

a. An explanation of the model

The goal of this linear programming model is to maximize the profit generated by a brewery. The profit is calculated as the revenue from selling beer and ale minus the cost of raw materials.

Here's how it's formulated:

**1. Decision Variables:**

A: the number of barrels of Ale to produce.

B: the number of barrels of Beer to produce.

C1: quantity of corn bought below 480 units for \$1.20

C2: quantity of corn bought between 480 to 980 units for \$2.00

C3: quantity of corn bought above 980 units for \$2.50

$C = C1 + C2 + C3$  = total quantity bought for corn

H1: quantity of hops bought below 160 units for \$7.75

H2: quantity of hops bought between 160 to 360 units for \$8.25

$H = H1 + H2$  = total quantity bought for hops

M1: quantity of malt bought below 1190 units for \$1.40

M2: quantity of malt bought between 1190 to 2190 units for \$1.50

M3: quantity of malt bought between 2190 to 3190 units for \$1.55

$M = M1 + M2 + M3$  = total quantity bought for Malt

**2. Objective Function:**

Maximize Profit = Revenue - Cost =  $(100A + 100B) - (1.2 \cdot C1 + 2 \cdot C2 + 2.5 \cdot C3 + 7.75 \cdot H1 + 8.25 \cdot H2 + 1.4 \cdot M1 + 1.5 \cdot M2 + 1.55 \cdot M3)$

**3. Constraints:**

a. Quantities purchased = Quantities for produce:

$C1 + C2 + C3 = 15B + 5A$

$H1 + H2 = 4B + 4A$

$M1 + M2 + M3 = 20B + 35A$

b. Buying availability constraints:

$H1 + H2 \leq 360$

$M1 + M2 + M3 \leq 3190$

c. Price segments constraints

$C1 \leq 480$

$C2 \leq 980 - 480$

$H1 \leq 160$ ,

$H2 \leq 360 - 160$

$M1 \leq 1190$

$M2 \leq 2190 - 1190$

$M3 \leq 3190 - (2190 - 1190)$

d. Non-negativity constraints:  
A, B, C1, C2, C3, H1, H2, M1, M2, M3  $\geq 0$

b. The solution to the model

(complete code in 440\_HW2\_Q1.ipynb)

```
Status: Optimal
Optimal A: 37.0
Optimal B: 53.0
Optimal C1: 480.0
Optimal C2: 500.0
Optimal C3: 0.0
Optimal H1: 160.0
Optimal H2: 200.0
Optimal M1: 1190.0
Optimal M2: 1000.0
Optimal M3: 165.0
Optimal Profit: $1112.25
```

c. An explanation of what the solution shows

The solution shows that the optimal solutions are:

1. Purchasing 37 units of ale and 53 units of beer
2. Purchasing 480 units of Corn at \$1.20 each (C1), 500 units at \$2.00 each (C2), and none at \$2.50 each (C3).
3. Purchasing 160 units of Hops at \$7.75 each (H1) and 200 units at \$8.25 each (H2).
4. Purchasing 1190 units of Malt at \$1.40 each (M1), 1000 units at \$1.50 each (M2), and 165 units at \$1.55 each (M3).

The optimal total profit obtained from this production plan is \$1112.25.

d.

The solution given by model is:

```
Status: Optimal
Optimal A: 12.0
Optimal B: 28.0
Optimal C1: 480.0
Optimal C2: 0.0
Optimal C3: 0.0
Optimal H1: 160.0
Optimal H2: 0.0
Optimal M1: 980.0
Optimal M2: 0.0
Optimal M3: 0.0
Optimal Profit: $812.0
```

The model made the same decision as the original model with approximately the same profit.

e. the cost of corn goes down to \$1.20 for any quantity

Status: Optimal  
Optimal A: 0.0  
Optimal B: 90.0  
Optimal C1: 0.0  
Optimal C2: 0.0  
Optimal C3: 1350.0  
Optimal H1: 160.0  
Optimal H2: 200.0  
Optimal M1: 1190.0  
Optimal M2: 610.0  
Optimal M3: 0.0  
Optimal Profit: \$1909.0

Due to the price change, the cost of corn becomes lower, so it's more profitable to produce more corn in this scenario. Therefore the quantity of corn goes from 980 in model a,b,c to 1350. With more corn, the model chose to produce more beer, which needs 10 more units of corn than ale.

f. make 3 more units of Ale

The model in part a,b,c gives the optimal to make 37 ale and 53 beer, so in this question the model needs to make 40 ale, giving the solution below:

Status: Optimal  
Optimal A: 40.0  
Optimal B: 50.0  
Optimal C1: 480.0  
Optimal C2: 470.0  
Optimal C3: 0.0  
Optimal H1: 160.0  
Optimal H2: 200.0  
Optimal M1: 1190.0  
Optimal M2: 1000.0  
Optimal M3: 210.0  
Optimal Profit: \$1102.5

- From the solution, the model adjusts to produce 3 less barrels of beer due to raw materials constraints. The increase in the production of ale by 3 units has led to adjustments in the quantities of raw materials. To meet the higher Ale production requirement, the model suggests purchasing less corn and more malts than the solution in model a,b,c. This is due to the reason that the production of ale needs 10 less units of corn while it needs 15 more units of malt compared to beer. This change is necessary to ensure that there are enough raw materials to produce the additional Ale units.
- From a business perspective, this result indicates that if we need to produce 3 more units of Ale, necessary adjustments in the quantities of raw materials are needed to accommodate this change.

- However, it's important to note that this adjustment may impact costs and profit margins. Purchasing more raw materials, especially at different price points, can affect the overall cost structure of production. Businesses should evaluate the cost implications of such changes and assess whether the increased production aligns with their profit goals and market demand.