

PHSX815_Project2: Attack of the Cosmic Muons

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1 Introduction

Muons lifetimes can be measure using a scintillator, which traps the incoming muon and electron from the muon decay. The muon and the decay electron each produce two short light pulses. The time between these light pulses are measured to calculate the Muon decay time. A simulation for this scenario can help for students who have to collect data for four weeks before they can even attempt to distinguish between the positive and negative muon lifetime.

According to literature, an experiment recording the lifetimes of muon should give an exponential plot. However, this graph should be a result of superposition of two separate exponential plots. In the last simulation, we explored how close negative and positive lifetimes can be to still be distinguishable. This project we will explore whether the fraction of negative muons and positive muons make any noticeable difference to the average lifetimes of muons.

2 Hypotheses

Average muon lifetime is about $2.2\mu\text{s}$. The average value is a result of both positive and negative muon lifetime. According to the literature, the negative muon has a slightly lower lifetime than the positive muon. For the sake of this project we will assume that the negative muon has a lifetime of $2.2\mu\text{s}$, and the positive has a lifetime of $2.4\mu\text{s}$. Let's define a p as the fraction of negative muons. So if half of the N decays are negative muons, $p = 0.5$. This also means that our the fraction of positive muons will be $(1-p)$.

$$H_0 : p = 0.5$$

$$H_1 : p \neq 0.5$$

This hypothesis just tests what type of fraction of negative and positive muons will give us the colloquially observed superposition exponential plot.

3 Code and Experimental Simulation

The decay of the muon can be simulated using the exponential probability sampling. Exponential sampling is controlled using the rate parameter, λ . The rate parameter is related to any decay lifetime τ by the following relation:

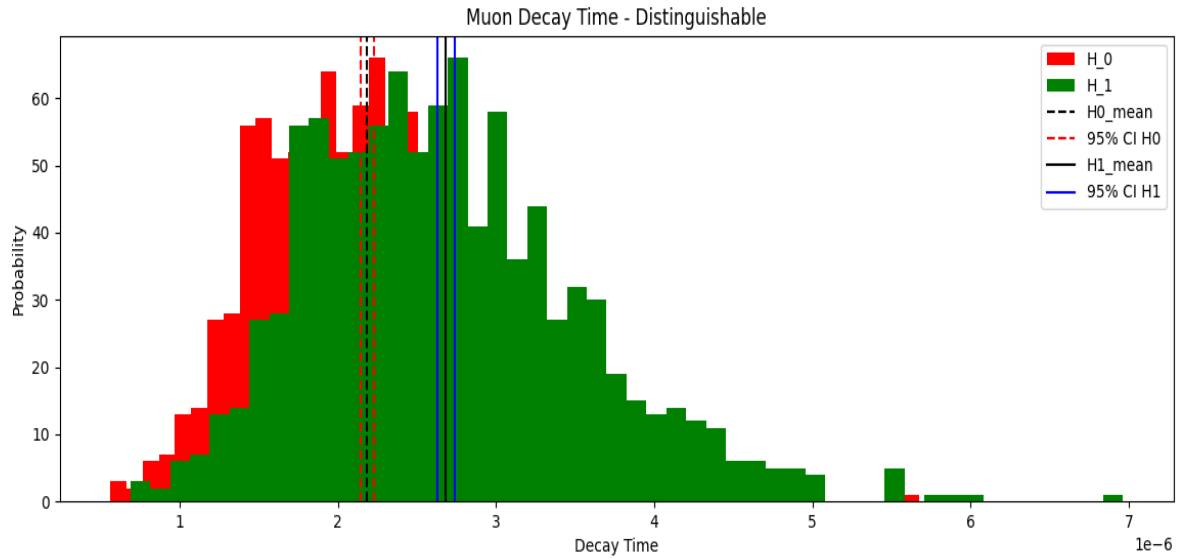


Figure 1: In the last simulation, we explored how close negative and positive lifetimes can be to still be distinguishable. To test the hypothesis, the code was used to plot the normal normal distribution for $\tau_p = 2.2 \mu s$ and null hypothesis of $\tau_n = 2.7 \mu s$. To further help the analysis the 95 % confidence interval was plotted for each of the histogram. Visually, it seems that the two lifetimes can be much closer and still be distinguishable. Hence, the negative muon lifetime is smaller than $2.7 \mu s$.

$$\lambda = \frac{1}{\tau}$$

The code will give you an option to set the values for: negative muon lifetime, positive muon lifetime, and the p. The code uses categorical random sampling to decide the number of positive and negative muon decays in N total events. The categorical sampling just uses the p and (1-p) values as the probability for negative and positive muons respectively.

To test our hypothesis, I ran the simulation for two scenarios with 100000 events each:

1. $p = 0.5$
2. $p = 0.2$

The code as default takes 10 measurements each event and writes all the recordings into a text file. The next step is to plot these values. Since the exponential plot is not easily discernable, The code considers the average of each event, so the final plot is Erlang.

4 Analysis

After plotting the two histograms together with their averages and the quadrants, it looks like the that the change in fraction p does not make significant change to the averages. However, we do start to notice that the plot has two distinct peaks for $p = 0.3$. However, for $p = 0.5$ there is no noticeable distinction. As we discussed earlier, a realistic simulation of muon decay would be superposition of two plots with an average of $2.2 \mu s$.

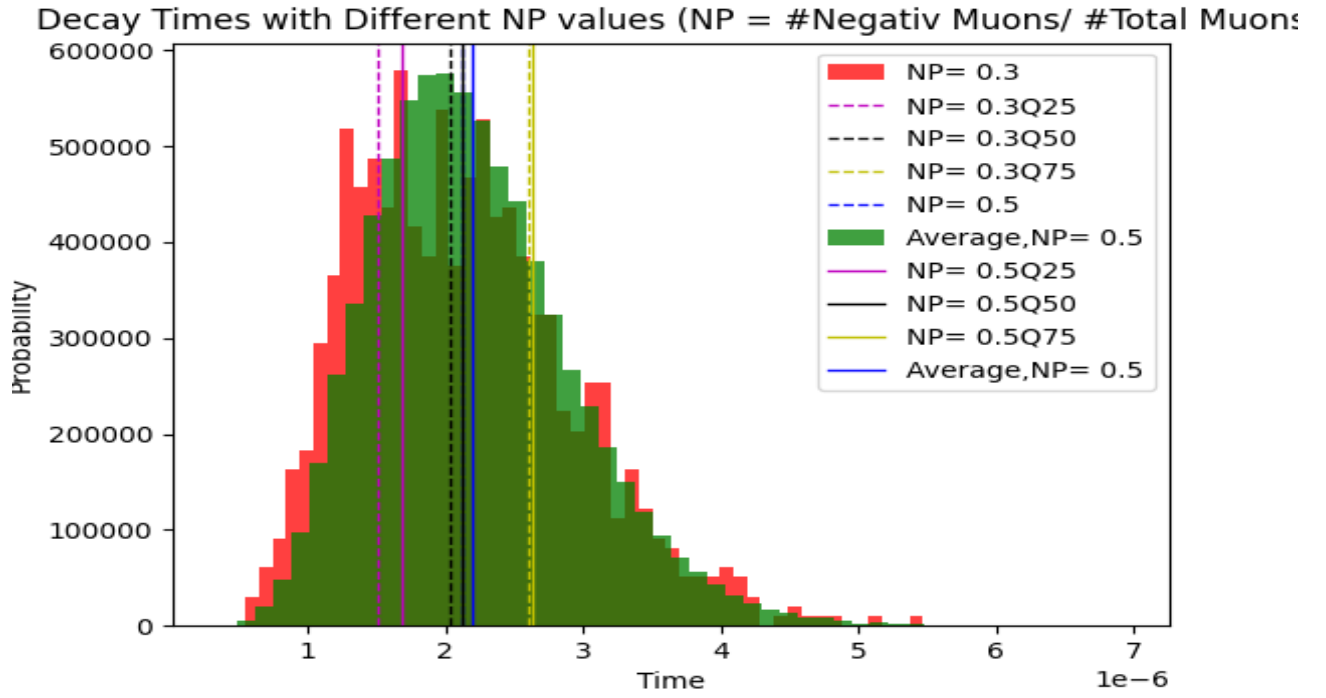


Figure 2: In the last simulation, we explored how close negative and positive lifetimes can be to still be distinguishable. To test the hypothesis, the code was used to plot the normal normal distribution for $\tau_p = 2.2 \mu s$ and null hypothesis of $\tau_n = 2.7 \mu s$. To further help the analysis the 95 % confidence interval was plotted for each of the histogram. Visually, it seems that the two lifetimes can be much closer and still be distinguishable. Hence, the negative muon lifetime is smaller than $2.7 \mu s$.

5 Conclusion

The simulation suggests that we should reject the null hypothesis in favor of alternative hypothesis. The simulation shows that in N events the fraction of positive muon and negative muons is different. The simulation is almost ready to be used in labs for students. An analysis of posterior distribution might be helpful feature to add before that happens.