# Revenue Management

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# Revenue Management

*Revenue Management is a set of strategies and tactics to manage the allocation of capacity to different classes of customers with different prices over time in order to maximize revenue.*

- Pricing
- Capacity Allocation
- Network Management

# Pricing

# Value-Based Pricing

- In the value-based pricing, the price is set based on customers’ valuation, i.e., *willingness to pay*.

- Firms try to estimate customer valuation using *market research methods*, such as customer survey and focus groups.

- The customer’s decision to buy a product depends on the customer’s valuation as well as how customer react to a price (i.e., *customer behavior*).

# Customer Buying Behavior

- Assumption 1: Customer is able to assign a monetary value to a product and service. This value is called *Reservation Price* or *Maximum Willingness to Pay*

- Assumption 2: Customer will buy the product when the price of the product is less than customer's reservation price, i.e., when consumer surplus is positive, where

  *Consumer Surplus = Reservation Price – Selling Price*

  Customer will never purchase a product that yields negative consumer surplus.

- Assumption 3: In choosing between different products with positive consumer surplus, customer will buy the product that *maximizes consumer surplus*.

# Price Response Function

- ➢ Suppose the is reservation price of the population is randomly distributed with a cumulative distribution function $X \sim F(x)$.

- ➢ If the firm sets a price of $p = \$2000$, what is the probability that a randomly chosen customer buy the product?

$$\Pr\left(\textit{Reservation Price} \geq p\right) = 1 - \Pr\left(X \leq p\right) = 1 - F\left(p\right).$$

- ➢ If the firm sets a price of $p = \$2000$, how many customers among the 1000 customers will buy the trip?

$$D\left(p\right) = 1000(1 - F\left(p\right))$$

- ➢ $D(p)$ is known as *Price Response Function*, also known as demand curve in Economics.

# Price Response Function

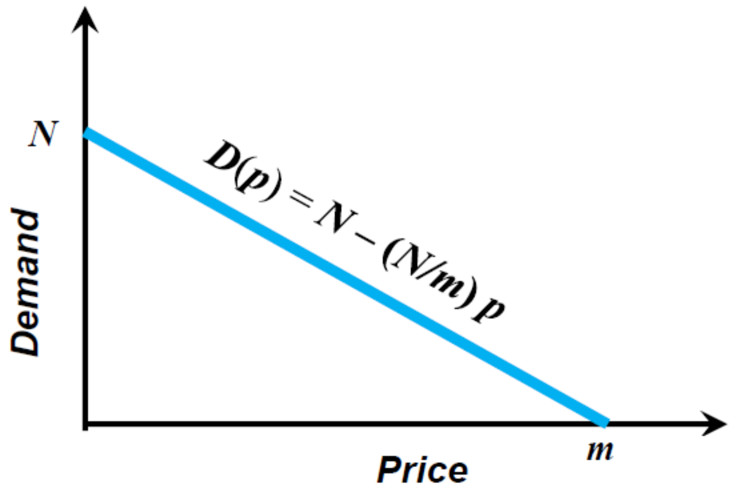
➢ Observation: If customers reservation price in a population of size $N$ is uniformly distributed between 0 and $m$,

$$f(x) = \frac{1}{m} \quad for\ 0 \leq x \leq m$$

Then the price response function is

$$D(p) = N - \left(\frac{N}{m}\right)p,$$

Which is a linear function commonly used in economics.



# Maximizing Revenue

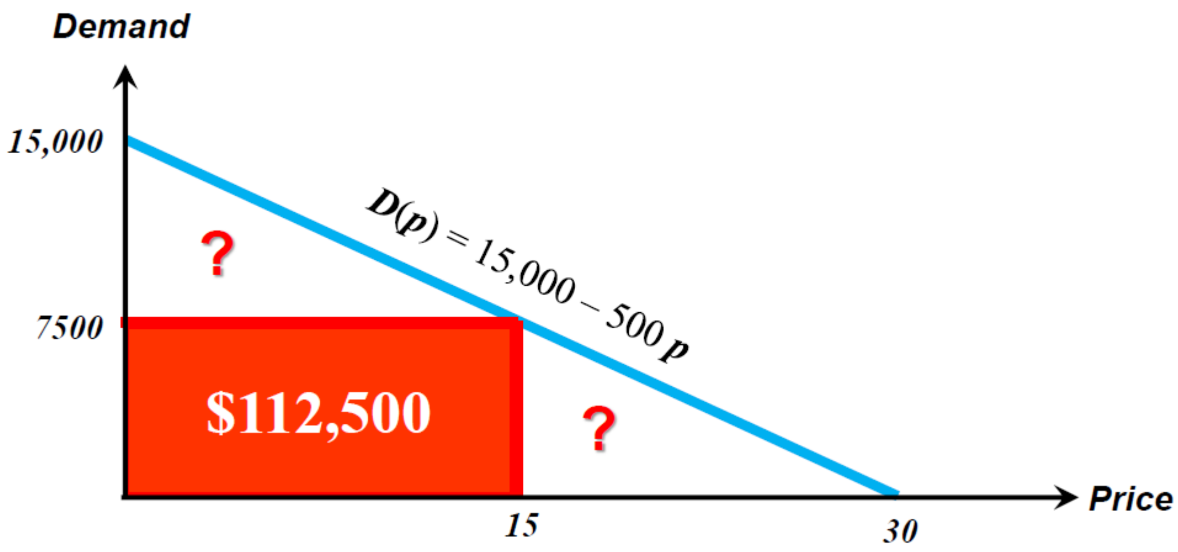
- The price response function for a particular product that a firm sells is well estimated by the following linear function:

$$D(p) = 15000 - 500p$$

- What is the price that maximizes the firm's revenue?

$$R = pD(p) = 15000p - 500p^2$$

$$p^* = 15, \;\; R^* = 15 * 7500 = \$112{,}500$$



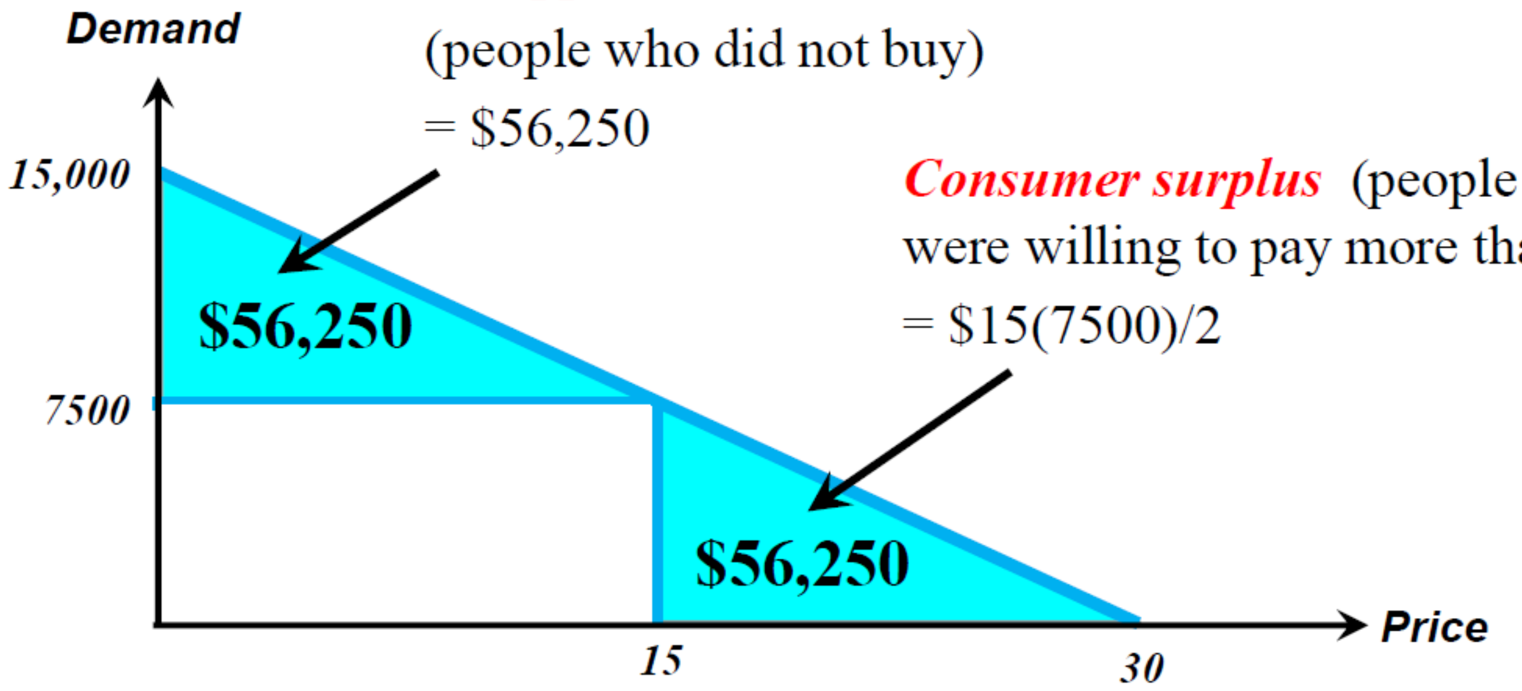
# Maximizing Revenue

$$\textit{Total revenue} = p\,D(p)$$

$$= \$15(7500) = \$112{,}500$$

***Untapped market***
(people who did not buy)
= \$56,250

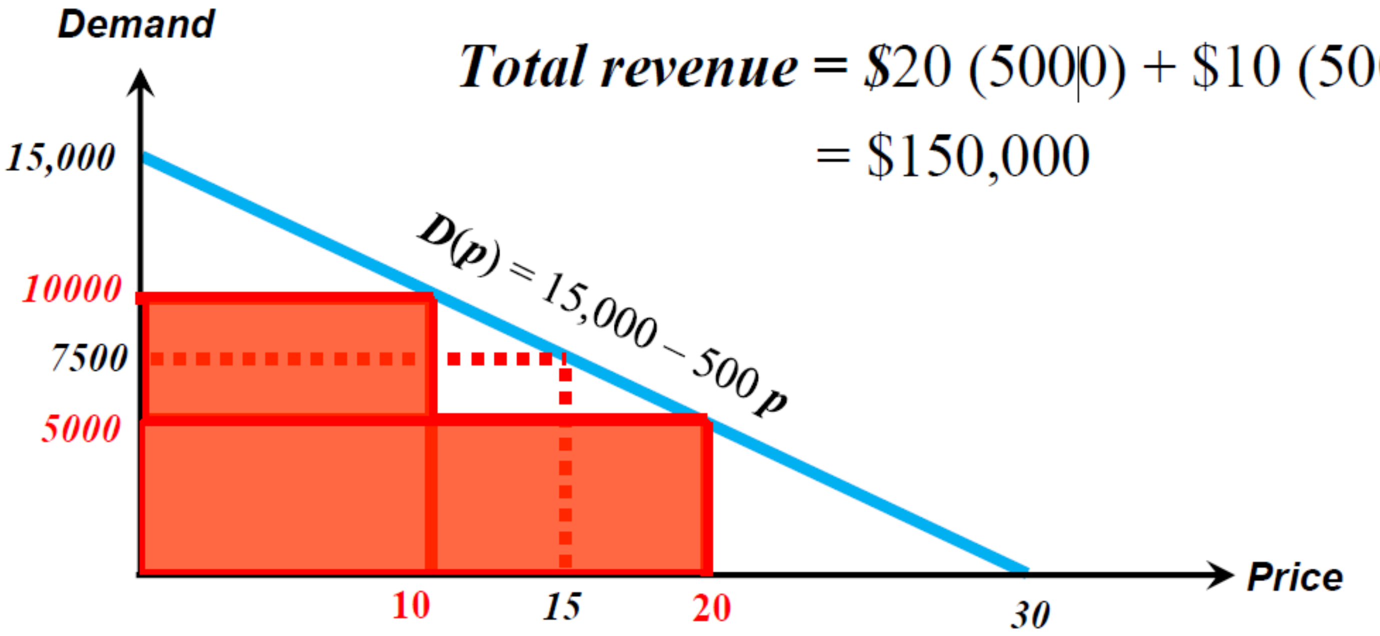
***Consumer surplus*** (people who were willing to pay more than \$15)
= \$15(7500)/2



# Maximizing Revenue

- **Question**: Is there any other pricing strategy that can lead to a higher revenue?

- **Answer:** Price discrimination. Offering two different prices: First offering the price of $20 and sell to 5000 customers. Then offering the price of $10 and sell to another 5000 customers.




$$\textit{Total revenue} = \$20\ (5000) + \$10\ (5000) = \$150{,}000$$

# Price Discrimination

- First-Degree Price Discrimination: Firm identifies perfectly the RP for all consumers and prices accordingly. (Ideal for firm, but impossible to do)
- Second-Degree Price Discrimination: Firm identifies imperfectly the RP of the buyers through a form of *self-selection*.
  - Offer a menu of choices to the buyer from which the buyer selects.
  - Buyers with low RPs will choose the inexpensive offerings on the menu.
  - The trick is to encourage those with a high RP not to choose the cheap option, but choose the expensive items from the menu.
- Third-Degree Price Discrimination: Firm tries to identify imperfectly the RP of buyers using some observable signal from buyers. (Age, Income, Employment Status, Location, Purchase History, Negotiation)

# Second-Degree Discrimination
## *Versioning*

➢ All Laser printers are fast. The manufacturers adds a line of code to slow them down, so they can offer both a slow and fast printer.

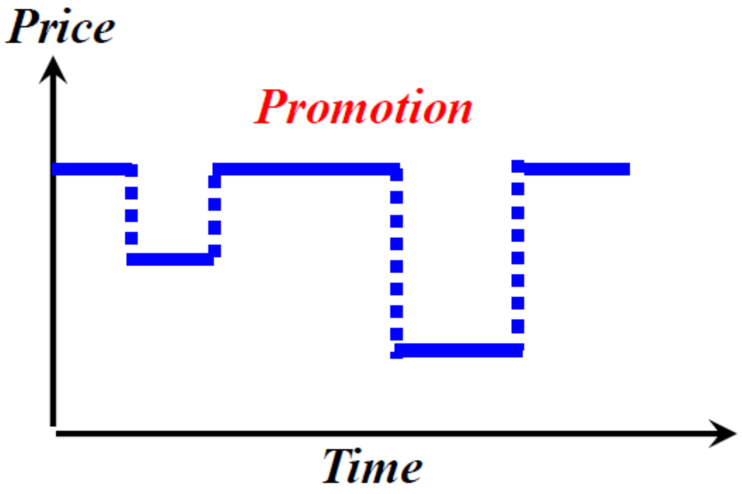
➢ Student version of a software programs are simply the professional versions with some features disabled.

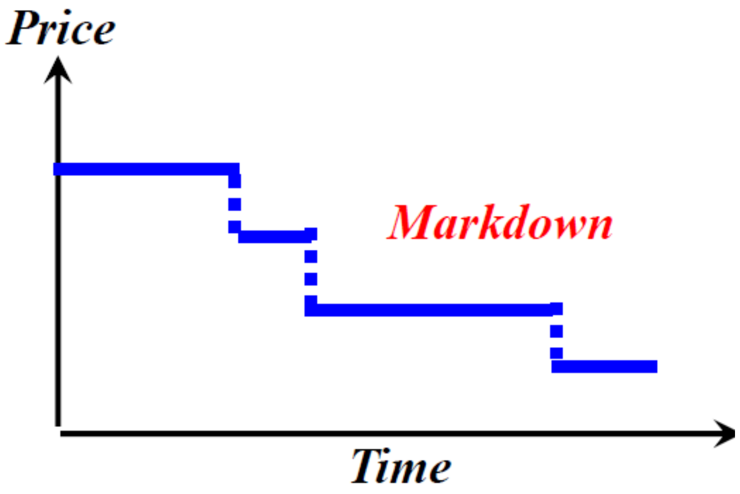

➢ Hardback versus paperback books. Hardback itself is viewed by many buyers to be a superior product to the paperback.

# Second-Degree Discrimination
## *Markdowns*

Retailers use two main mechanisms to provide discounts to customers:

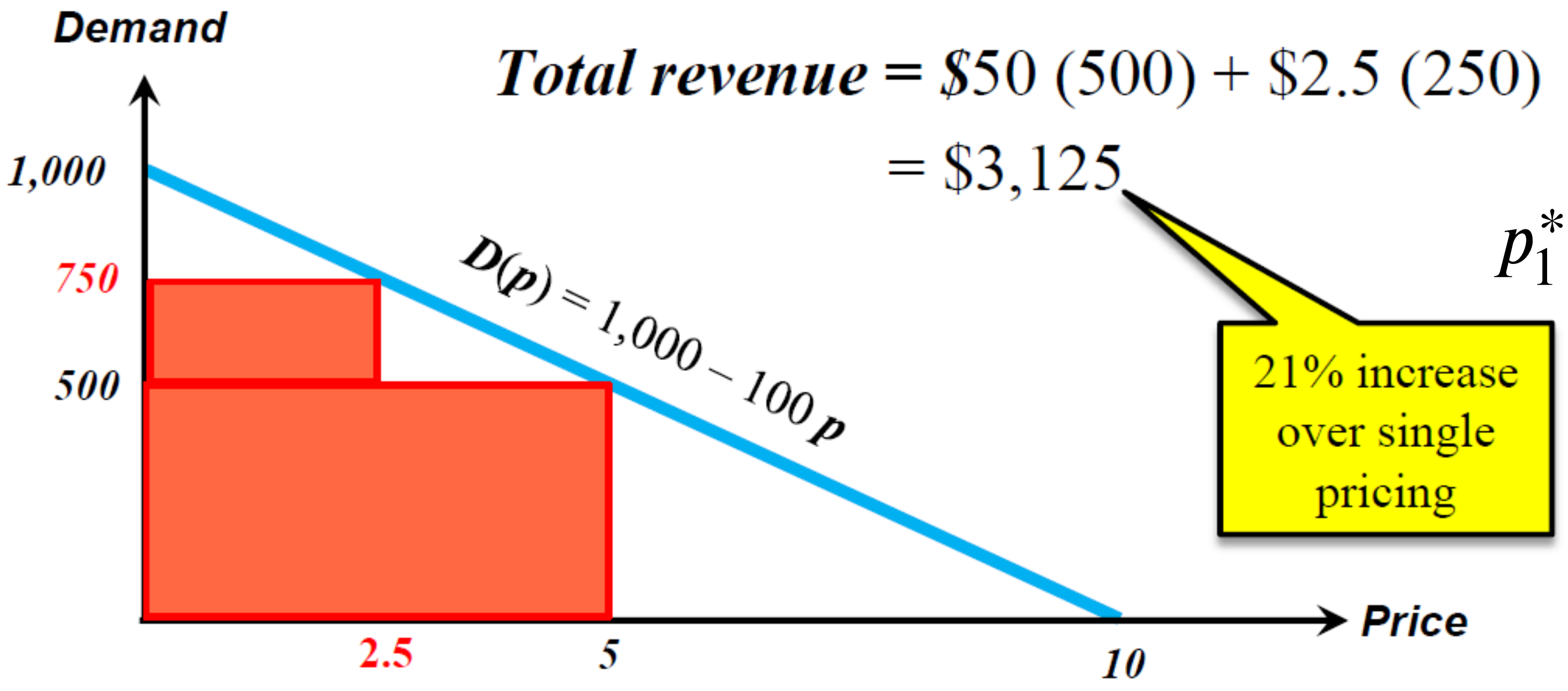
- ➢ Promotion: Temporary reduction in price. E.g., memorial days sales, two-for-one, coupon
- ➢ Markdown: Permanent reduction in price to clear inventory before it becomes obsolete.



# Second-Degree Discrimination
## *Markdowns*

**Question:** How does markdowns increase revenue? If the retailer decides to only do markdown once, what are the optimal original and markdown price?




$p_1^* = 6.67,\ p_2^* = 3.33$

Why?

# Second-Degree Discrimination

## *Markdowns*

- A retailer has 160 jeans and has four months to sell before he needs to clear the shelf space for the next fashion Jeans.
- The retailer is planning to establish a list price at the beginning of the first month, and then mark the jeans down at the beginning of each of the next three months.
- Unsold jeans at the end of four months will be sold to another outlet store for $5 a pair.
- Demand in each of the four months are as follows:

$$D_1(p) = 120 - 1.5p$$

$$D_2(p) = 90 - 1.5p$$

$$D_3(p) = 80 - 1.5p$$

$$D_4(p) = 50 - 2p$$

# Second-Degree Discrimination
## *Markdowns*

Decision Variables: $p_i$ for $i = 1,2,3,4$

Maximize: $\sum_{i=1}^{4} p_i D_i(p_i) + 5(160 - \sum_{i=1}^{4} p_i D_i(p_i))$

Subject to:

$$\sum_{i=1}^{4} D_i(p_i) \leq 160$$

$$p_1 \geq p_2 \geq p_3 \geq p_4 \geq 5$$

Solve this with gurobi as an exercise!

# Capacity Allocation

# Capacity Allocation

- Airlines: How to allocate seats of a single-leg flight to different customer classes?

- Hotels: How to allocate rooms in a hotel for a single day to different classes of customers?

- Car Rentals: How to allocate vehicles to different classes of customers in a single day?

# Revenue Management

## *Leisure and Business Customers*

➢ High Towers Hotel has a total of 100 rooms that it offers to both leisure and business customers.

➢ Hotel offers a $175 discount fare for a midweek stay for leisure and $240 for business customers.

| **Leisure Customers** | **Business Customers** |
|---|---|
| Highly price sensitive | Less price sensitive |
| Book earlier | Book later |
| More flexible to departure and arrival times | Less flexible |

# Revenue Management

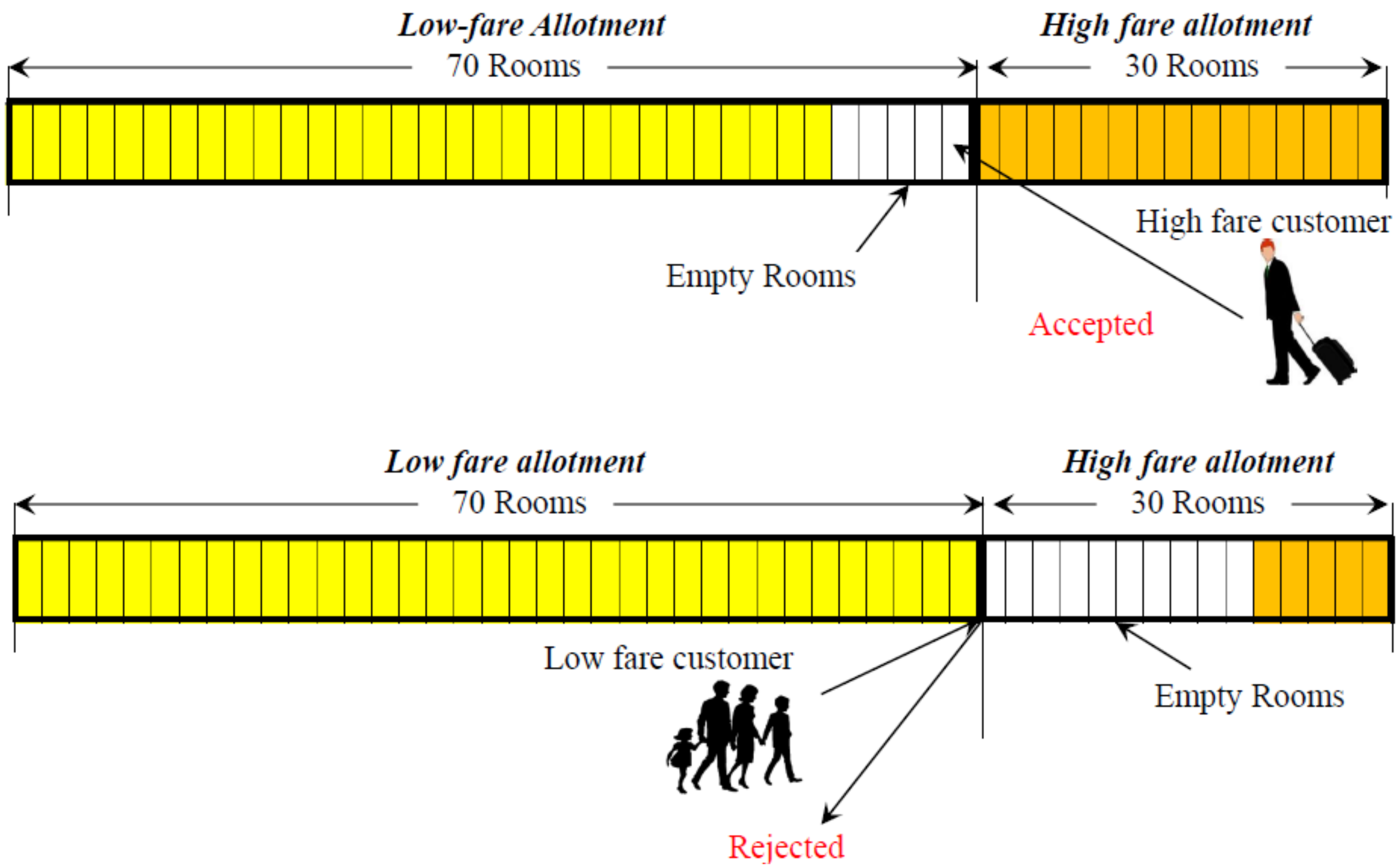
## *Leisure and Business Customers*

- Question: The hotel accepts reservations according to First-Come-First-Served. Do you agree with this booking policy?
- Answer: No. This policy may result in all 100 rooms being reserved one week before the date, mostly by leisure travelers since they often book early.

- Nested Booking:
  - The business customers can also use the allotment of the leisure customers
  - Business class will never rejected as long as there is a room available
  - Leisure customers are rejected if their allotment is closed.

# Revenue Management

## *Leisure and Business Customers*



# Revenue Management
## *Leisure and Business Customers*

Definition: *Booking Limit* for a fare is the maximum number of reservations allowed at that fare and lower.

Definition: *Protection Level* for a fare is the number of rooms set aside for that fare or higher.

Assumption 1: All low-fare customers arrive before high-fare customers.

Assumption 2: The demand for each fare class is independent of other classes.

What discount booking limit maximizes the hotel's revenue?

# Revenue Management

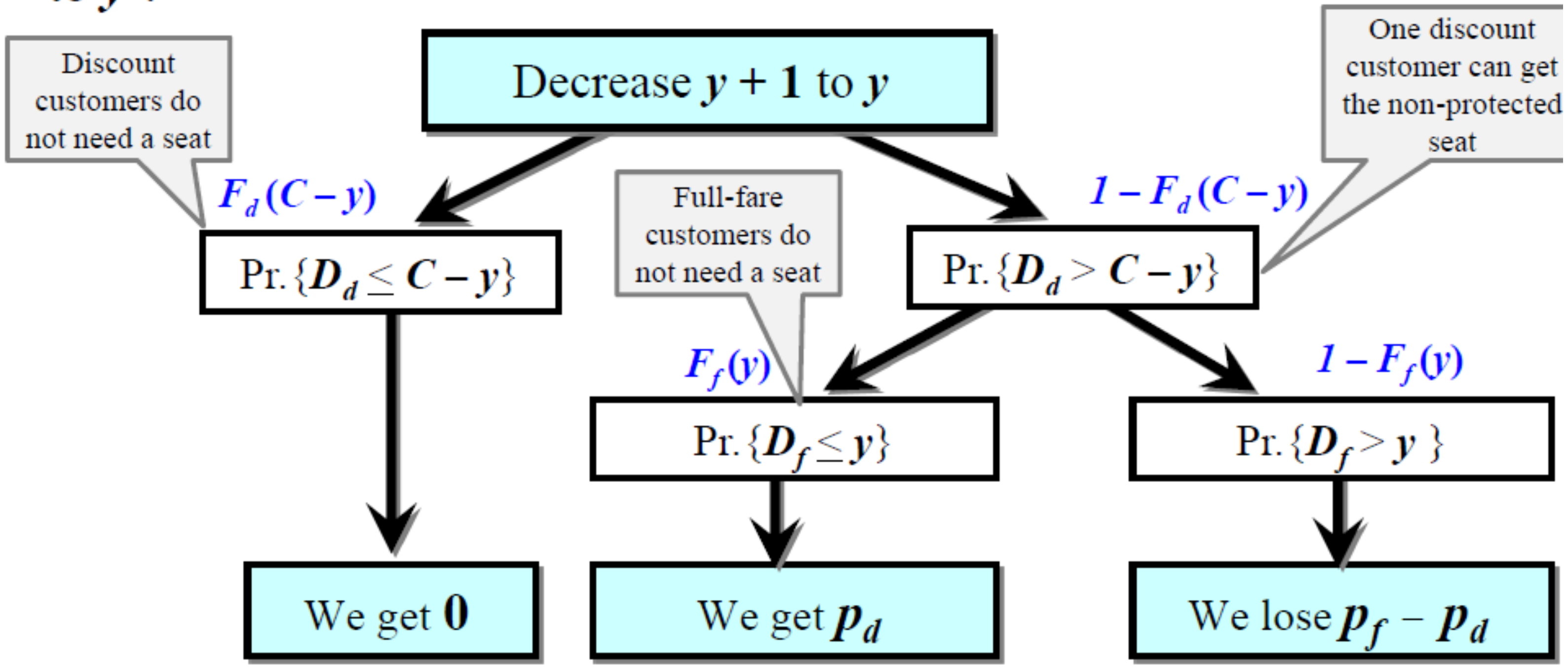
## *Leisure and Business Customers*

- System has capacity of $C = 100$
- Price of a full-fare is $p_f = \$240$ and price of a discount is $p_d = \$175$
- Demand for ***discount*** class, $D_d$, has probability distribution $f_d(x)$ with cumulative probability distribution $F_d(x) = \Pr\{D_d \leq x\}$.
- Demand for ***full-fare*** class, $D_f$, has probability distribution $f_f(x)$ with cumulative probability distribution $F_f(x) = \Pr\left\{ D_f \leq x \right\}$.
- Discount booking limit is $b$. Protection level for full-fare is $y = C - b$.

# Revenue Management

## *Leisure and Business Customers*

**Question:** Suppose the protection level is $\boldsymbol{y}+\mathbf{1}$. Should we reduce it to $\boldsymbol{y}$ ?




$$= F_d(C-y)\times 0 + \left[1-F_d(C-y)\right]\left\{F_f(y)p_d - \left[1-F_f(y)\right](p_f-p_d)\right\}$$

$$= \left[1-F_d(C-y)\right]\left\{F_f(y)p_f - (p_f-p_d)\right\}$$

# Revenue Management

## *Leisure and Business Customers*

**Solution:** Decrease the protection level from $y + 1$ to $y$, if

$$F_f(y) > 1 - \frac{p_d}{p_f} = 1 - \frac{175}{240} = 0.271\,.$$

**Discrete Distribution**: Scan from the top of the table for cumulative distribution of full fare toward the bottom until you find the first value of $x$ with a cumulative value $F_f(x)$ greater than or equal to critical ratio.

**Gaussian Distribution**: Once you have mean and std for large sample size.

| Demand for Full Fare | $F_f(x)$ |
|---|---|
| 0 -- 20 | 0.033 |
| 21 | 0.067 |
| 22 | 0.092 |
| 23 | 0.133 |
| 24 | 0.183 |
| 25 | 0.242 |
| $\boxed{y = 26}$ | $\boxed{0.283}$ |
| $y + 1 =$ 27 | 0.300 |
| 30 | 0.458 |
| 31 | 0.517 |
| 32 | 0.558 |
| 33 | 0.592 |
| 34 | 0.633 |
| 35 | 0.692 |
| 36 | 0.758 |
| 37 | 0.792 |
| 38 | 0.833 |
| 39 | 0.883 |
| 40 | 0.933 |
| 41 -- 60 | 1.000 |

# Revenue Management

## *Overbooking*

- High Towers Hotel has a total of 100 rooms that it offers to both leisure and business customers.
- Hotel offers a $175 discount fare for a midweek stay
- Historical data about the number of customers who book a room but fail to show is as follow.
- The cost of arranging alternative accommodation for an overbooked customer is $300.
- How many rooms the hotel should overbook?

| Number of No-Shows | Probability |
|---|---|
| 0 | 0.05 |
| 1 | 0.1 |
| 2 | 0.15 |
| 3 | 0.3 |
| 4 | 0.2 |
| 5 | 0.15 |
| 6 | 0.05 |

# Revenue Management

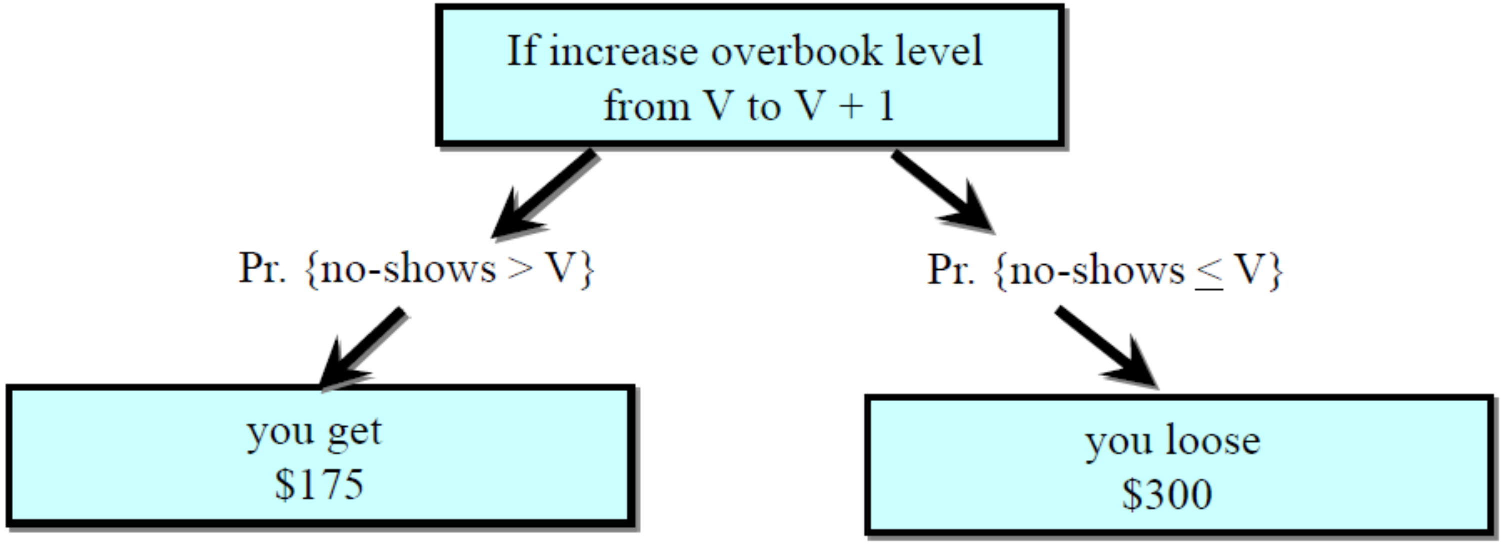
*Overbooking*

Capacity of the Hotel = 100 rooms

Number of overbooked rooms = V

Total reservations = 100 + V

**Question:** What is the optimal overbooking level V?



# Revenue Management

## *Overbooking*

Expected Profit from one ***additional overbook*** $= \$175 \text{ Pr. } \{\text{no-show} > V\}$
$- \; \$300 \text{ Pr. } \{\text{no-show} \le V\}$

$$= \$175 \text{ Pr. } \{\text{no-show} > V\} - \$300 \text{ Pr. } \{\text{no-show} \le V\} > 0$$

Increase the overbooking level from $V$ to $V + 1$ if:

$$\text{Pr.}\{no\text{-}show \le V\} < \frac{\$175}{\$175 + \$300} = 0.368$$

| Number of No-Shows | Probability | Cumulative |
|---|---|---|
| 0 | 0.05 | 0.05 |
| 1 | 0.1 | 0.15 |
| 2 | 0.15 | 0.3 |
| 3 | 0.3 | 0.6 |
| 4 | 0.2 | 0.8 |
| 5 | 0.15 | 0.95 |
| 6 | 0.05 | 1 |

Optimal overbooking = 3 Rooms