Modelling Equilibrium for a Multi-criteria Selfish Routing Network Equilibrium Flow Problem

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Abstract

The selfish routing of network flow often considers a single objective, namely travel time or travel distance, and optimisation models are often guided by the principle of user equilibrium (UE). A more challenging approach is to consider multiple objectives simultaneously, for example, distance, travel time and pollution. We present a multi-criteria problem, whereby we consider fuel consumption in addition to travel time on a simple parallel network, we first consider scalarisation approaches, where the preferences are included a priori, such as the weighted sum model, and consider the issues with preserving user equilibrium.

Further models and analysis are then presented which circumvent these issues through changes to the edge parameters, allowing user equilibrium to be preserved, but fuel consumption managed.

Keywords: Network flow, User equilibrium, Traffic assignment, Multi-criteria optimisation

1. Introduction

The concept of selfish routing arises naturally in many real-world scenarios where agents wish to route their flow through a network as cheaply as possible for a given commodity (origin and destination pair). Examples include traffic, computing, mechanical, electrical and logistics networks. The most recognisable of these to the layperson is traffic networks, whereby each vehicle wishes to route itself between two points and research into this area has been ongoing since the example given by Pigou in the 1920s [1].

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The classic *traffic assignment problem* (TAP) utilises the eponymous Wardrop principles of traffic assignment, introduced in 1952, which have provided the bedrock for the assignment stage of the majority of transport models [2].

Wardrop's first principle, *user equilibrium* (UE), states that journey times on all used paths between an origin and a destination pair (referred to as a commodity) are equal, and less than those which would be experienced by a single vehicle on any unused route. It effectively reflects the selfish behaviour of travellers, postulating that individuals wish to minimise their own travel time. The second principle, *system optimal* (SO), assigns traffic to minimise the average travel time and can be viewed as a cooperative way to make gains holistically; however, individuals may suffer when compared to the journey time experienced subject to user equilibrium [3]. Many extensions of TAP have been stated, including numerous cases in which Wardrop's first principle, user equilibrium, has been augmented with additional criteria, so-called multi-criteria problems [4, 5, 6, 7]. In general, these have sought to minimise a weighted sum objective, where all criteria are converted into units known as *value of time* (VOT).

In this paper, we consider a multi-criteria extension of a simplified single origin/destination pair connected with parallel edges, with the aim of alleviating fuel consumption. We discuss the issues with the multi-criteria approach requiring conversion to VOT, mainly when such a conversion does not make sense. Finally, we investigate the effects of changing the parameters of the edges within the network to reduce the overall fuel consumption, negating the need for central control.

2. Preliminaries

2.1. *Traffic Assignment Problem [TAP]*

Given a network $G = (V, E)$ and k commodities, for each commodity i there is an amount of traffic r_i to be routed, let the set of used paths by commodity i be denoted by $\mathcal{P}_i$ and the set of all used paths is then $\mathcal{P} = \bigcup_i \mathcal{P}_i$. Let f_P and x_e be the flow on path $P \in \mathcal{P}$, and edge $e \in E$ respectively. Finally, denote by $\mathbf{x} = (x_e)_{e \in E}$, the vector of flows on edges. An instance of TAP is then given by the triple (G, r, t), where r is the associated vector of commodity demands and t is the vector of travel time functions. The fixed demand user equilibrium problem can be stated as the following optimisation problem [8]:

$$\text{minimise} \qquad U(\mathbf{x}) \stackrel{\text{def}}{=} \sum_{e \in E} \int_0^{x_e} t_e(\omega) d\omega \tag{1}$$

subject to

$$\sum_{P \in \mathcal{P}_i} f_P = r_i, \ \forall i \in \{1, \dots, k\} \tag{2}$$

$$f_P \geq 0, \ \forall P \in \mathcal{P} \tag{3}$$

$$\sum_{P \in \mathcal{P} : e \in P} f_P = x_e, \ \forall e \in E, \tag{4}$$

subject to the three assumptions:

A1. The network is strongly connected.

A2. The traffic demand r_i is positive for all $i \in \{1, \ldots, k\}$.

A3. The travel time function $t_e : [0, \infty) \to (0, \infty)$ only depends on the flow on edge e, is twice differentiable, and satisfies the conditions $t_e^{'}(x_e) > 0$ and $t_e^{''}(x_0) \geq 0$, hence it is strictly increasing and convex.

To find a solution to Wardrop's second principle one can replace the objective function $U(\mathbf{x})$ with the objective function $T(\mathbf{x}) \stackrel{\text{def}}{=} \sum_{e \in E} x_e t_e(x_e)$, i.e. minimise the total travel time. It can be shown that the Hessian matrices of $U(\mathbf{x})$ and $T(\mathbf{x})$ are positive-definite and thus the functions are strictly convex. The feasible region, defined by the linear equality constraints (2) and non-negativity constraints (3), is convex, thus resulting in a unique global minimum for both optimisation problems [8, 9].

2.2. User equilibrium on a single commodity with parallel edges

Consider a multigraph G consisting of a single commodity (pair of vertices) connected by parallel edges. For this specific problem, the set of paths correspond to the set of edges. This condition simplifies the optimisation problems and allows a solution via conventional convex optimisation, or in the discrete case, dynamic programming [10].

The graph given in Fig. 1 consists of a set of m edges $E = \{1, 2, \ldots, m\}$, that connect, in parallel, the single commodity between the starting and terminal pair of nodes (s, t).

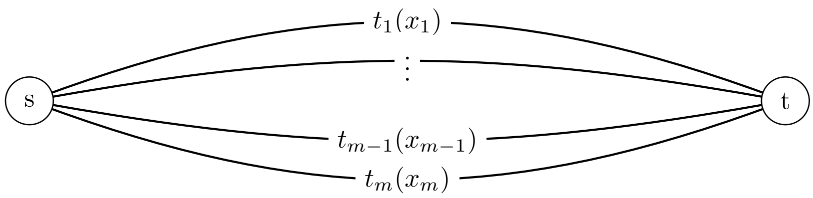


Figure 1: Single commodity with parallel edges.

For a demand r, the solution to the user equilibrium problem $\mathbf{x}^*_{\mathbf{UE}}$ displayed in Fig. 1 can be found by solving the following optimisation problem:

$$\text{minimise} \qquad U(\mathbf{x}) \stackrel{\text{def}}{=} \sum_{e \in E} \int_0^{x_e} t_e(\omega) d\omega \tag{5}$$

subject to

$$\sum_{e \in E} x_e = r \tag{6}$$

$$x_e \geq 0, \qquad \forall e \in E. \tag{7}$$

2.3. Travel time function

Traditionally, the most common choice for the travel time function has been a curve satisfying the assumption A3, fitted by the Bureau of Public Roads [11]

$$t_e(x_e) = a_e \left(1 + 0.15 \left(\frac{x_e}{c_e}\right)^4\right), \tag{8}$$

where a_e and c_e are the free-flow time and the practical capacity of edge e derived from congestion conditions, respectively, and x_e is the edge volume. Here 0.15 and 4 are parameters are calibrated to data.

2.4. Speed as a function of edge flow

Denoting by d_e the length of edge e, the average speed at free-flow m_e is given by the formula $m_e = \frac{d_e}{a_e}$. The speed s_e as a function of the edge flow x_e is then given by

$$s_e(x_e) = \frac{d_e}{t_e(x_e)} = \frac{a_e m_e}{a_e \left(1 + b\left(\frac{x_e}{c_e}\right)^n\right)} = \frac{m_e}{1 + b\left(\frac{x_e}{c_e}\right)^n}. \tag{9}$$

Given that $t_e(x_e)$ is positive, continuous and strictly increasing, for a fixed d_e the function $s_e(x_e)$ is strictly decreasing on the interval $[0, \infty)$, from the maximum $s_e(0) = \frac{d_e}{s_e(0)}$ to the minimum $\lim_{x_e \to \infty} s_e(x_e) = 0$.

2.5. Fuel consumption curve

Fuel consumption can be modelled by a strictly convex quadratic $y = Ax^2 + Bx + C$ adapted from [12], whose coefficients satisfy $A > 0$, $C > 0$, and $-2\sqrt{AC} < B < 0$, as in Fig. 2. This function attains a minimum for $x^* = -\frac{B}{2A}$, whose value is $y^* = C - \frac{B^2}{4A}$.

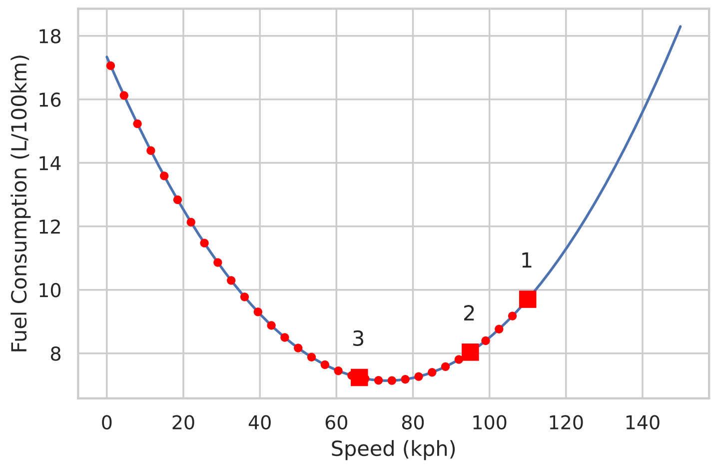


Figure 2: Fuel consumption curve $F_e(s_e)$ for $A = 0.0019$, $B = -0.2784$ and $C = 17.337$. The dotted lines indicate $F_e(s_e(x_e))$ (from right to left). The squares show $(m_e, F_e(m_e))$ for $m_1 = 110$, $m_2 = 95$ and $m_3 = 65$ kph (see Table 1).

This model is largely consistent with the fuel consumption of an engine: larger for extreme values of speed, and optimal for moderate speed. For simplicity we assume that all vehicles in this model have the same parameters A, B, C. The speed $s_e(x_e)$ of a car travelling on edge e decreases from $m_e = s_e(0)$ (free-flow) to nearly 0, as the number of vehicles x_e increases. Correspondingly, the fuel consumption is given by

$$F_e(s_e(x_e)) = A[s_e(x_e)]^2 + B[s_e(x_e)] + C. \tag{10}$$

We have two cases. If $m_e \leq x^* = -\frac{B}{2A}$ (maximum allowed speed below optimum speed for fuel consumption) as in Fig. 2 squares 1 and 2, then the fuel consumption of individual vehicles will only increase with traffic. For $x^* = -\frac{B}{2A}$ the fuel consumption actually decreases as the traffic builds up to a moderate value, after which it increases as seen in Fig 2 square 3. The total fuel consumption on an edge e with x_e vehicles is then given by $x_e F_e(s_e(x_e))$, while the total fuel consumption of the network across all edges is

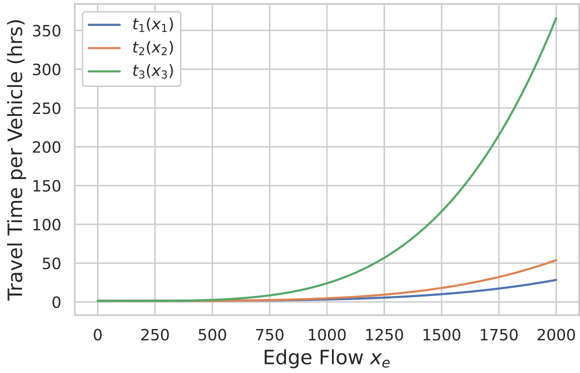
$$F(\mathbf{x}) = \sum_{e \in E} \frac{d_e}{100} \cdot x_e F_e(s_e(x_e)). \tag{11}$$

To demonstrate the interplay between the travel time, speed and fuel consumption experienced by a single vehicle, the edge functions $t_e(x_e), s_e(x_e), F_e(s_e), x_e F_e(s_e)$ are plotted in Figs. 3a, 3b, 3c, 3d respectively, for the values of m_e, c_e and d_e given in Table 1. Fig. 3c illustrates that the edges $1, 2$, both have the characteristic of exhibiting a minimum for fuel consumption for a demand $x_e > 0$ (the Goldilocks speed zone which is optimal for fuel consumption is below the free-flow speed). On edge 3 fuel consumption strictly increases as vehicles can never make it into the Goldilocks zone, hence the optimum fuel consumption is achieved for $x_e = 0$ (free-flow). In comparison the total fuel consumption (Fig. 3d) along an edge is strictly increasing; however, it is noticeable from $x_1 F_1(x_1)$ and $x_2 F_2(x_2)$ that the rate of change slows for a demand $500 \leq x_e \leq 1250$, corresponding with the region in which the speed enters the Goldilocks zone.

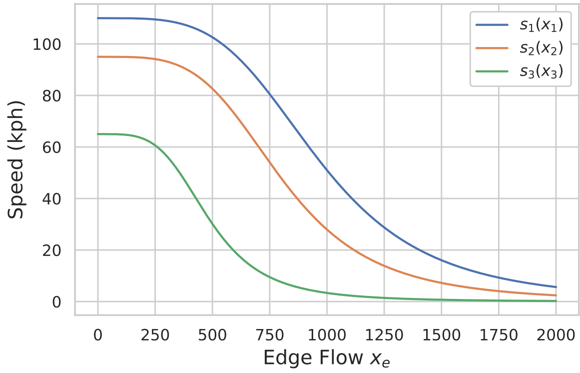
One must also be careful drawing conclusions for this simplified model as the saturation of the edge could, in reality (or a more complicated simulation), lead to increased congestion (and thus fuel consumption) because of braking, over-revving, etc. However, it does support the fundamental idea that in order to reduce fuel consumption, vehicles must drive within a given range, too slow or too fast resulting in increased fuel consumption. The free-flow speed values in our model given in Table 1 correspond to common speed limits in the United Kingdom: 70, 60 and 40 miles per hour, respectively.

Edge (e)	m_e	d_e	c_e
1	110	160	600
2	95	130	500
3	65	80	300

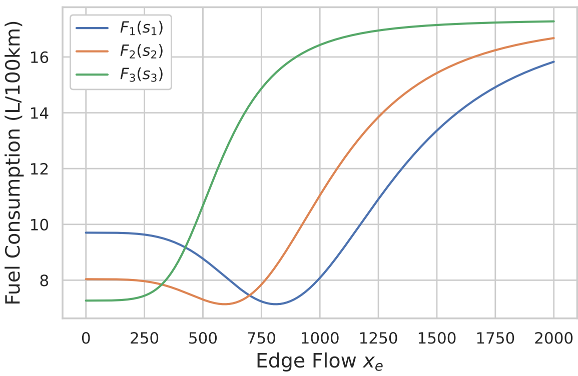
Table 1: Edge travel time parameters.



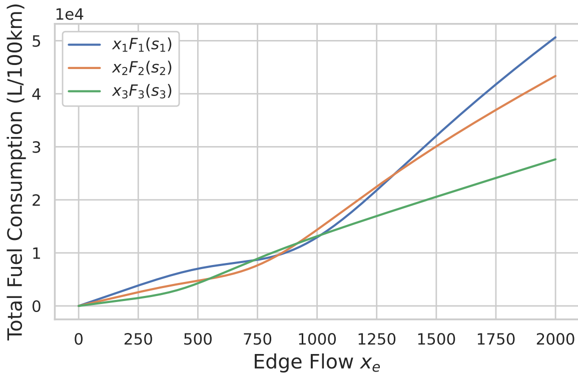
(a) Travel time per vehicle (hrs) - $t_e(x_e)$



(b) Speed (kph) - $s_e(x_e)$



(c) Fuel consumption per vehicle (L/100km) - $F_e(s_e(x_e))$



(d) Total fuel consumption on edge (L/100km) - $x_eF_e(s_e(x_e))$

Figure 3: Interplay between edge flow functions.

3. Proposed Modelling Approaches

3.1. Weighted sum model

To examine the interplay between the total travel time $T(\mathbf{x})$ and fuel consumption $F(\mathbf{x})$, given the differences in scale, we consider a normalised weighted sum model [13]. To this aim, given a scalar function $\phi : D \to (0, \infty)$, it will be convenient to consider the normalisation $\tilde{\phi} : D \to [0, 1)$ defined by

$$\tilde{\phi}(\mathbf{x}) = 1 - \frac{\inf_{\mathbf{y}\in D}(\phi(\mathbf{y}))}{\phi(\mathbf{x})}, \tag{12}$$

where it is understood that $\inf_{\mathbf{y}\in D}(\phi(\mathbf{y}))$ is computed a priori.

One may consider the weighted sum involving $\tilde{T}(\mathbf{x})$ and $\tilde{F}(\mathbf{x})$, given by

$$\lambda\tilde{T}(\mathbf{x}) + (1-\lambda)\tilde{F}(\mathbf{x}), \quad \lambda \in [0, 1], \tag{13}$$

and try to minimise its value. Clearly, for $\lambda = 1$, the total travel time $T(\mathbf{x})$ is minimised, while for $\lambda = 0$ the total fuel consumption is minimised.

Another weighted sum involving normalised values of $U(\mathbf{x})$ (user equilibrium) and $F(\mathbf{x})$ (total fuel consumption) can be defined as

$$\lambda\tilde{U}(\mathbf{x}) + (1-\lambda)\tilde{F}(\mathbf{x}), \quad \lambda \in [0,1]. \tag{14}$$

Whilst this is valid as an objective, it ceases to make sense in the context of user equilibrium as the introduction of fuel consumption will have the effect of shifting the optimal solution away from user equilibrium (i.e. the travel times will not be equal according to Wardrop's first principle), it will in some senses act as central control. As aforementioned in the introduction, this raises the question as to how one might model user equilibrium with a competing objective, one idea that has been employed is by using the concept of VOT; however, this requires a conversion which is dependent on the opinions of individuals and careful consideration should be given as to whether one should include such normative (value) judgements.

3.2. Constraining by total fuel consumption

An obvious extension to the problem is to include a constraint that limits fuel consumption.

$$\text{minimise} \qquad U(\mathbf{x}) \stackrel{\text{def}}{=} \sum_{e\in E} \int_0^{x_e} t_e(\omega)d\omega \tag{15}$$

subject to

$$\sum_{e\in E} x_e = r \tag{16}$$

$$\sum_{e\in E} F_e(s_e(x_e)) \leq C \tag{17}$$

$$x_e \geq 0, \qquad \forall e \in E. \tag{18}$$

The optimisation problem defined above can be viewed as a side constrained model [8], which satisfies a generalised Wardrop principle; however, we encounter difficulties with it's interpretation as the constraints placed on the problem by the fuel consumption are not necessarily something an individual driver would consider when selfishly routing. In effect it will result in a new optimal point where travel times are now unequal, thus again we violate Wardrop's first principle.

3.3. Incentives to change behaviour

To overcome the issues presented in sections 3.1 and 3.2, methods must be examined whereby individuals are coerced into making a different route choice which results in lower total fuel consumption. In a sense, similar to incentives in the market economy, we wish to somehow leverage the selfish nature of individuals (preserving Wardrop's first principle) to reduce fuel consumption. We must attempt to provide an answer to the question; why has anyone got any incentive to change routes?

Two possible approaches that can be considered:

- Change the network, e.g. add/remove new edges (i.e. Network (Infrastructure) Design).
- Adapt the controllable parameters of the network, e.g. change the free-flow speed limits of existing roads (edges).

In the remainder of this paper, we consider the second approach; however, the first approach is a rich area of research with much work done in network design under budgetary constraints utilising such techniques as bi-level optimisation and mathematical programs with equilibrium constraints (MPEC), an example of which is given in [14].

4. Models

We present two approaches, the first a weighted sum model which can be used to analyse the interplay between total travel time and total fuel consumption; the second utilises a change in the travel time function, allowing a search to be undertaken for the set of free-flow speeds for which solutions are feasible for maintaining user equilibrium but are bounded by the total fuel consumption.

4.1. Weighted sum model

As previously noted form a normalised weighted sum and minimise:

$$\begin{aligned} \text{minimise} \quad & \lambda\tilde{T}(\mathbf{x}) + (1-\lambda)\tilde{F}(\mathbf{x}) && (19) \\ \text{subject to} \quad & && \\ & \sum_{e\in E} x_e = r && (20) \\ & x_e \geq 0, \qquad \forall e \in E. && (21) \end{aligned}$$

Given the definition in equation (12), the normalised minimum values of $\tilde{F}(\mathbf{x})$ and $\tilde{T}(\mathbf{x})$ are 0. The utopian (ideal) point $\mathbf{z} \in \mathbb{R}^2$ (best time, lowest fuel consumption) is defined as

$$\mathbf{z} = \left(\min_x(\tilde{F}(\mathbf{x})), \min_x(\tilde{T}(\mathbf{x}))\right) = (0,0).$$

If $\mathbf{x}^*_\lambda$ is the optimal solution for a given value of λ, then we denote the solution which is nearest to the utopian point by $\mathbf{x}^*_{\lambda_0}$. This satisfies the condition

$$\mathbf{x}^*_{\lambda_0} = \min_\lambda \left(\left\| \left(\tilde{F}(\mathbf{x}^*_\lambda), \tilde{T}(\mathbf{x}^*_\lambda)\right) - \mathbf{z} \right\|_2 \right) = \min_\lambda \left(\left\| \left(\tilde{F}(\mathbf{x}^*_\lambda), \tilde{T}(\mathbf{x}^*_\lambda)\right) \right\|_2 \right). \tag{22}$$

4.2. Adjusting the edge free-flow speed

Here we define the optimisation problem by adjusting the travel time function to allow different values of the free-flow time a_e (note the distance d_e is fixed and cannot change). Utilising the relationship $m_e = \frac{d_e}{a_e}$, we have $a_e = \frac{d_e}{m_e}$ and the travel time function can be written as

$$t_e(x_e) = \frac{d_e}{m_e}\left(1 + b\left(\frac{x_e}{c_e}\right)^n\right). \tag{23}$$

The values m_e can now be manipulated and, for a given value the [TAP] optimisation problem can be solved. The optimisation problem is considered as a sub-problem of the main search which seeks to minimise the total fuel consumption by manipulating the values of m_e.

Algorithm 1 Steps for generating global minimum UE solutions

Step 1 $\rightarrow$ Set m_e, $\forall e \in E$

Step 2 $\rightarrow$ Solve user equilibrium sub-problem

Step 3 $\rightarrow$ Store solution

Step 4 $\rightarrow$ Repeat 1-3 until all desired combinations of $\{m_e\}_{e\in E}$ have been explored.

In Step 2 of algorithm 1, m_e is fixed and the sub-problem is just a specific case of [TAP] and has a unique global solution.

It should be noted that this method is not tractable in larger networks as the number of configurations is $|E|^k$, where k is the number of choices m_e for a given edge e; however, it is easily parallelisable and does allow exploration into the pattern of behaviour.

5. Results and Discussion

In the first part of this section we present results, generated for the weighted sum model which demonstrates the interplay between travel time and fuel consumption. We then present the results, utilising the same network, of adjusting the edge free-flow speed limits m_e to analyse the impact this has on the total travel time and total fuel consumption for solutions solved for user equilibrium.

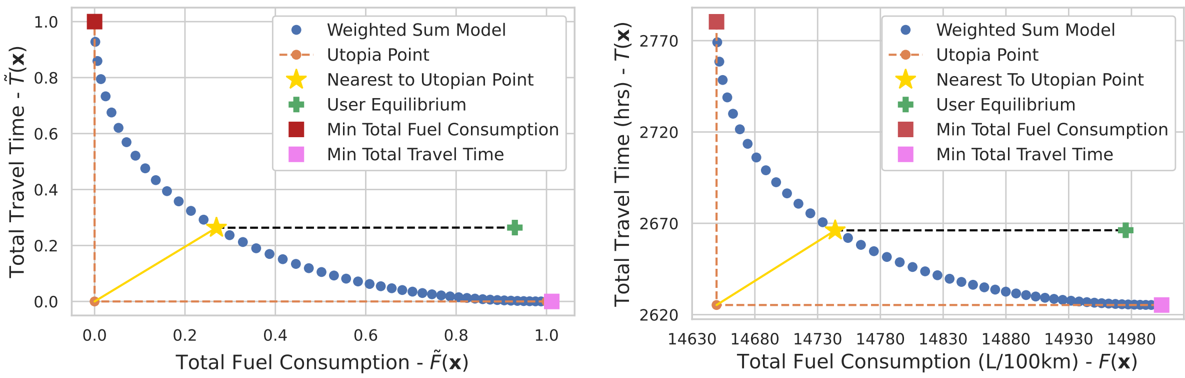
Results were generated using the 3 edge parallel network, with values of d_e and c_e given by Table 1, and for a demand value $r = 1500$, i.e. 1500 vehicles should be routed across the network. The fuel consumption curve (10) had values of $A = 0.0019$, $B = -0.2784$ and $C = 17.337$ and is depicted in Fig. 2.

5.1. Weighted sum model

The results of the optimisation for values of $\lambda \in [0, 1]$ and the three configurations of $\mathbf{m} = (m_1, m_2, m_3)$, $\mathbf{m}^1, \mathbf{m}^2, \mathbf{m}^3$ are given in Table 2. Fig. 4 shows a comparison of the normalised results for $\mathbf{m}^1$ and the results in the original scale. Fig. 5 plots results for $\mathbf{m}^1, \mathbf{m}^2, \mathbf{m}^3$ in the original scale, in addition to the weighted sum model, the minimimum total travel time, minimum fuel consumption and user equilibrium are also plotted. The point $(F(\mathbf{x}^*_{\lambda 0}), T(\mathbf{x}^*_{\lambda 0}))$ is also included as a means of measuring how far the user equilibrium is from a more idealised point in the Pareto set.

$\mathbf{m}$	$\mathbf{x}$	x_1	x_2	x_3	$T(\mathbf{x})$	$F(\mathbf{x})$	λ_0	$\phi(\mathbf{x})$
$\mathbf{m}^1 = (110, 80, 65)$	$\mathbf{x}^*_{\lambda_0}$	715.02	450.7	334.28	2666.13	14744.0	0.3	11120.64
	$\mathbf{x}^*_{UE}$	661.79	444.67	393.54	2666.21	14975.43	-	-
$\mathbf{m}^2 = (95, 95, 95)$	$\mathbf{x}^*_{\lambda_0}$	499.22	585.28	415.51	2470.71	13957.74	0.14	12349.56
	$\mathbf{x}^*_{UE}$	425.83	583.19	490.98	2622.48	14077.74	-	-
$\mathbf{m}^3 = (110, 80, 40)$	$\mathbf{x}^*_{\lambda_0}$	732.13	495.34	272.53	2941.01	15287.75	0.34	11089.86
	$\mathbf{x}^*_{UE}$	764.47	567.52	168.01	3044.31	15441.731	-	-

Table 2: Comparison of weighted sum Model solutions for $\mathbf{m}^1, \mathbf{m}^2$ and $\mathbf{m}^3$.



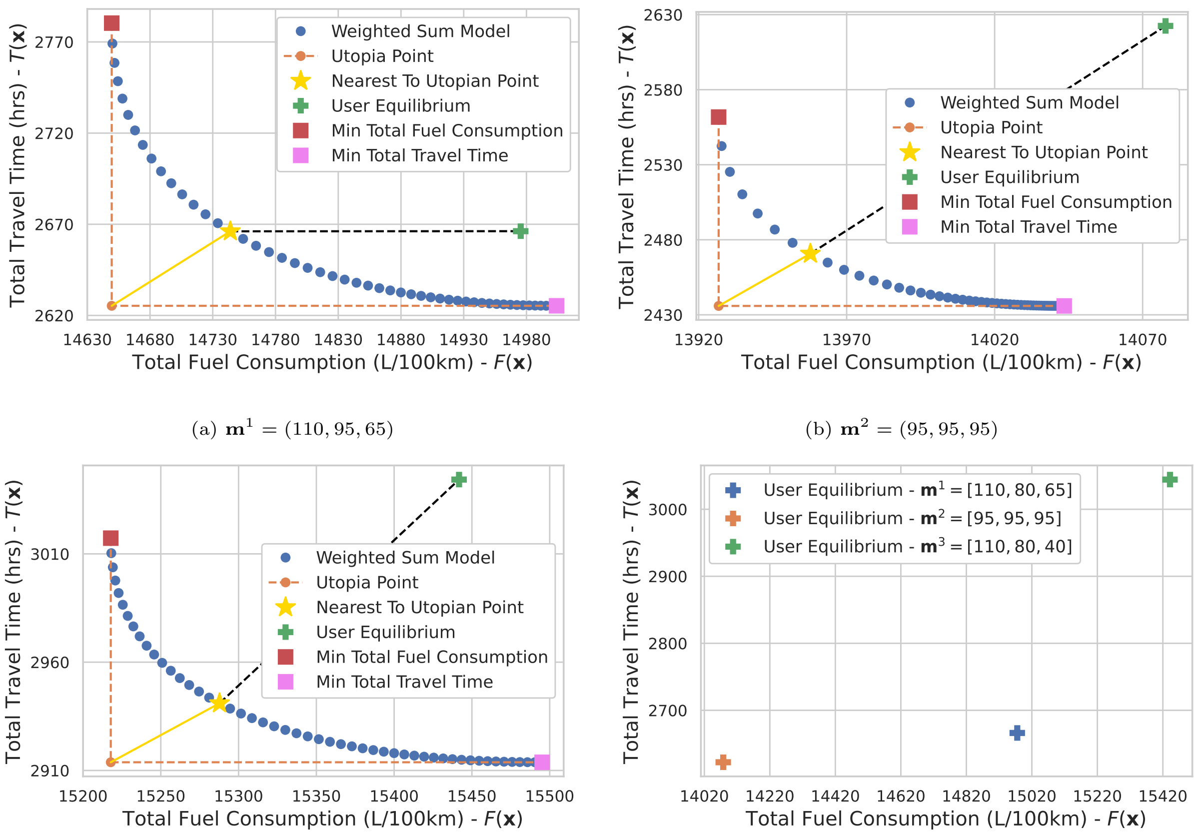
(a) Normalised plot for $\mathbf{m}^1 = (110, 95, 65)$

(b) Original scale plot $\mathbf{m}^1 = (110, 95, 65)$

Figure 4: Comparison of normalised and original scale plots.

The Pareto set has extreme values corresponding to the minimum values of $T(\mathbf{x})$ and $U(\mathbf{x})$ and in each case the user equilibrium solution is dominated by solutions of the weighted sum model. This is not surprising as the average time drivers experience by selfishly routing (user equilibrium) can never do better than the minimum total travel time found by minimising the total travel time or the minimum fuel consumed by minimising fuel consumption. Whilst it can be the case that for certain demand levels, i.e. extremely low or extremely high, the total travel time for selfishly routing is equal to the minimum total travel time; the solution to user equilibrium, at best would correspond to a point in the Pareto set.

We can also measure the inefficiency of the user equilibrium solution $\mathbf{x}^*_{\mathbf{UE}}$ for a given configuration by comparing it to the solution that is nearest, in Euclidean distance, to the solution within the Pareto set $\mathbf{x}^*_{\lambda_0}$. One should be careful not to conclude that a configuration where the user equilibrium solution is close to



(a) $\mathbf{m}^1 = (110, 95, 65)$

(b) $\mathbf{m}^2 = (95, 95, 95)$

(c) $\mathbf{m}^3 = (110, 80, 40)$

(d) $\mathbf{m}^1$ 'vs' $\mathbf{m}^2$ 'vs' $\mathbf{m}^3$

Figure 5: Weighted sum model for 3 different configurations of $\mathbf{m}$.

this point is better (they are different problems for each configuration). On inspection of Figs. 5a, 5b and 5c, the user equilibrium solution $\mathbf{x}^*_{\mathbf{UE}}$ that is furthest from the solution $\mathbf{x}^*_{\lambda_0}$ is given by configuration $\mathbf{m}^2$ (Fig. 5b); however, Fig. 5d clearly shows that this solution dominates both configurations $\mathbf{m}^1$ and $\mathbf{m}^3$.

5.2. Adjusting the edge free-flow speed

We used a simple iterative search to generate a set of user equilibrium solutions, furthermore, we identified the Pareto set for the solutions that are efficient for total travel time and fuel consumption. For each solution, the variance of the edge travel time functions, which have positive flow (i.e. those used), was also generated to demonstrate the validity of the user equilibrium claim. The maximum value of the variance over all solutions was 3.87×10^{-10}.

The results are displayed in Fig. 6 and clearly show the trade-off between total travel time and total fuel consumption. Points 1, 2, 3 on the graph indicate the increase in allowable speeds on the edges; at point 1 for low speeds the total fuel consumption and total travel time are both high, as we increase the speeds and approach point 2, we enter the Goldilocks zone of the fuel consumption curve, thus the total fuel consumption is minimised. Finally, as we approach point 3, speeds increase and we leave the Goldilocks zone, allowing the total travel time to be lowered, but severely impacting the total fuel consumption.

Two additional points are plotted showing how a reasonable adjustment in the values of m_e can result in a new user equilibrium solution that strictly dominates in terms of both total travel time and total fuel consumption. Figs. 7a and 7b plot the total fuel consumption and the total travel time against the mean

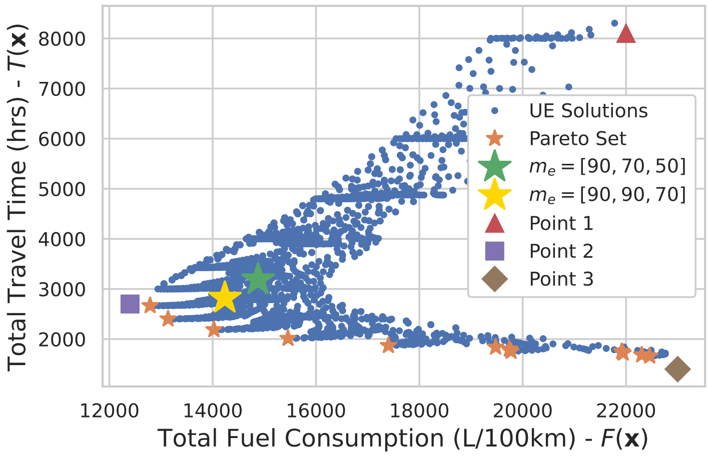


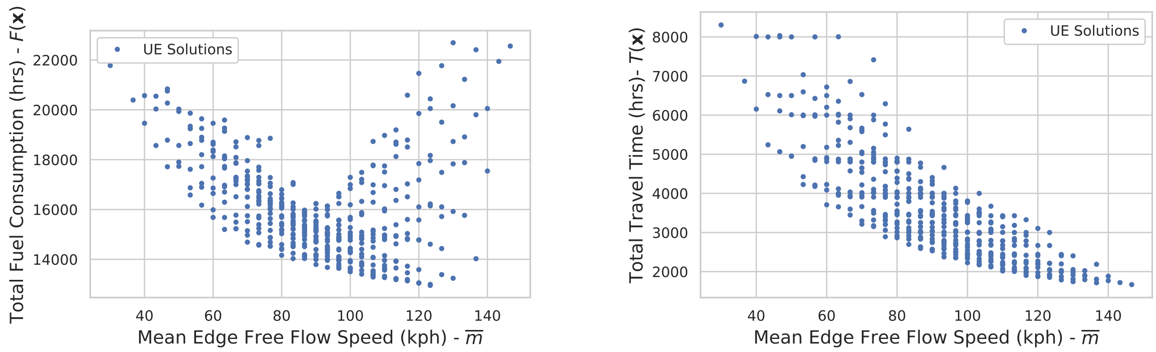
Figure 6: Total Fuel Consumption 'vs' Total Travel time for varying free-flow speeds m_e.

edge speed $\overline{m} = \frac{1}{|E|} \sum_{e \in E} m_e$, respectively. Each of the figures are intuitive with the expected behaviour, in the case of Fig. 7a, the total fuel consumption, it can be seen that this mirrors the fuel consumption curve $F_e(s_e)$, when the mean speed is too low or too high we see an increase in the total fuel consumption, the best results are obtained between the values of $\overline{m} = 60$ and $\overline{m} = 80$. In Fig. 7b, the total travel time can be seen to decrease as the speed limit is progressively lifted.

6. Concluding Remarks and Future Work

In this paper we considered a multi-criteria extension of a simplified single origin/destination pair connected with parallel edges to alleviate fuel consumption. We discussed the issues with the multi-criteria approach of converting criteria to Value of Time, i.e. when such a conversion can be deemed difficult to make sense of within the context of selfish routing.

Further, we presented two approaches, the first allowed the interplay between the competing objectives of total fuel consumption and total travel time to be explored; the second, through the manipulation of the free-flow speed of an edge, demonstrated the Pareto set of solutions and the possibility of non-invasive traffic planning to reduce the total fuel consumption whilst still maintaining a respectable total travel time under



(a) Mean edge free-flow speed $\overline{m}$ 'vs' Total Fuel Consumption

(b) Mean edge free-flow speed $\overline{m}$ 'vs' Total Travel Time

Figure 7: Decomposition of user equilibrium solutions plotted against mean edge speed $\overline{m}$.

the notion of selfish routing, i.e. adjusting the existing speed limits to reach the desired goal.

To explore this approach further one can use a larger test network that is not limited to parallel edges, e.g. Sioux Falls; development of a more tractable approach and an investigation into the effect that the changes to free-flow speeds have on routing and congestion through the use of mesoscopic/microscopic simulations.

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