Problem 4

L 1.) $\overset{\text{cov}}{C}(X, Y) = E[(X-\mu_x)(Y-\mu_y)]$

$\Rightarrow C(aX, bY) = E[a(X-\mu_x)\,b(Y-\mu_y)]$

$= E[abXY - abX\mu_y - abY\mu_x + ab\mu_x\mu_y]$

$= ab\,E[(X-\mu_x)(Y-\mu_y)] = ab\,C(X, Y)$

L 2.) $C(X+Y, Z) = E[(X+Y-\mu_{X+Y})(Z-\mu_Z)]$

$= E[(X+Y-\mu_X-\mu_Y)(Z-\mu_Z)]$

$= E[XZ - X\mu_Z + YZ - Y\mu_Z - \mu_X Z + \mu_X\mu_Z - \mu_Y Z + \mu_Y\mu_Z]$

$= E[(XZ - X\mu_Z - \mu_X Z + \mu_X\mu_Z)\ (YZ - Y\mu_Z - \mu_Y Z + \mu_Y\mu_Z)]$

$= E[X(Z-\mu_Z) - \mu_X(Z-\mu_Z) + Y(Z-\mu_Z) - \mu_Y(Z-\mu_Z)]$

$= E[(X-\mu_X)(Z-\mu_Z) + (Y-\mu_Y)(Z-\mu_Z)]$

$= E[(X-\mu_X)(Z-\mu_Z)] + E[(Y-\mu_Y)(Z-\mu_Z)]$

$= C(X, Z) + C(Y, Z)$