
Two unlikely tales

A league table

Partway through a round-robin soccer tournament involving five teams, all official match records were accidentally destroyed. The parts that could be entered with certainty from memory are shown in the table below (for and against are total goals for and total goals against each team). In the complete tournament, each team was supposed to play each of the others once.

Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B			1		0	5
C	2		1		7	4
D		1		0	2	2
E		2		0	5	2

The league commissioner claims that the complete table can be reconstructed from this information, including the individual games played to date and the results (including score) in those.

Are they correct?

Hats, hats, hats

I have five very clever, but unimaginatively named, friends: *A*, *B*, *C*, *D* and *E*. Recently I happened to notice them come into a room and sit down in a line of five chairs also labelled *A* to *E* from front to back. Not all the chair labels matched the names of the persons sitting in them. Each person could only see the people further forward in the line. Having a bag containing three black hats and three white hats to hand, this seemed too good an opportunity to miss. I showed them the bag (and the hats in it) and placed a hat on each person's head in such a way that they could not see the colour of their own hat (or the hats remaining in the bag). Then I asked the traditional question: "*Does anyone know what colour hat they're wearing?*" After a brief, but significant, pause, *C* and *D* simultaneously said "*I do*", and hearing that, *B* said "*So do I*". After that came silence. The fun over, I packed things up and noticed that *A*'s hat was the same colour as the one left in the bag.

In what order were they sitting?

Task

Answer the questions above in a way that is clear and convinces the reader that you are correct. Some things to watch out for:

- The reader is intelligent, but not interested in solving the problem on their own.
- You must not presuppose that there is a unique answer (i.e., “this works therefore it is correct” is not convincing).
- Do not make any assumptions beyond the facts stated in the text. For instance, “part way through a round-robin tournament” means only that each team has played every other team at most once – it’s perfectly possible (though inconsistent with the data) that one team might already have played all their games and the others have played only against that one team.

This task is much more about presenting your answers effectively than it is about getting the answers.

Objectives

1.1, 1.2, 2.1, 2.5, 2.6, 2.7, 4.8.

(Individual)