# COS 485 — Homework 7

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## Problem 1

In this problem we categorize the following sorting algorithms into the various genres of algorithms discussed in this course.

| **Algorithm** | **Technique** | **Justification** |
|---|---|---|
| Insertion sort | Iterative Improvement | Go through each element starting at position 0. Shift all of the elements greater than the current one to the left. |
| Selection sort | Greedy | Find the minimum value, swap it with the item at position 0. |
| Bubble sort | Greedy | If the next value is smaller than the current one, swap them. |
| Quicksort | Divide and Conquer | Divide the problem size recursively, sort at the bottom, then sort on your way up. Note that Quicksort *can* be implemented with a randomized partition. |
| Merge sort | Divide and Conquer | Divide the problem size recursively, sort at the bottom, then sort on your way up. |
| Heap sort | Greedy | Larger values go higher (maxheap), or smaller values go higher (minheap) |

## Problem 2

In this problem we are asked to explain why this flow is valid, although not a maximum flow.

This flow is not optimal as we have not saturated any of the forward edges, and there is a backwards edge, namely **A** — **B**, which has been saturated even though it should not have any flow whatsoever.

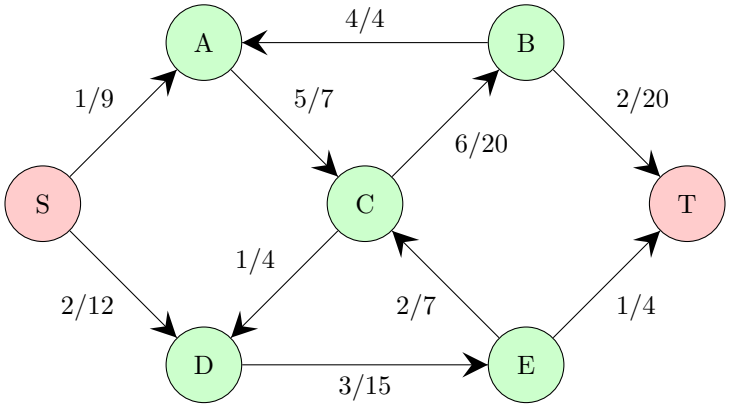



Figure 1: Flow graph with non-maximal flow

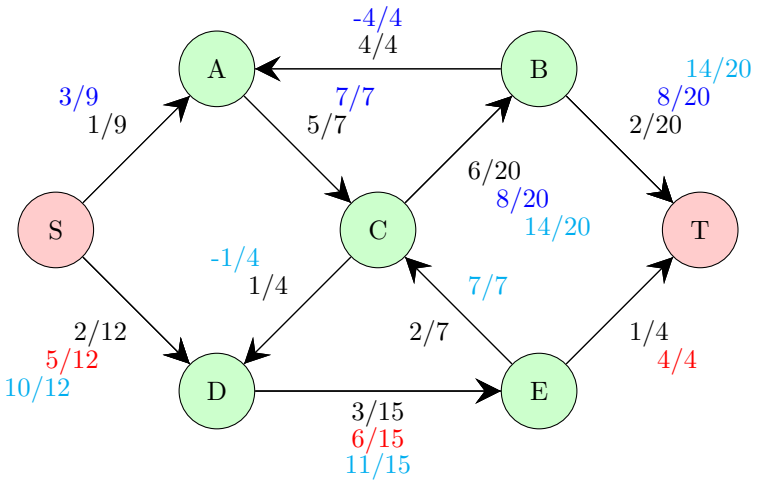
## Problem 3

In this problem we are to find a set of augmenting paths which will make the flow in Figure 1 maximal.

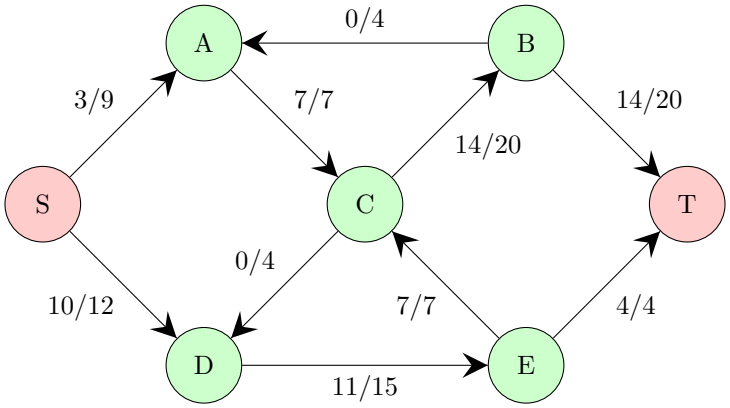
We consider the following set of augmenting paths:

| Path | Flow Added |
|---|---|
| **S — D — E — T** | 3 |
| **S — A — C — B — T** | 6 |
| **S — D — E — C — B — T** | 6 |

These paths correspond to the following changes to the flow of the graph.



The flow of this graph is maximal as domonstrated by the following minimum cut of forward paths. **A** — **C**, **E** — **C**, and **E** — **T** are all saturated forward paths. These paths span the graph and thus form a minimum cut. The sum of the flow of these three paths is 18, and the flow of the graph is now 18 as well. Below is the final flow of the graph.



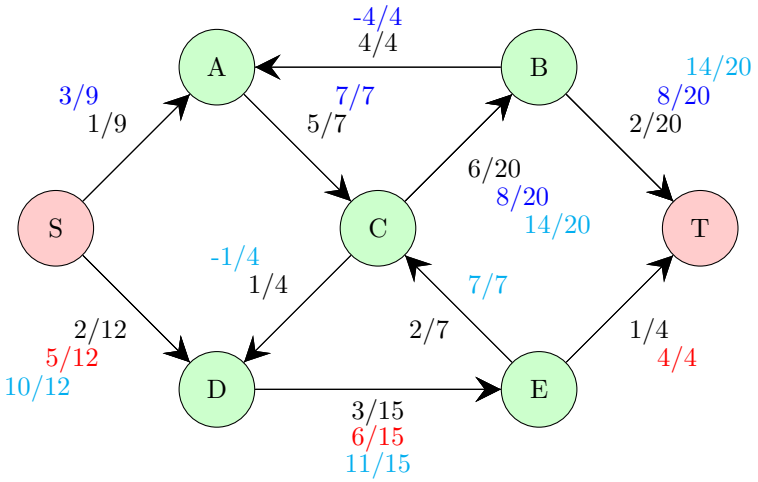

## Problem 3

In this problem we are to find a set of augmenting paths which will make the flow in Figure 1 maximal.

We consider the following set of augmenting paths:

| **Path** | **Flow Added** |
|---|---|
| **S** — **D** — **E** — **T** | 3 |
| **S** — **A** — **C** — **B** — **T** | 6 |
| **S** — **D** — **E** — **C** — **B** — **T** | 6 |

These paths correspond to the following changes to the flow of the graph.

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