Dynamic modelling of a SCARA robot.

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ABSTRACT

This paper describes a method for modelling industrial robots that considers dynamic approach to manipulation systems motion generation, obtaining the complete dynamic model for the mechanic part of the robot and taking into account the dynamic effect of actuators acting at the joints.

For a four degree of freedom SCARA robot we obtain the dynamic model for the basic (minimal) configuration, that is, the three degrees of freedom that allow us to place the robot end effector in a desired point, using the Lagrange Method to obtain the dynamic equations in matrix form. The manipulator is considered to be a set of rigid bodies interconnected by joints in the form of simple kinematic pairs. Then, the state space model is obtained for the actuators that move the robot joints, uniting the models of the single actuators, that is, two DC permanent magnet servomotors and an electrohydraulic actuator.

Finally, using a computer simulation program written in FORTRAN language, we can compute the matrices of the complete model.

1. INTRODUCTION

There are various methods, based on the manipulator systematic modelling, for robot motion control[2,3]. The problem of distributing motion to individual degrees of freedom is thus resolved. In this paper we will consider dynamic approach to manipulation systems motion generation.[1]

The manipulator is considered as a dynamic system modelled by dynamic equations of the mechanical part, and the state space model of actuators in the joints. Usually, when a SCARA robot is modelled in dynamic form, the effect of the translational element and that of the gripper are despised because of their low masses and inertias. In our case, because of we obtain the model with the aim of design a control algorithm, we can not simplify the model in this way. So, our model comprises the whole basic configuration. This form allows us to design a control law that compensates for all non-linear effects. We will also obtain the dynamic model for the mechanical part of the robot, despising the third link (that we will call simplified model), and then, we will compare both models.

An advantage is that dynamic motion synthesis can be carried out optimally with respect to some dynamic performance index (energy, time...) together with the constraints wich are also dynamical in nature, such as constraints on maximal driving torques or forces, or maximal input signals to the actuators[1].

2. MANIPULATOR AND ACTUATORS DESCRIPTION

The robot modelled is the DAIV-1, a SCARA type developped in the Cathedra of System Engineering at the E.T.S.I.I., University of Valladolid. It is an articulated robot with four degrees of freedom, three rotations and one translation. It moves in the X-Y plane (refered to the coordinate frame placed in the robot base) with a vertical displacement along the Z axis, and that allows the wrist to rotate about this last axis.

For the aim of studying the robot dynamic behaviour when it performs a positioning task or describes a defined trajectory in its working space, we will obtain the dynamical model for the basic configuration, that is, two rotational degrees of freedom, joints one and two driven by DC permanent magnet servomotors, that allow the end effector to be positioned in the horizontal plane (we call it, X-Y plane), and a translational degree of freedom along the Z axis, joint three, driven by an electrohydraulic servovalve. We will obtain also the simplified model for the mechanical part, that is, the two rotational degrees of freedom, joints one and two.

We will use three operational coordinates refered to the coordinate frame placed at the robot base, we call it x,y,z. Also, we will use three generalized coordinates in order to know the relative positions among the different robot elements, they are also state variables for our model. Generalized coordinates that we use are: (See Fig. 1)

θ_1 , is the rotation angle described by element one, refered to the X axis in the coordinate frame, placed in the robot base. Values for θ_1 are from 0 to 200 degrees.

θ_2 , gives us the relative position of element two refered to element one. Values for θ_2 are from 0 to 160 degrees.

q_3 , is the vertical displacement of the robot end effector. Values for this displacement are comprised between 0 and 150 mm., but a descendent displacement of the end effector involves a negative value for q_3, so values for the variable q_3 are comprised between 0 (home position) and -150.

The end effector position in the cartesian space is determined using the operational coordinates. Because we don't take into account the fourth degree of freedom, we don't need operational or generalized coordinates for the end effector orientation.

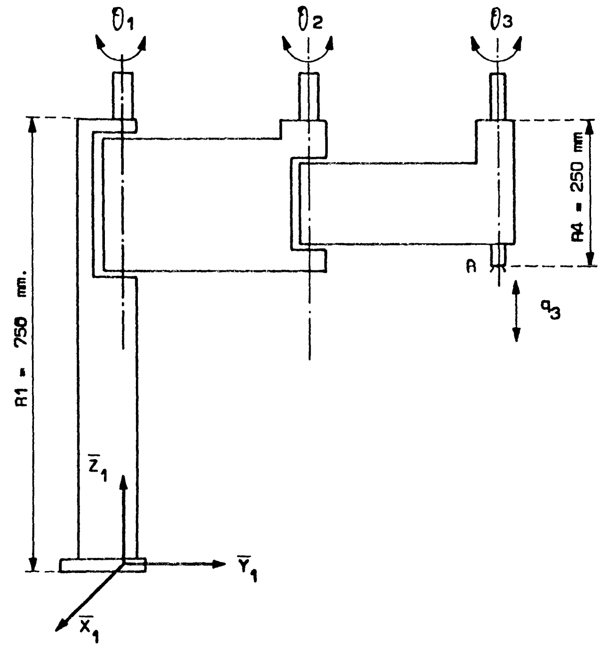

Fig. 1 DAIV-1 SCARA robot.

In its home position , the robot has its two horizontal elements aligned along the X axis. Angles θ_1 and θ_2 take positive values from this position. The maximun value for z coordinate is 500 mm., that corresponds with q_3=0. This is the higher point that the robot end effector can reach, refered to the base coordinate frame.

Joints one and two are driven by DC servomotors with three control loops, in position, velocity and current. The servomotor used for both joints is the MAVILOR Mod. 301, wich main features are a quick response, small weight and a low moment of rotor inertia. Joint three is a translational one, we use in it an electrohydraulic servomechanism that includes a piston-servovalve ensemble. The servovalve is a MOOG series 76 with a four ways distributor. The servoactuator is a MOOG E 851.

3. MANIPULATOR SIMPLIFIED DYNAMIC MODEL

Dynamic equations of an n degree of freedom manipulator describe the motion of the system, that is, position, velocity and acceleration, given the driving torques or forces acting at manipulator joints. They represent a set of n non linear, second order differential equations[3].

Dynamic equations can be obtained by applying any of the different methods known from classical mechanics : Lagrange's equations, Newton-Euler equations, etc. We have choose Lagrange's equations because of its advantages[4] : they don't take into account the reaction forces and torques among the elements, and they deal with scalar magnitudes instead of vectorial magnitudes.

In order to obtain the dynamic simplified model[7] for the mechanical part of the robot we will consider it as a set of two rigid bars, that we call bar one and bar two, that can rotate over an horizontal plane. Bar one rotates about a fixed axis O1 (joint one of the robot) and bar two rotates about axis O2 (joint two), placed at the end of bar one. (See Fig. 2).

In order to obtain the dynamic equations by using Lagrange Method we need obtain the kinetic energy of the system and its potential energy. Because the particular mechanical structure of the robot, its structural rigidity and the small loads that it can manipulate (maximum 1 Kg.) we don't take into account the effect of the potential energy.

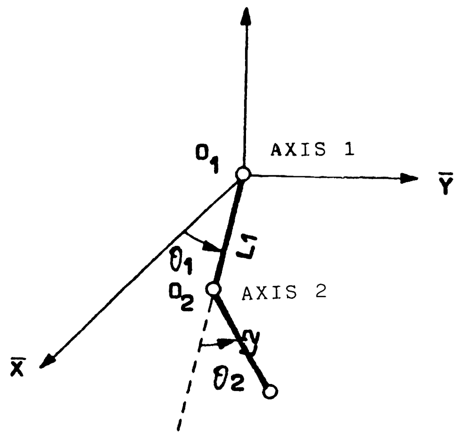

Fig. 2. Schematic representation

The kinetic energy for bar one is :

$$E_{c1} = \frac{1}{2} I_1 \dot{\theta}_1^2 + \frac{1}{2} I_{R1} (n_1 \dot{\theta}_1)^2 \qquad (1)$$

where I_1 is the moment of inertia of bar one about it rotation axis, including the corresponding part of joint two and that of the reductor, together with the inertia of the position sensor. I_{R1} is the moment of rotor inertia of motor one comprising the corresponding part of the reductor and n_1 is the reduction rate of motor one. The effect of friction at the joints will be take into account in the actuators model.

Now, we obtain kinetic energy for bar two. It includes bar two, the position sensor, actuator two, and element three, the gripper and the load. The kinetic energy is :

$$E_{c2} = \frac{1}{2} M_2 (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} (\dot{\theta}_1 + \dot{\theta}_2)^2 I_2 + \frac{1}{2} M_{R2} (l_1 \dot{\theta}_1)^2 + \frac{1}{2} I_{R2} (n_2 \dot{\theta}_2)^2 \qquad (2)$$

$\dot{x}, \dot{y}$ are mass center linear velocities along the X,Y axes, I_2 is the Moment of Inertia of element two about an paralel axis to Z axis, placed at its mass center, M_2 is the mass of element two, comprising the third element, the gripper and the load, M_{R2} is the rotor mass of motor in joint two, I_{R2} is the moment of rotor inertia of motor two and the corresponding part of the reductor, n_2 is the reduction rate in joint two, and l_1 is the length of bar one.

Calling 'd' to the distance from O_2 to the mass center of bar two, the values for x,y in function of the generalized coordinates are :

$$\begin{aligned} x &= l_1 \cos \theta_1 + d \cos (\theta_1 + \theta_2) \\ y &= l_1 \sin \theta_1 + d \sin (\theta_1 + \theta_2) \end{aligned} \qquad (3)$$

Introducing these values in (2) and adding (1) plus (2) we obtain the total kinetic energy. The expression is :

$$\begin{aligned} E_{ct} = {} & \frac{1}{2} \dot{\theta}_1^2 (I_1 + I_2 + M_2 d^2 + n_1 I_{R1} + (M_{R2} + M_2) l_1^2 + 2 M_2 l_1 d \cos \theta_2) + \frac{1}{2} \dot{\theta}_2^2 (I_2 + n_2^2 I_{R2} + M_2 d^2) + \\ & + \dot{\theta}_1 \dot{\theta}_2 (I_2 + M_2 d^2 + M_2 l_1 d \cos \theta_2) \end{aligned} \qquad (4)$$

we will call : $I_3 = I_2 + M_2 d^2$

$I_4 = I_1 + I_3 + n_1 I_{R1} + (M_{R2} + M_2) l_1^2$

$$I_5 = M_2\, l_1\, d$$

$$I_6 = I_3 + n_2^2\, I_{R2}$$

and we obtain the expression for the kinetic energy in the form:

$$E_{ct} = \frac{1}{2}\,\dot{\theta}_1^2\,(I_4 + 2\,I_5 \cos\theta_2) + \frac{1}{2}\,\dot{\theta}_2^2\, I_6 + \dot{\theta}_1\dot{\theta}_2\,(I_3 + I_5 \cos\theta_2) \qquad (5)$$

To obtain the dynamic equations according to Lagrange Method we apply the equation :

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right] - \frac{\partial L}{\partial q_i} = F_{qi} \qquad (6)$$

where L is now the kinetic energy (E_{ct}), q_i is the generalized coordinate in joint i, and F_{qi} is the generalized torque acting at link i.

The dynamic equations that we obtain are :

$$(I_4+2I_5\cos\theta_2)\ddot{\theta}_1 - 2I_5 \sin\theta_2\,\dot{\theta}_1\dot{\theta}_2 + (I_3+I_5\cos\theta_2)\ddot{\theta}_2 - I_5 \sin\theta_2\,\dot{\theta}_2^2 = P_{m1}$$

$$I_6\,\ddot{\theta}_2 + (I_3+I_5\cos\theta_2)\ddot{\theta}_1 + I_5 \sin\theta_2\,\dot{\theta}_1^2 = P_{m2} \qquad (7)$$

We can express these equations in matrix form: $P_m = H(q)\,\ddot{q} + h(q,\dot{q})$, where H(q) is the Inertia matrix of the model and $h(q,\dot{q})$ is the vector of centripetal and Coriolis forces. These matrices are, in our simplified model :

$$H(q) = \begin{vmatrix} I_4+2I_5\cos\theta_2 & I_3+I_5\cos\theta_2 \\ I_3+I_5\cos\theta_2 & I_6 \end{vmatrix} \qquad h(q,\dot{q}) = \begin{vmatrix} -2I_5\sin\theta_2\,\dot{\theta}_2\dot{\theta}_1 - I_5\sin\theta_2\,\dot{\theta}_2^2 \\ I_5\sin\theta_2\,\dot{\theta}_1^2 \end{vmatrix}$$

With the simplifications made, this dynamic model is easy to obtain and, depending on the complexity of the task to be performed, it can be usefull as a dynamic representation of the manipulator. The problem is that we can not know the dynamic behaviour of the vertical element and it influence on the system. If we want design an effective descentralized control for positioning the end effector, we must consider the dynamic influence of the third element.

4. MANIPULATOR BASIC CONFIGURATION DYNAMIC MODEL

Now, we take into account the influence of the vertical element.[6] We will consider element three as a rigid bar that describes a translation along an axis paralell to Z axis, and placed at the end of bar two. (See Fig. 3)

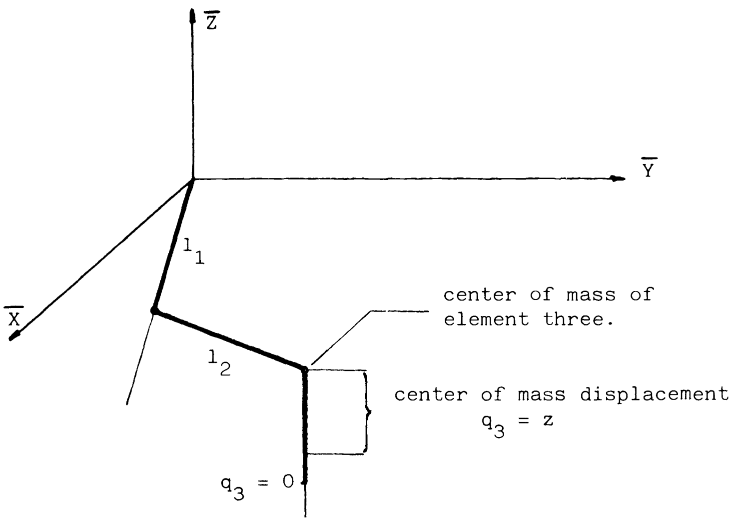

Fig. 3. Basic configuration schematic representation

We must obtain the kinetic energy for the whole system. From the previous model we know the kinetic energy for bars one and two.(5). We can use this expression making two changes: M_2 is now the mass of bar two without the mass of element three, and I_2 changes in the same way. Now, bar three includes the end effector and the load influence. The home position for this bar involves a zero value for the height of its mass center respect to the Z axis. The independient variable in this expression is the generalized coordinate q_3. The expression for the kinetic energy of bar three is :

$$E_{c3} = \frac{1}{2} M_{p3} (x^2+y^2+z^2) + \frac{1}{2} (\dot{\theta}_1+ \dot{\theta}_2)^2 I'_3 + \frac{1}{2} M_{t3} (D \dot{\Psi})^2 = (A)+(B)+(C) \quad (8)$$

I'_3 is the moment of inertia of bar three about a longitudinal axis along its mass center, M_{t3} is the total mass of element three including the gripper and the load mass, M_{p3} is M_{t3} without the servovalve mass and without the link part solidary with bar two. Since $z=q_3$ we obtain :

$$(A) = \frac{1}{2} M_{p3} (\dot{\theta}_1^2(l_1^2+l_2^2+2l_1l_2 \cos \theta_1)+\dot{\theta}_2^2 l_2^2+\dot{\theta}_1\dot{\theta}_2(2l_1l_2 \cos \theta_2+2l_2^2)+\dot{q}_3^2) \quad (9)$$

In expression (C), D is the distance from joint one to the end of bar two, and Ψ is the angle between X axis and this fictious line (D). The value for angle Ψ is $\Psi=\theta_1+(\theta_2/2)$ and the value for D is

$$D = \sqrt{l_1^2+l_2^2+2l_1l_2\cos \theta_2} \quad (10)$$

Introducing these values in (C) we obtain:

$$(C) = \frac{1}{2} M_{t3} (D \dot{\theta}_1^2 + D \dot{\theta}_1\dot{\theta}_2 + D \frac{1}{4} \dot{\theta}_2^2) \quad (11)$$

If we call :

$$I_8 = M_{p3}(l_1^2+l_2^2)+I'_3+I_{10} \qquad I_{11} = 2M_{p3}l_2^2+2I'_3+I_{10}$$

$$I_9 = M_{p3}l_2^2+I'_3 \qquad I_{12} = M_{p3}l_1l_2$$

$$I_{10}= M_{t3}(l_1^2+l_2^2) \qquad I_{13} = M_{t3}l_1l_2$$

Including these values in expression (8) we obtain :

$$E_{c3} = \frac{1}{2}(I_8+2I_{12}\cos \theta_1+2I_{13}\cos \theta_3)\dot{\theta}_1^2 + \frac{1}{2}(I_9+\frac{1}{4}I_{10}+\frac{1}{2}I_{13}\cos \theta_2)\dot{\theta}_2^2 + $$
$$+ \frac{1}{2}(I_{11}+2(I_{12}+I_{13})\cos \theta_2)\dot{\theta}_1\dot{\theta}_2+ \frac{1}{2}M_{p3}\dot{q}_3^2 \quad (12)$$

The expression for the complete kinetic energy is $E_C=E_{ct}+E_{c3}=(5)+(12)$

$$E_C = \frac{1}{2}(I_{14}+2I_{15}\cos \theta_2+2I_{12}\cos \theta_1)\dot{\theta}_1^2+\frac{1}{2}(I_{16}+\frac{1}{2}I_{13}\cos \theta_2)\dot{\theta}_2^2+\frac{1}{2}(I_{17}+I_{18}\cos \theta_2)\dot{\theta}_1\dot{\theta}_2+\frac{1}{2}M_{p3}\dot{q}_3^2 \quad (13)$$

where we have called :

$$I_{14} = I_4 + I_8 \qquad I_{16} = I_6 + I_9 +\frac{1}{4}I_{10} \qquad I_{18} = I_5 +2(I_{12}+I_{13})$$

$$I_{15} = I_5 + I_{13} \qquad I_{17} = I_3 + I_{11}$$

Now, we apply Lagrange Method in order to obtain the dynamic equations, as we have made in the previous case, we don't take into account the effect of gravity. Dynamic equations that we obtain are :

$$P_{m1} = (I_{14}+2I_{15}\cos \Theta_2+2I_{12}\cos \theta_1)\ddot{\theta}_1+\frac{1}{2}(I_{17}-I_{18}\cos \theta_2)\ddot{\theta}_2-2I_{15}\sin \theta_2 \dot{\theta}_1\dot{\theta}_2-\frac{1}{2}I_{18}\sin \theta_2 \dot{\theta}_2^2$$

$$P_{m2} = (I_{16}+\frac{1}{2}I_{13}\cos \theta_2)\ddot{\theta}_2+\frac{1}{2}(I_{17}+I_{18}\cos \theta_2)\ddot{\theta}_1+I_{15}\sin \theta_2 \dot{\theta}_1^2-\frac{1}{4}I_{13}\sin \theta_2 \dot{\theta}_2^2$$

$$F_t = M_{p3}\ddot{q}_3$$

These equations in matrix form are: $P_m=H(q)\ddot{q} + h(q,\dot{q})$

$$H(q) = \begin{vmatrix} I_{14}+2I_{12}\cos \theta_1+2I_{15}\cos \theta_2 & \frac{1}{2}(I_{17}+I_{18}\cos \theta_2) & 0 \\ \frac{1}{2}(I_{17}+I_{18}\cos \theta_2) & I_{16}+\frac{1}{2}I_{13}\cos \theta_2 & 0 \\ 0 & 0 & M_{p3} \end{vmatrix}$$

$$h(q,\dot{q}) = \begin{vmatrix} -2I_{15}\sin \theta_2 \dot{\theta}_1\dot{\theta}-\frac{1}{2}I_{18}\sin \theta_2 \dot{\theta}_2^2 \\ I_{15}\sin \theta_2 \dot{\theta}_1^2-\frac{1}{4}I_{13}\sin \theta_2 \dot{\theta}_2^2 \\ 0 \end{vmatrix}$$

We have obtained a non-linear model that relates the velocities and accelerations in the links with the driving torques and force acting at they. The model shows a strong coupling between joints one and two, while joint three has no coupling with joints one and two because of the robot particular geometry.

If we compare models obtained for the basic configuration and that of the simplified structure, we can see that the introduction of link three modifies the relations between torques and accelerations in links one and two. For some desired values of $\ddot{\theta}_1$ and $\ddot{\theta}_2$, the values obtained for Pm1 and Pm2 are different for each model. So, altough there is not direct coupling between link three and links one and two, it is no valid obtain the simplified model and then enlarge the model matrices H(2*2) and h(2*1), in order to include element three, adding in H a line and a row with zeros, with $H(3,3)=M_{p3}$, and adding a zero to form h(3*1). In the same way we can not obtain the matrices ofthe simplified model as a subset of the complete model matrices.

The third order model takes into account the whole dynamic behaviour of the system when it performs a positioning task. This allows us to design a descentraliced control in the basis of this model.

5. ACTUATORS MODEL

Because of its low inductance , we use the second order state space model for DC motors driving joints one and two[5]. The model, in matrix form is $\dot{x}=Ax+bu+fP_m$, where x is the state vector $x = (\theta_i,\dot{\theta}_i)^T$, and A,b,f are matrices given by :

$$A = \begin{vmatrix} 0 & 1 \\ 0 & -\frac{Kd}{J} - \frac{K^2}{RJ} \end{vmatrix} \qquad b = \begin{vmatrix} 0 \\ \frac{K}{RJn} \end{vmatrix} \qquad f = \begin{vmatrix} 0 \\ -\frac{1}{Jn^2} \end{vmatrix}$$

where R is the rotor resistence, J is the moment of rotor inertia reduced to the output shaft, K is the motor constant, Kd is the viscous friction coefficient, n is the reduction rate, u is the voltage input to the motor and P_m is the driving torque reduced to the output shaft.

The electrohydraulic actuator is usually modelled by a set of non linear differential equations of the fifth order, but with servovalves with a wide bandwidth we can use the third order linearized model[1]. described by the following matrix equation: $\dot{x}=Ax+bi+fF_t$ when x is the state vector, $x=(l,\dot{l},p)^T$, and A,b,f are matrices given by

$$A = \begin{vmatrix} 0 & 0 & 0 \\ 0 & -\frac{B}{m} & \frac{S}{m} \\ 0 & -\frac{4Sb}{V} & -\frac{4b(Kc+C)}{V} \end{vmatrix} \qquad b = \begin{vmatrix} 0 \\ 0 \\ \frac{4Kqb}{V} \end{vmatrix} \qquad f = \begin{vmatrix} 0 \\ -\frac{1}{m} \\ 0 \end{vmatrix}$$

p is the load preasure, l and $\dot{l}$ are linear velocity and acceleration of the piston, i is the input current to the servovalve, B is the viscous friction coefficient, m is the piston mass, S is the piston area,C is the oil leakage coefficient, b is the oil compressibility coefficient, F_t is the external force of the piston, and Kq, Kc are servovalve characteristics.

6. NUMERICAL APPLICATION

We show, as numerical application the dynamic model obtained for the DAIV-1 robot. We will show manipulator dynamic model for both, the simplified and the basic configuartion.

The mechanic parameters for the simplified configuration are:

I_1 = 1.1 Kg m^2	M_2 = 12.15 Kg	n_1 = 100
I_2 = 0.4 Kg m^2	d = 0.225 m	n_2 = 110
M_1 = 12 Kg	M_{R2} = 1 Kg	I_{R1}= 4.5E-4 Kg m^2
l_1 = 0.4 m	l_2 = 0.4 m	I_{R2}= 4.5E-4 Kg m^2

Introducing these values, the dynamic model obtained is :

$$\begin{vmatrix} P_{m1} \\ P_{m2} \end{vmatrix} = \begin{vmatrix} 5.9876+2.19\cos\theta_2 & 2.7386+1.095\cos\theta_2 \\ 2.7386+1.095\cos\theta_2 & 8.1836 \end{vmatrix} \begin{vmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{vmatrix} + \begin{vmatrix} -2.19\sin\theta_2\,\dot\theta_1\dot\theta_2 - 1.095\sin\theta_2\dot\theta_2^2 \\ 1.095\sin\theta_2\,\dot\theta_1^2 \end{vmatrix}$$

For the basic configuration we have $I_2 = .3$ Kg m^2 and $M_2 = 10$ Kg, and we need other parameters as:

$I'_3 = 0.76$ Kg m^2 $\qquad M_{p3} = 1.15$ Kg $\qquad M_{t3} = 2.15$ Kg

With these values, the basic configuration dynamic model is :

$$\begin{vmatrix} P_{m1} \\ P_{m2} \\ F_t \end{vmatrix} = \begin{vmatrix} 8.904+0.048\cos\theta_1+2.491\cos\theta_2 & 0.772+0.818\cos\theta_2 & 0 \\ 0.772+0.818\cos\theta_2 & 6.449+0.172\cos\theta_2 & 0 \\ 0 & 0 & 1.15 \end{vmatrix} \begin{vmatrix} \ddot\theta_1 \\ \ddot\theta_2 \\ \ddot q_3 \end{vmatrix} +$$

$$+ \begin{vmatrix} -2.491\sin\theta_2\,\dot\theta_1\dot\theta_2 - 0.818\sin\theta_2\,\dot\theta_2^2 \\ 1.245\sin\theta_2\,\dot\theta_1^2 - 0.086\sin\theta_2\,\dot\theta_2^2 \\ 0 \end{vmatrix}$$

As we can see, the equivalent terms in both models, that is, the terms that relate the same variables, have different values. H(1,1) is higher in the basic configuration model than in the simplified model, while H(1,2)=H(2,1) and H(2,2) are lower for the basic configuration model.

So, the third order model shows a lower interaction between elements one and two because, in this configuration, element two has less mass and inertia than in the simplified configuration. The same reason explains the diminution on the value of H(2,2). The amount on the value of H(1,1) is due to the dynamic effect of element three.

The values obtained for the elements of $h(q,\dot q)$ are also different but there is only a small difference between these equivalent elements for the two vectors. Because of we have despised the effect of gravity in our study, vector h is very similar for both models.

We have simulated the dynamic behaviour of the robot using both models and the results show small differences for the values of P_{m1} while the difference between values for P_{m2} is higher. The third order model represents with more fidelity the manipulator dynamic system and allows us to know the driving generalized torques that we must apply in order to obtain a desired positioning trajectory, while this can not be made by using the simplified model.

Now we can obtain the complete dynamic model for the basic configuration joining the manipulator mechanical model obtained with the state model of the actuators. This last model results in the form $\dot x = A\,x + B\,u + F\,P_m$ where x is the state vector (7*1) that includes the generalized coordinates, its derivatives and the piston load preassure.

We can present the manipulator dynamic equations in the form $P_m = H_m(s)\,\dot s + h(s)$, where s is x without its final element (p) and $H_m(s) = (0|H(q))^1$. By eliminating the driving torque vector between this equation and that of the actuators we obtain :

$$\dot x = Ax + Bu + FH_m(s)\dot s + Fh(s) \qquad (14)$$

By introducing $F_m = (FH_m(s)|0)$ the model becomes :

$$\dot x = Ax + Bu + F_m\dot x + Fh \qquad (15)$$

The model of the complete manipulation system is obtained in the form :

$$\dot x = \bar A(x) + \bar B(x)\,u \qquad (16)$$

where $\quad \bar A(x)=(I-F_m(x))^{-1}(Ax+Fh(x))$ is the system matrix.

$\bar B(x)= (I-F_m(x))^{-1}B$ is the gain matrix.

Model (16) relates the state variables with the control vector values. The main problem is that obtaining numeric values for matrices $\bar A$ and $\bar B$ is very complex. We have developped a FORTRAN program that gives us these values starting from values for the state vector in a desired time instant. Below is shown an example obtained for the model developped.

State Vector :

.25E+00
.40E+00
-.80E-01
.20E+00
.20E+00
.15E+00
.70E+07

System Matrix :

.6000000E+00
.6000000E+00
.4500000E+00
-.1278295E+02
-.1577155E+02
.1624270E+06
.3738000E+04

Gain Matrix :

.6000000E+00	.0000000E+00	.0000000E+00
.6000000E+00	.0000000E+00	.0000000E+00
.4500000E+00	.0000000E+00	.0000000E+00
-.1077502E+02	-.2767705E+00	.0000000E+00
-.1602565E+02	.2864963E+01	.0000000E+00
.1624270E+06	.0000000E+00	.0000000E+00
.3738000E+04	.0000000E+00	.1916400E+06

7. CONCLUSIONS

We have obtained the manipulator dynamic model for two different configurations, a simplified one (second order model) and the basic configuration (third order model). The comparison made shows the advantages of the basic configuration model that takes into account the influence of the robot vertical element: it represents the manipulator dynamic system with more fidelity and allows us to obtain the torques and forces to be applyed to the robot links in order to perform a desired positioning trajectory.

Adding the actuators model we can obtain a complete dynamic model, caracterized by mean of two matrices (the system matrix and the gain matrix) that we can easily compute for a desired value of the state vector. This allows us to design an efficient descentralized control system.

8. REFERENCES

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