

# Federated Learning Algorithms

From easy to more complicated

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# Acknowledgements

- PhD students of mine
  - Tom Overman, ESAM
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  - Jay Xu, CS (graduated, at Meta)

## 1. Introduction, Motivation and Setting

Horizontal, Vertical, Hybrid Federated Learning

## 2. Federated Average (FedAvg)

Variants to cope with non-IID

## 3. Vertical Setting

## 4. Hybrid Setting

# INTRODUCTION

# WHY CAN'T WE JUST CENTRALIZE THE DATA?

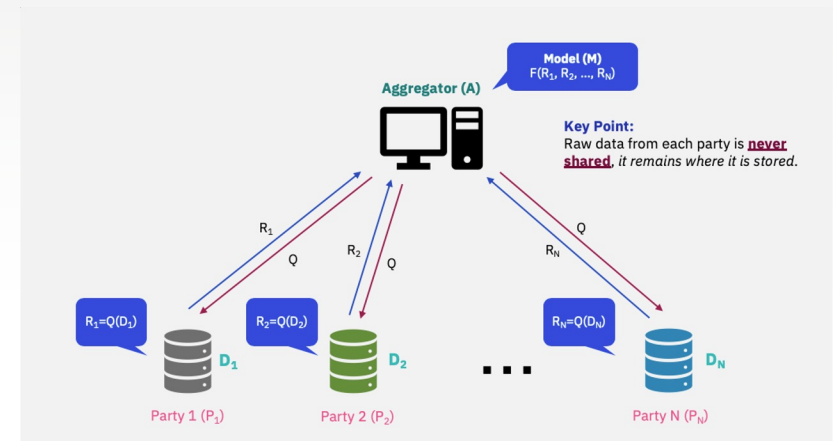
- Sending the data may be **too costly**
  - Self-driving cars are expected to generate several TBs of data a day
  - Some wireless devices have limited bandwidth/power
- Data may be considered **too sensitive**
  - We see a growing public awareness and regulations on data privacy
  - Keeping control of data can give a competitive advantage in business

# HOW ABOUT EACH PARTY LEARNING ON ITS OWN?

- The local dataset may be **too small**
  - Sub-par predictive performance (e.g., due to overfitting)
  - Non-statistically significant results (e.g., medical studies)
- The local dataset may be **biased**
  - Not representative of the target distribution

# Federated Learning

- Data is held on group of clients which can communicate with central server
- Build central model on server without ever accessing data from clients
  - ▢ Only send model weights
    - Other aspects that cannot be used to reconstruct data
- Goals of FL
  - ▢ Communication efficient
  - ▢ Robust to partial device participation
  - ▢ Resistant to model degradation with heterogeneous (non-IID) data



# Why Federated Learning?



A consortium of hospitals

Shared patients



Telecommunication network providers

Stations do not share data  
Customers roam



Smart phones of a provider

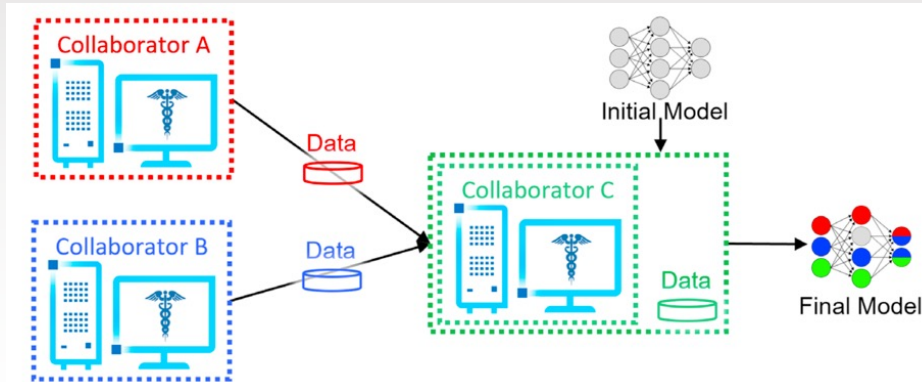
User data not to be shared



Data from cars of drivers

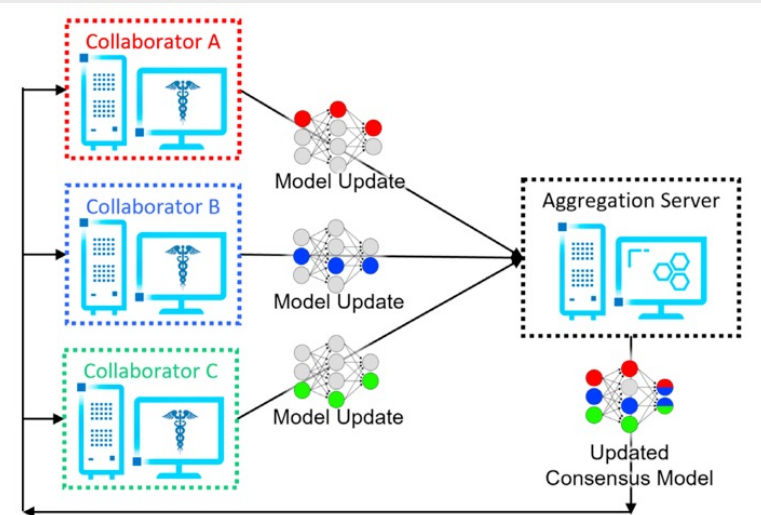
Car manufacturers  
Insurance companies





## Centralized Learning

Centralized learning is a traditional machine learning approach where all data is collected and stored in a central repository or server.



## Federated Learning

Learning technique that trains an algorithm via multiple independent sessions, each using its own dataset.

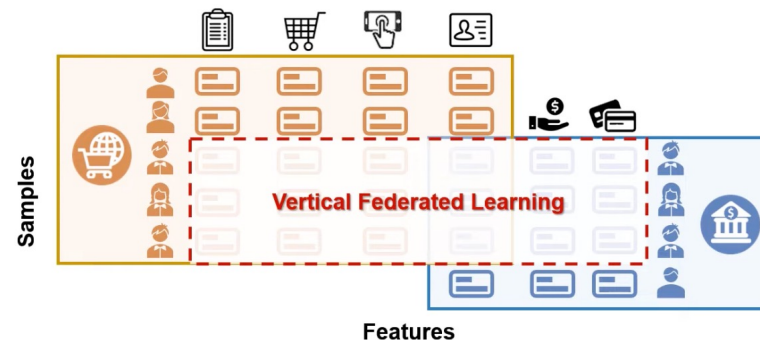
	Centralized Learning	Federated Learning
Data Storage	All training data is collected and stored in a central repository or server. Model training occurs on this central server using the entire dataset.	Data remains decentralized and is stored locally on edge devices or in various locations. Model training is performed locally on the edge devices without sharing raw data with a central server.
Data Privacy	The central server has access to the entire dataset, which can pose privacy and security concerns, especially when dealing with sensitive or private data.	Data remains on the local device and is not directly shared with a central server. Only model updates (gradients) are transmitted, and they are often anonymized and encrypted.
Scalability	Centralized learning may not be well-suited for edge computing environments, as it typically requires significant computational resources on the central server.	Federated learning is more suitable for edge computing, as it distributes the computation to the edge devices, reducing the central server's computational load.
Regulatory Compliance	Centralized learning may face more regulatory challenges related to data privacy, particularly in regions with strict data protection laws.	Federated learning can help organizations comply with data privacy regulations more effectively by keeping data local and minimizing data transmission.

**Dataset** has samples and feature spaces

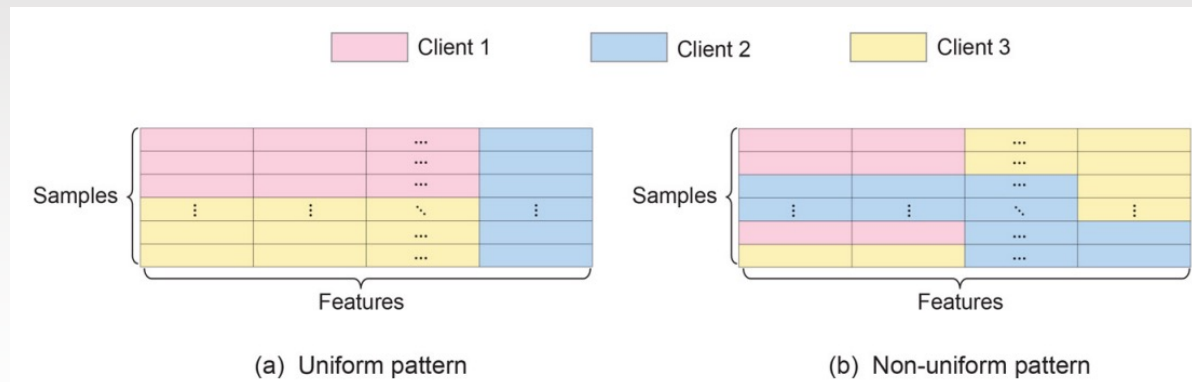
- **Sample** (Row) : single, individual data point or observation in a dataset
- **Feature** (Column) : characteristic or property of a data sample that is used to describe the data sample (also known as an attribute)

**Horizontal Federated Learning** : Intersecting features, different samples (Different hospitals have different patients but they may collect same information from their patients; Images of the same resolution collected at different places)

**Vertical Federated Learning** : Different features, intersecting samples (Shopping mall and bank may have same customers but they collect different information; Chips goes through different processes, each process captures its own features)



[9B] Comprehensive Analysis of Privacy Leakage in Vertical Federated Learning During Prediction ([https://www.youtube.com/watch?v=WHZ\\_gKAUoXl](https://www.youtube.com/watch?v=WHZ_gKAUoXl))



Haiping Gao, Songyang Ge, Tsung-Hui Chang, FedHD: Communication-efficient federated learning from hybrid data, Journal of the Franklin Institute, nVolume 360, Issue 12, 2023,

**Hybrid Federated Learning** : A client may observe only part of the data samples with only part of the data features.

**Example 1**: Joint financial prediction(they want to predict the customers' purchasing capacities of financial products) of a customer by 1)**banks**, 2)**trust and invest cooperation**, and 3)**stock services**. The three entities respectively serve just a fraction of the customers and have only their partial records.

**Example 2** : Different types of chips can go through partially same processes(same wafer fabrication, but different lithography...). Furthermore, there will be multiple rooms for same process, which also capture the same features.

# Horizontal/vertical/hybrid?



A consortium of hospitals

Shared patients



Telecommunication network providers

Stations do not share data  
Customers roam



Smart phones of a provider

User data not to be shared



Data from cars of drivers

Car manufacturers  
Insurance companies

# ALGORITHMS FOR IID HORIZONTAL FL

**FedAvg**(Federated Averaging) is a fundamental algorithm in federated learning that allows multiple devices or edge servers to collaboratively train a global machine learning model while keeping their data decentralized and private.

**Algorithm 1** FederatedAveraging. The  $K$  clients are indexed by  $k$ ;  $B$  is the local minibatch size,  $E$  is the number of local epochs, and  $\eta$  is the learning rate.

**Server executes:**

```

initialize  $w_0$ 
for each round  $t = 1, 2, \dots$  do
     $m \leftarrow \max(C \cdot K, 1)$ 
     $S_t \leftarrow$  (random set of  $m$  clients)
    for each client  $k \in S_t$  in parallel do
         $w_{t+1}^k \leftarrow \text{ClientUpdate}(k, w_t)$ 
     $m_t \leftarrow \sum_{k \in S_t} n_k$ 
     $w_{t+1} \leftarrow \sum_{k \in S_t} \frac{n_k}{m_t} w_{t+1}^k$  // Erratum4
    
```

**ClientUpdate**( $k, w$ ): // Run on client  $k$

```

 $\mathcal{B} \leftarrow$  (split  $\mathcal{P}_k$  into batches of size  $B$ )
for each local epoch  $i$  from 1 to  $E$  do
    for batch  $b \in \mathcal{B}$  do
         $w \leftarrow w - \eta \nabla \ell(w; b)$ 
    return  $w$  to server
    
```

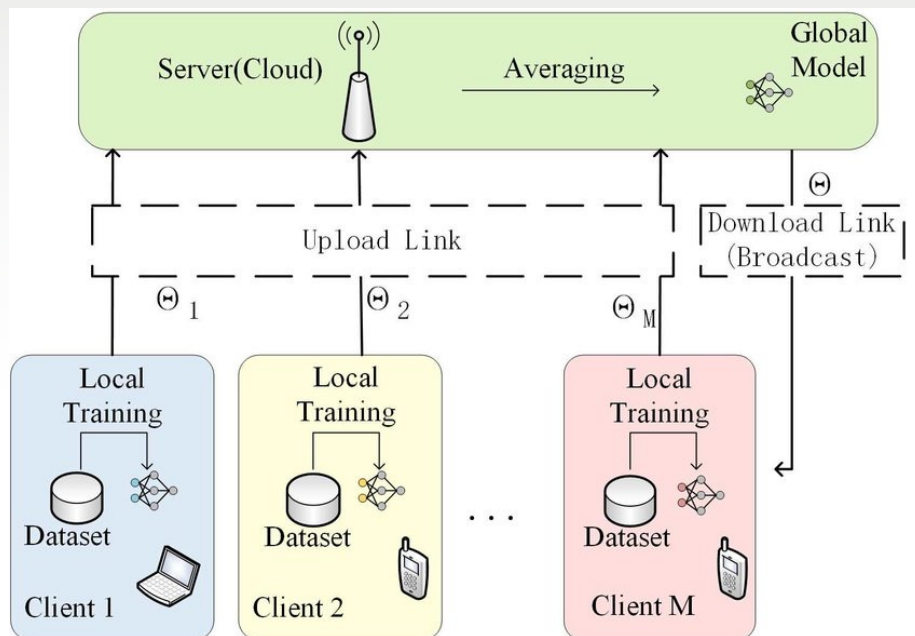
**Initialization:** Initially, there is a global model that starts as an arbitrary model. Each participating device or edge server has its own local copy of this global model.

**Local Model Training:** Each device or edge server independently trains its local model using its own private data. This training occurs locally and does not involve sharing the raw data with a central server.

**Model Update:** After local training, each device sends the model to the server.

**Aggregation:** The models from all participating devices are securely transmitted to a central server. The central server aggregates these models by averaging them.

**Global Model Update:** The averaged model becomes the updated global model. The global model now incorporates the collective knowledge from all the devices without any individual device sharing its private data.



**Local Model Training:** Which local training algorithm to pick? How many rounds of local training do we force the local clients to do before the model aggregation?

**Model Update:** How to express the local updates? (Privacy Issue)

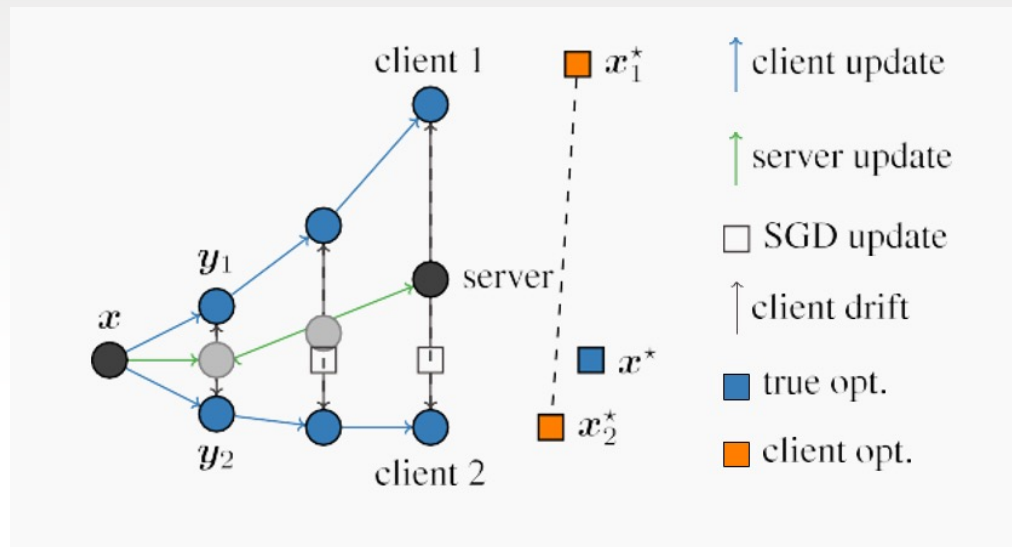
**Aggregation:** Which aggregation to pick?

What setting of FL is FedAvg amenable to?

Yang, Y., Zhang, Z., & Yang, Q. (2021). Communication-efficient federated learning with binary neural networks. *IEEE Journal on Selected Areas in Communications*, 39(12), 3836-3850.

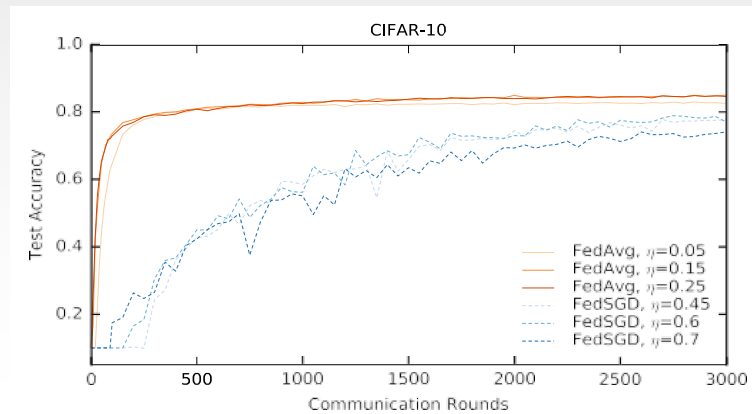


# Non-IID



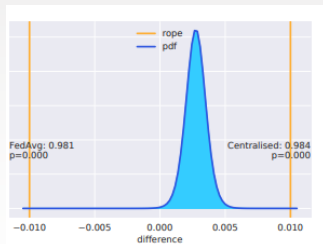
[Karimireddy et al., 2020])

# FedAvg [MCMAHAN ET AL., 2017]

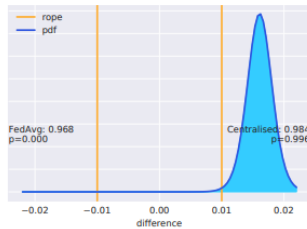


- FedAvg with  $E > 1$  allows to reduce the number of communication rounds
  - Often the bottleneck in FL (especially in the cross-device setting)
- Empirically achieves better generalization than parallel SGD with large mini-batch
- Convergence to the optimal model can be guaranteed for i.i.d. data [Stich, 2019] [Woodworth et al., 2020]
  - Issues arise in strongly non-i.i.d. case

# FedAvg vs Centralized



(d) FedAvg vs. Centr. i.i.d.



(j) FedAvg vs. Centr. non-i.i.d.

- Several variants of MNIST
- Posterior based on t-test
- $p$  is area outside of rope
  - Probability the algorithm is better
- FedAvg good proxy in iid
  - Performance degrades in non-iid

A Performance Evaluation of Federated Learning Algorithms,  
Adrian Nilsson, Simon Smith, Gregor Ulm, Emil Gustavsson, Mats Jirstrand

# Setting

- IID vs non-IID
  - Distribution of data on clients – features and labels
- $w_{t+1} = \frac{1}{K} \sum_k w_t^k$ 
  - Poor performance in non-IID cases
- $w_{t+1} = \frac{1}{n} \sum_k n_k w_t^k$ 
  - $n_k$  = number of samples at client  $k$
  - Is it available?

# Convergence

- Clients optimally solve
  - Works for non-convex problems
  - Convex case makes sense only if clients stop before convergence
    - Fix number of epochs
- General
  - Perform a fix number of epochs at clients
  - Convergence to optimality in the convex case under assumptions
    - Not trivial to establish
    - IID and non-IID cases

# Convergence of FedAvg with Non-IID data

Despite client drift, FedAvg is guaranteed to converge to the optimum for convex functions and the right choice of decreasing learning rate

$$\mathbb{E}[F(\mathbf{w}_T)] - F^* \leq \frac{\kappa}{\gamma + T - 1} \left( \frac{2B}{\mu} + \frac{\mu\gamma}{2} \mathbb{E}\|\mathbf{w}_1 - \mathbf{w}^*\|^2 \right), \quad (4)$$

where

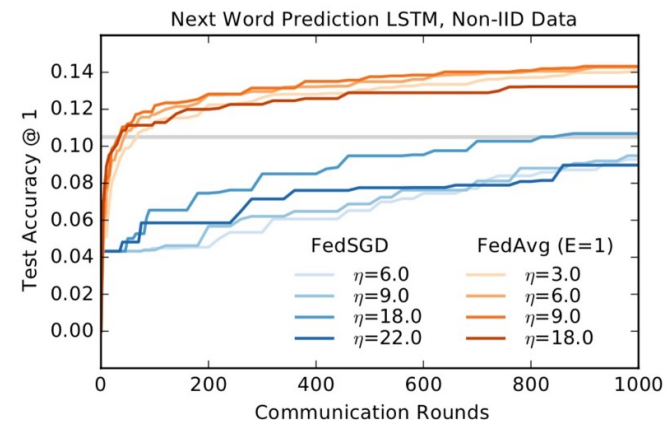
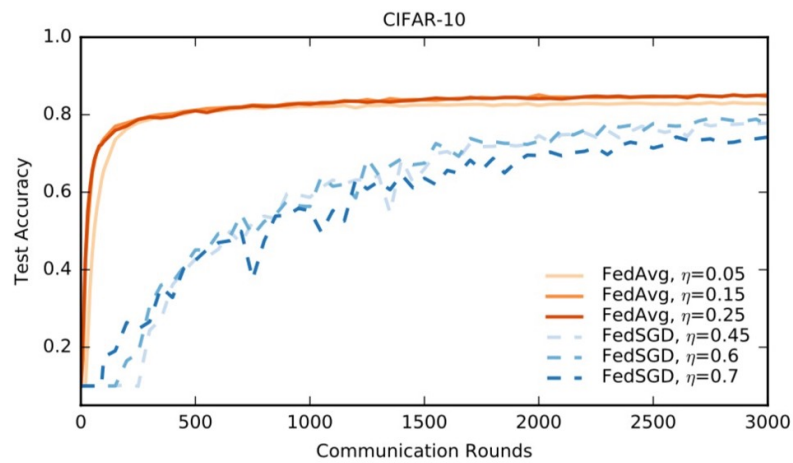
$$B = \sum_{k=1}^N p_k^2 \sigma_k^2 + 6L\Gamma + 8(E-1)^2 G^2. \quad (5)$$

Also guaranteed stationary point for smooth, non-convex objectives

# FedSGD

- Apply distributed gradient computation
  - Instead of models getting communicated, gradients are communicated
- Loop
  - For each client  $k$ 
    - Select mini batch  $B_k$
    - Compute  $\omega_k = \frac{1}{|B_k|} \nabla f_k(w; B_k)$
    - Send to server  $\omega_k$
  - Server
    - Receive all  $\omega_k$
    - $w = w - \eta \sum_k \omega_k$
    - Send  $w$  to each client
- More communication than in FedAvg
- Privacy
  - Gradients reveal more information
  - More susceptible to attacks

# Comparison





# Other Aspects of FL

- How to select validation/test data
  - Particularly in non-iid
- Normalization/standardization
- Full client participation vs partial
- Descriptive statistics sharing

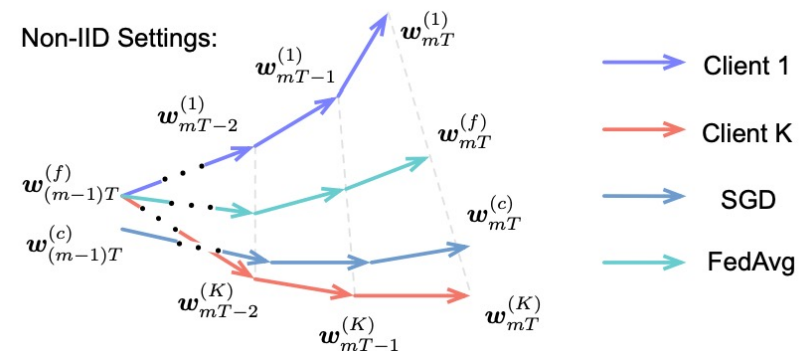
# ALGORITHMS FOR NON-IID HORIZONTAL FL

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# Motivation

- How non-IID challenges FedAvg's aggregation
  - $w_t = \sum_{k \in \mathcal{K}} \frac{n_k}{n} w_t^k$ 
    - Client updates  $w_t^k$  are learned from non-IID data, thus can be highly skewed. Averaging updates does not yield a similar model as training on global data. Weighted averaging can make it worse if a highly skewed client is large
- Question
  - Can each **local model** be trained on data that are **representative** of the global distribution at the time of aggregation, without sharing data?



Zhao, Y., Li, M., Lai, L., Suda, N., Cavin, D., & Chandra, V. (2018). Federated learning with non-iid data. arXiv preprint arXiv:1806.00582.

# RADFed – Randomized Aggregation Delayed

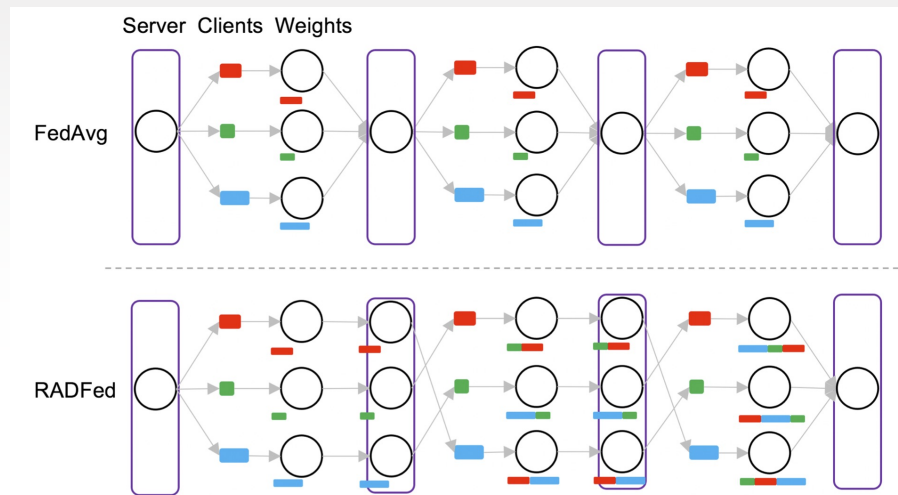


Fig. 1: A comparison of aggregation between RADFed and FedAvg. Colored blocks represent the data on clients. Each color represents a class. The length of each block shows the size of the data. Each circle represents a model's weights. Colored blocks under each circle show the data used to train a certain set of weights.

- FedAvg:

- 3 aggregations
- At each aggregation, each set of local model weights is trained on data of a **different** class

- RADFed:

- 1 aggregation
- Each set of local model weights is trained on data of **all three** classes (i.e., more **representative** of the global distribution)
- **Bonus:** same (or similar) number of samples trained at aggregation
- Cost: fewer aggregations but the same total number of communication rounds.

# RADFed – Randomized Aggregation Delayed

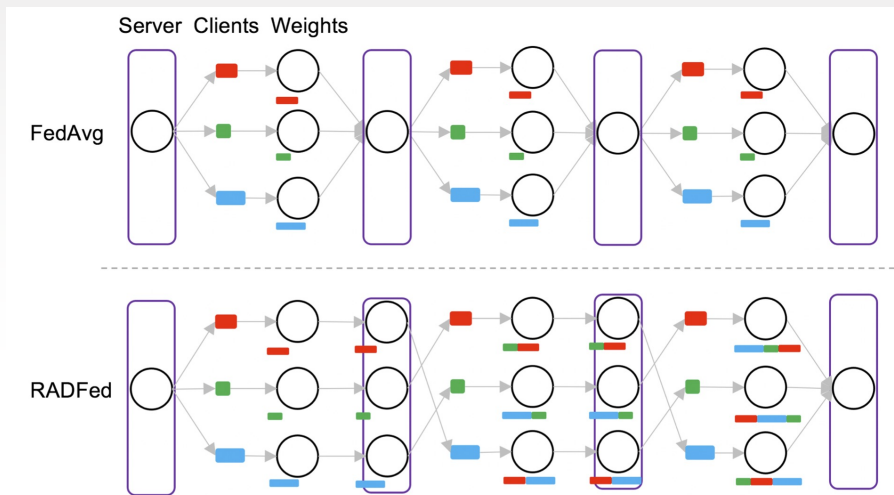


Fig. 1: A comparison of aggregation between RADFed and FedAvg. Colored blocks represent the data on clients. Each color represents a class. The length of each block shows the size of the data. Each circle represents a model's weights. Colored blocks under each circle show the data used to train a certain set of weights.

## Algorithm 1 RADFed

$K$  clients participate in training;  $C$  is the fraction of clients participating in each training round;  $T$  is the number of training iterations and  $S$  is the number of redistributing iterations.

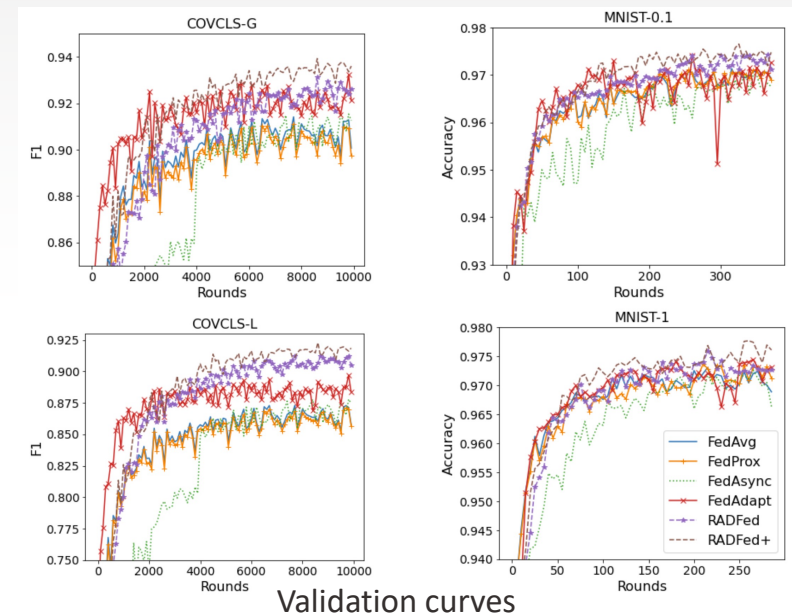
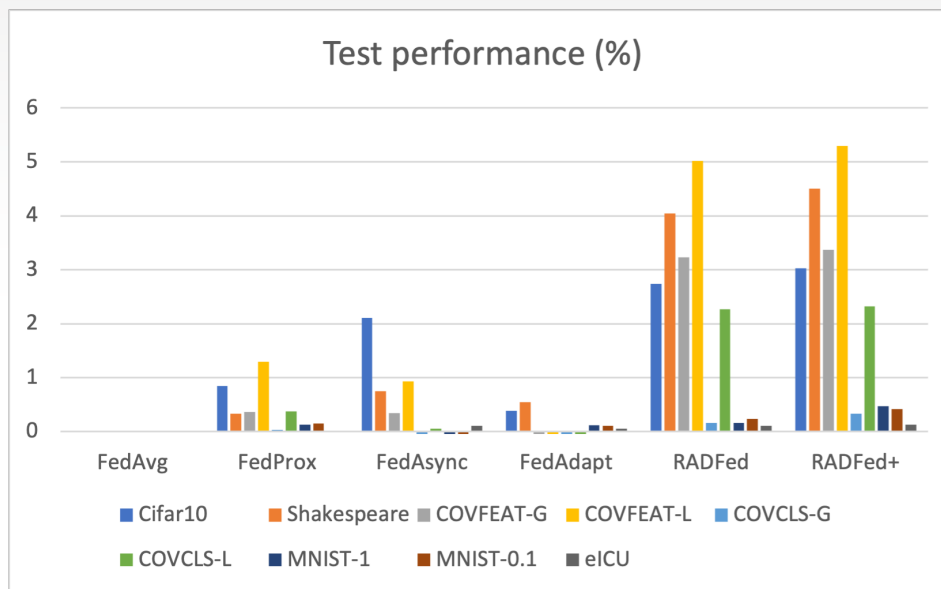
**Server executes:**

```

1: initialize  $w_1$ 
2:  $m \leftarrow \max(C \cdot K, 1)$ 
3: for each round  $t = 1, 2, \dots, T$  do
4:    $w_t^k \leftarrow w_t$ , for  $k = 1, 2, \dots, m$ 
5:    $\bar{w}_t \leftarrow (w_t^1, \dots, w_t^m)$ 
6:   for each redistributing iteration  $s = 1, 2, \dots, S$  do
7:      $U \leftarrow$  uniformly sample  $m$  training clients
8:     for  $i = 1, 2, \dots, m$  do
9:        $\bar{w}_{s+1}^i \leftarrow \text{ClientUpdate}(U_i, \bar{w}_s^i)$ 
10:    end for
11:  end for
12:   $w_{t+1} \leftarrow \frac{1}{m} \sum_{i=1}^m \bar{w}_{S+1}^i$  Bonus: no sharing client size
13: end for
14: return  $w_{T+1}$ 
  
```

# Performance

- Using Ray to simulate distributed training for over 100 clients

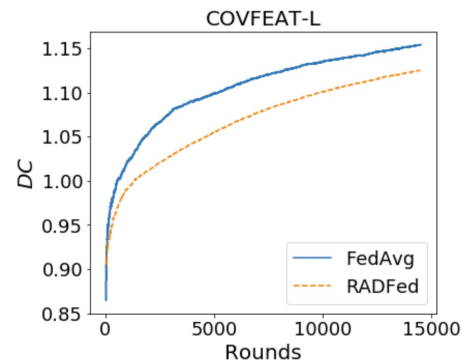


# Lower Divergence

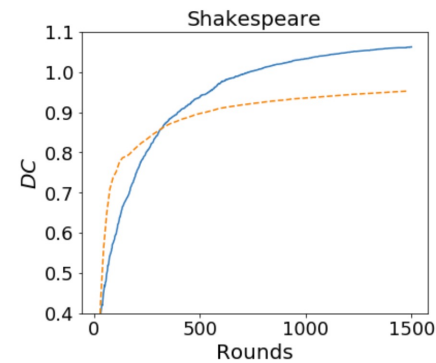
- Divergence between the Centralized and federated models (DC)<sup>5</sup>

- $$DC(t) = \frac{||w_{FL}^t - w_C^t||}{||w_C^t||}$$

- $w_{FL}^t$ : the weights of the global model in federated training at the  $t$ -th round
- $w_C^t$ : the weights in centralized training



(a)



(b)

<sup>5</sup> Zhao, Y., Li, M., Lai, L., Suda, N., Civin, D., & Chandra, V. (2018). Federated learning with non-iid data. arXiv preprint arXiv:1806.00582.

# Two Ideas from Standard Optimization

- Stochastic hedging
  - Force optimization variables/parameters to agree
- Variance reduction (SVRG)

## Procedure SVRG

**Parameters** update frequency  $m$  and learning rate  $\eta$

**Initialize**  $\tilde{w}_0$

**Iterate:** for  $s = 1, 2, \dots$

$$\tilde{w} = \tilde{w}_{s-1}$$

$$\tilde{\mu} = \frac{1}{n} \sum_{i=1}^n \nabla \psi_i(\tilde{w})$$

$$w_0 = \tilde{w}$$

**Iterate:** for  $t = 1, 2, \dots, m$

Randomly pick  $i_t \in \{1, \dots, n\}$  and update weight

$$w_t = w_{t-1} - \eta(\nabla \psi_{i_t}(w_{t-1}) - \nabla \psi_{i_t}(\tilde{w}) + \tilde{\mu})$$

**end**

**option I:** set  $\tilde{w}_s = w_m$

**option II:** set  $\tilde{w}_s = w_t$  for randomly chosen  $t \in \{0, \dots, m-1\}$

**end**

Johnson, Rie, and Tong Zhang.

"Accelerating stochastic gradient descent using predictive variance reduction."  
In Advances in Neural Information Processing Systems, pp. 315-323. 2013



# Other FedAvg-Style Algorithms

## - SCAFFOLD

- Uses control variables on clients and server to reduce the client drift
- Improves convergence in non-IID settings

## - FedProx

- Adds a proximal term to each local objective function to be minimized that serves to minimize the drift of the local weights from the global weights

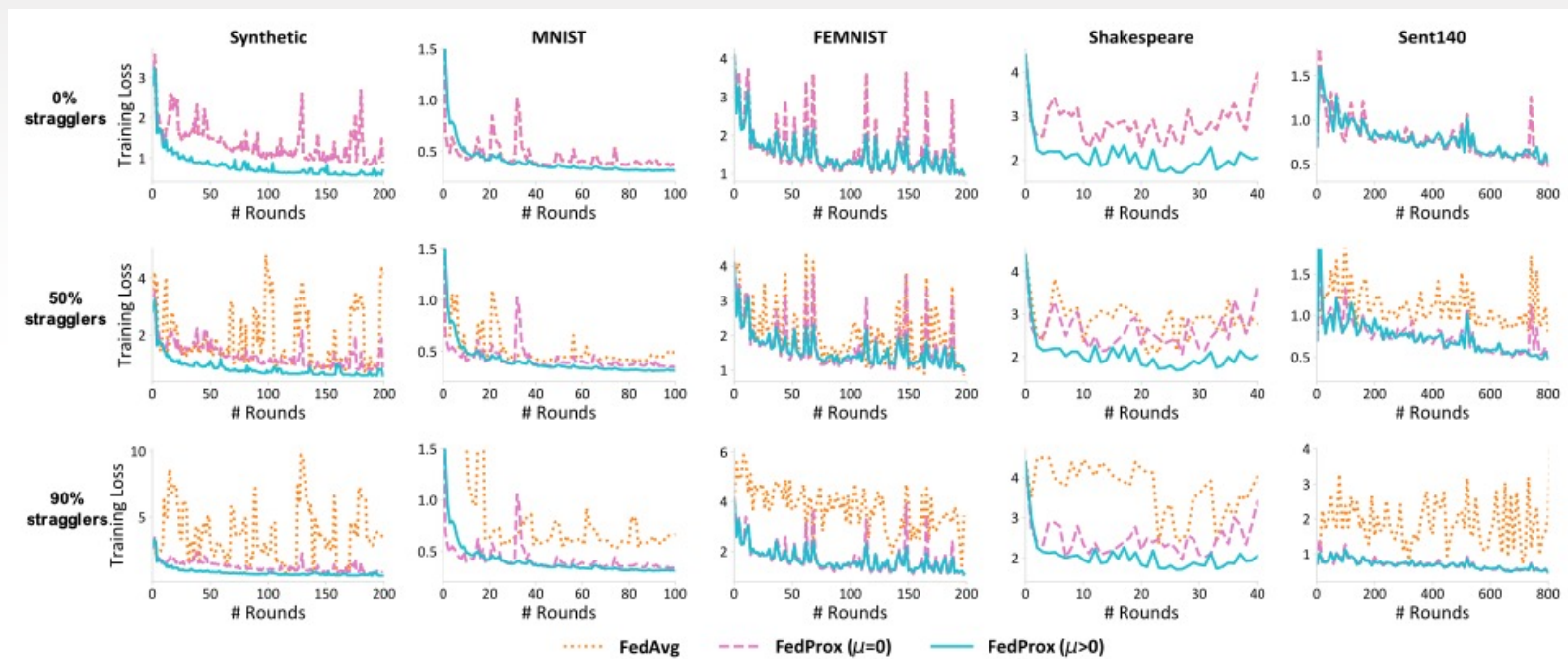
## - FedSAM

- Uses sharpness aware minimization (SAM) methods on each local optimization problem to help convergence to a wider minima on the global problem

# FedProx [Li, et al. 2020]

- Incentivize clients to produce the same model (weights)
- Same framework as FedAvg
- Clients solve 'regularized' problem
  - $loss = F_k(w) + \frac{\mu}{2} ||w - w^t||^2$
  - Minimize original loss + coordination incentive
  - $w^t$  = current global solution obtained/maintained by the server

# FedProx



- SCAFFOLD [Karimireddy et al, 2019]

[Algorithm]














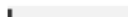
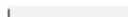




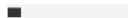
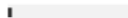

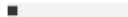
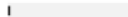

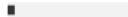
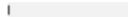






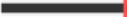
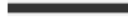
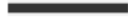



- global server has global model  $x$  and global control  $c$
- local user  $k$  has local model  $w_k$  and local control  $c_k$
- for initialization, set  $c = c_k = 0$

For each round,

1. global server broadcasts  $(x, c)$  to some fraction of local users  $\mathcal{K}$
2. local user  $k$  initialize local model as  $w_k \leftarrow w$
3. for  $E$  times (for  $i = 1, \dots, E$ )
  - a. compute approximate gradient(mini-batch gradient)  $g_k(w_k)$
  - b. update the local model  $w_k \leftarrow w_k - \eta_l(g_k(w_k) - c_k + c)$
  - c. update local control  $c_k^+ \leftarrow$  (i)  $g_k(w)$ , or (ii)  $c_k - c + \frac{1}{E\eta_l}(w - w_k)$
  - d. send  $(\Delta w_k, \Delta c_k) = (w_k - w, c_k^+ - c_k)$  and set  $c_k \leftarrow c_k^+$
4. global server aggregates local updates as  $w \leftarrow w + \eta_g \frac{1}{|\mathcal{K}|} \sum_{k \in \mathcal{K}} \Delta w_k, \quad c \leftarrow c + \frac{1}{N} \sum_{k \in \mathcal{K}} \Delta c_k$

- First round equals FedAvg
- Borrows SVRG ideas
- Requires fewer rounds of communication
- Not affected by data heterogeneity or client sampling

# SCAFFOLD

	Epochs	0% similarity (sorted)		10% similarity		100% similarity (i.i.d.)	
		Num. of rounds	Speedup	Num. of rounds	Speedup	Num. of rounds	Speedup
SGD	1	317 	(1×)	365 	(1×)	416 	(1×)
SCAFFOLD1		<b>77</b> 	(4.1×)	62 	(5.9×)	60 	(6.9×)
	5	152 	(2.1×)	20 	(18.2×)	10 	(41.6×)
	10	286 	(1.1×)	16 	(22.8×)	7 	(59.4×)
	20	266 	(1.2×)	<b>11</b> 	(33.2×)	<b>4</b> 	(104×)
FEDAVG	1	258 	(1.2×)	74 	(4.9×)	83 	(5×)
	5	428 	(0.7×)	34 	(10.7×)	10 	(41.6×)
	10	711 	(0.4×)	25 	(14.6×)	6 	(69.3×)
	20	1k+ 	(< 0.3×)	18 	(20.3×)	<b>4</b> 	(104×)
FEDPROX	1	1k+ 	(< 0.3×)	979 	(0.4×)	459 	(0.9×)
	5	1k+ 	(< 0.3×)	794 	(0.5×)	351 	(1.2×)
	10	1k+ 	(< 0.3×)	894 	(0.4×)	308 	(1.4×)
	20	1k+ 	(< 0.3×)	916 	(0.4×)	351 	(1.2×)

# FedSAM [Caldarola et al, 2022]

- Flat minima generalize better

- Empirical and theoretical evidence

- Flatness

- Smallest eigenvalue of Hessian
- Sharpness  $\max_{\|\epsilon\|_p \leq \rho} \mathcal{L}_{\mathcal{D}}(\theta + \epsilon) - \mathcal{L}_{\mathcal{D}}(\theta)$

$$\min_{\theta \in \mathbb{R}^d} \max_{\|\epsilon\|_p \leq \rho} \mathcal{L}_{\mathcal{D}}(\theta + \epsilon) + \lambda \|\theta\|_2^2$$

- First order Taylor approximation  $\bar{\epsilon}(\theta) =$

$$\arg \max_{\|\epsilon\|_2 \leq \rho} \mathcal{L}_{\mathcal{D}}(\theta) + \epsilon^T \nabla_{\theta} \mathcal{L}_{\mathcal{D}}(\theta) = \rho \frac{\nabla_{\theta} \mathcal{L}_{\mathcal{D}}(\theta)}{\|\nabla_{\theta} \mathcal{L}_{\mathcal{D}}(\theta)\|_2}$$

$$\min_{\theta} L_D(\theta + \bar{\epsilon}(\theta))$$

- Problem?
- FedSAM: clients use such updates