MATH 3150 Assignment 1 Spring 2025

Instructions: Typeset your solutions using LaTeX and upload the final pdf file to CANVAS. The submission system only accepts pdfs, and will close after the due date. You are encouraged to work with others on homework, but writing should be separate and reflect individual understanding.

1. Use mathematical induction to prove the following statements.
   (a) (10 pts) Consider the sequence of real numbers
   $$x_1 = 1 \quad \text{and} \quad x_{n+1} = \sqrt{1 + 2x_n}\ , \quad \text{for } n \in \mathbb{N}.$$
   Then, $x_n < 4$ for all $n \in \mathbb{N}$.

   (b) (10 pts) For all positive integers n, the number $4^n - 1$ is divisible by 3.

2. (20 pts) Use the Rational Zeros Theorem to find all rational solutions, if any, to the equation
   $$p(x) = 6x^4 - x^3 + 5x^2 - x - 1 = 0.$$
   Explain your reasoning.

3. Determine whether each of the following numbers is rational. Explain your reasoning.
   (a) (10 pts) $\sqrt{3 - \sqrt{5}}$

   (b) (10 pts) $\log_2(5)$

4. (20 pts) Let r be real number. Show that if $|r - 2| \leq 1$, then $r^2 - 4r + 3 \leq 0$.

5. Let A and B be nonempty subsets of $\mathbb{R}$ so that $a \leq b$ for every $a \in A$ and for every $b \in B$.
   (a) (5 pts) Show that A is bounded from above and that B is bounded from below.

   (b) (10 pts) Show that $\sup(A) \leq \inf(B)$.

   (c) (5 pts) Provide an example of such sets A and B where $A \cap B \neq \emptyset$.