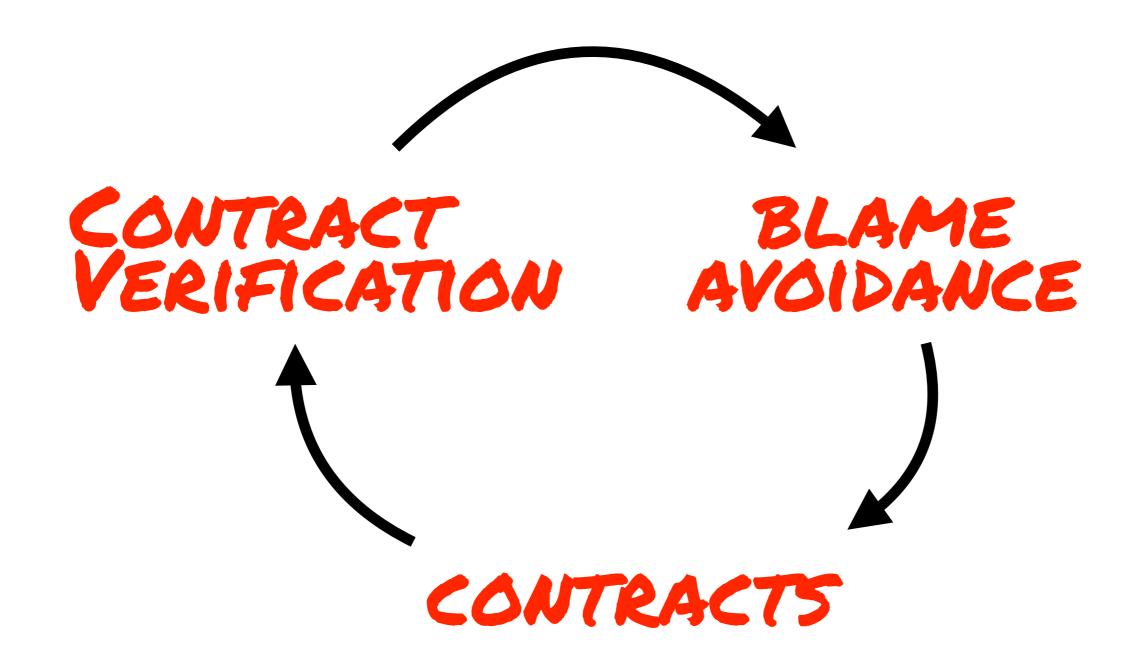
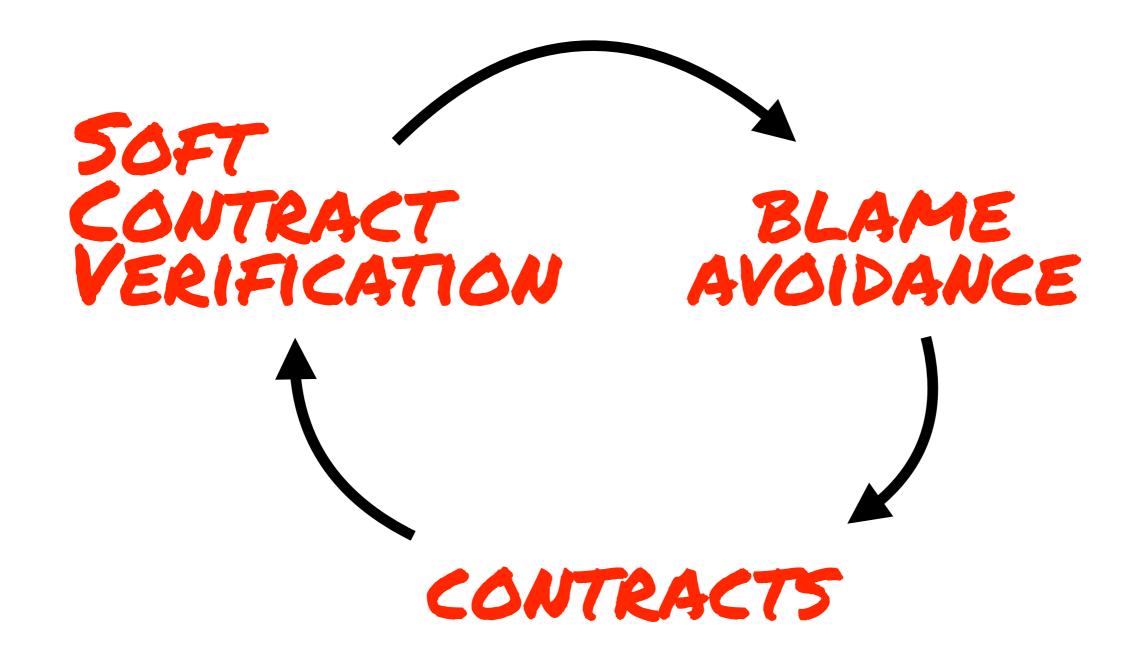


BLAME AVOIDANCE



CONTRACTS





SOFT CONTRACT VERIFICATION

Phuc C. Nguyen

Sam Tobin-Hochstadt

David Van Horn



SOFT CONTRACT VERIFICATION:

- FIRST-CLASS, DEPENDENT, RECURSIVE, DISJ., 4 CONJ. CONTRACTS
- LIFTS FIRST-ORDER SMT SOLVER WITHOUT ENCODINGS
- COMPETITIVE W/ BOTH LIGHT + HEAVY WEIGHT VERIFICATION TECHNIQUES
- WORKS EVEN WITHOUT APPARENT CONTRACTS







```
;; Terms
M ::= X N / + - * =
       (\lambda ([X : T] ...) M)
       (M M \dots)
       (if0 M M M)
       (C \stackrel{\triangle}{=} M)
;; Contracts
C ::= M
       (C \dots -> C)
```



```
(/ 10 (if0 7 2 3))
     (/103)
```



```
(/ 10 (if0 7 2 0))
      (/100)
(err "Divide by zero")
```







```
((\lambda ((f : (nat -> nat))) (f 3))
   (\lambda ((x : nat)) (+ x (/ 10 x))))
                   B
((\lambda ((x : nat)) (+ x (/ 10 x))) 3)
                   B
            (+ 3 (/ 10 3))
                   01
                 (+ 3 3)
                   01
```



```
(pos? 4 7)
(if0 (pos? 7) 7 blame)
   (if0 0 7 blame)
```



```
(pos? 4 0)
(if0 (pos? 0) 0 blame)
          01
    (if0 1 0 blame)
         blame
```



```
((zero? -> pos?) Φ (λ ((x : nat)) (+ 1 x)))

(λ ((x : nat))
(pos? Φ ((λ ((x : nat)) (+ 1 x))
(zero? Φ x))))
```

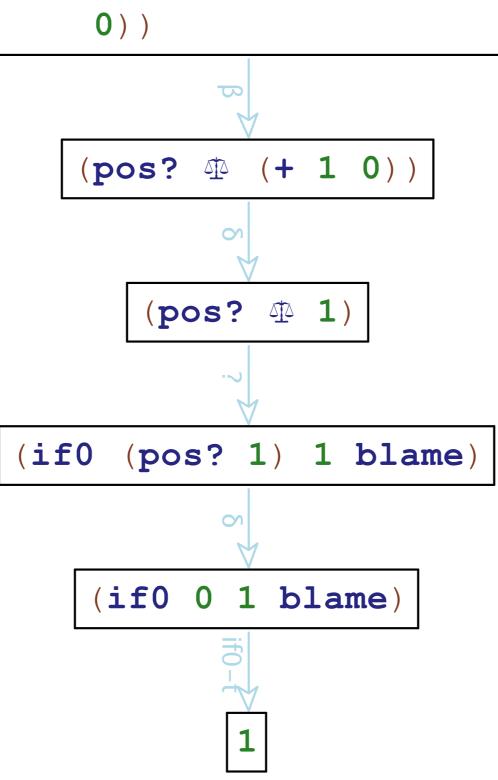


```
(((zero? -> pos?) \oplus (\lambda ((x : nat)) (+ 1 x)))
 0)
     ((\lambda ((x : nat))
         (pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
                    (zero? 4 x))))
      0)
                          TO
       (pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
                  (zero? 4 0)))
      (pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
                  (if0 (zero? 0) 0 blame)))
                          01
```



```
(pos? Φ ((λ ((x : nat)) (+ 1 x))
```







```
(((zero? -> pos?) \oplus (\lambda ((x : nat)) (+ 1 x)))
 7)
     ((\lambda (x : nat))
         (pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
                    (zero? 4 x))))
      7)
                         B
       (pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
                  (zero? 4 7)))
      (pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
                  (if0 (zero? 7) 7 blame)))
                          01
```

```
B
(pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
           (zero? 4 7)))
(pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
           (if0 (zero? 7) 7 blame)))
                   01
 (pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
            (if0 1 7 blame)))
(pos? \Phi ((\lambda ((x : nat)) (+ 1 x))
           blame))
                 blame
```

CPCF

SYMBOLIC CPCF

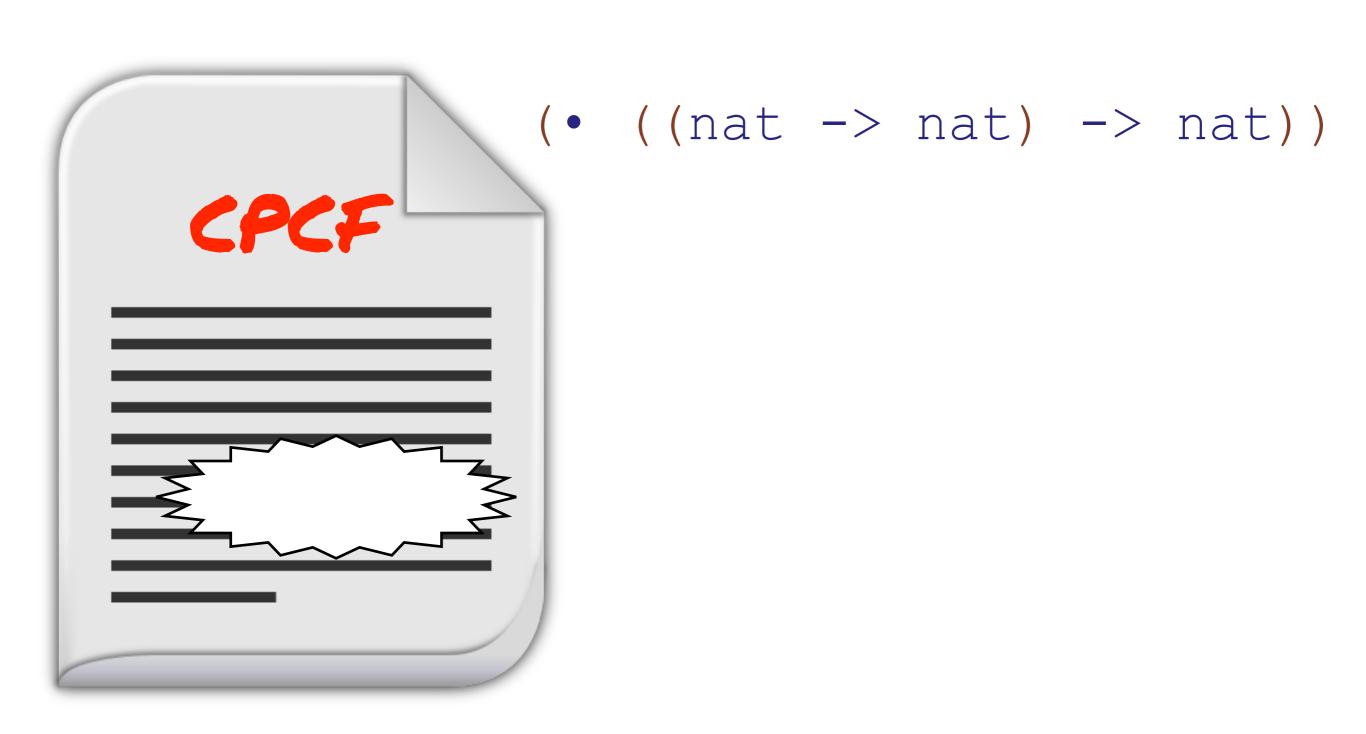




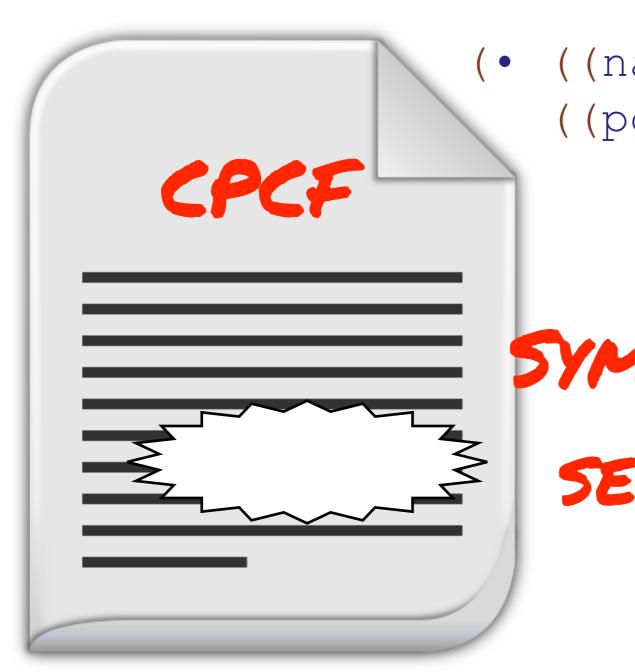




```
(λ ([f : (nat -> nat)])
(f 3))
```

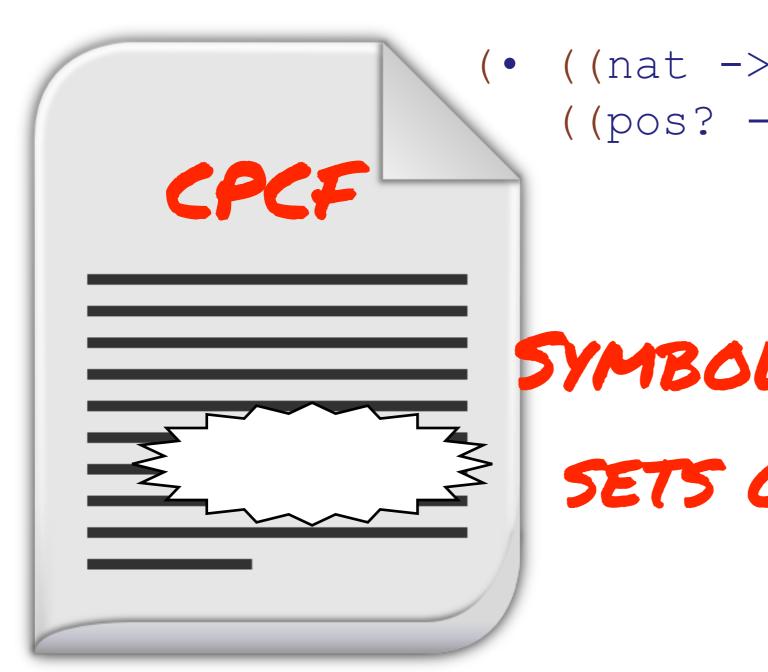






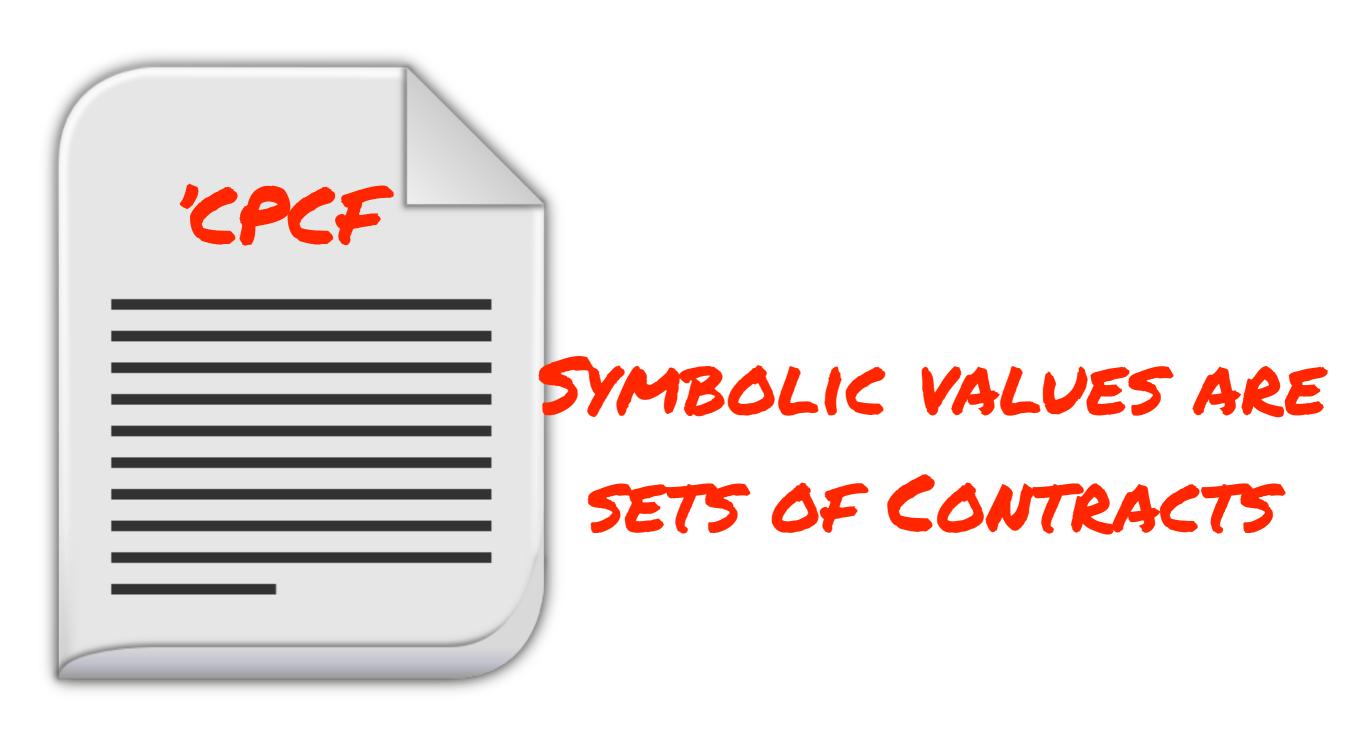
SYMBOLIC VALUES ARE
SETS OF CONTRACTS

ABSTRACTION



SYMBOLIC VALUES ARE
SETS OF CONTRACTS

ABSTRACTION





SOUNDNESS:

ALL CONCRETIZATIONS ARE APPROXIMATED BY 'CPCF

- 1) CONTRACTS AS SYMBOLIC VALUES
- 2) CHECKS REFINE VALUES
- 3) REFINEMENTS INFLUENCE COMPUTATION
- 4) PARTITION VERIFICATION W/ BLAME
- 5) HEAP OF INVARIANTS -> SMT



```
(/ 10 (if0 7 2 3))
```



```
(/ (• nat) (if0 7 2 3))
```



```
(/ (• nat) (if0 7 2 3))
     (/ (• nat) 3)
        (• nat)
```

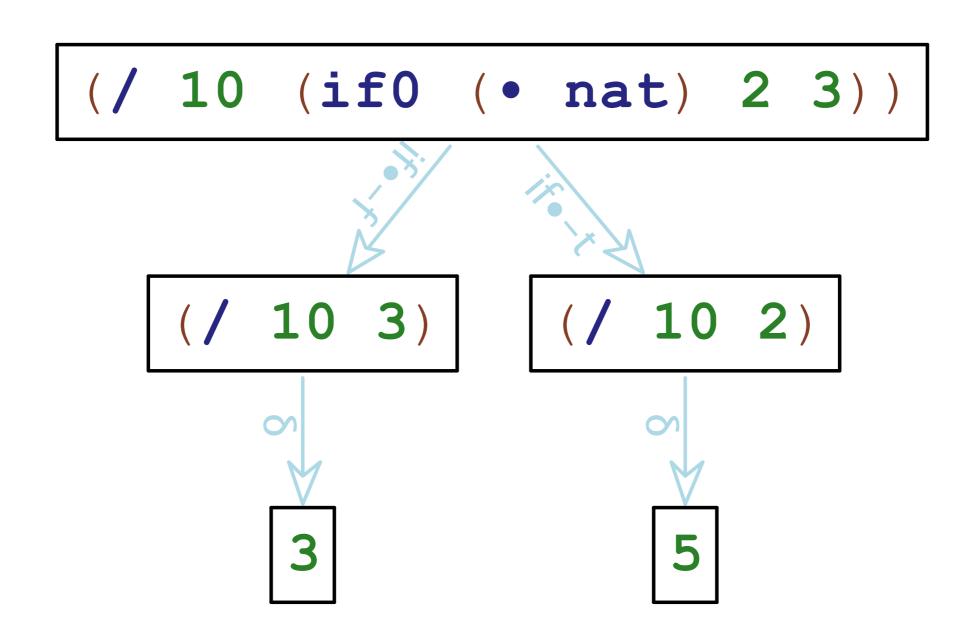


```
(/ 10 (if0 7 2 3))
```



```
(/ 10 (if0 (• nat) 2 3))
```







(add1 7)



```
((• (nat -> nat)) 7)
```



(• nat)



```
((• (nat -> nat)) 7)
((\lambda (y : nat)) \Omega) 7)
                            (• nat)
           B
```



```
( (λ ((f : (nat -> nat))) (f 3))
(λ ((x : nat)) (+ x (/ 10 x)))
```



```
( (• ((nat -> nat) -> nat))
(λ ((x : nat)) (+ x (/ 10 x))))
```



```
( (• ((nat -> nat) -> nat))
(λ ((x : nat)) (+ x (/ 10 x)))
```



```
((• ((nat -> nat) -> nat))
                                     (\lambda ((x : nat)) (+ x (/ 10 x)))
                      ((\lambda ((y : nat)) \Omega)
                       ((\lambda ((x : nat)) (+ x (/ 10 x)))
                                                                 (• nat)
                        (• nat)))
                        ((\lambda ((y : nat)) \Omega)
                          (+ (• nat) (/ 10 (• nat))))
((\lambda ((y : nat)) \Omega)
                                                 ((\lambda ((y : nat)) \Omega)
 (+ (• nat) (err "Divide by zero")))
                                                  (+ (• nat) (• nat)))
                                                  ((\lambda ((y : nat)) \Omega)
        (err "Divide by zero")
                                                   (• nat))
```



(pos? 4 7)



(pos? 4 (• nat))



```
(pos? 4 (• nat))
(if0 (pos? (• nat)) (• nat pos?) blame)
   (if0 (• nat) (• nat pos?) blame)
           blame
                   (• nat pos?)
```

- 1) CONTRACTS AS SYMBOLIC VALUES
- 2) CHECKS REFINE VALUES
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(pos? 4 7)



```
(pos? 4 (• nat pos?))
```



```
(pos? ♠ (• nat pos?))

(• nat pos?)
```



```
(/ 10 (• nat pos?))

(• nat)
```

- 1) CONTRACTS AS SYMBOLIC VALUES
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```
((• ((nat -> nat) -> nat)
((pos? -> pos?) -> pos?))
(λ ((x : nat)) (+ x (/ 10 x)))
```

(• nat pos?)

```
((• ((nat -> nat) -> nat)
                        ((pos? -> pos?) -> pos?))
                    (\lambda ((x : nat)) (+ x (/ 10 x)))
   ((\lambda ((y : nat)) \Omega)
    ((\lambda (x : nat)) (+ x (/ 10 x)))
                                               (• nat pos?)
     (• nat pos?)))
((\lambda ((y : nat)) \Omega)
 (+ (• nat pos?) (/ 10 (• nat pos?))))
       ((\lambda ((y : nat)) \Omega)
        (+ (• nat pos?) (• nat)))
            ((\lambda ((y : nat)) \Omega)
             (• nat))
```

'CPCF



```
((• ((nat -> nat) -> nat))
((pos? -> pos?) Φ (λ ((x : nat)) (+ x (/ 10 x)))))
```



```
((• ((nat -> nat) -> nat))
             ((pos? -> pos?) \oplus (\lambda ((x : nat)) (+ x (/ 10 x))))
         ((• ((nat -> nat) -> nat))
          (\lambda (x : nat)) (pos? \Phi ((\lambda (x : nat)) (+ x (/ 10 x)))
                                      (pos? 4 x)))))
(at)) (pos? \Phi ((\lambda ((x : nat)) (+ x (/ 10 x)))
                                                            (• nat)
                (pos? 4 x))))
```

 $(t)) \Omega$

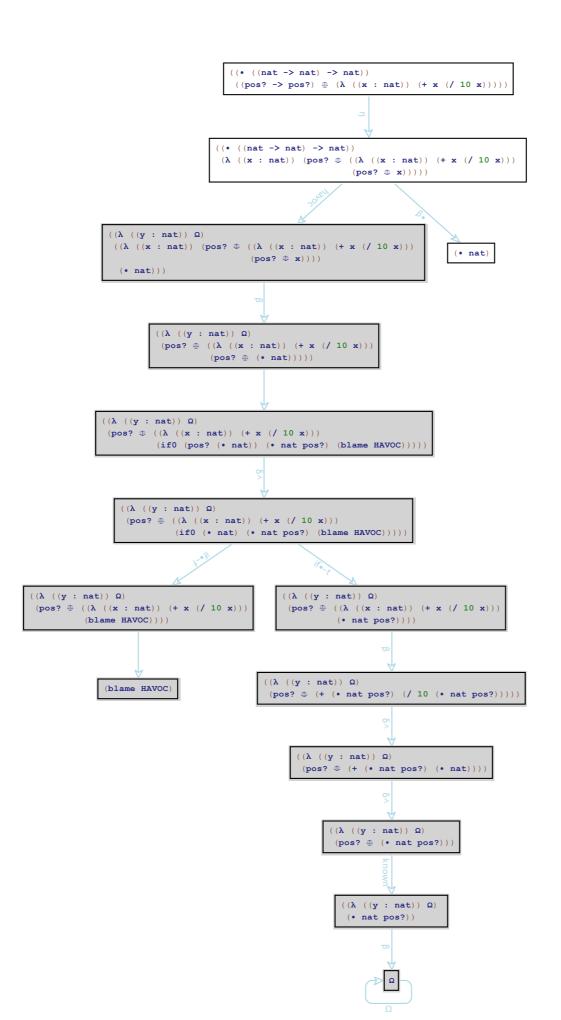


```
((λ ((y : nat)) Ω)
(pos? Φ ((λ ((x : one of the context))))
(** nat pos
```

```
(blame HAVOC)
```

```
((\lambda ((y : nat)) \Omega)
 (pos? 4 (• nat pos?)))
             known
   ((\lambda ((y : nat)) \Omega)
    (• nat pos?))
             7
```







- 1) CONTRACTS AS SYMBOLIC VALUES
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```
((λ ((x : nat)) (if0 x 1 (/ 10 x))) (• nat))
```



```
((λ ((x : nat)) (if0 x 1 (/ 10 x))) (• nat))

(if0 (• nat) 1 (/ 10 (• nat)))
```



```
((\lambda ((x : nat)) (if0 x 1 (/ 10 x))) (\bullet nat))
                          TO
           (if0 (• nat) 1 (/ 10 (• nat)))
              (/ 10 (• nat))
(err "Divide by zero")
                              nat)
```



```
(((λ ((x : nat)) (if0 x 1 (/ 10 x))) (• nat))
#())
```



```
(((\lambda ((x : nat)) (if0 x 1 (/ 10 x))) (• nat)) #())
```



1-011 CPCF

```
(1
\#((1 \mapsto (\lambda \ ((x : nat)) \ (if0 \ x \ 1 \ (/ \ 10 \ x)))))
(2 \mapsto (nat \ zero?))))
```



```
ifo_F
```

```
((/ 10 (\& 2))
\#((1 \mapsto (\lambda ((x : nat)) (if0 x 1 (/ 10 x))))
(2 \mapsto (nat pos?))))
```

```
((• nat)
#((1 → (λ ((x : nat)) (if0 x 1 (/ 10 x))))
    (2 → (nat pos?))
    (3 → /)
    (4 → 10)))
```



```
(((\lambda ((x : nat)) (if0 x 1 (/ 10 x))) (• nat))
#(())

((if0 (& 2) 1 (/ 10 (& 2)))
#((1 → (\lambda ((x : nat)) (if0 x 1 (/ 10 x))))
(2 → nat)))
```

```
(1
\#((1 \mapsto (\lambda \ ((x : nat)) \ (if0 \ x \ 1 \ (/ \ 10 \ x)))))
(2 \mapsto (nat \ zero?))))
```

```
((• nat)
#((1 → (λ ((x : nat)) (if0 x 1 (/ 10 x))))
(2 → (nat pos?))
(3 → /)
(4 → 10)))
```

ESSENTIAL IDEAS:

- 1) CONTRACTS AS SYMBOLIC VALUES
- 2) CHECKS REFINE VALUES
- 3) REFINEMENTS INFLUENCE COMPUTATION
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ESSENTIAL IDEAS:

- 1) CONTRACTS AS SYMBOLIC VALUES
- 2) CHECKS REFINE LOCATIONS
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```
(((λ ((x : nat)) (if0 (= x 5) (= 5 x) 0)) (• nat))
#())
```



```
(((\lambda ((x : nat)) (if0 (= x 5) (= 5 x) 0)) (• nat))
#())
```

```
β
```

```
((if0 (= (& 2) 5) (= 5 (& 2)) 0)

#((1 → (λ ((x : nat)) (if0 (= x 5) (= 5 x) 0)))

(2 → nat)))
```



```
(((λ ((x : nat)) (if0 (= x 5) (= 5 x) 0)) (• nat))
#())

((if0 (= (& 2) 5) (= 5 (& 2)) 0)
#((1 → (λ ((x : nat)) (if0 (= x 5) (= 5 x) 0)))
(2 → nat)))
```

```
(2 \mapsto nat))
```



```
 \begin{array}{l} ((\mathbf{if0}\ 1\ (=5\ (\&\ 2))\ 0) \\ \#((1\ \mapsto\ (\lambda\ ((\mathbf{x}\ :\ \mathsf{nat}))\ (\mathbf{if0}\ (=\mathbf{x}\ 5)\ (=5\ \mathbf{x})\ 0))) \\ (2\ \mapsto\ \mathsf{nat}) \\ (3\ \mapsto\ =) \\ (4\ \mapsto\ (5\ (\lambda\ ((\mathbf{x}\ :\ \mathsf{nat}))\ (\mathsf{not}\ (=\mathbf{x}\ (\&\ 2)))))))) ) \end{array}
```



(**5** +



```
6. 137 8
```

```
O))))
```

```
((if0 0 (= 5 (& 2)) 0)

#((1 → (λ ((x : nat)) (if0 (= x 5) (= 5 x) 0)))

(2 → nat)

(3 → =)

(4 → (5 (λ ((x : nat)) (= x (& 2))))))
```



```
0)))
```

```
) ) )
```

```
 ((= 5 (\& 2)) 
 \#((1 \mapsto (\lambda ((x : nat)) (if0 (= x 5) (= 5 x) 0))) 
 (2 \mapsto nat) 
 (3 \mapsto =) 
 (4 \mapsto (5 (\lambda ((x : nat)) (= x (\& 2)))) 
 (5 \mapsto 0)))
```



```
"CPCF
```

```
O)))
```

```
 ((= 5 (\& 2)) 
 #((1 \mapsto (\lambda ((x : nat)) (if0 (= x 5) (= 5 x) 0))) 
 (2 \mapsto nat) 
 (3 \mapsto =) 
 (4 \mapsto (5 (\lambda ((x : nat)) (= x (\& 2)))) 
 (5 \mapsto 0)) )
```



```
"CPCF
```

```
O)))
```

```
((= 5 (\& 2)))
\#((1 \mapsto (\lambda ((x : nat)) (if0 (= x 5) (= 5 x) 0)))
(2 \mapsto nat)
(3 \mapsto =)
(4 \mapsto (5 (\lambda ((x : nat)) (= x (\& 2)))))
(5 \mapsto 0)))
```



```
x4=5
x4=x2
x5=0
x2=5?
```

x2, x4, x5 : nat

```
(((\lambda ((x : nat)) (if0 (= x 5) (= 5 x) 0)) (• nat))
#(())

((if0 (= (& 2) 5) (= 5 (& 2)) 0)
#((1 → (\lambda ((x : nat)) (if0 (= x 5) (= 5 x) 0)))
(2 → nat)))
```

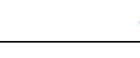








```
((= 5 (\& 2))
\#((1 \mapsto (\lambda ((x : nat)) (if0 (= x 5) (= 5 x) 0)))
(2 \mapsto nat)
(3 \mapsto =)
(4 \mapsto (5 (\lambda ((x : nat)) (= x (\& 2)))))
(5 \mapsto 0)))
```



ESSENTIAL IDEAS:

- 1) CONTRACTS AS SYMBOLIC VALUES
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BEYOND CPCF

$V::=U/\mathcal{C}$ $C,D::=X\mid C\mapsto \lambda X.C\mid \mathsf{flat}(E)$ $\mid \langle C,C\rangle\mid C\lor C\mid C\land C\mid$

```
\begin{array}{lll} P,Q & ::= & \vec{M}E \\ M,N & ::= & (\mathsf{module}\ f\ C\ V) \\ E,E' & ::= & f^\ell \mid X \mid A \mid E\ E^\ell \mid \mathsf{if}\ E\ E\ E \mid O\ \vec{E}^\ell \mid \mu X.E \\ & \mid & \mathsf{mon}_\ell^{\ell,\ell}(C,E) \\ U & ::= & n \mid \mathsf{tt} \mid \mathsf{ff} \mid (\lambda X.E) \mid \bullet \mid (V,V) \mid \mathsf{empty} \\ V & ::= & U/\mathcal{C} \\ C,D & ::= & X \mid C \mapsto \lambda X.C \mid \mathsf{flat}(E) \\ & \mid & \langle C,C \rangle \mid C \lor C \mid C \land C \mid \mu X.C \\ O & ::= & \mathsf{add1} \mid \mathsf{car} \mid \mathsf{cdr} \mid \mathsf{cons} \mid + \mid = \mid o? \mid \dots \\ o? & ::= & \mathsf{nat?} \mid \mathsf{bool?} \mid \mathsf{empty?} \mid \mathsf{cons?} \mid \mathsf{proc?} \mid \mathsf{false?} \\ A & ::= & V \mid \mathcal{E}[\mathsf{blame}_\ell^\ell] \end{array}
```

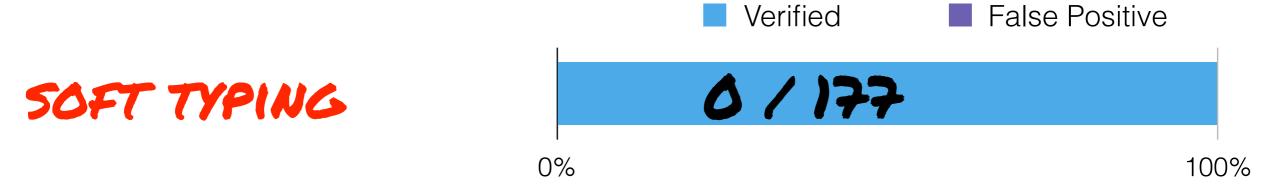
THEOREM: VERIFIED MODULES CAN'T BE BLAMED.

SOFT TYPING

OCCURRENCE
TYPING

H.O. RECURSION SCHEMES

DEPENDENT REFINEMENT TYPES



OCCURRENCE
TYPING

H.O. RECURSION SCHEMES

DEPENDENT REFINEMENT TYPES SOFT TYPING

Verified False Positive

O

100%

0/142

OCCURRENCE TYPING

H.O. RECURSION SCHEMES

DEPENDENT REFINEMENT TYPES SOFT TYPING

Verified

0%

False Positive

100%

OCCURRENCE TYPING

H.O. RECURSION **SCHEMES**

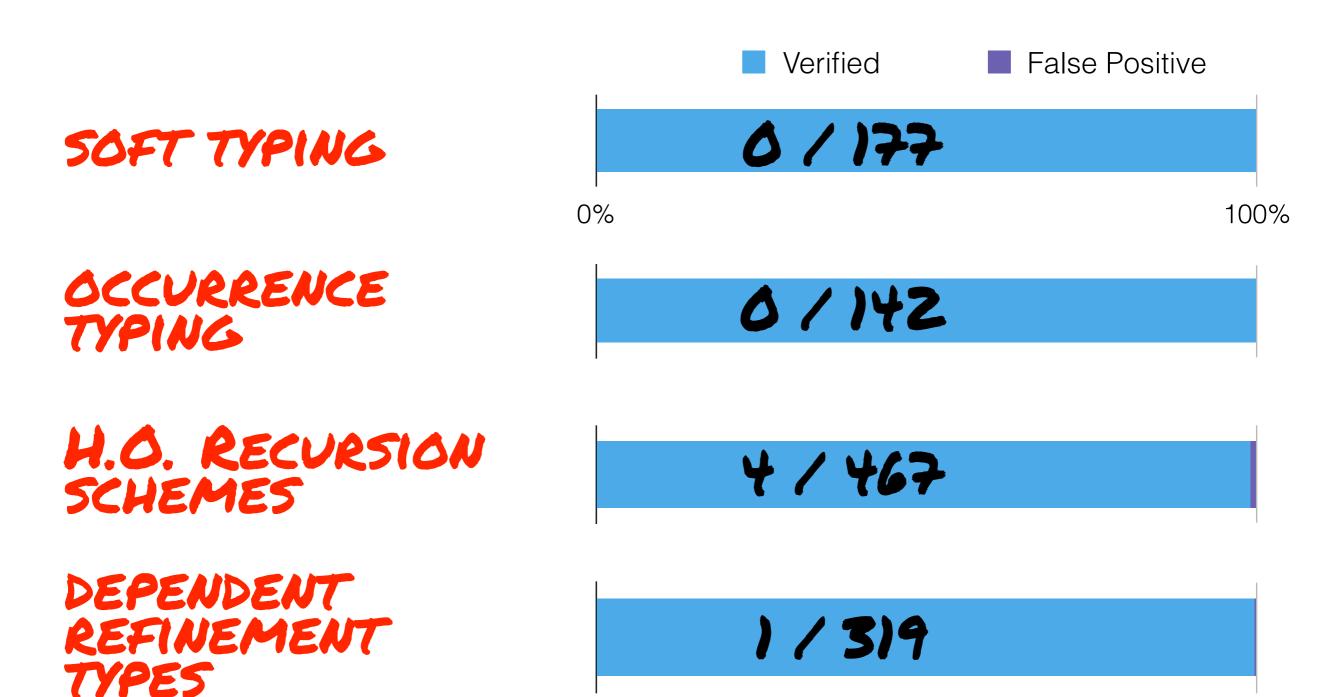
DEPENDENT REFINEMENT TYPES

0/142

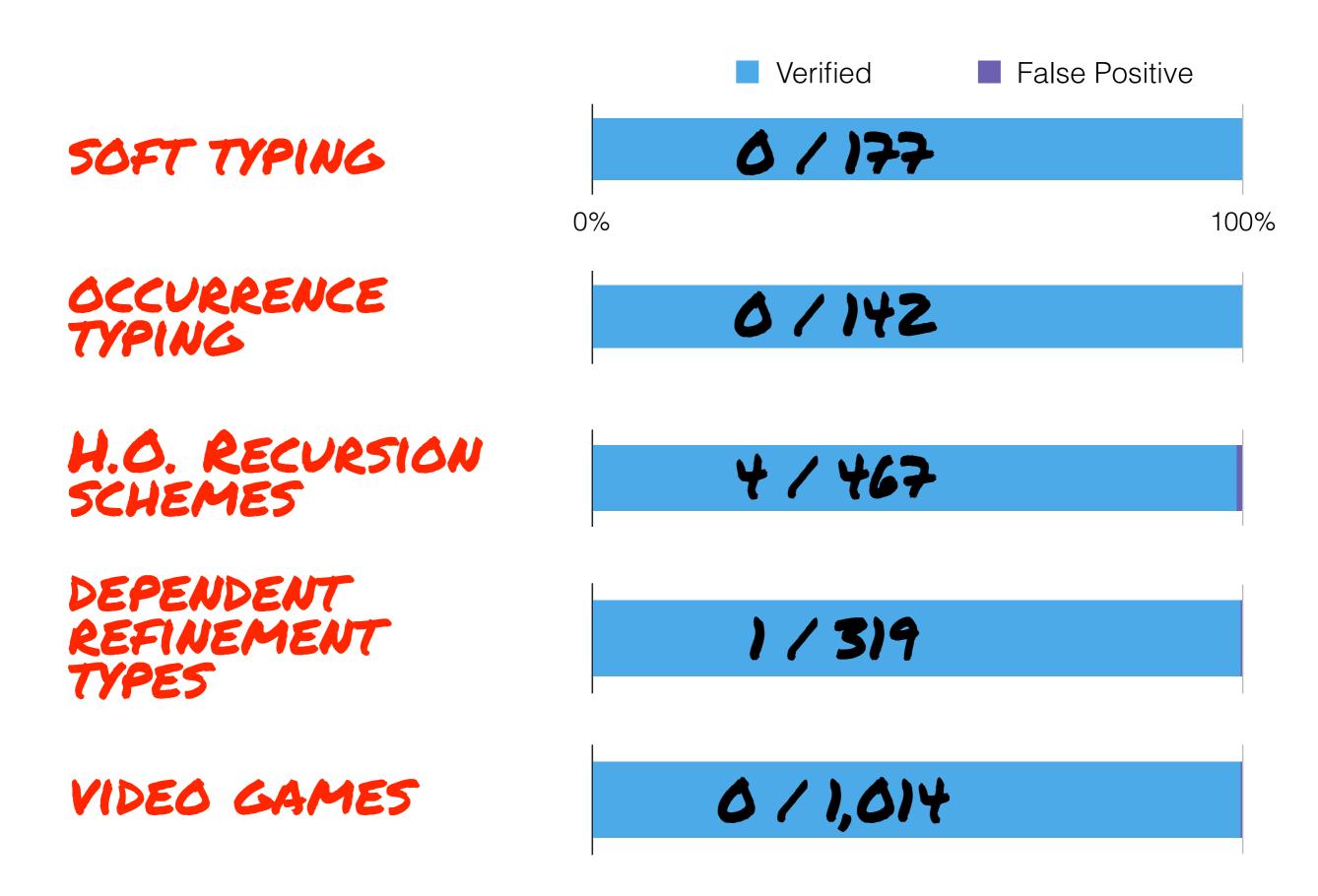
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VIDEO GAMES



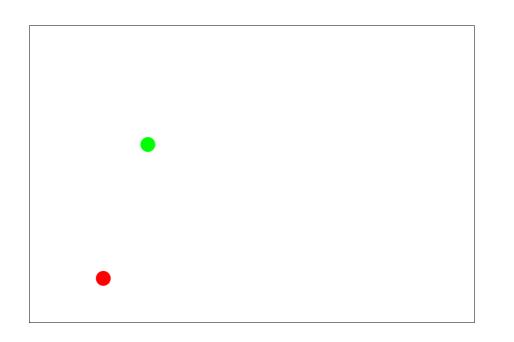
TAKE AWAY

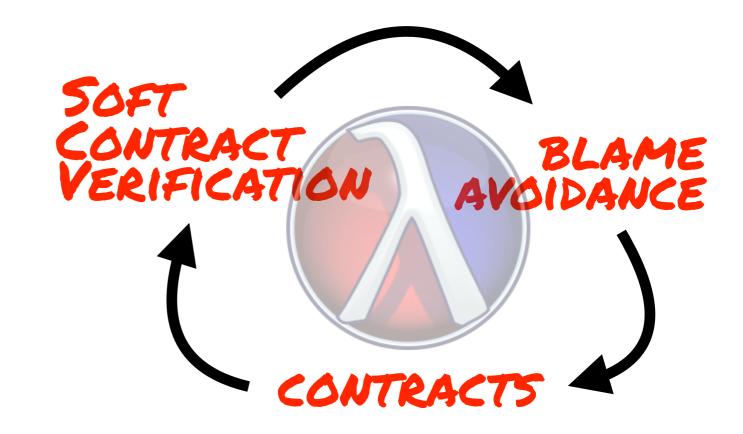


TAKE AWAY

- CONTRACTS FAIL AT FIRST ORDER; ONLY NEED A F.O. SOLVER FOR VERIFICATION
- RUNTIME ENFORCEMENT MECHANISMS => STATIC VERIFICATION TECHNIQUES







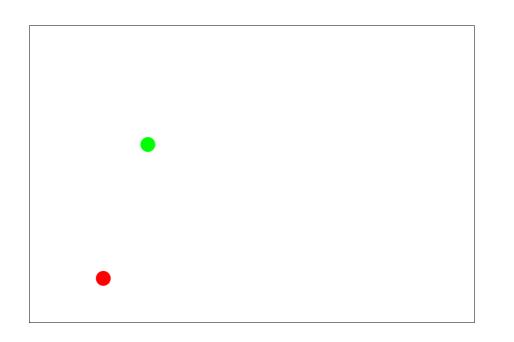
THEOREM:
VERIFIED MODULES
CAN'T BE BLAMED.

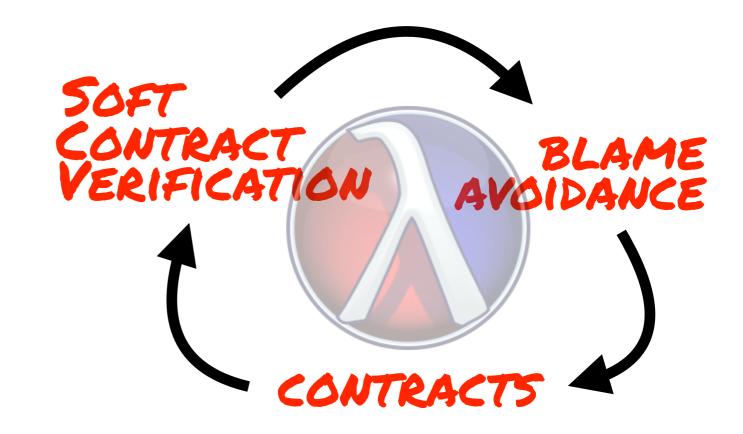
ESSENTIAL IDEAS:

- 1) CONTRACTS AS SYMBOLIC VALUES
- 2) CHECKE CONDITIONALS REFINE LOCATIONS
- 3) REFINEMENTS INFLUENCE COMPUTATION
- Y) PARTITION VERIFICATION W/ BLAME
- 5) HEAP OF INVARIANTS -> SMT



ter.ps/softcontract





THEOREM:
VERIFIED MODULES
CAN'T BE BLAMED.

ESSENTIAL IDEAS:

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