Minimum Time Control in SCARA Robot Simulation

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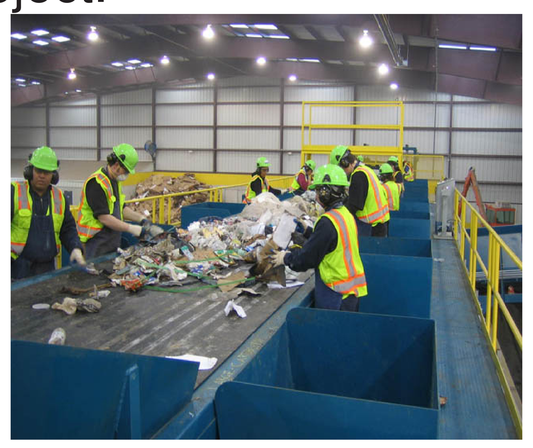
Introduction

Improving the efficiency of recycling through automation was the inspiration for our project.



We model the removal of objects from a conveyor belt with a MATLAB simulation of a SCARA robot.
We solve a constrained optimization problem to find optimal (minimum-time) path between starting and ending robot configurations.

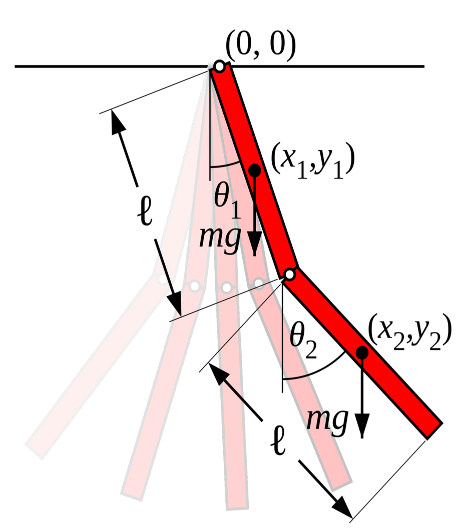
Double Pendulum Physics

We calculate the kinetic energy of our system as

$$E = \frac{1}{2}\dot{\theta}_1^2(I_4 + 2I_5\cos\theta_2) + \frac{1}{2}\dot{\theta}_2^2 I_6 + \dot{\theta}_1\dot{\theta}_2(I_3 + I_5\cos\theta_2)$$

where the I_j terms represent moments of inertia.
We ignore gravity and thus potential energy. The Euler-Lagrange equation then applies only to the kinetic energy above:

$$\frac{\partial E}{\partial \theta_j} - \frac{d}{dt}\frac{\partial E}{\partial \dot{\theta}_j} = 0$$



The Optimization Problem

$$\begin{aligned} &\min\ T \\ \text{s.t. } &\frac{dx(t)}{dt} = f(x(t), u(t)) \\ &x(0) = x_0 \\ &x(T) = x_T \\ &0 \le t \le T \end{aligned} \qquad \begin{aligned} &\min\ T \\ \text{s.t. } &\frac{x_{j+1} - x_j}{\Delta\tau} = Tf(x_j, u_j) \\ &x_1 = x_0 \\ &x_n = x_T \\ &1 \le j \le n \end{aligned}$$

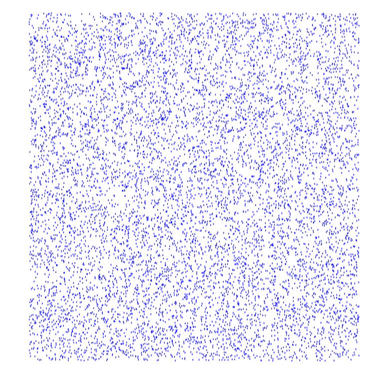
where

- T = total time taken by path
- t = time
- $x(t)$ = state at time t
- $u(t)$ = control at time t
- x_0 = initial state
- x_T = end state

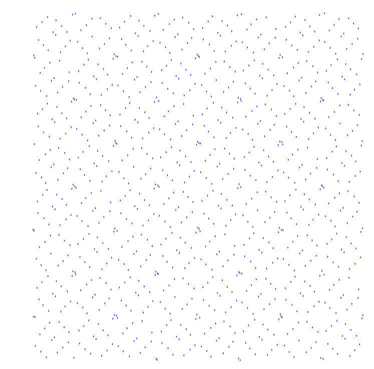
Discretized Precomputation

$$\mathbb{E}[f] = \frac{1}{M}\sum_{i=1}^{M} f(\xi_i)$$

Monte Carlo (MC)

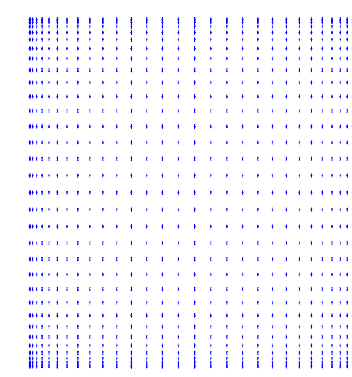


Quasi-Monte Carlo (QMC)

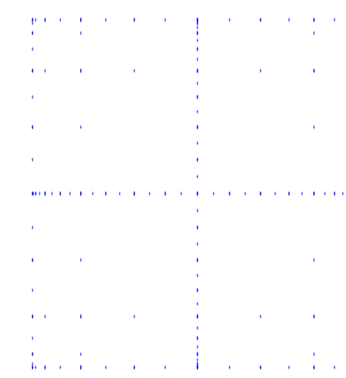


$$\mathbb{E}[f] = \sum_{i=1}^{M} \omega_i f(\xi_i)$$

Tensor Product (TP)



Sparse Grid (SG)



Optimizing the Optimizer

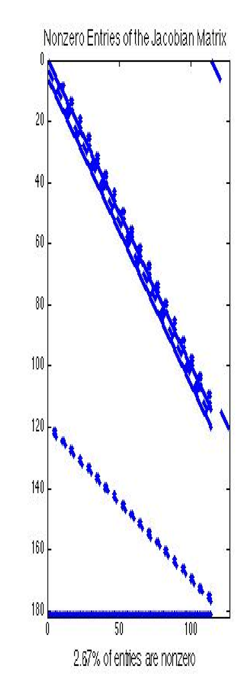
Represent f using hierarchical basis functions (with varying support):

$$f(\xi) \approx \sum_{\mathbf{l}} f_{\mathbf{l}}(\xi), \quad f_{\mathbf{l}}(\xi) = \sum_{\mathbf{i}\ \text{odd}} s_{\mathbf{l},\mathbf{i}}\phi_{\mathbf{l},\mathbf{i}}(\xi)$$

where d-dimensional functions $\phi_{\mathbf{l},\mathbf{i}}(\xi)$ are defined as

$$\phi_{\mathbf{l},\mathbf{i}}(\xi) = \prod_{j=1}^{d} \phi_{l_j,i_j}(\xi_j)$$

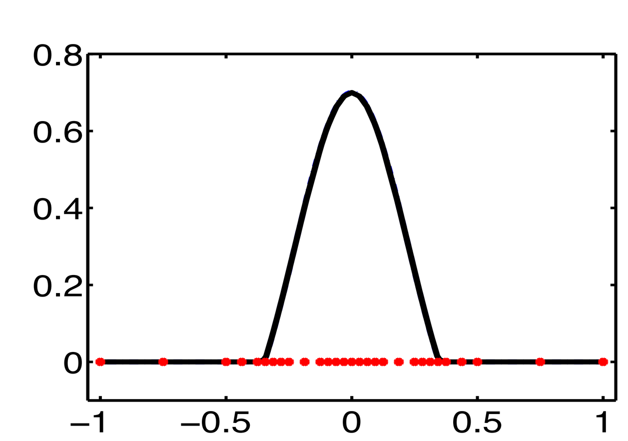
Coefficients $s_{\mathbf{l},\mathbf{i}}$ are called *hierarchical surpluses* as they represent hierarchichal increments between neighboring levels.



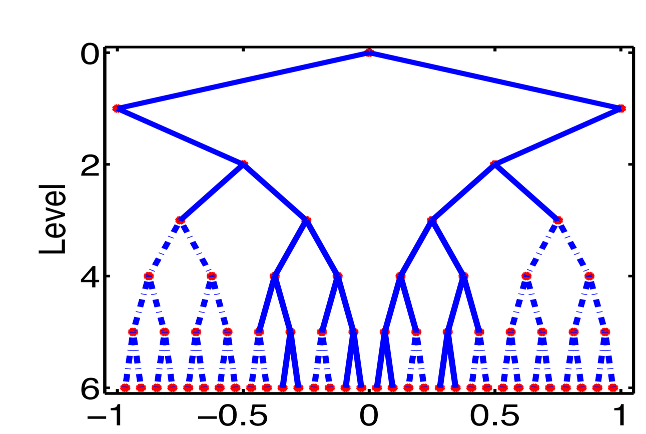
1D hierarchical piecewise linear basis

Optimality

Hierarchical surpluses $s_{\mathbf{l},\mathbf{i}}$ are a natural local error indicator. Thus, refining only points with relatively large surpluses we obtain an adaptive grid (with more points spent in non-smooth areas).

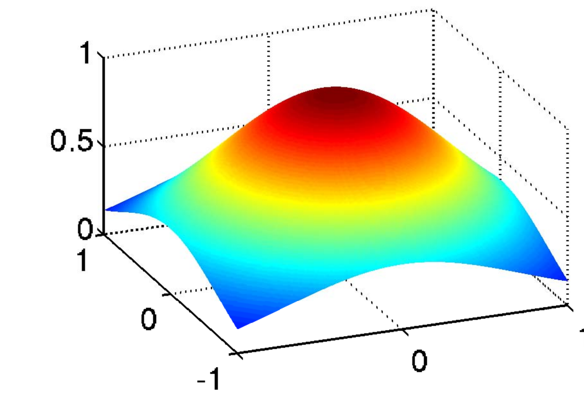


Adaptive grid

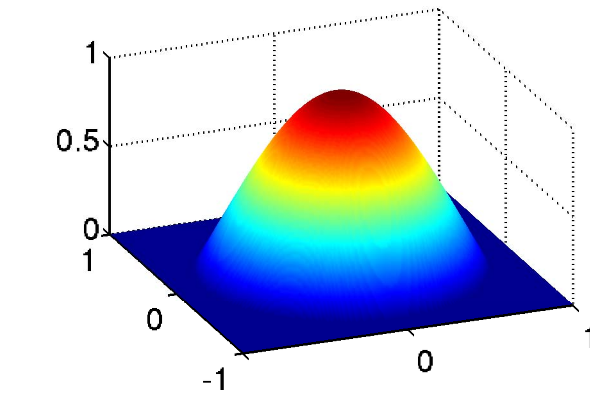


Adaptivity tree

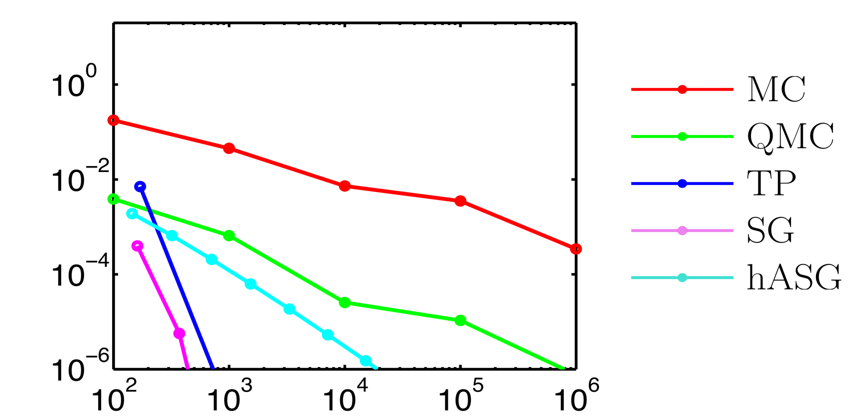
Dog Pics

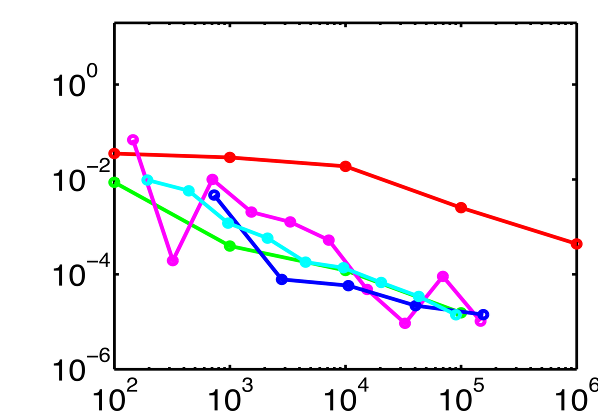


$$f_1(\xi) = \exp\left(-\sum_{i=1}^{d}\xi_i^2\right)$$

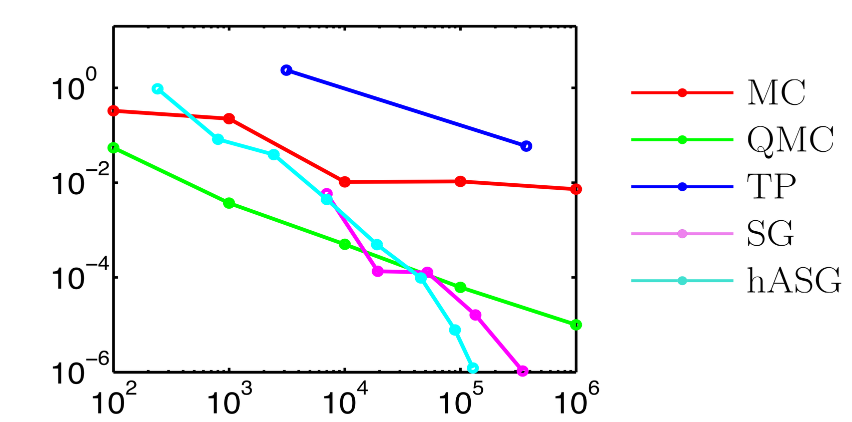


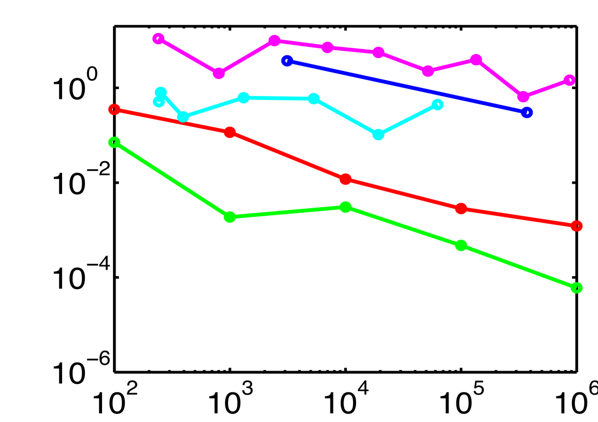
$$f_2(\xi) = 2\max\left\{f_1(\xi) - \tfrac{1}{2}, 0\right\}$$





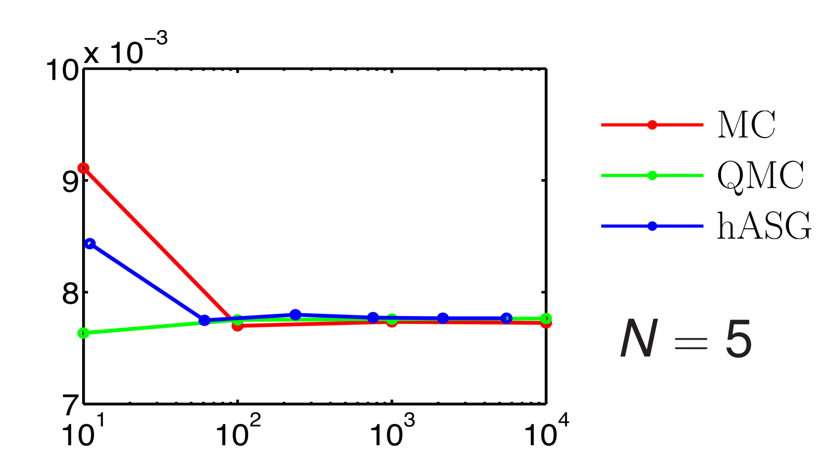
Absolute errors in integral vs the number of samples for $d = 2$

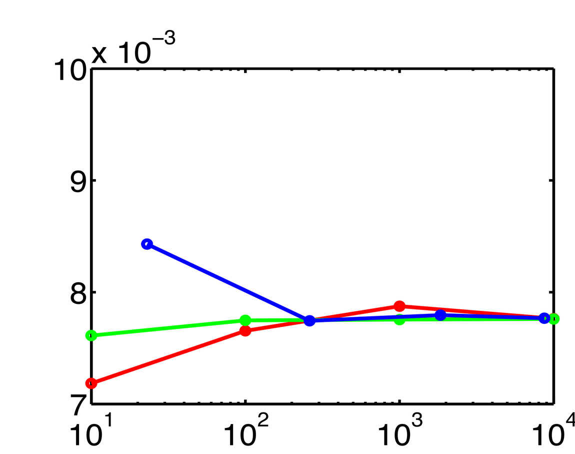




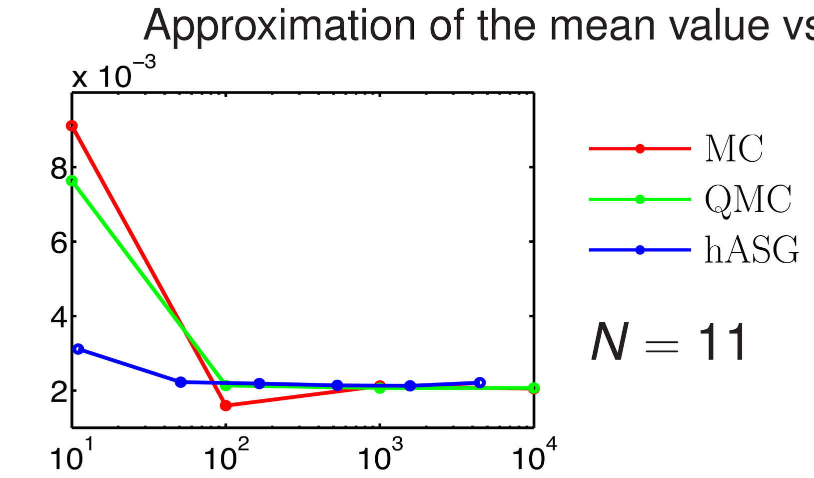
Absolute errors in integral vs the number of samples for $d = 5$

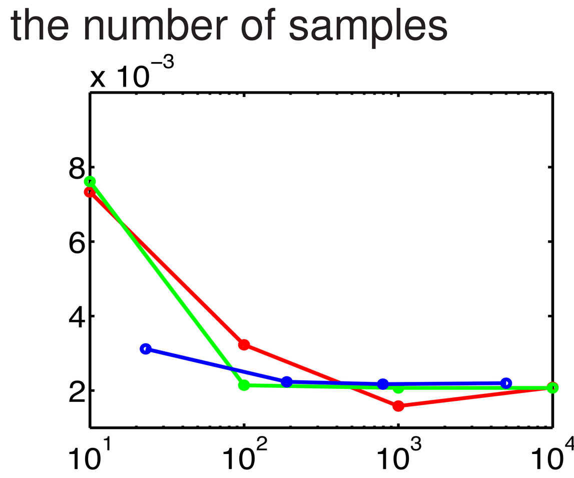
Results





Approximation of the mean value vs the number of samples





Approximation of the semi-deviation vs the number of samples

Future work

- Finding the reason for occasional failure of optimizing with MATLAB's fmincon
- Investigating why the minimum time path is asymmetric from belt to goal and vice versa

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