COS 485 — Homework 6

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Problem 1

In this problem we are asked to make a decision tree argument about the lower limit on the worst case number of comparisons needed to find a key in a sorted list.

Let us assume our list has N items. We assume that these items are in sorted order. There are N possible solutions, namely keys, from which we must choose one. Thus the height of the decision tree must be large enough to cover the search space of N items. Since we have sorted the items we have a binary decision, either an item is greater than its predecessor or it is less than its predecessor. Thus we have a binary tree where each node can have at most two children, the height of the tree can be calculated by solving the following equation.

$$N \leq 2^h \implies \lg(N) \leq h \implies h \geq \lg(N)$$

Since the height of the decision tree is equal to the number of needed comparisons, we have that there can be a minimum of $\lg N$ comparisons in the worst case to find a particular key in a sorted list of N elements.

Problem 2

In this problem we consider the case of one miserly king who has demanded a weighing of N coins with the understanding that one coin is counterfeit and lighter than the others.

The algorithm to efficiently solve this is as follows:

1. Divide coins into three parts A, B, and C. A and B will be divided into problems of size $\left\lfloor \frac{n}{3} \right\rfloor$, and the remaining problem of size $\left\lfloor \frac{n}{3} \right\rfloor \leq S \leq \left\lceil \frac{n}{3} \right\rceil + 1$ are placed in C.
2. Weigh two of the piles.
   If $A = B$, then throw out A and B and keep C

If $A < B$, then throw out B and C and keep A
If $B < A$, then throw out A and C and keep B

3. Repeat

In the end, if N wasn't divisible by 3, then we will have either 1 or two extra coins to deal with. In either case, however, continue our process of comparisons until there is nothing left. If the amount is not evenly divisible by 3, we may do 2 more comparisons than normal.

In the worst case, we do $\lceil \log_3(n) \rceil + 1$ comparisons.