POLICY GRADIENT

Peaking popularity
Already the most widely used

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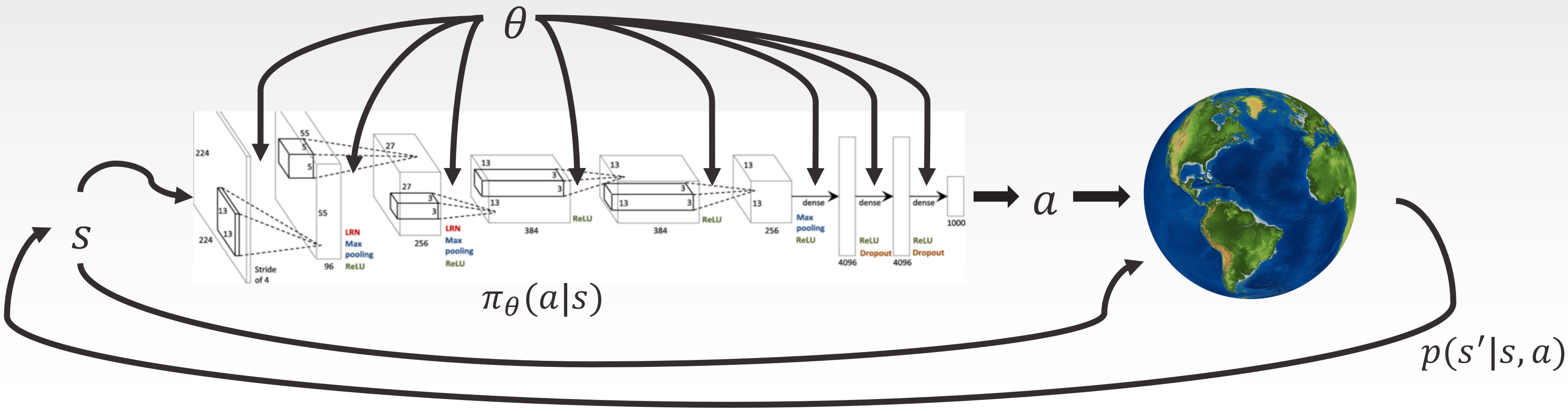
Outline

- Formulation
- Algorithm
- Issues
- Bias

FORMULATION

Episodes



$$\underbrace{p_\theta(s_1, a_1, \dots, s_T, a_T)}_{\pi_\theta(\mathcal{T})} = p(s_1) \prod_{t=1}^{T} \pi_\theta(a_t|s_t) p(s_{t+1}|s_t, a_t)$$

$$\theta^* = \arg\max_\theta E_{\mathcal{T} \sim \pi_\theta(\mathcal{T})} \left[\sum_t r(s_t, a_t) \right]$$

Episodes

$$\theta^* = \arg\max_\theta E_{\tau\sim\pi_\theta(\tau)}\left[\sum_t r(s_t, a_t)\right]$$

$$\theta^* = \arg\max_\theta E_{(s,a)\sim p_\theta(s,a)}\left[\sum_t r(s,a)\right]$$

infinite horizon case

$$\theta^* = \arg\max_\theta \sum_{t=1}^{T} E_{(s_t,a_t)\sim p_\theta(s_t,a_t)}[r(s_t, a_t)]$$

finite horizon case

Telescoping

$$\theta^* = \underset{\theta}{\operatorname{argmax}}\, \underbrace{E_{\tau\sim\pi_\theta(\tau)}\left[\sum_t r(s_t, a_t)\right]}_{J(\theta)}$$

- Assuming everything is finite
  - Otherwise an approximation

$$J(\theta) = E_{\tau\sim\pi_\theta(\tau)}\left[\sum_t r(s_t, a_t)\right] = \sum_i p(\tau^i) \sum_t r(s_t^i, a_t^i)$$

sum over samples from π_θ

ALGORITHM

Gradient Optimization

- $J(\theta)$ not convex
- Gradient optimization still applicable

$$\theta = \theta + \alpha \, \nabla J(\theta)$$

  - α learning rate
- Challenge to find the gradient
- $g(\theta) = E_{z\sim Q} f(\theta, z)$
  - $\nabla g(\theta) = E_{z\sim Q} \nabla_\theta f(\theta, z)$
- What about $g(\theta) = E_{z\sim Q(\theta)} f(\theta, z)$?

Direct Policy Differentiation

$$\underbrace{\theta^* = \arg\max_{\theta} E_{\tau\sim\pi_\theta(\tau)}\left[\sum_t r(s_t, a_t)\right]}_{J(\theta)}$$

a convenient identity

$$\pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) = \pi_\theta(\tau)\frac{\nabla_\theta \pi_\theta(\tau)}{\pi_\theta(\tau)} = \nabla_\theta \pi_\theta(\tau)$$

$$J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[\underbrace{r(\tau)}_{\sum_{t=1}^{T} r(s_t, a_t)}] = \int \pi_\theta(\tau) r(\tau) d\tau$$

$$\nabla_\theta J(\theta) = \int \nabla_\theta \pi_\theta(\tau) r(\tau) d\tau = \int \pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau)\, r(\tau) d\tau = E_{\tau\sim\pi_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau)\, r(\tau)]$$

Direct Policy Differentiation

$$\theta^* = \underset{\theta}{\text{argmax}}\, J(\theta)$$

$$J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[r(\tau)]$$

$$\underbrace{p_\theta(s_1, a_1, \dots, s_T, a_T)}_{\pi_\theta(\tau)} = p(s_1)\prod_{t=1}^{T}\pi_\theta(a_t|s_t)p(s_{t+1}|s_t, a_t)$$

log of both sides

$$\log\pi_\theta(\tau) = \log p(s_1) + \sum_{t=1}^{T}\log\pi_\theta(a_t|s_t) + \log p(s_{t+1}|s_t, a_t)$$

$$\nabla_\theta J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[\nabla_\theta \underbrace{\log\pi_\theta(\tau)}_{}\, r(\tau)]$$

$$\nabla_\theta\left[\cancel{\log p(s_1)} + \sum_{t=1}^{T}\log\pi_\theta(a_t|s_t) + \cancel{\log p(s_{t+1}|s_t, a_t)}\right]$$

$$\boxed{\nabla_\theta J(\theta) = E_{\tau\sim\pi_\theta(\tau)}\left[\left(\sum_{t=1}^{T}\nabla_\theta\log\pi_\theta(a_t|s_t)\right)\left(\sum_{t=1}^{T}r(s_t, a_t)\right)\right]}$$

Algorithm

$$J(\theta) = E_{\tau \sim p_\theta(\tau)}\left[\sum_t r(s_t, a_t)\right] \approx \frac{1}{N}\sum_i \sum_t r(s_t^i, a_t^i)$$

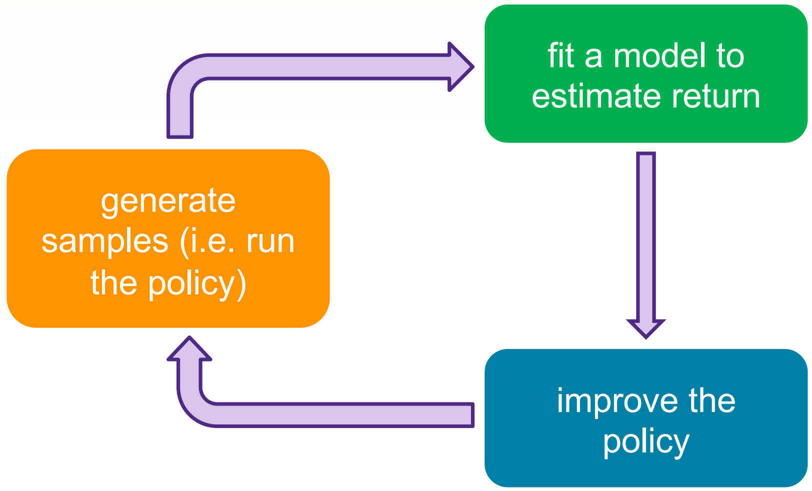
$$\nabla_\theta J(\theta) = E_{\tau \sim \pi_\theta(\tau)}\left[\left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\right)\left(\sum_{t=1}^{T} r(s_t, a_t)\right)\right]$$

$$\nabla_\theta J(\theta) \approx \underline{\frac{1}{N}\sum_{i=1}^{N}}\left(\sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\right)\underline{\left(\sum_{t=1}^{T} r(s_t^i, a_t^i)\right)}$$

$$\underline{\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)}$$

REINFORCE algorithm:

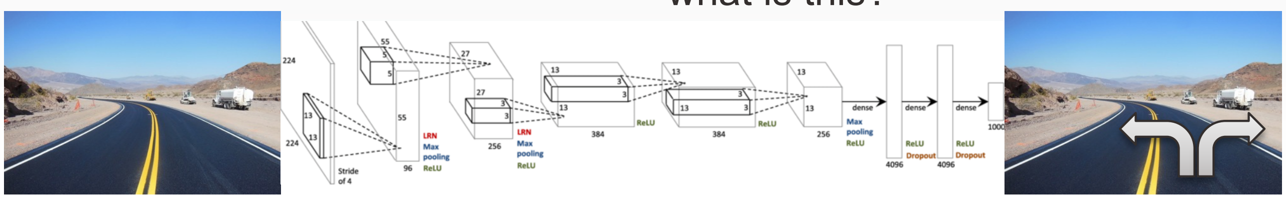
1. sample $\{\tau^i\}$ from $\pi_\theta(a_t|s_t)$ (run the policy)
2. $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_i\left(\sum_t \nabla_\theta \log \pi_\theta\left(a_t^i|s_t^i\right)\right)\left(\sum_t r\left(s_t^i|a_t^i\right)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



Algorithm

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\right)\left(\sum_{t=1}^{T} r(s_t^i, a_t^i)\right)$$

what is this?



s_t $\pi_\theta(a_t|s_t)$ a_t

- Network produces softmax
- Take log as “loss” function
  - Standard back propagation

Multi-step MDP

- Replace instantaneous reward r with long-term value $Q^{\pi}(s,a)$
- For discounted objective
  - Any differentiable policy $\pi_{\theta}(a|s)$
  - Can be shown

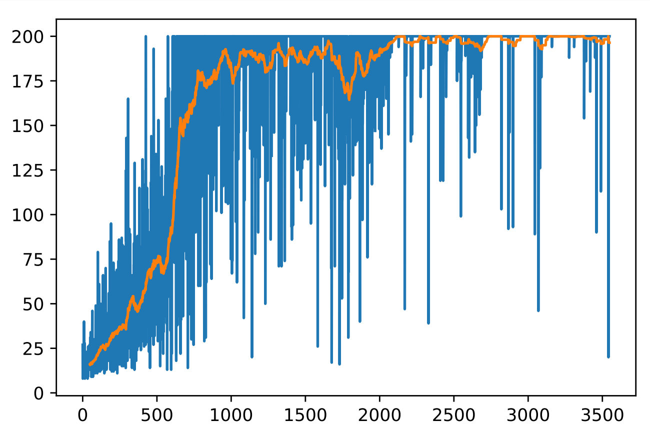
$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}}[\nabla_{\theta} \log \pi_{\theta}(a|s) Q^{\pi_{\theta}}(s,a)]$$

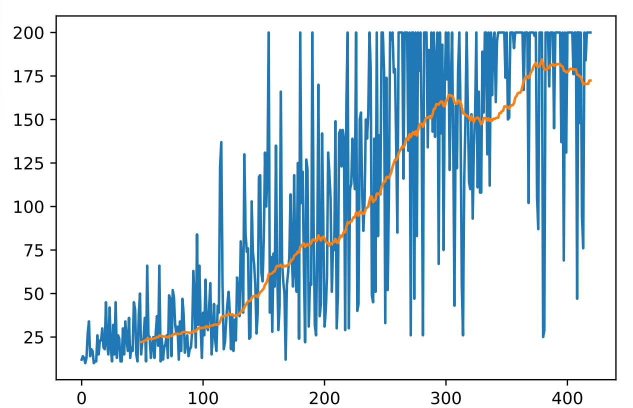
ENHANCEMENTS

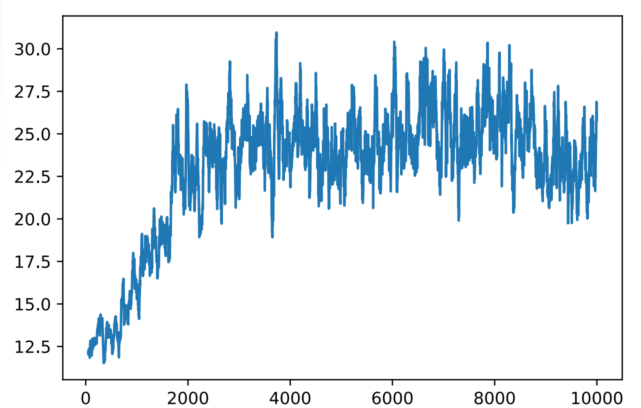
Variance of Policy-Gradient

- Known to have high variance

Created by Ghani Ebrahimi







Baselines

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log \pi_\theta\left(\tau^i\right)\left[r\left(\tau^i\right) - b\right]$$

a convenient identity

$$\pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) = \nabla_\theta \pi_\theta(\tau)$$

- Let *b* (baseline) be constant
  - Independent of τ

$$E[\nabla_\theta \log \pi_\theta(\tau) b] = \int \pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) b \, d\tau = \int \nabla_\theta \pi_\theta(\tau) b \, d\tau = b\nabla_\theta \int \pi_\theta(\tau) \, d\tau = b\nabla_\theta 1 = 0$$

  - Subtracting a baseline is unbiased in expectation
  - Candidate

$$b = \frac{1}{N}\sum_{i=1}^{N} r\left(\tau^i\right)$$

Reducing Variance

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\right)\left(\sum_{t=1}^{T} r(s_t^i, a_t^i)\right)$$

- Policy at time t' cannot affect reward at time t when $t < t'$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\underbrace{\left(\sum_{t'=1}^{T} r\left(s_{t'}^i, a_{t'}^i\right)\right)}_{\substack{\text{“reward to go”} \\ \hat{Q}_{i,0}}}$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\left(\sum_{t'=t}^{T} r\left(s_{t'}^i, a_{t'}^i\right)\right)$$

REINFORCE algorithm

Off-policy PG and Importance Sampling

- Recall $\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(a|s) Q^{\pi_\theta}(s,a)]$
  - At time *t*
    - *Q* matters
    - *Q* captures reward only from *t* onwards
- Scientific justification for truncation

Analyzing Variance

$$\text{Var}[x] = E[x^2] - E[x]^2$$

$$\nabla_\theta J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau)(r(\tau) - b)]$$

$$\text{Var} = E_{\tau\sim\pi_\theta(\tau)}[(\nabla_\theta \log \pi_\theta(\tau)(r(\tau) - b))^2] - \underbrace{E_{\tau\sim\pi_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau)(r(\tau) - b)]^2}_{[E_{\tau\sim\pi_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau) r(\tau)]]^2}$$

$$\frac{d\text{Var}}{db} = \frac{d}{db} E[g(\tau)^2(r(\tau) - b)^2] = \frac{d}{db}(E[g(\tau)^2 r(\tau)^2] - 2E[g(\tau)^2 r(\tau) b] + b^2 E[g(\tau)^2])$$

$$= -2E[g(\tau)^2 r(\tau)] + 2bE[g(\tau)^2] = 0$$

$$b = \frac{E[g(\tau)^2 r(\tau)]}{E[g(\tau)^2]} \longleftarrow$$

This is just expected reward, but weighted by gradient magnitudes!

Optimal constant baseline

Reducing Variance Using a Baseline

- Good baseline
  - State value function $B(s) = V^{\pi_\theta}(s)$
- Policy gradient

$$A^{\pi_\theta}(s,a) = Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s)$$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a)\, A^{\pi_\theta}(s,a)]$$

- Advantage function $A^{\pi_\theta}(s,a)$
  - For optimal policy advantage function always non-positive

Estimating Advantage Function

- Two function approximators and two parameter vectors

$$V_v(s) \approx V^{\pi_\theta}(s)$$
$$Q_w(s,a) \approx Q^{\pi_\theta}(s,a)$$
$$A_{w,v}(s,a) = Q_w(s,a) - V_v(s)$$

- θ fixed; w, v learnable
  - Updating both value functions by ideas from Q-learning
- Alternative to have a single network for advantage

Policy Gradient Algorithms

- Many equivalent forms

$$\begin{aligned}\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a)\, r_t] && \text{REINFORCE}\\ &= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a)\, Q^w(s,a)] && \text{Q Actor-Critic}\\ &= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a)\, A^{w,v}(s,a)] && \text{Advantage Actor-Critic}\end{aligned}$$

- Equality for expectation
- When sampling and approximating functions
  - Different algorithms

Algorithm

- Loop
  1. sample $\{\tau^i\}$ from $\pi_\theta(a_t|s_t)$ (run the policy)
  2. $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_i\left(\sum_t \nabla_\theta \log \pi_\theta\left(a_t^i|s_t^i\right)\right)\left(\sum_t A_{w,v}(s_t^i, a_t^i)\right)$
  3. $\theta \leftarrow \theta + \alpha\nabla_\theta J(\theta)$
  4. Create set: episode τ^i and each $\left(s_t^i, a_t^i\right)$
  5. Adjust w, v based on MSE (recomputing the advantage function)

- Only value and Q functions approximation used
  - MSE based only on states and policy (actions)

UPDATING FUNCTIONS

Functional Approximation of V

- Updating the value function approximation
- At state *s*
  - Have reward plus future based on approximate value function
    - Based on optimal action
  - Have value of approximate value function
- Match them
  - L2 loss

fitted value iteration algorithm:

1. set $y_i \leftarrow \max_{a_i}(r(s_i, a_i) + \gamma E[V_v(s_i')])$
2. set v $\leftarrow \mathrm{argmin}_{v'} \frac{1}{2} \sum_i (V_{v'}(s_i) - y_i)^2$

Functional Approximation of Q

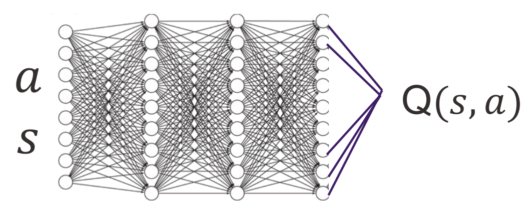
fitted value iteration algorithm:

1. set $y_i^1 \leftarrow \max_{a_i}(r(s_i, a_i) + \gamma E[V_v(s_i')])$
2. set $v \leftarrow \mathrm{argmin}_{v'} \frac{1}{2}\sum_i \left(V_{v'}(s_i) - y_i^1\right)^2$

fitted Q iteration algorithm:

1. set $y_i^2 \leftarrow r(s_i, a_i) + \gamma E[V_v(s_i')]$ ⟵ approximate $E[V_v(s_i')] \approx \max_{a'} Q_w(s_i', a_i')$
2. set $w \leftarrow \mathrm{argmin}_{w'} \frac{1}{2}\sum_i \left(Q_{w'}(s_i, a_i) - y_i^2\right)^2$

Doesn't require simulation of actions!



parameters w

Value Iteration with Fitted Q-factor

- Observed data are trajectories
  - Formally, $\mathrm{U} = \{(s_i, a_i, s_i', r_i) | i \in N\}$
- Loop
  - Sample $S \subseteq U$
  - For $i \in S$
    - $y_i \leftarrow r_i + \gamma \max_{a_i'} Q_w(s_i', a_i')$
  - Set $w \leftarrow \mathrm{argmin}_{w'} \frac{1}{2} \sum_{i \in S} (Q_{w'}(s_i, a_i) - y_i)^2$

Online Version

full fitted Q-iteration algorithm:

1. collect dataset $\{(s_i, a_i, s_i', r_i)\}$ using some policy

$K\times$

2. set $y_i \leftarrow r(s_i, a_i) + \gamma \max_{a_i'} Q_w(s_i', a_i')$
3. set $w \leftarrow \arg\min_{w'} \frac{1}{2} \sum_i (Q_{w'}(s_i, a_i) - y_i)^2$

online Q iteration algorithm:

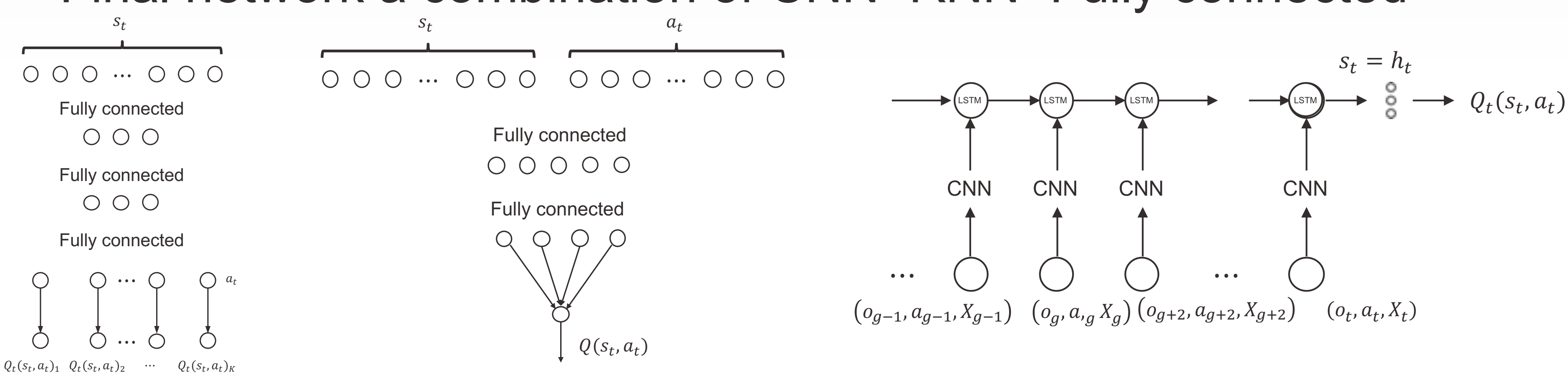
1. take some action a_i and observe (s_i, a_i, s_i', r_i)
2. $y_i = r(s_i, a_i) + \gamma \max_{a'} Q_w(s_i', a_i')$
3. $w \leftarrow w - \alpha \frac{dQ_w}{dw}(s_i, a_i)(Q_w(s_i, a_i) - y_i)$

Overall Algorithm

- Loop
  1. sample $\{\tau^i\}$ from $\pi_\theta(a_t|s_t)$ (run the policy)
  2. $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_i\left(\sum_t \nabla_\theta \log \pi_\theta\left(a_t^i \middle| s_t^i\right)\right)\left(\sum_t A_{w,v}(s_t^i, a_t^i)\right)$
  3. $\theta \leftarrow \theta + \alpha\nabla_\theta J(\theta)$
  4. Create set: episode τ^i and each $\left(s_t^i, a_t^i\right)$
  5. For all t, i present in episodes in step 4
     1. Let $y_{ti}^1 \leftarrow \max_{a_i}(r\left(s_t^i, a_t^i\right) + \gamma E\left[V_v\left(s_t^{i\prime}\right)\right])$.
     2. Let $y_{ti}^2 \leftarrow r\left(s_t^i, a_t^i\right) + \gamma E\left[V_v\left(s_t^{i\prime}\right)\right]$. // OR $y_{ti}^2 \leftarrow r\left(s_t^i, a_t^i\right) + \gamma \max_{a_i'} Q_w(s_{ti}^{i\prime}, a_{ti}^{i\prime})$
  6. Value function update: set $v \leftarrow \operatorname{argmin}_{v'} \frac{1}{2}\sum_{ti}\left(V_{v'}\left(s_t^i\right) - y_{ti}^1\right)^2$
  7. Q-factor update: set $w \leftarrow \operatorname{argmin}_{w'} \frac{1}{2}\sum_{ti}\left(Q_{w'}\left(s_t^i, a_t^i\right) - y_{ti}^2\right)^2$

Deep Q-Learning

- Deep network used to model Q factor
- State modeled by RNN
  - Possibly combined with CNN if images involved
- Final network a combination of CNN+RNN+Fully connected



Exploration-Exploitation

- Exploration-Exploitation tradeoff
- Have visited part of the state space and found a reward of 100
  - Is this the best we can hope for?
  - Should we keep ‘pounding’ the visited states and figure out actions that lead to 100?
  - Perhaps there are other states that we have not yet visited that lead to even higher reward
- Exploitation
  - Should we stick with what we know
  - Find a good policy with respect to this knowledge
    - Risk of missing out on a better reward somewhere else
- Exploration
  - Should we look for states with more reward
    - At risk of wasting time and getting some negative reward

Exploration-Exploitation Epsilon-Greedy

- Exploitation
  - Follow your current best policy
  - Be greedy
- Exploration
  - Deviate to explore
  - Strategy
    - Greedy policy vector a
    - Perturb it by epsilon
      $a + \epsilon$
    - ϵ random vector
    - Has to be probability vector

- Common exploration
  - Probability(a)
    - $1 - \epsilon$ if a = optimal (argmax)
    - $\epsilon / (|\mathcal{A}| - 1)$ otherwise

Exploration-Exploitation

- Sample from one policy
  - Exploration
  - Behavior policy
- Update and optimization different policy
  - Exploitation
  - Learning policy
- Same policies: on-policy learning
- Two different policies: off-policy learning

Off-policy vs On-policy

- On-policy algorithm
  - Learn the policy being executed by the agent
  - One policy
- Off-policy algorithm
  - Evaluate a policy from samples generated by a different policy
    - Target policy
  - Learn policy independent of policy taken by agent (behavior policy)
  - Two policies

Off-policy Framework

- $\pi_{\theta'}$ is target policy
  - Gradient with respect to this policy
  - Gradient at this point
- Loop
  - Sample episode based on policy π_θ
    - // Behavior policy
  - Perform gradient step at $\pi_{\theta'}$
    - // Adjusts $\pi_{\theta'}$
- Challenge
  - The two policies should be somehow related
    - Idea: gradient taken with respect to a function that involves π_θ

Policy Gradient and On-policy

$$\theta^* = \underset{\theta}{\operatorname{argmax}}\, J(\theta)$$

$$J(\theta) = E_{\tau \sim \pi_\theta(\tau)}[r(\tau)]$$

$$\nabla_\theta J(\theta) = \underline{E_{\tau \sim \pi_\theta(\tau)}}[\nabla_\theta \log \pi_\theta(\tau) r(\tau)]$$

Same policy

- Neural networks change only slightly with each gradient step
- On-policy learning inefficient

Must have this

REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(a_t|s_t)$
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta\left(a_t^i \middle| s_t^i\right)\right) \left(\sum_t r(s_t^i, a_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Off-policy PG and Importance Sampling

$$\theta^* = \underset{\theta}{\operatorname{argmax}} J(\theta)$$

$$J(\theta') = E_{\tau \sim \pi_{\theta'}(\tau)}[r(\tau)]$$

We have samples from some $\pi_\theta(\tau)$

$$J(\theta') = E_{\tau \sim \pi_\theta(\tau)}\left[\frac{\pi_{\theta'}(\tau)}{\pi_\theta(\tau)} r(\tau)\right]$$

$$\pi_\theta(\tau) = p(s_1) \prod_{t=1}^{T} \pi_\theta(a_t|s_t) p(s_{t+1}|s_t, a_t)$$

$$\frac{\pi_{\theta'}(\tau)}{\pi_\theta((\tau)} = \frac{\cancel{p(s_1)} \prod_{t=1}^{T} \pi_{\theta'}(a_t|s_t) \cancel{p(s_{t+1}|s_t, a_t)}}{\cancel{p(s_1)} \prod_{t=1}^{T} \pi_\theta(a_t|s_t) \cancel{p(s_{t+1}|s_t, a_t)}} = \frac{\prod_{t=1}^{T} \pi_{\theta'}(a_t|s_t)}{\prod_{t=1}^{T} \pi_\theta(a_t|s_t)}$$

Importance sampling

$$E_{x \sim p(x)}[f(x)] = \int p(x) f(x) dx$$

$$= \int \frac{q(x)}{q(x)} p(x) f(x) dx$$

$$= \int q(x) \frac{p(x)}{q(x)} f(x) dx$$

$$= E_{x \sim q(x)}\left[\frac{p(x)}{q(x)} f(x)\right]$$

Off-policy PG and Importance Sampling

$$\theta^* = \underset{\theta}{\operatorname{argmax}} J(\theta)$$

$$J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[r(\tau)]$$

Convent identity

$$\pi_\theta(\tau)\nabla_\theta \log \pi_\theta(\tau) = \nabla_\theta \pi_\theta(\tau)$$

Estimate the value of some *new* parameters θ':

$$J(\theta') = E_{\tau\sim\pi_\theta(\tau)}\left[\frac{\pi_{\theta'}(\tau)}{\pi_\theta(\tau)} r(\tau)\right]$$

Depends on θ'

$$\nabla_{\theta'} J(\theta') = E_{\tau\sim\pi_\theta(\tau)}\left[\frac{\nabla_{\theta'}\pi_{\theta'}(\tau)}{\pi_\theta(\tau)} r(\tau)\right] = E_{\tau\sim\pi_\theta(\tau)}\left[\frac{\pi_{\theta'}(\tau)}{\pi_\theta(\tau)} \nabla_{\theta'} \log \pi_{\theta'}(\tau) r(\tau)\right]$$

At $\theta = \theta'$

$\nabla_\theta J(\theta) = E_{\tau\sim\pi_\theta(\tau)}[\nabla_\theta \log \pi_\theta(\tau) r(\tau)]$ - Original policy gradient

Off-policy PG and Importance Sampling

$$\theta^* = \underset{\theta}{\operatorname{argmax}} J(\theta)$$

$$J(\theta) = E_{\tau \sim \pi_\theta(\tau)}[r(\tau)]$$

$$\frac{\pi_{\theta'}(\tau)}{\pi_\theta(\tau)} = \frac{\prod_{t=1}^T \pi_{\theta'}(a_t|s_t)}{\prod_{t=1}^T \pi_\theta(a_t|s_t)}$$

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim \pi_\theta(\tau)}\left[\frac{\pi_{\theta'}(\tau)}{\pi_\theta(\tau)} \nabla_{\theta'} \log \pi_{\theta'}(\tau) r(\tau)\right]$$

$$= E_{\tau \sim \pi_\theta(\tau)}\left[\left(\prod_{t=1}^T \frac{\pi_{\theta'}(a_t|s_t)}{\pi_\theta(a_t|s_t)}\right)\left(\sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(a_t|s_t)\right)\left(\sum_{t=1}^T r(s_t, a_t)\right)\right]$$

$$= E_{\tau \sim \pi_\theta(\tau)}\left[\sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(a_t|s_t) \underbrace{\left(\prod_{t'=1}^t \frac{\pi_{\theta'}(a_{t'}|s_{t'})}{\pi_\theta(a_{t'}|s_{t'})}\right)}_{\text{future has no effect on present}} \underbrace{\left(\sum_{t'=t}^T r(s_{t'}, a_{t'})\right)}_{\text{at } t \text{ past reward does not matter (sunk cost)}}\right]$$

future has no effect on present

at *t* past reward does not matter (sunk cost)

Off-policy PG and Importance Sampling

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim \pi_\theta(\tau)}\left[\sum_{t=1}^{T} \nabla_{\theta'} \log \pi_{\theta'}(a_t|s_t) \underbrace{\left(\prod_{t'=1}^{t} \frac{\pi_{\theta'}(a_{t'}|s_{t'})}{\pi_\theta(a_{t'}|s_{t'})}\right)}_{\text{exponential in } T\ldots} \left(\sum_{t'=t}^{T} r(s_{t'}, a_{t'})\right)\right]$$

$$\theta^* = \underset{\theta}{\operatorname{argmax}} \sum_{t=1}^{T} E_{(s_t,a_t)\sim p_\theta(s_t,a_t)}[r(s_t, a_t)]$$

$$J(\theta) = \sum_{t=1}^{T} E_{(s_t,a_t)\sim p_\theta(s_t,a_t)}[r(s_t, a_t)] = \sum_{t=1}^{T} E_{s_t \sim p_\theta(s_t)}\left[E_{a_t\sim\pi_\theta(a_t|s_t)}[r(s_t, a_t)]\right]$$

$$J(\theta') = \sum_{t=1}^{T} E_{s_t \sim p_\theta(s_t)}\left[\cancel{\frac{p_{\theta'}(s_t)}{p_\theta(s_t)}} E_{a_t\sim\pi_\theta(a_t|s_t)}\left[\frac{\pi_{\theta'}(a_t|s_t)}{\pi_\theta(a_t|s_t)} r(s_t, a_t)\right]\right]$$

Ignore this part

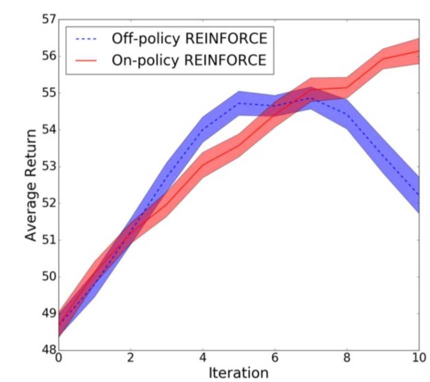
Perform importance sampling for states and actions

Off-policy PG Algorithm

Off-policy REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(a_t|s_t)$ // behavior policy
2. $\nabla_{\theta'} J(\theta') \approx \sum_i \left(\sum_t \nabla_{\theta'} \log \pi_{\theta'}(a_t^i|s_t^i)\, r(s_t^i, a_t^i) / \pi_\theta(a_t^i|s_t^i) \right)$
3. $\theta' \leftarrow \theta' + \alpha \nabla_{\theta'} J(\theta')$ // target policy

- Choice of behavior policy
  - Epsilon greedy with respect to learned policy
  - Policy that minimizes the variance of learned policy
    - Details complicated



Hanna and Stone 2018: Towards a Data Efficient Off-Policy Policy Gradient

Practical Version

- Replace reward with advantage

Off-policy actor-critic algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(a_t|s_t)$ // behavior policy
2. $\nabla_{\theta'} J(\theta') \approx \sum_i \left(\sum_t \nabla_{\theta'} \log \pi_{\theta'}\left(a_t^i \middle| s_t^i\right) A^{\pi_\theta}(s_t^i, a_t^i) / \pi_\theta\left(a_t^i \middle| s_t^i\right) \right)$
3. $\theta' \leftarrow \theta' + \alpha \nabla_{\theta'} J(\theta')$ // target policy

Policy Gradient in Practice

- Remember that the gradient has high variance
  - Not the same as supervised learning
    - Much more uncertainty
  - Gradients will be really noisy
- Consider using much larger batches
- Tweaking learning rates is very hard
  - Adaptive step size rules like ADAM help

Advantages of Policy-Based RL

- Advantages:
  - Good convergence properties
  - Effective in high-dimensional or continuous action spaces
  - Learns stochastic policies
- Disadvantages:
  - Typically converge to a local rather than global optimum
  - Evaluating a policy is typically inefficient and high variance