COS 485 — Homework 6

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Problem 1

In this problem we are asked to make a decision tree argument about the lower limit on the worst case number of comparisons needed to find a key in a sorted list.

Let us assume our list has N items. We assume that these items are in sorted order. There are N possible solutions, namely keys, from which we must choose one. Thus the height of the decision tree must be large enough to cover the search space of N items. Since we have sorted the items we have a binary decision, either an item is greater than its predecessor or it is less than its predecessor. Thus we have a binary tree where each node can have at most two children, the height of the tree can be calculated by solving the following equation.

$$N \leq 2^h \implies \lg(N) \leq h \implies h \geq \lg(N)$$

Since the height of the decision tree is equal to the number of needed comparisions, we have that there can be a minimum of $\lg N$ comparisions in the worst case to find a particular key in a sorted list of N elements.

Problem 2

In this problem we consider the case of one miserly king who has demanded a weighing of N coins with the understanding that one coin is counterfeit and lighter than the others.

The algorithm to effeciently solve this is as follows:

1. divide coins into three parts A, B, and C. A and B to be divided into a multiple of three, and the rest are left in C
2. weigh two of them
   if $A = B$, then throw out A and B and keep C
   if $A < B$, then throw out B and C and keep A
   if $B < A$, then throw out a and C and keep B
3. repeat

In the end, N wasn't divisible by 3, then we will have either 1 or two extra to deal with. If there is one extra, then we do two comparisons. If there are two extra left then we still do two comparisons.

In the worst case, we do $\lceil \log_3 N \rceil + 1$ comparisons.

ALTERNATE

In this problem, we are given N coins and asked to find a lightweight counterfeit amid them. We are instructed that this can only be done by weighing them. The scale we are given indicates the three following cases:

1. Side A is heaviest.
2. Side B is heaviest.
3. Both sides are of equal weight.

To solve this problem, we divide our coins into problems of size $\left\lceil \frac{n}{3} \right\rceil$, and arbitrarily pick two of the three available piles to weigh. There are two possible results of this weighing:

1. The two piles are equal in size. Throw them both out; none are fakes. Keep the third pile we didn't weigh, as it must contain a fake.
2. The two piles are not equal in size. Keep the lightest one, as it contains the fake, and throw out both the heavy one we weighed and the one we didn't weigh.

Repeat this process of dividing up into thirds, keeping discarding the two heavy piles, until only the counterfeit coin remains.