MIS 381N Homework 2

Sam Malcolm (stm2334)

Problem 1

If tortes and apple pies are represented as x_1 and x_2 respectively, then our problem becomes:

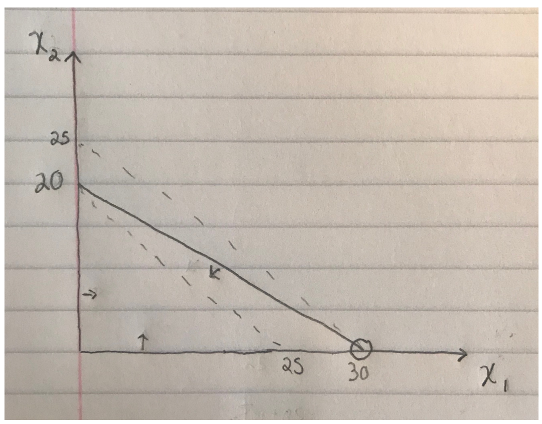
Choose x_1, x_2

To maximize $4x_1 + 5x_2$

Subject to the constraints:

$$\begin{cases} 2x_1 + 3x_2 & \leq 60 \\ x_1, x_2 & \geq 0 \end{cases} \tag{1}$$

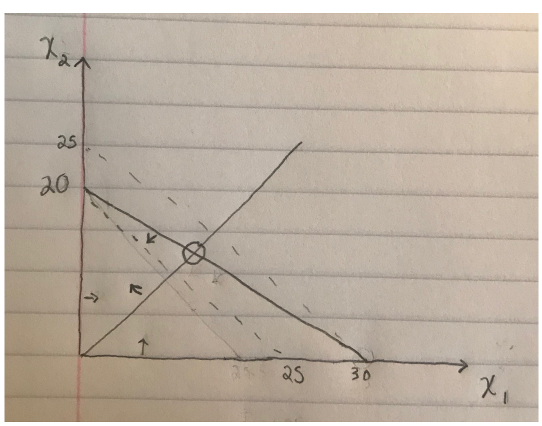
This can be represented graphically as:



To earn the most points, Max should eat 30 tortes and 0 pies for a score of 120 points.

If Max wants to eat as many pies as tortes, we add an additional constraint of:

$$\begin{cases} x_1 & \leq x_2 \end{cases} \tag{2}$$



We see an updated optimal solution where $2x_1 + 3x_2 = 60$ intersects $x_1 = x_2$. Setting $x_1 = x_2$ and solving, we find that Max should eat 12 tortes and 12 apple pies earning him a new score of 108 points. This decreases his point total by 12 points.

Problem 2

a)

If wheat and corn are represented as x_1 and x_2 respectively, then our problem becomes:

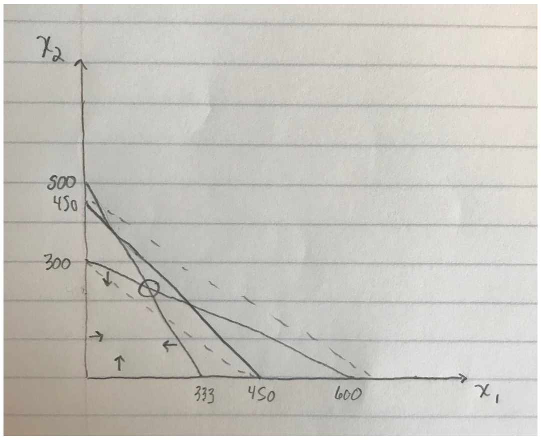
Choose x_1, x_2

To maximize $2000x_1 + 3000x_2$

Subject to the constraints:

$$\begin{cases} x_1 + x_2 & \leq 450 \\ 3x_1 + 2x_2 & \leq 1000 \\ 2x_1 + 4x_2 & \leq 1200 \\ x_1, x_2 & \geq 0 \end{cases} \tag{3}$$

This can be represented graphically as:



We observe that the solution is located where $3x_1 + 2x_2 = 1000$ intersects with $2x_1 + 4x_2 = 1200$. Solving algebraically we find that the solution is to farm 200 acres of wheat and 200 acres of corn.

b)

```r
library(lpSolve)
A = matrix(c(1,3,2,1,2,4),3,2)
C = c(2000,3000)
b = c(450,1000,1200)
dir = rep("<=",3)

solve = lp("max",C,A,dir,b)
print(solve$solution)
```

```
## [1] 200 200
```

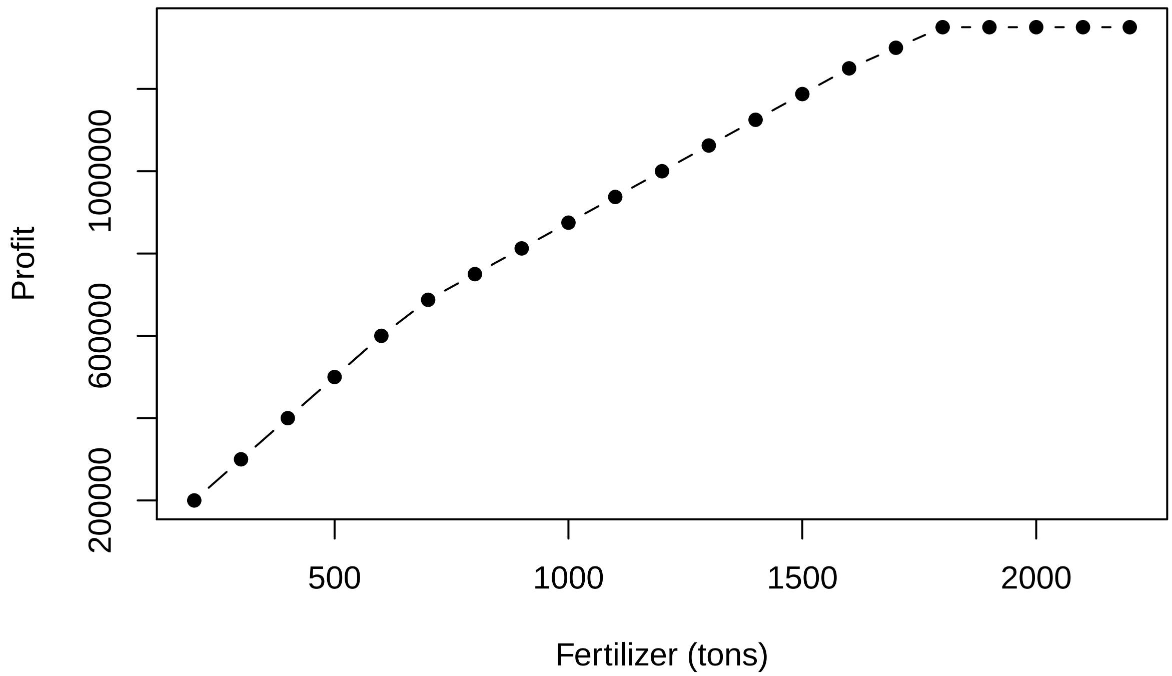
c)

```r
profit = rep(0,21)
wheat = rep(0,21)
corn = rep(0,21)
fert = seq(200,2200,by=100)
for (i in 1:21){
  b = c(450,1000,fert[i])
  solve = lp('max',C,A,dir,b)
  wheat[i] = solve$solution[1]
  corn[i] = solve$solution[2]
  profit[i] = 2000*wheat[i] + 3000*corn[i]
}

production_table = data.frame(fert, wheat, corn, profit)
print(production_table)
```

```
##    fert wheat  corn  profit
## 1   200   100   0.0  200000
## 2   300   150   0.0  300000
## 3   400   200   0.0  400000
## 4   500   250   0.0  500000
## 5   600   300   0.0  600000
## 6   700   325  12.5  687500
## 7   800   300  50.0  750000
## 8   900   275  87.5  812500
## 9  1000   250 125.0  875000
## 10 1100   225 162.5  937500
## 11 1200   200 200.0 1000000
## 12 1300   175 237.5 1062500
## 13 1400   150 275.0 1125000
## 14 1500   125 312.5 1187500
## 15 1600   100 350.0 1250000
## 16 1700    50 400.0 1300000
## 17 1800     0 450.0 1350000
## 18 1900     0 450.0 1350000
## 19 2000     0 450.0 1350000
## 20 2100     0 450.0 1350000
## 21 2200     0 450.0 1350000
```

```r
plot(fert,profit,xlab = 'Fertilizer (tons)', ylab = 'Profit',type="b",pch = 16)
```



The farmer discontinues wheat production at 1,800 tons of fertilizer. The farmer discontinues corn production until 700 tons of fertilizer are available.

Generally, we observe that at lower levels of fertilizer, the farmer produces more wheat. At higher levels of fertilizer, the farmer produces more corn. Profit grows in a relatively consistent trend until leveling off when 1,800 tons of fertilizer become available (due to the finite size of the farm).

Problem 3

If the investment for each opportunity is represented as x_1,x_2,x_3,x_4,x_5 respectively, then we must:

Choose x_1,x_2,x_3,x_4,x_5

to maximize $13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$

subject to the constraints:

$$\begin{cases} 11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 & \leq 40 \\ 3x_1 + 6x_2 + 5x_3 + 1x_4 + 34x_5 & \leq 20 \\ x_1, x_2, x_3, x_4, x_5 & \leq 1 \\ x_1, x_2, x_3, x_4, x_5 & \geq 0 \end{cases}$$

This can be rewritten in the following matrix form:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 13 \\ 16 \\ 16 \\ 14 \\ 39 \end{bmatrix} \quad A = \begin{bmatrix} 11 & 53 & 5 & 5 & 29 \\ 3 & 6 & 5 & 1 & 34 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 40 \\ 20 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

We can then maximize $\mathbf{C}^T\mathbf{x}$ subject to $A\mathbf{x} \leq \mathbf{b}$ and $x \geq 0$

Problem 4

If corn, milk, and bread be represented by x_1,x_2,x_3 respectively, then we must:

Choose x_1,x_2,x_3

to minimize $0.18x_1 + 0.23x_2 + 0.05x_3$

subject to the constraints:

$$\begin{cases} 72x_1 + 121x_2 + 65x_3 & \geq 2000 \\ 72x_1 + 121x_2 + 65x_3 & \leq 2250 \\ 107x_1 + 500x_2 & \geq 5000 \\ 107x_1 + 500x_2 & \leq 50000 \\ x_1, x_2, x_3 & \leq 10 \\ x_1, x_2, x_3 & \geq 0 \end{cases}$$

This can be rewritten in the following matrix form:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0.18 \\ 0.23 \\ 0.05 \end{bmatrix} \quad A = \begin{bmatrix} -72 & -121 & -65 \\ 72 & 121 & 65 \\ -107 & -500 & 0 \\ 107 & 500 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -5000 \\ 50000 \\ -2000 \\ 2250 \\ 10 \\ 10 \\ 10 \end{bmatrix}$$

We can then solve using r

```
C = c(.18,.23,.05)
A = matrix(c(-72, 72,-107,107,1,0,0,-121,121,-500,500,0,1,0,-65,65,0,0,0,0,1),7,3)
b = c(-2000,2250,-5000,50000,10,10,10)
dir = rep("<=",7)
s = lp('min',C,A,dir,b)
print(s$status)
```

```
## [1] 0
```

```
print(s$solution)
```

```
## [1]  1.944444 10.000000 10.000000
```

Problem 5

If Forest Unit 1 Years 1,2,3 and Forest Unit 2 Years 1,2,3 are represented by x_1,x_2,x_3,y_1,y_2,y_3 respectively, then we must:

Choose x_1,x_2,x_3,y_1,y_2,y_3

to maximize $x_1 + x_2 + x_3 + y_1 + y_2 + y_3$

subject to the constraints:

$$
\begin{cases}
x_1 + y_1 & \geq 1.2 \\
x_2 + y_2 & \geq 1.5 \\
x_3 + y_3 & \geq 2 \\
x_1 + y_1 & \leq 2 \\
x_2 + y_2 & \leq 2 \\
x_3 + y_3 & \leq 3 \\
x_1 + \frac{1}{1.3}x_2 + \frac{1}{1.4}x_3 & \leq 2 \\
y_1 + \frac{1}{1.2}y_2 + \frac{1}{1.6}y_3 & \leq 3
\end{cases}
$$

This can be rewritten in the following matrix form:

$$
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad A = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1/1.3 & 1/1.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/1.2 & 1/1.6 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -1.2 \\ -1.5 \\ -2 \\ 2 \\ 2 \\ 3 \\ 2 \\ 3 \end{bmatrix}
$$

We can then solve using r

```
C=rep(1,6)
A=matrix(c(-1,0,0,1,0,0,1,0,0,-1,0,0,1,0,1/1.3,0,0,0,-1,0,0,1,1/1.4,0,-1,0,0,1,0,0,0,1,0,-1,0,0,1,0,0,1/
b=c(-1.2,-1.5,-2,2,2,3,2,3)
dir=rep("<=",8)
s = lp("max",C,A,dir,b)
print(s$status)
```

```
## [1] 0
```

```
print(s$solution)
```

```
## [1] 0.4615385 2.0000000 0.0000000 1.1250000 0.0000000 3.0000000
```

Harvest the following weight from Forest Unit 1: Year 1: 0.4615385 tons Year 2: 2 tons Year 3: 0 tons

Harvest the following weight from Forest Unit 2: Year 1: 1.125 tons Year 2: 0 tons Year 3: 3 tons