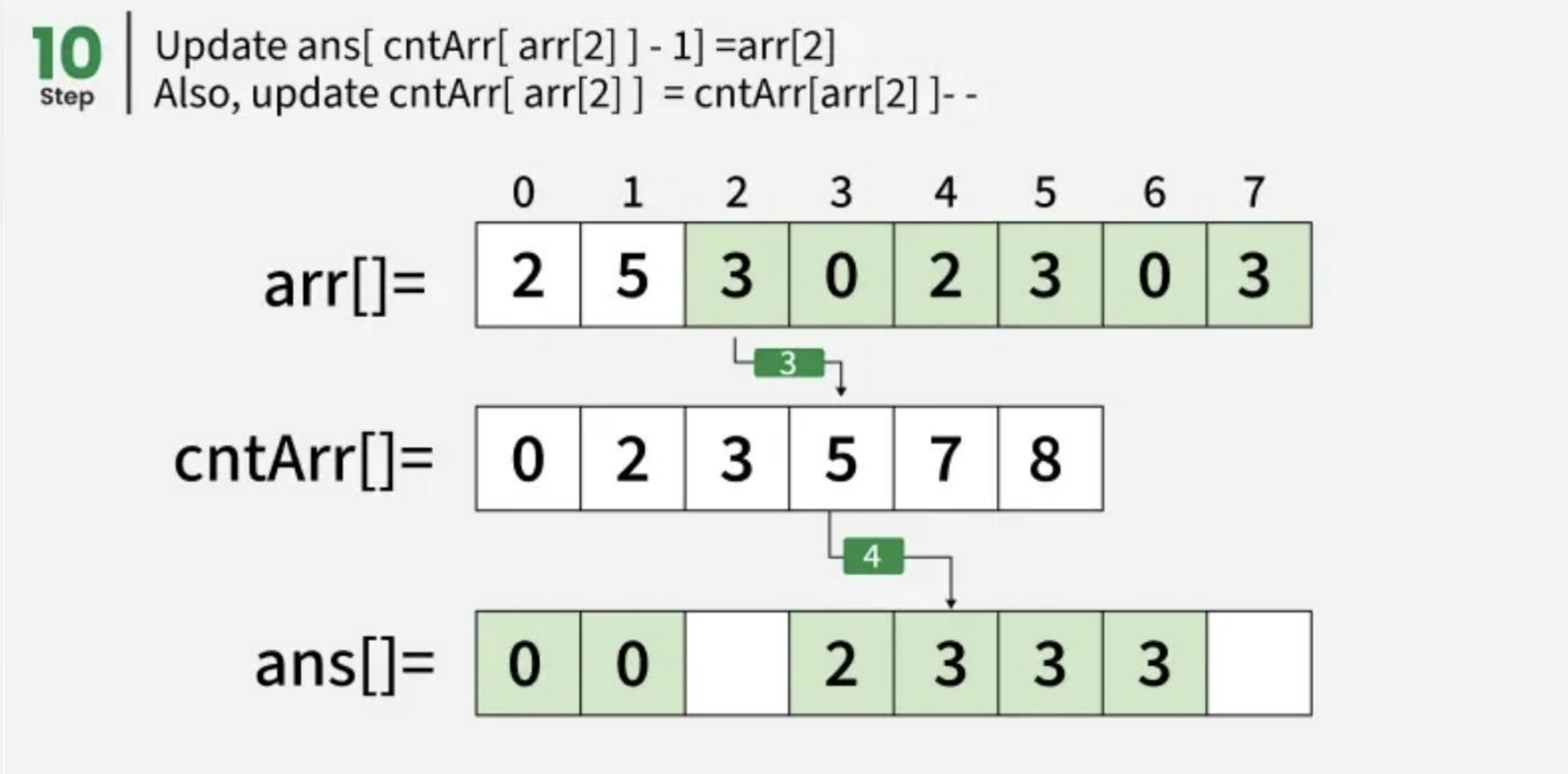


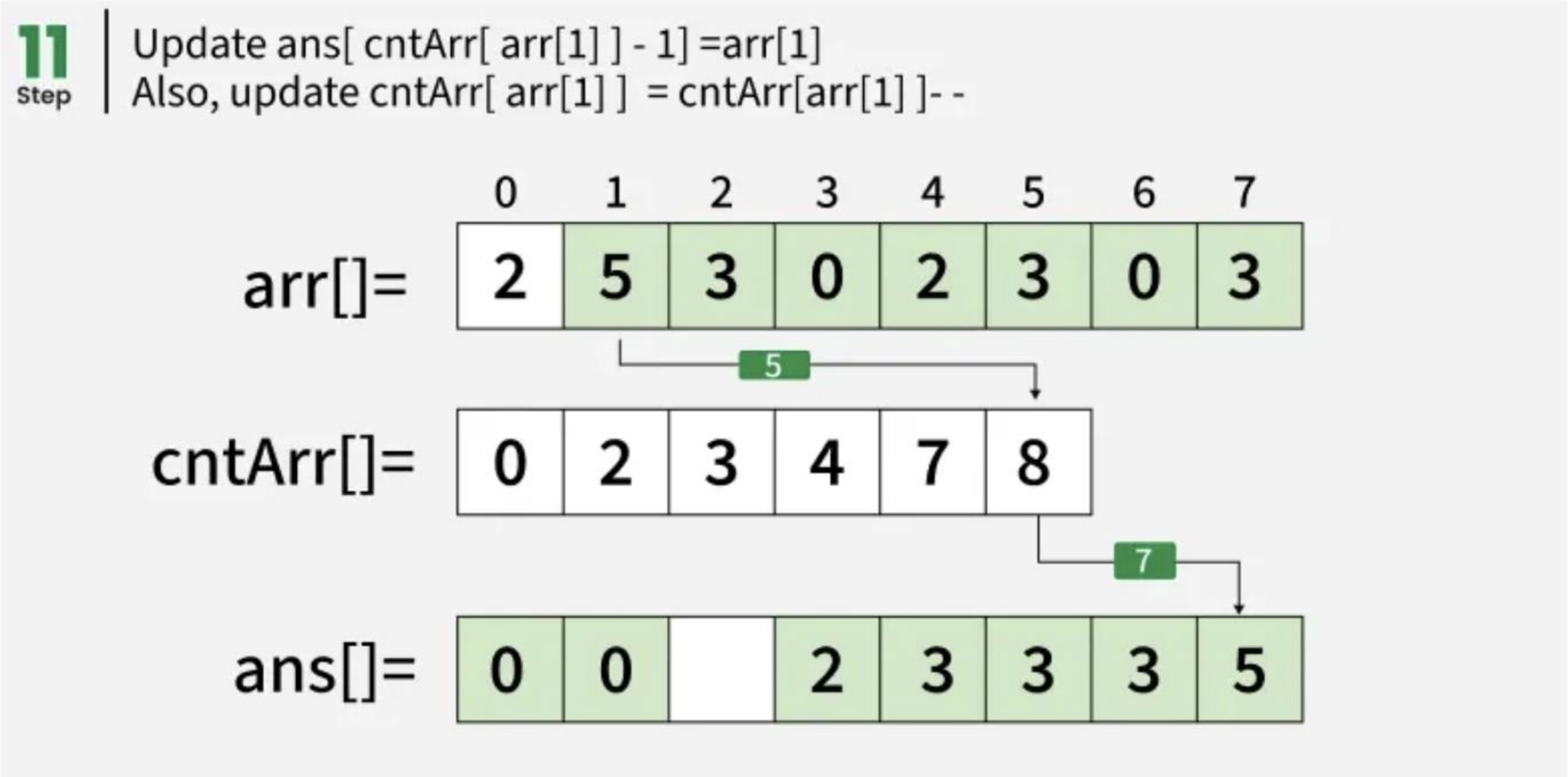

06
Step
Update ans[cntArr[arr[6]] - 1] =arr[6]
Also, update cntArr[arr[6]] = cntArr[arr[6]]- -
0 1 2 3 4 5 6 7
arr[]=
2 5 3 0 2 3 0 3
0
cntArr[]=
2 2 4 6 7 8
1
ans[]=
0 3

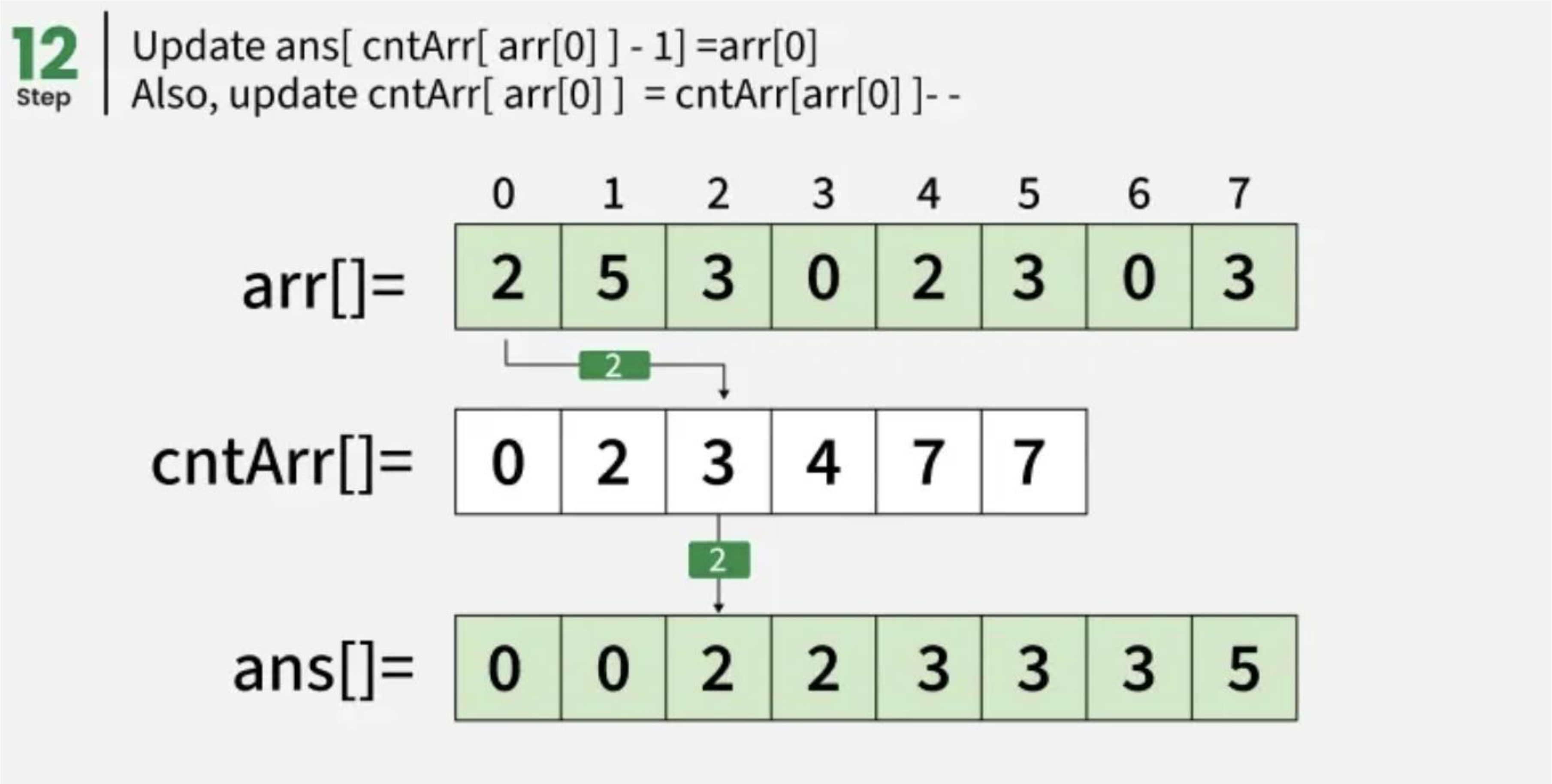

07
Step
Update ans[cntArr[arr[5]] - 1] =arr[5]
Also, update cntArr[arr[5]] = cntArr[arr[5]]- -
0
1
2
3
4
5
6
7
arr[]=
2
5
3
0
2
3
0
3
3
cntArr[]=
1
2
4
6
7
8
5
ans[]=
0
3
3


08
Step
Update ans[cntArr[arr[4]] - 1] =arr[4]
Also, update cntArr[arr[4]] = cntArr[arr[4]]- -
0 1 2 3 4 5 6 7
arr[]=
2 5 3 0 2 3 0 3
2
cntArr[]=
1 2 4 5 7 8
3
ans[]=
0 2 3 3


09
Step
Update ans[cntArr[arr[3]] - 1] =arr[3]
Also, update cntArr[arr[3]] = cntArr[arr[3]]- -
0 1 2 3 4 5 6 7
arr[]= 2 5 3 0 2 3 0 3
0
cntArr[]= 1 2 3 5 7 8
0
ans[]= 0 0 2 3 3


10
Step
Update ans[cntArr[arr[2]] - 1] =arr[2]
Also, update cntArr[arr[2]] = cntArr[arr[2]]- -
0 1 2 3 4 5 6 7
arr[]=
2 5 3 0 2 3 0 3
3
cntArr[]=
0 2 3 5 7 8
4
ans[]=
0 0 2 3 3 3


11
Step
Update ans[cntArr[arr[1]] - 1] =arr[1]
Also, update cntArr[arr[1]] = cntArr[arr[1]]- -
0 1 2 3 4 5 6 7
arr[]=
2 5 3 0 2 3 0 3
5
cntArr[]=
0 2 3 4 7 8
7
ans[]=
0 0 2 3 3 3 5



12
Step
Update ans[cntArr[arr[0]] - 1] =arr[0]
Also, update cntArr[arr[0]] = cntArr[arr[0]]- -
0 1 2 3 4 5 6 7
arr[]=
2 5 3 0 2 3 0 3
2
cntArr[]=
0 2 3 4 7 7
2
ans[]=
0 0 2 2 3 3 3 5

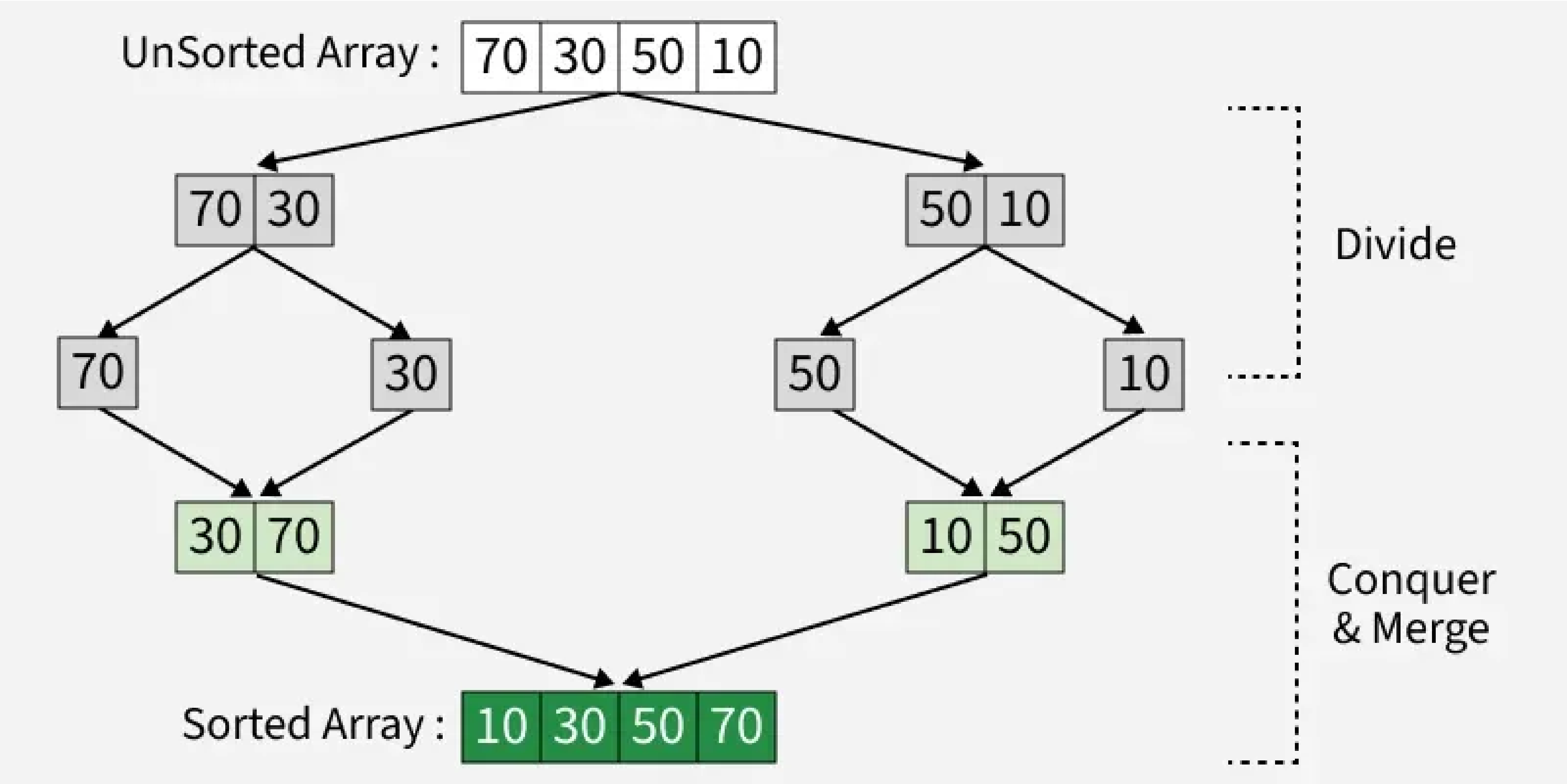
Complexity

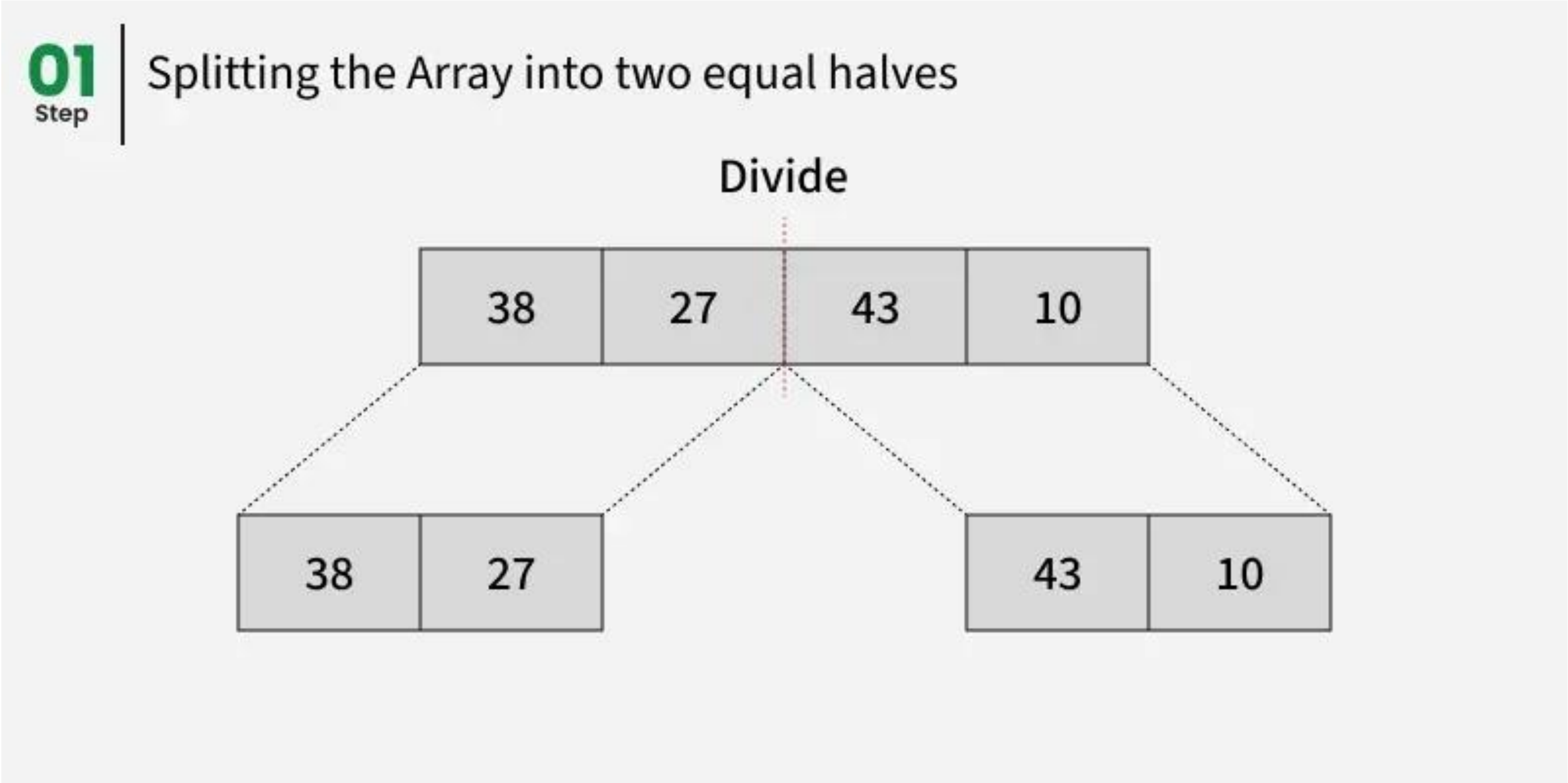
- **Time Complexity**: O(N+M) in all cases, where **N** and **M** are the size of **inputArray[]** and **countArray[]** respectively.
- **Auxiliary Space:** O(N+M), where **N** and **M** are the space taken by **outputArray[]** and **countArray[]** respectively.

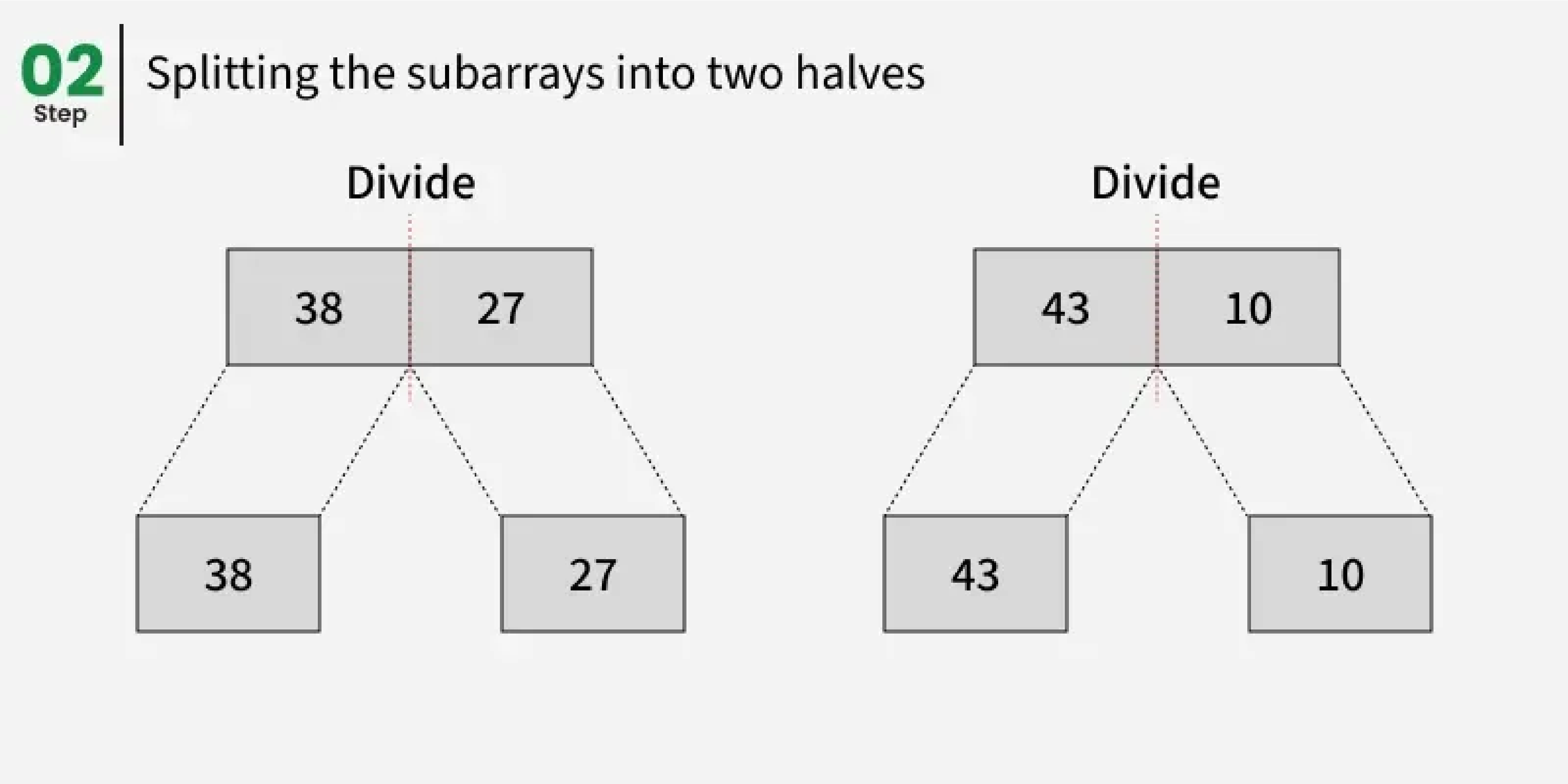
```
COUNTING-SORT(A, k)
    // A is the input array
    // k is the maximum value in A

    Create array count[0..k] initialized to 0
    Create array output of same length as A

    // Step 1: Count occurrences
    for i = 0 to length(A)-1
        count[A[i]] = count[A[i]] + 1

    // Step 2: Cumulative count
    for i = 1 to k
        count[i] = count[i] + count[i - 1]

    // Step 3: Build output array (stable)
    for i = length(A)-1 downto 0
        output[count[A[i]] - 1] = A[i]
        count[A[i]] = count[A[i]] - 1

    return output
```

Merge Sort

- **Merge sort** is a popular sorting algorithm known for its efficiency and stability.
- It follows the ***Divide and Conquer approach***.
- It works by dividing the input array into ***two halves*** and ***recursively sorting*** each half.
- Finally merging them back together to obtain the sorted array.

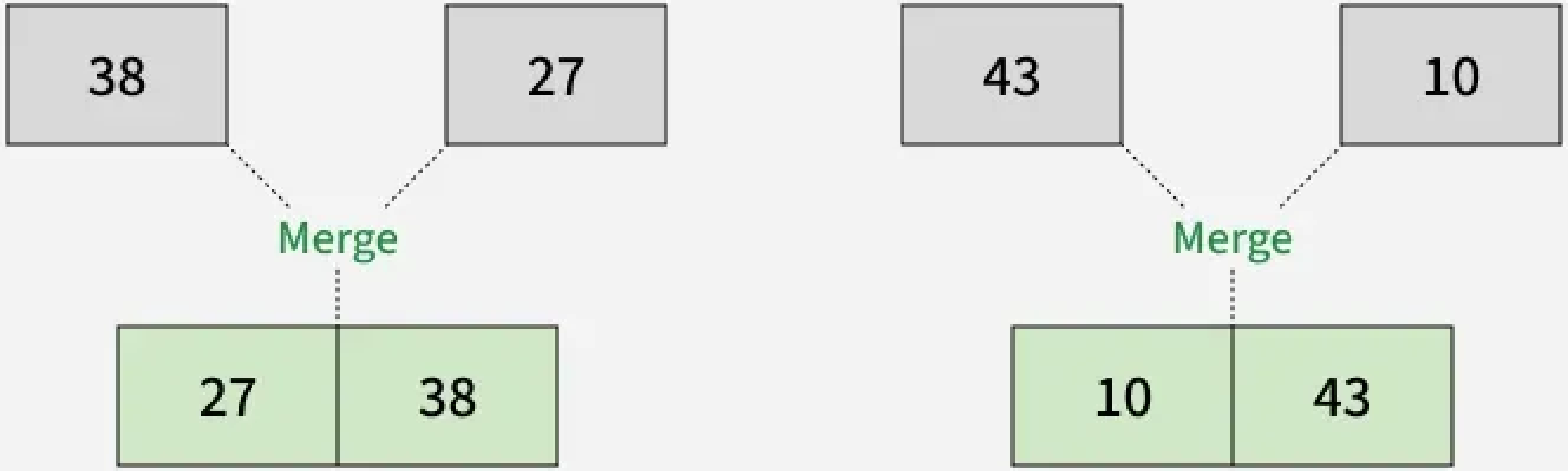

UnSorted Array :
70 30 50 10
70 30
50 10
70
30
50
10
Divide
30 70
10 50
Conquer
& Merge
Sorted Array :
10 30 50 70

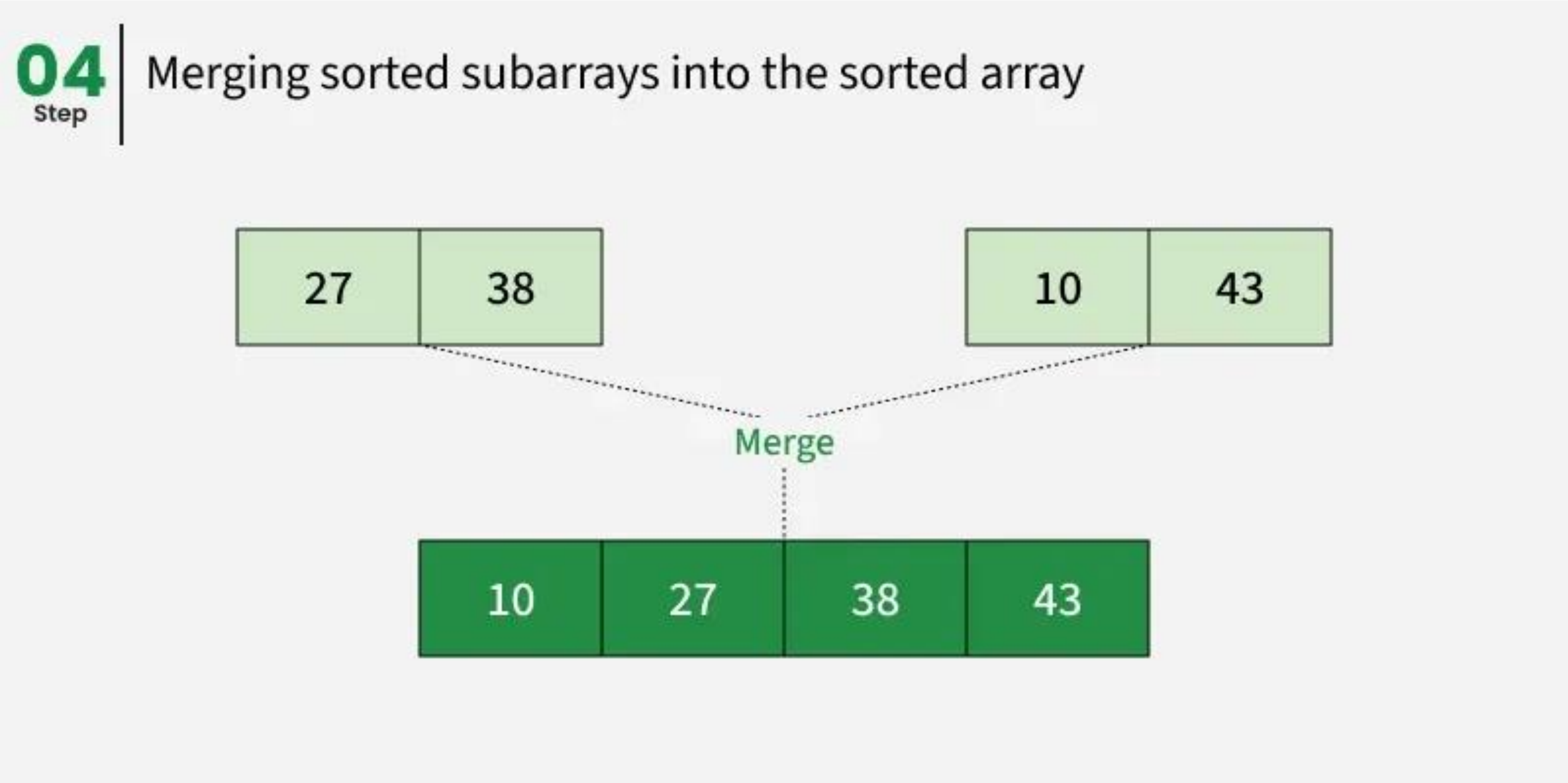

01
Step
Splitting the Array into two equal halves
Divide
38
27
43
10
38
27
43
10


02
Step
Splitting the subarrays into two halves
Divide
38
27
38
27
Divide
43
10
43
10

03 Step

Merging unit length cells into sorted subarrays




04
Step
Merging sorted subarrays into the sorted array
27
38
10
43
Merge
10
27
38
43

Complexity

- **Time Complexity:**
  - **Best Case:** O(n log n), When the array is already sorted or nearly sorted.
  - **Average Case:** O(n log n), When the array is randomly ordered.
  - **Worst Case:** O(n log n), When the array is sorted in reverse order.
- **Auxiliary Space:** O(n), Additional space is required for the temporary array used during merging.

```
MergeSort(arr, left, right):
    if left < right:
        mid = (left + right) / 2
        MergeSort(arr, left, mid)        # Sort left half
        MergeSort(arr, mid + 1, right)   # Sort right half
        Merge(arr, left, mid, right)     # Merge both halves

Merge(arr, left, mid, right):
    Create temporary arrays L[] and R[]
    Copy data into L[] = arr[left...mid]
                   R[] = arr[mid+1...right]
    i = 0, j = 0, k = left
    while (i < len(L) and j < len(R)):
        if L[i] <= R[j]:
            arr[k] = L[i]
            i = i + 1
        else:
            arr[k] = R[j]
            j = j + 1
        k = k + 1
    Copy remaining elements of L[], if any
    Copy remaining elements of R[], if any
```

References

- **Chapter 10: Data Structures using C** by E. Balagurusamy
- Visit the site for live visualization: https://visualgo.net/

Thank You