MLDS 440 - LAB 01

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Algebraic Formulation

Standard form

Decision variable:	$x \in \mathbb{R}^d$
Maximize:	$c^\top x$
Subject to:	$Ax \leq b$
	$x \geq 0$

where $c \in \mathbb{R}^d$, $A \in \mathbb{R}^{d_c \times d}$ and $b \in \mathbb{R}^{d_c}$. $Ax \leq b$ means $(Ax)_i \leq b_i$ for all $i = 1, \ldots, d_c$.

Any LP problem can be converted in this form. Why?

Food manufacture problem

The single-period problem

- A product is manufactured by refining raw oils and blending them together. The raw oils are of two categories: Vegetable oils (VEG1, VEG2) and Non-vegetable oils (OIL1, OIL2, OIL3). Each oil can be purchased at the prices below ($/ton):

VEG1	VEG2	OIL1	OIL2	OIL3
110	120	130	110	115

- The final product sells at $150 per ton.
- Vegetable oils and non-vegetable oils require different production lines for refining, it is not possible to refine more than 200 tons of vegetable oils and more than 250 tons of non-vegetable oils.
- There is no loss of weight in the refining process and the cost of refining may be ignored.

Food manufacture problems

The single-period problem

- There is a technological restriction of hardness on the final product. In the units in which hardness is measured, this must lie between 3 and 6. It is assumed that hardness blends linearly and that the hardnesses of the raw oils are

VEG1	VEG2	OIL1	OIL2	OIL3
8.8	6.1	2.0	4.2	5.0

- What buying and manufacturing policy should the company pursue in order to maximize profit?

LP formulation

VEG1	VEG2	OIL1	OIL2	OIL3	Product
x1	x2	x3	x4	x5	Y

Maximize:

$$-110x_1 - 120x_2 - 130x_3 - 110x_4 - 115x_5 + 150y$$

subject to:

$$\begin{aligned}
&x_1 + x_2 && \leq 200 \\
&x_3 + x_4 + x_5 && \leq 250 \\
&8.8x_1 + 6.1x_2 + 2x_3 + 4.2x_4 + 5x_5 - 6y && \leq 0 \\
&-8.8x_1 - 6.1x_2 - 2x_3 - 4.2x_4 - 5x_5 + 3y && \leq 0 \\
&x_1 + x_2 + x_3 + x_4 + x_5 - y && = 0 \\
&x_1, x_2, x_3, x_4, x_5, y && \geq 0
\end{aligned}$$

Food manufacture problems

The multi-period problem

Now, each oil may be purchased for immediate delivery (January) or bought on the future market for delivery in a subsequent month. Prices at present and in the futures market:

$\{p_{ij}\}_{i=1,\ldots,6,\ j=1,\ldots,5}$

	VEG1	VEG2	OIL1	OIL2	OIL3
Jan	110	120	130	110	115
Feb	130	130	110	90	115
Mar	110	140	130	100	95
Apr	120	110	120	120	125
May	100	120	150	110	105
Jun	90	100	140	80	135

Food manufacture problems

The multi-period problem

- It is possible to store up to 1000 tons of each raw oil for use later. The cost of storage for vegetable and non-vegetable oil is $5 per ton per month. The final product cannot be stored, nor can refined oils be stored.

- At present, there are 500 tons of each type of raw oil in storage. It is required that these stocks will also exist at the end of June.

LP formulation

Decision variables

- For month i: how much to produce? y_i
- For month i and type j of oil:
  - How much to buy? b_{ij}
  - How much to use? u_{ij}
  - How much in the storage at the end of month j? s_{ij}
  - Anything to say about s ?

LP formulation

Objective

$$-\sum_{i=1}^{6}\sum_{j=1}^{5} p_{ij}b_{ij} - \sum_{i=1}^{5}\sum_{j=1}^{5} 5s_{ij} + \sum_{i=1}^{6} 150y_i$$

LP formulation

Constraints

- Production capacities

$$u_{i1} + u_{i2} \leq 200, \quad u_{i3} + u_{i4} + u_{i5} \leq 250 \quad \text{for } i = 1,\ldots,6$$

- Hardness

$$3y_i \leq 8.8u_{i1} + 6.1u_{i2} + 2u_{i3} + 4.2u_{i4} + 5u_{i5} \leq 6y_i \quad \text{for } i = 1,\ldots,6$$

- Continuity

$$\sum_{j=1}^{5} u_{ij} - y_i = 0 \quad \text{for } i = 1,\ldots,6$$

- Storage linking:

$$s_{(i-1)j} + b_{ij} - u_{ij} - s_{ij} = 0 \quad \text{for } i = 1,\ldots,6 \text{ and } j = 1,\ldots,5$$

Fix $s_{0j} = s_{6j} = 500 \quad \text{for } j = 1,\ldots,5.$

- Storage Capacity

$$s_{ij} \leq 1000 \;\; \text{for } i = 1,\ldots,6 \;\; \text{and } j = 1,\ldots,5$$