

Samuel Terry - 678635

Etude 13, Two Tales - Hats, hats, hats

All of my hat sequences will be going from left (front/A) to right (back/E).

A B C D E

←----- (They are facing this direction)

Given the limited information almost everyone is able to determine what color hat is on their head. But we are able to determine everyone's hat color and the order they are sitting in. We know there are three white hats and three black hats and that after hats are handed out one will be placed into a bag. This means that all possible outcomes consist of either three white hats and two black hats OR three black hats and two white hats. Below are all the combinations using three white hats and two black hats (and the inverse).

W W W B B	B B B W W
W W B B W	B B W W B
W B B W W	B W W B B
B B W W W	W W B B B
B W W W B	W B B B W
B W B W W	W B W B B
B W W B W	W B B W B
W B W B W	B W B W B
W B W W B	B W B B W
W W B W B	B B W B W

There is some information we know.

The person in seat A can never know what color hat is on their head because they can't see anyone else's hat.

Every person except A can see the color of the hat in front of them.

Now, in this particular example the first two people to know the colors of their hat were seat C and seat D. This means that seat E didn't know the color of his hat. So let's look at some of our possible hat sequences and what information they would tell seat E.

If seat E sees three hats of the same color in any order they will know their hat color right away. This is because there are at most three hats of a certain color. If E sees three white hats then he knows right away that he has a black hat on his head and vice-versa. Because seat E doesn't know the color of their hat right away, we know that they didn't see three hats of the same color in front of them. This rules out any sequences where there are three hats of the same color in front of seat E. For example, W W B W B or B B B W W.

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This leaves us with the following possible scenarios.

W W B B W	B B W W B
W B B W W	B W W B B
B B W W W	W W B B B
B W B W W	W B W B B
B W W B W	W B B W B
W B W B W	B W B W B

Now, in all of these scenarios there is no way for seat E to know the color of their hat because they see two black hats and two white hats in front of them. We know that seat D and seat C know the color of the hats on their heads. But how is this possible? **Without knowing** the color of the hat in seat E, the person in seat D can only know what is in front of them if they see three of the same color. If they see two whites and a black or vv (vice-versa) they have no way of knowing which color there is three of or two of in the scenario.

Now, if seat D knows the color of their hat (they see three of the same color in front of them), then how would seat C know their color at the exact moment as seat D. Seat C would be seeing two hats of the same color : W W / B B. If seat C doesn't know the color of the hat(s) behind them and they don't see three hats (they only see two) of the same color in front of them, there is no way to know what the color of the hat on their head is.

Because seat C and seat D spoke simultaneously. Seat C's answer wasn't influenced by anything seat D could see. So, it isn't possible for seat C to know the color of their hat at the same time as seat D, if seat D sees three hats of the same color in front of them. This rules out all scenarios where seat D sees three hats of the same color.

This leaves us with the following scenarios.

W W B B W	B B W W B
W B B W W	B W W B B
B B W W W	W W B B B
B W B W W	W B W B B
B W W B W	W B B W B
W B W B W	B W B W B

In our scenario seat C and D speak simultaneously after a brief but significant pause (based on what seat E knows/doesn't know). Now, if seat E sees some combination of two white hats and two black hats they can't know their hat color. This means seat C and D are aware that seat E doesn't know his hat color, which means seat E sees two black hats and two white hats in some order. With this information we know that

Seat D sees : two white hats and a black in some order and vv.

Seat C sees : two white, two black, one white one black, one black one white.

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With seats D and C knowing their hat colors, seat B was also able to determine the color of their hat. If seat E doesn't know their color then D and C know E sees two black and two white. IF seat D and seat C see two white they know they are two black and vv.

If seat D sees two black or two white in front, they know their hat color, but if seat C sees W B or B W they aren't sure of their hat color. And so seats D AND C need to see two hats of the same color in front of them to determine their own hat colors simultaneously.  
This gives us the following scenarios.

W W B B W	B B W W B
B B W W W	W W B B B

Seat D and seat C were able to determine the color of their hats because seat E didn't know their own color. Because D and C were able to determine the color of their hats simultaneously, we know they both saw two hats of the same color in front of them. And so seat B was able to use this information to know the color of their hat, knowing that their hat color would be the same as the person in front of them. Unfortunately for the person in seat A, they aren't able to determine their hat color based on this information because they don't know the colors of anyone's hats. But seat A would know that their hat color is the same as seat B's hat color. The scenarios stay the same.

W W B B W	B B W W B
B B W W W	W W B B B

As the game wraps up we find out that A's hat was the same color as the hat in the bag. This means that A can't have been in seat E. This is because seat E sees two white hats and two black hats, which means seat E has the third hat of some color. And so seats A and B had black hats while the rest had white hats and vv. This gives us the following scenarios.

B B W W W
W W B B B

Of these scenarios how are we able to figure out the seating order? Well, A is in seat A, B is in seat B and E is in seat E. But we are told in our scenario that not all chair labels matched the names of the person sitting in them. And so C needs to be sitting in seat D and D needs to be sitting in seat C and so the final sitting order is.

A B D C E

Seat1 > seat2 > seat3> seat4> seat5