COS 485 — Homework 5

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Problem 1

In this problem, we are asked to do out the N-queens problem by hand with N=9. Below is a table containing our solution. For each queen we have assigned a color as follows:

queen 1 — red
queen 2 — blue
queen 3 — green
queen 4 — yellow
queen 5 — cyan
queen 6 — teal
queen 7 — magenta
queen 8 — olive

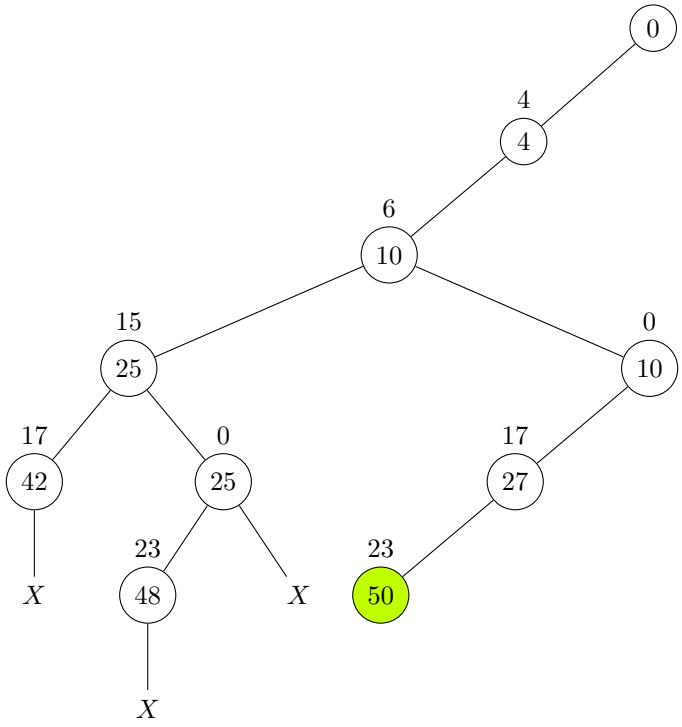
·	·	Q_1	·	·	·	·	·	·
·	x_1	x_1	x_1	·	·	·	·	Q_2
x_1	Q_3	x_1	·	x_1	·	·	x_2	x_2
x_3	x_3	x_1	·	·	x_1	x_2	Q_4	x_2
Q_5	x_3	x_1	x_3	·	x_2	x_1	x_4	x_2
x_5	x_3	x_1	Q_6	x_2	x_4	·	x_1	x_2
x_5	x_3	x_1	x_2	x_4	x_3	Q_7	x_4	x_1
x_5	x_3	x_1	x_4	Q_8	x_6	x_3	x_4	x_2
x_5	x_2	x_1	x_6	x_5	x_8	x_6	x_3	x_2

In the above table we demonstrate the use of the Monte Carlo approach to generate an estimate of the total cost to solve the N-queens problem with N=9. In the below table we give the row sums for all of the options, and then the total sum which represents the overall cost to solve the problem:

Row #	**Choices**	**Cost**
1	9	9
2	6	$9 \cdot 6$
3	4	$9 \cdot 6 \cdot 4$
4	3	$9 \cdot 6 \cdot 4 \cdot 3$
5	2	$9 \cdot 6 \cdot 4 \cdot 3 \cdot 2$
6	2	$9 \cdot 6 \cdot 4 \cdot 3 \cdot 2 \cdot 2$
7	1	$9 \cdot 6 \cdot 4 \cdot 3 \cdot 2 \cdot 2 \cdot 1$
8	1	$9 \cdot 6 \cdot 4 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1$
9	0	0
Total		**9999**

Problem 2

In this problem we demonstrate the use of "The Backtracking Algorithm for the Sum of Subsets problem"from Section 5.4 in the text. We utilized the algorithm to find the combination of the elements in the set {4,6,15,17,23,26,31} that sum to 50.



As requested, we are only showing the path to the first combination that sums to 50.

Problem 3

In this problem we explain a way the algorithm in Exercise 5.19 could be implemented.

```
Let there be an array where we will store the colors we create

Let there be an adjacency list for this graph

Let there be a count of uncolored nodes

for each node in the adjacency list
    if the count of uncolored nodes is zero
        stop
    if the node has not been colored
        pick a new color
        add that color to the array of possible colors
        color that index with the new color
        decrement the count of uncolored nodes
        for each node in the adjacency list
            if this node is not adjacent to any nodes with the current color
                color this node
                decrement the count of uncolored nodes
```

Using the above implementation, the algorithm would take the following amount of time for an arbitrary graph with N nodes.

The outermost loop will iterate N times in the worst case, thus in the worst case we will have N colors. This would happen if we were handed a completely connected graph. Our algorithm visits each node in the graph, and at each node, it then visits every other node in the graph. Thus, our traversal requires $\Theta(N^2)$ operations; however, at each of these visits, we must perform an $O(N)$ adjacency check in order to determine if we should color that node. This means we have, based on experience, an $O(N^3)$ algorithm in the worst case.

Since we wish for a more instructive analysis of this algorithm, we are going to analyze at a higher level of granularity. Thus, we may note that the number of times we go through the outer loop is dependent on the number of colors needed to completely color the graph. The number of colors needed is dependent on the level of connectedness in the graph. Thus, if it takes K colors to completely color the graph, then our algorithm is $\Theta(K \cdot N^2)$.

Problem 4