

COS 485 — Homework 6

Samuel Barton Benjamin Montgomery

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Problem 1

In this problem we are asked to make a decision tree argument about the lower limit on the worst case number of comparisons needed to find a key in a sorted list.

Let us assume our list has N items. We assume that these items are in sorted order. There are N possible solutions, namely keys, from which we must choose one. Thus the height of the decision tree must be large enough to cover the search space of N items. Since we have sorted the items we have a binary decision, either an item is greater than its predecessor or it is less than its predecessor. Thus we have a binary tree where each node can have at most two children, the height of the tree can be calculated by solving the following equation.

$$N \leq 2^h \implies \lg(N) \leq h \implies h \geq \lg(N)$$

Since the height of the decision tree is equal to the number of needed comparisons, we have that there can be a minimum of $\lg N$ comparisons in the worst case to find a particular key in a sorted list of N elements.

Problem 2

In this problem we consider the case of one miserly king who has demanded a weighing of N coins with the understanding that one coin is counterfeit and lighter than the others.

The algorithm to efficiently solve this is as follows:

1. divide coins into three parts A , B , and C . A and B to be divided into a multiple of three, and the rest are left in C
2. weigh two of them
 - if $A = B$, then throw out A and B and keep C
 - if $A < B$, then throw out B and C and keep A
 - if $B < A$, then throw out A and C and keep B
3. repeat

In the end, N wasn't divisible by 3, then we will have either 1 or two extra to deal with. If there is one extra, then we do two comparisons. If there are two extra left then we still do two comparisons.

In the worst case, we do $\lceil \log_3 N \rceil + 1$ comparisons.