Minimum Time Control in SCARA Robot Simulation

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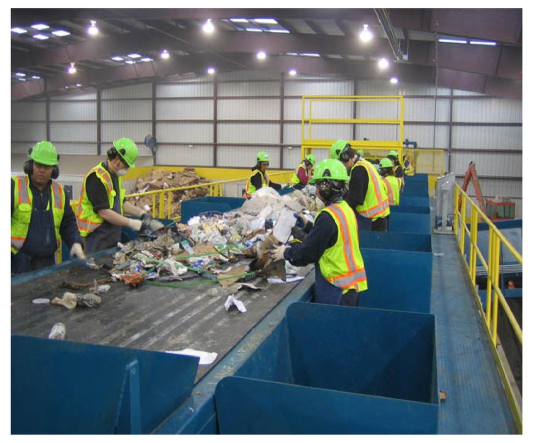
Introduction

Improving the efficiency of recycling through automation was the inspiration for our project.



We model the removal of objects from a conveyor belt with a MATLAB simulation of a SCARA robot.
We solve a constrained optimization problem to find the optimal path between starting and ending configurations.

Double Pendulum Physics

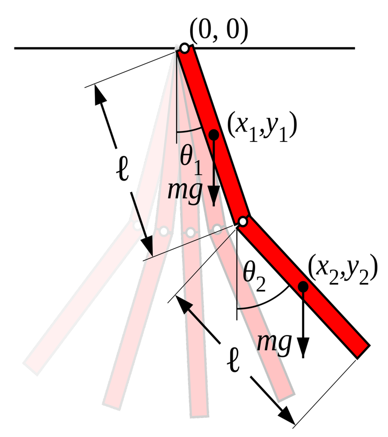
We calculate the kinetic energy of our system as

$$E = \frac{1}{2}\dot{\theta}_1^2(I_4 + 2I_5\cos\theta_2) + \frac{1}{2}\dot{\theta}_2^2 I_6 + \dot{\theta}_1\dot{\theta}_2(I_3 + I_5\cos\theta_2)$$

where the I_j terms represent moments of inertia.
We ignore gravity and thus potential energy. The Euler-Lagrange equation then applies only to the kinetic energy above:

$$\frac{\partial E}{\partial \theta_j} - \frac{d}{dt}\frac{\partial E}{\partial \dot{\theta}_j} = u$$

where u is the vector of torque at the actuators



The Optimization Problem

Original Problem

$$\begin{aligned} &\min\ T \\ &\text{s.t.}\ \frac{dx(t)}{dt} = f(x(t), u(t)) \\ &\qquad x(0) = x_0 \\ &\qquad x(T) = x_T \\ &\qquad t \in [0, T] \end{aligned}$$

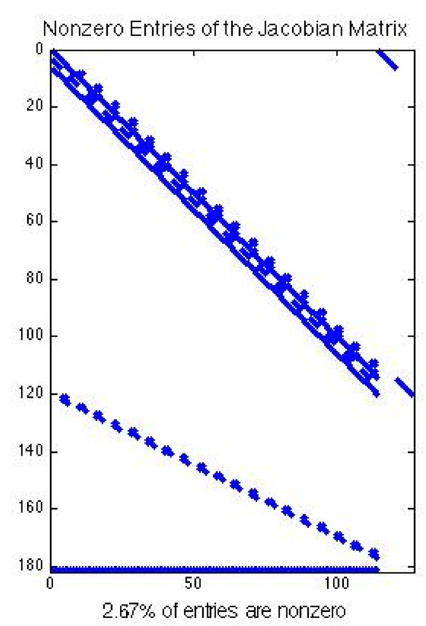
Discretized Problem

$$\begin{aligned} &\min\ T \\ &\text{s.t.}\ \frac{x_{j+1} - x_j}{\Delta\tau} = Tf(x_j, u_j) \\ &\qquad x_1 = x_0 \\ &\qquad x_n = x_T \\ &\qquad 1 \le j \le n \end{aligned}$$

where

- T = total time taken by path
- t = time
- $x(t)$ = state at time t
- $u(t)$ = control at time t
- x_0 = initial state
- x_T = end state

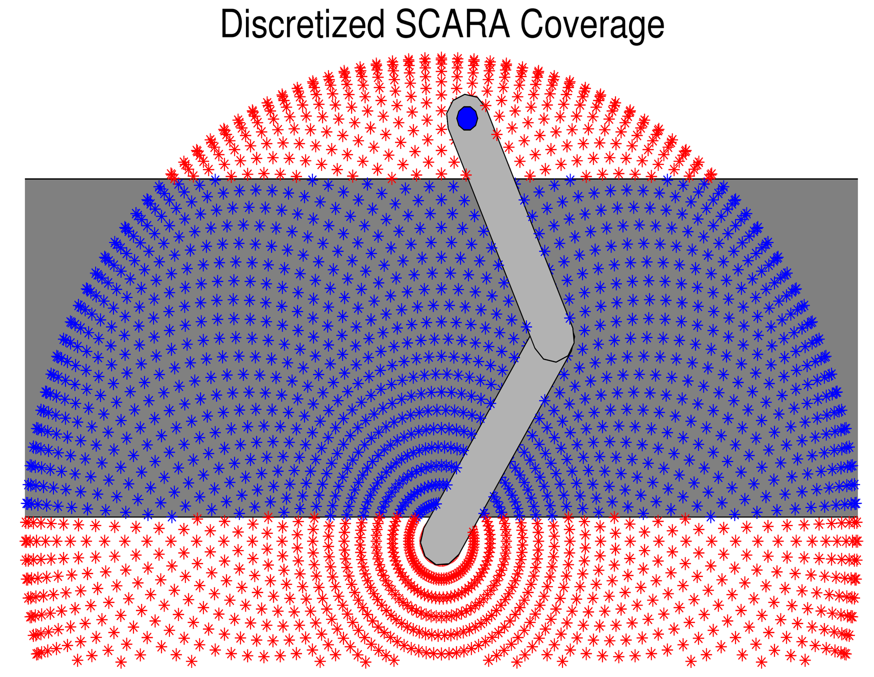
Optimizing the Optimizer



To accelerate MATLAB's fmincon solver for our discretized optimization problem, we precompute the Jacobian matrix

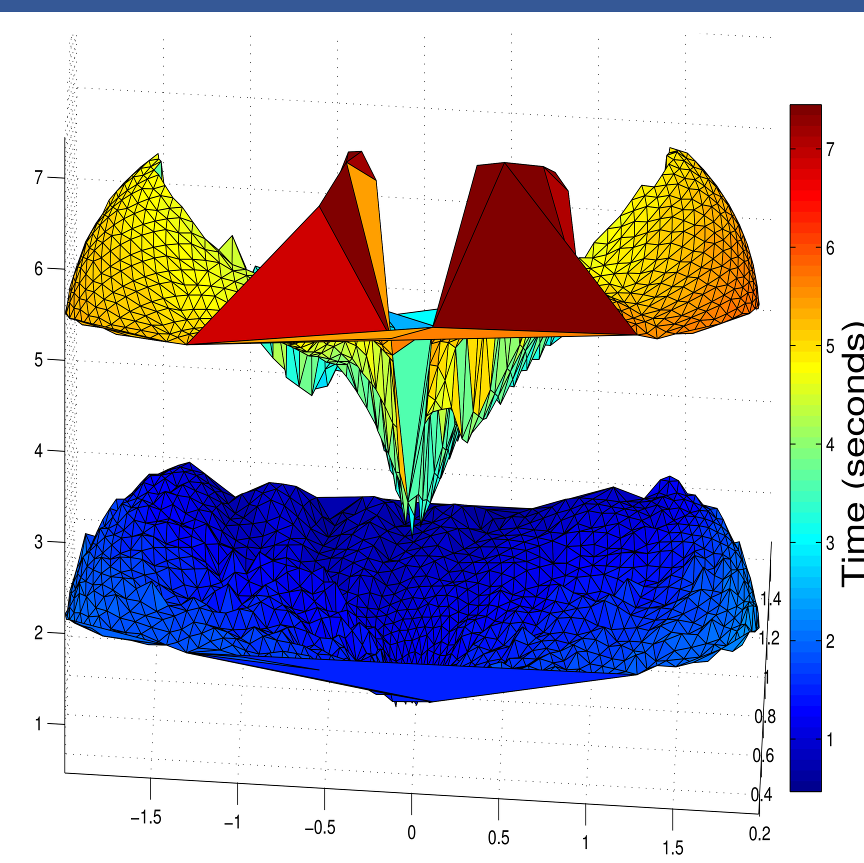
Discretized Precomputation

Discretized SCARA Coverage



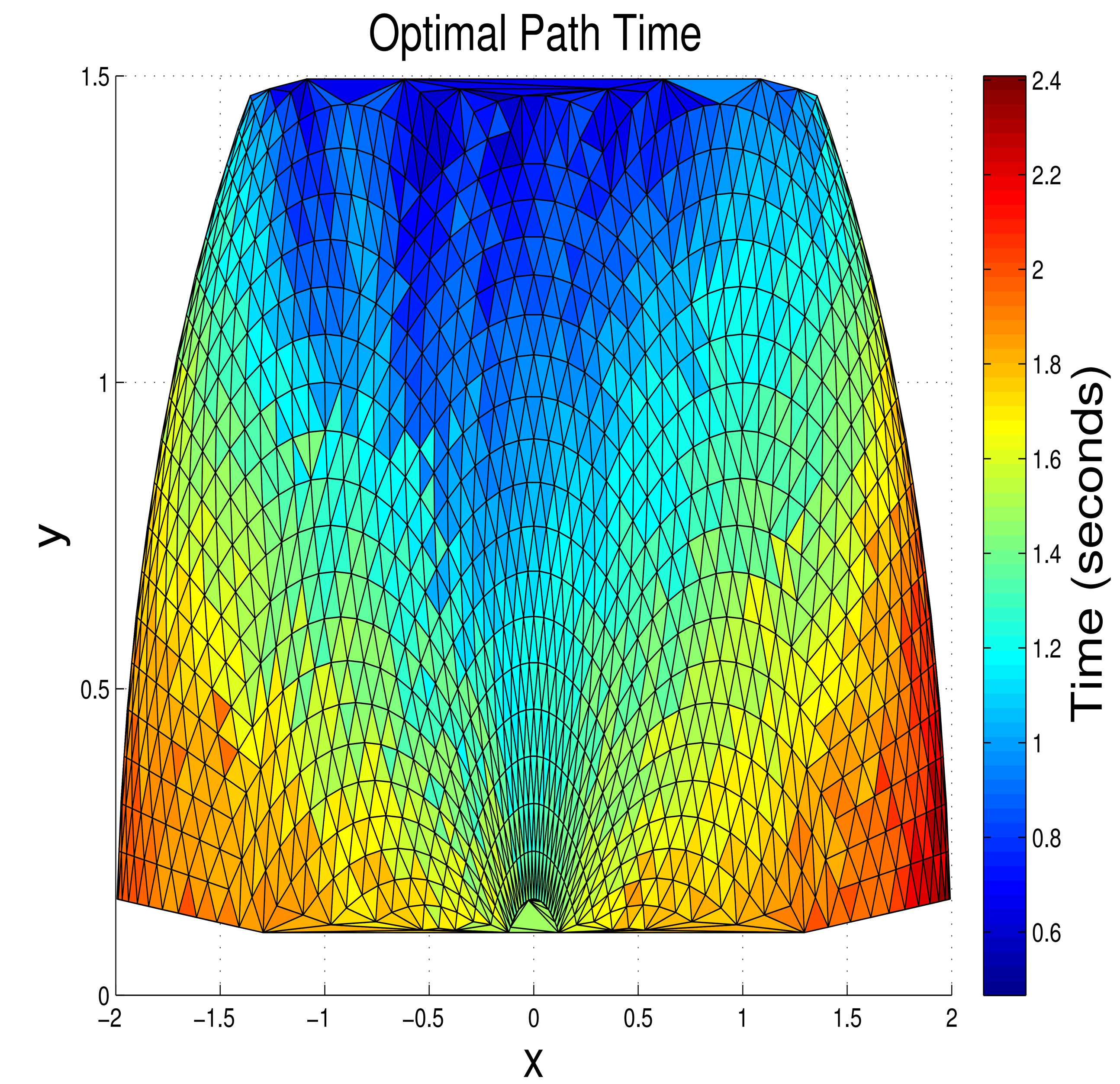
In order to run the simulation efficiently in real time, we precompute the optimal paths between a finite number of goal regions and points on the belt.

Optimality



Above are the suboptimal times achieved with PD (proportional derivative) controllers, below are the superior MinTimeControl results.
At each point on the belt, it takes less time to travel to a given goal region with an optimal path than with a corresponding PD controller

Results



Variation in optimal time to reach a fixed goal point given a starting robot configuration

Future work

- Finding the reason for occasional failure of optimizing with MATLAB's fmincon
- Investigating why the minimum time path is asymmetric from belt to goal and vice versa

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