

COS 485 — Homework 6

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April 18th, 2017

Problem 1

In this problem we are asked to make a decision tree argument about the lower limit on the worst case number of comparisons needed to find a key in a sorted list.

Let us assume our list has N items. We assume that these items are in sorted order. There are N possible solutions, namely keys, from which we must choose one. Thus the height of the decision tree must be large enough to cover the search space of N items. Since we have sorted the items we have a binary decision, either an item is greater than its predecessor or it is less than its predecessor. Thus we have a binary tree where each node can have at most two children, the height of the tree can be calculated by solving the following equation.

$$N \leq 2^h \implies \lg(N) \leq h \implies h \geq \lg(N)$$

Since the height of the decision tree is equal to the number of needed comparisons, we have that there can be a minimum of $\lg N$ comparisons in the worst case to find a particular key in a sorted list of N elements.

Problem 2

In this problem we consider the case of one miserly king who has demanded a weighing of N coins with the understanding that one coin is counterfeit and lighter than the others.

The algorithm to efficiently solve this is as follows:

1. Divide coins into three parts A , B , and C where the size each is $\lfloor \frac{n}{3} \rfloor$. If N is not divisible by 3, then we will put the 1 or two additional coins in D and set them aside.
2. Weigh two of A , B , or C .
 - If $A = B$, then throw out A and B and keep C
 - If $A < B$, then throw out B and C and keep A
 - If $B < A$, then throw out A and C and keep B
3. Repeat until only one of A , B , and C remain
4. If N is divisible by 3, then we are done. Otherwise, we have either 1 or 2 elements in D to consider. Compare whichever of A , B , or C remains with 1 of the elements in D using the above logic.

In the worst case, N is not divisible by 3 and has a remainder of 2. Thus there are 2 elements in D and we do $\lceil \log_3(n) \rceil$ comparisons to end up with 1 element from A , B , or C and two elements from D to consider. We make one additional comparison with 1 element from D and the remaining element from the prior comparisons, and can then determine which coin is the counterfeit. Thus in the worst case our algorithm will have a time complexity equal to the number of comparisons which is

$$T_n = \lfloor \log_3(n) \rfloor + 1 = \lceil \log_3(n) \rceil$$