Minimum Time Control in SCARA Robot Simulation

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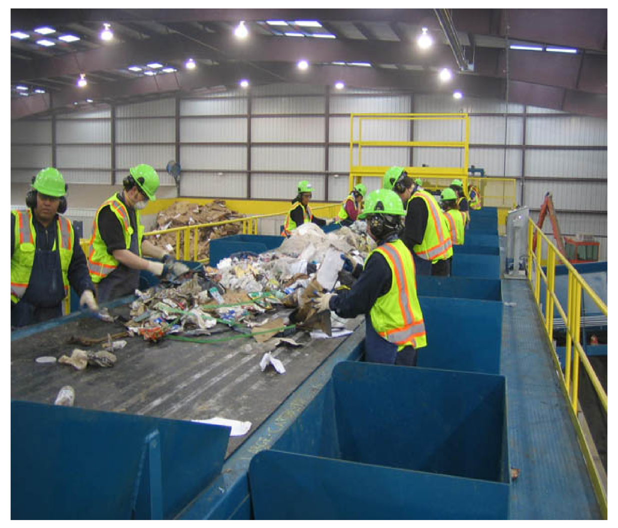
Introduction

- Motivation: Improve the efficiency of recycling through computer automation.
- Goal: Model the optimal removal of objects from a conveyor belt with a MATLAB simulation of a SCARA robot.
- Implementation: Solve a constrained optimization problem to find the optimal path between starting and ending robot configurations.



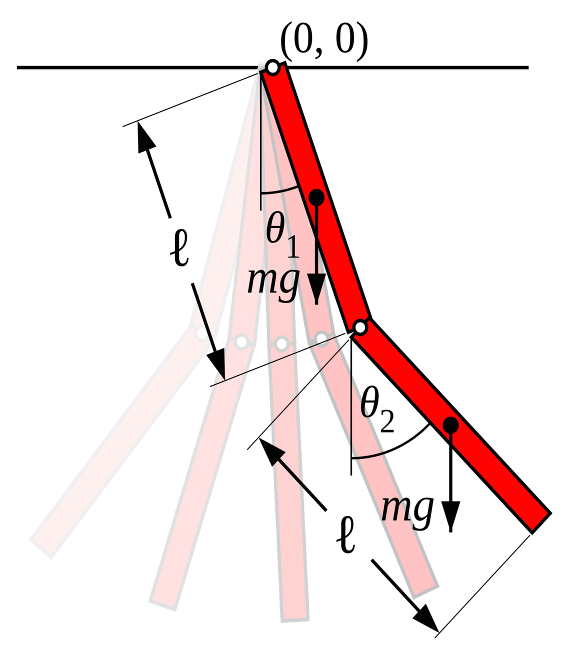
SCARA Robot: Operating Principle

- The Euler-Lagrange equation constrains kinetic energy and thus the possible system states:

$$\frac{\partial E}{\partial \theta_j} - \frac{d}{dt}\frac{\partial E}{\partial \dot{\theta}_j} = u_j$$

- u_j is the torque at joint j and E is the kinetic energy.

Double Pendulum



The Optimization Problem

Continuous Form

$$\min\ T$$
$$\text{s.t.}\ \frac{dx(t)}{dt} = f(x(t), u(t))$$
$$x(0) = x_0$$
$$x(T) = x_T$$
$$t \in [0, T]$$
$$\|u(t)\|_\infty \le M$$

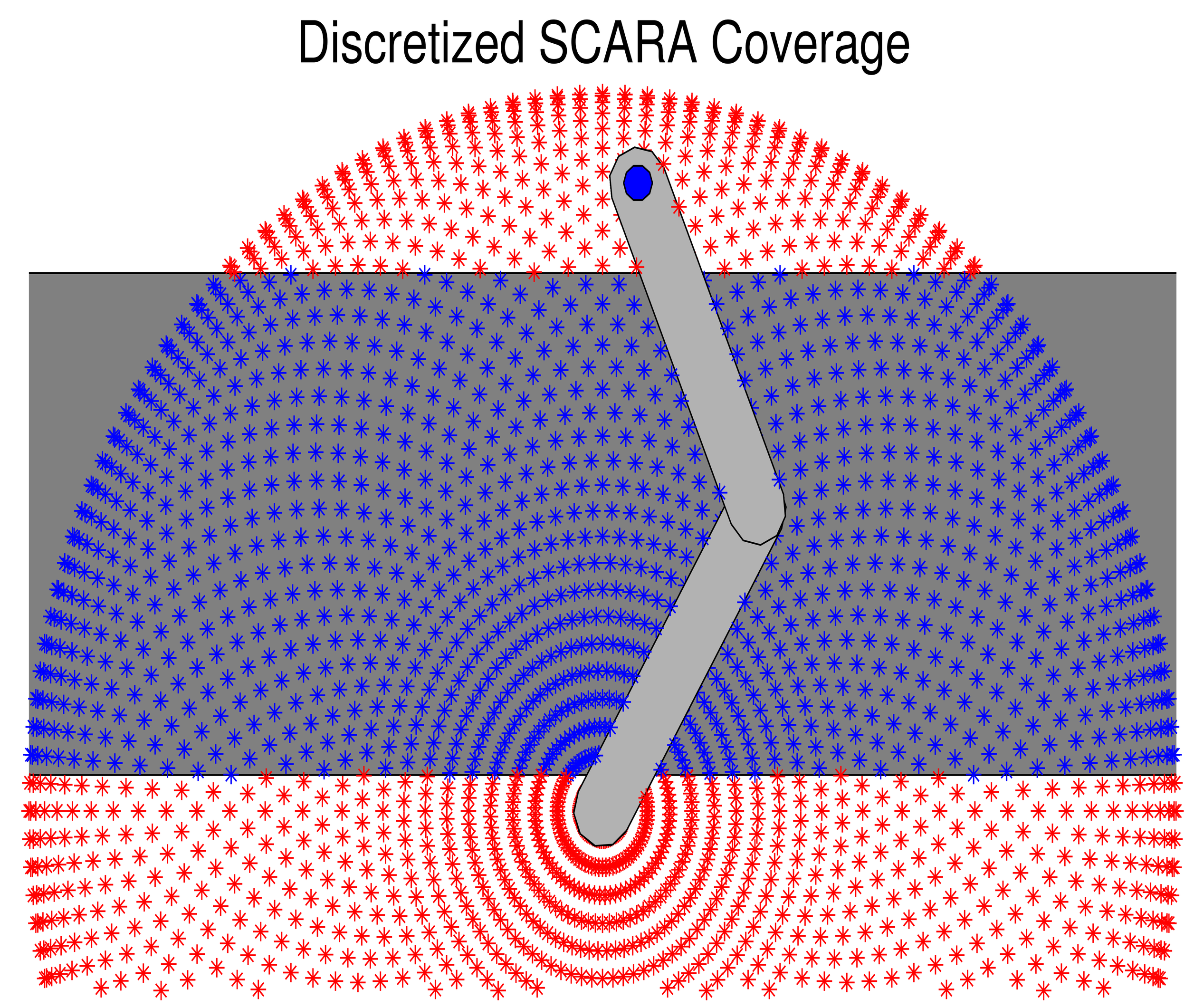
- T = total time taken by path
- t = time
- $f(t)$ = Lagrangian dynamics

Discrete Form

$$\min\ T$$
$$\text{s.t.}\ \frac{x_{i+1} - x_i}{\Delta\tau} = Tf(x_i, u_i)$$
$$x_1 = x_0$$
$$x_n = x_T$$
$$1 \le i \le n$$
$$\|u_i\|_\infty \le M$$

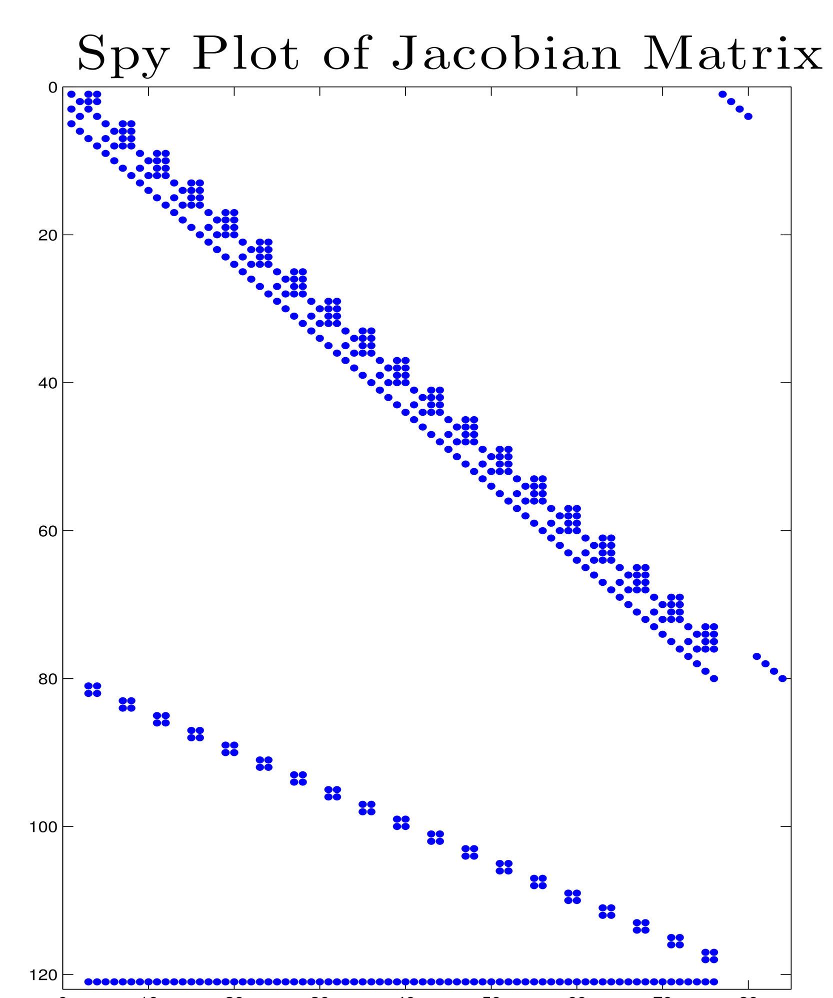
- $x(t) = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]$
- $u(t)$ = control
- x_0 = initial state
- x_T = end state
- M = torque bound

Discretized Precomputation



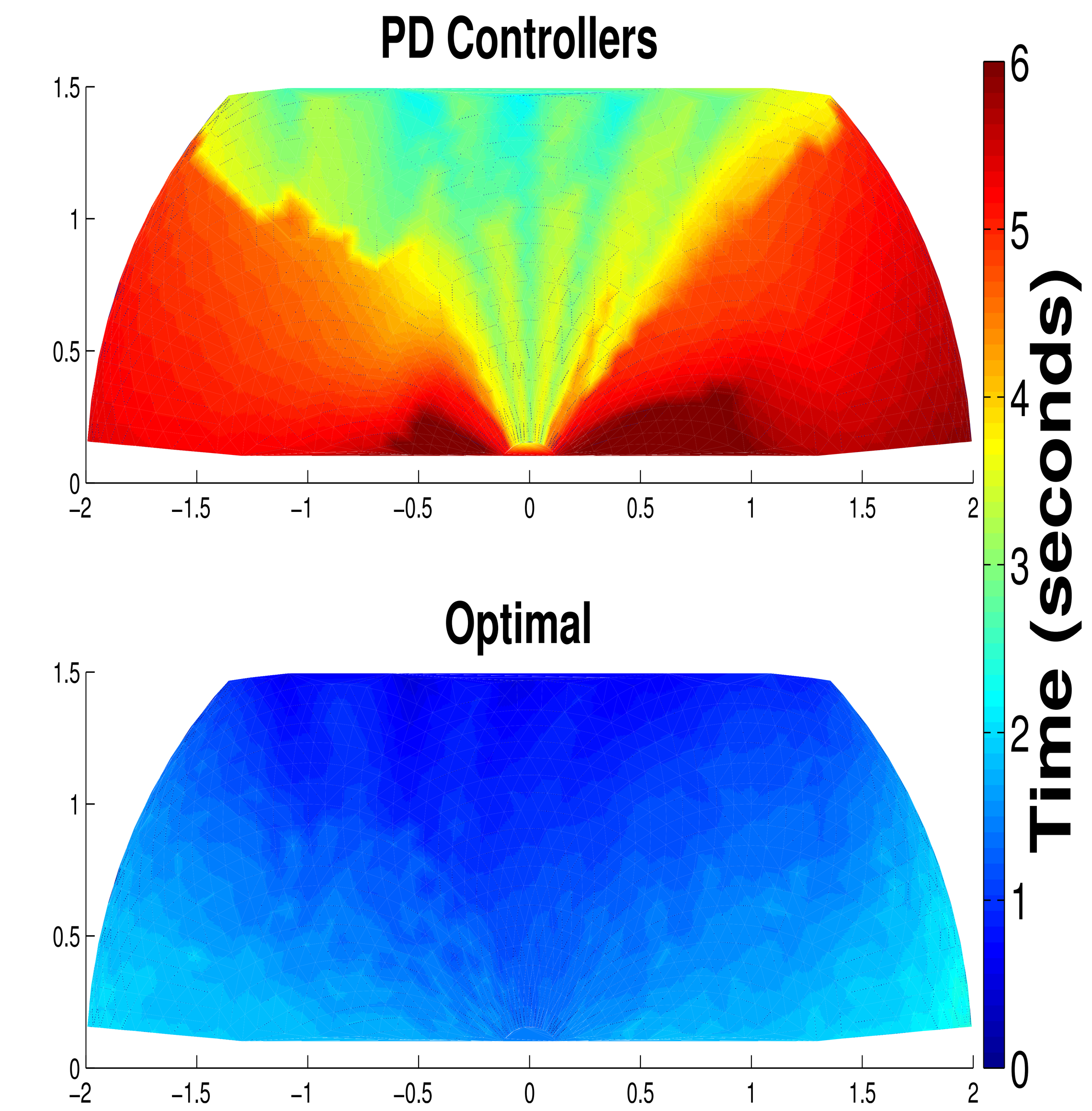
- Problem: The robot needs to make decisions in real time, but calls to fmincon for nonlinear optimization are expensive.
- Solution: Precompute and store optimal paths between a fixed number of radially spaced points.

Optimizing the Optimization

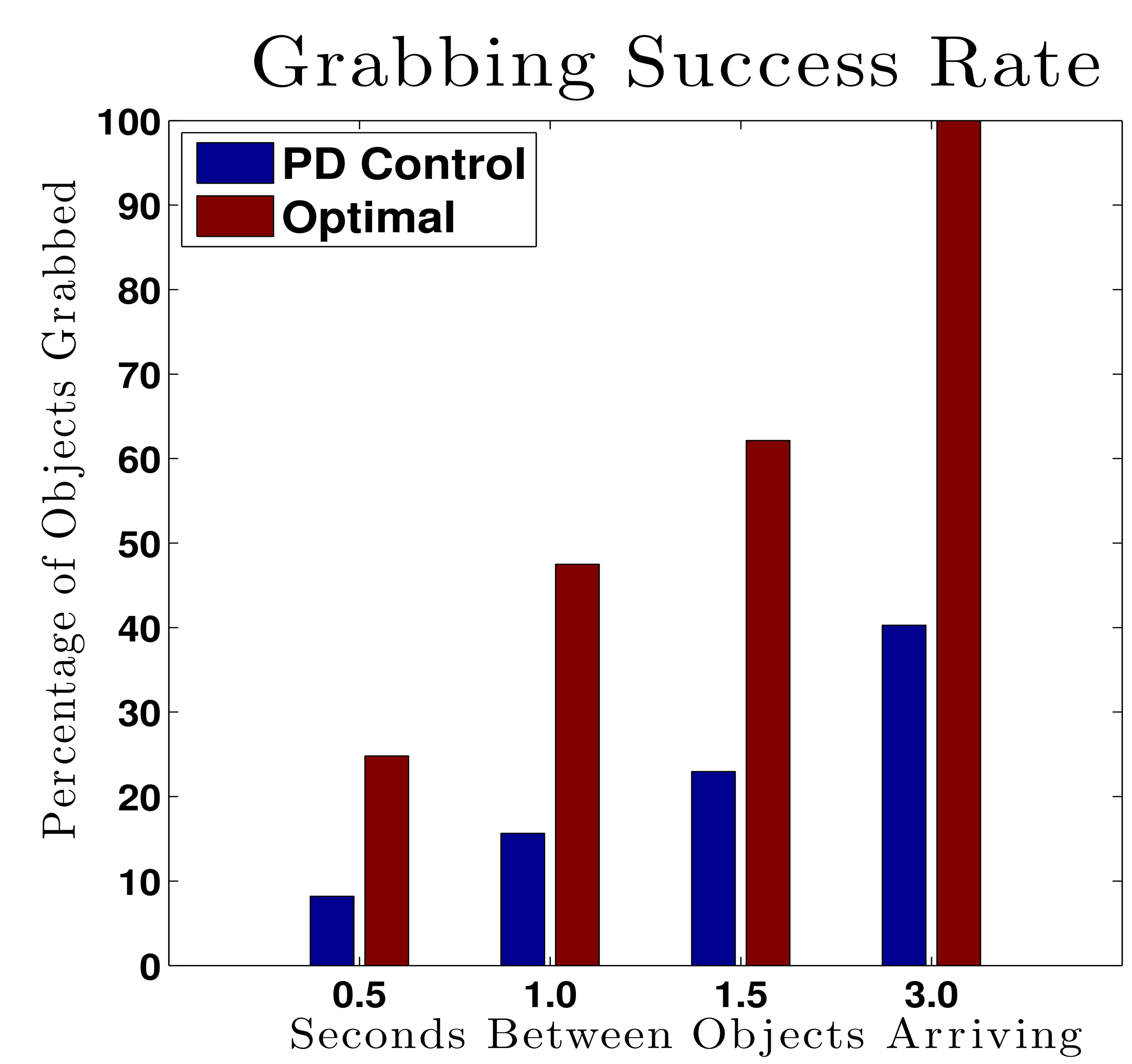


- Problem: fmincon assumes the Jacobian matrix of our system is dense, but it is actually sparse, with 2.67% of entries are nonzero.
- Solution: Only process nonzero entries in the matrix, giving a tenfold increase in speed.

Results



- The plots above represent the time required for the robot to move from a location on the belt to a goal region.
- Above is the suboptimal path times with PD controllers.
- Below are the optimal path times with fmincon.



Future Work

- Finding the reason for occasional failure of optimizing with MATLAB's fmincon.
- Investigating better ways of incorporating mod 2π arithmetic.

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