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MAE 162A Final Project: Horizontal Platform Mechanism

(1) Mechanism Dimensions

The Horizontal Platform Mechanism is composed of two main vector loops: a four-bar linkage on the left, and another loop to describe the rest of the mechanism's linkages. The first step in determining the dimensions was to look at the four-bar linkage. Taken alone, this kinematic chain resembles a Chebyshev linkage, of which the ratios of all links are proportionally related. The purpose and advantage of a Chebyshev linkage is that it will keep the midpoint of the coupler (link 3) vertically stable for a range of input angles. Therefore, if this linkage is used, the left leg of the platform will be satisfied. The appropriate ratios for a Chebyshev linkage are:

$$r_1 : r_2 : r_3 = 4 : 5 : 2$$

and $r_2 = r_4$. Specifying any of these lengths will define the others. I chose to specify $r_1 = 810\text{mm}$ to fit within the 850mm horizontal limit at the bottom. The four bar Chebyshev linkage is then:

$r_1 = 810\text{mm}, r_2 = r_4 = 1012.5\text{mm}, r_3 = 405\text{mm}$

The Chebyshev linkage was constructed as a sketch in SolidWorks. The rest of the mechanism was added to the sketch and experimented with until the platform appeared to stay flat over a range of input angles. See the sketch and assembly construction of the mechanism below:

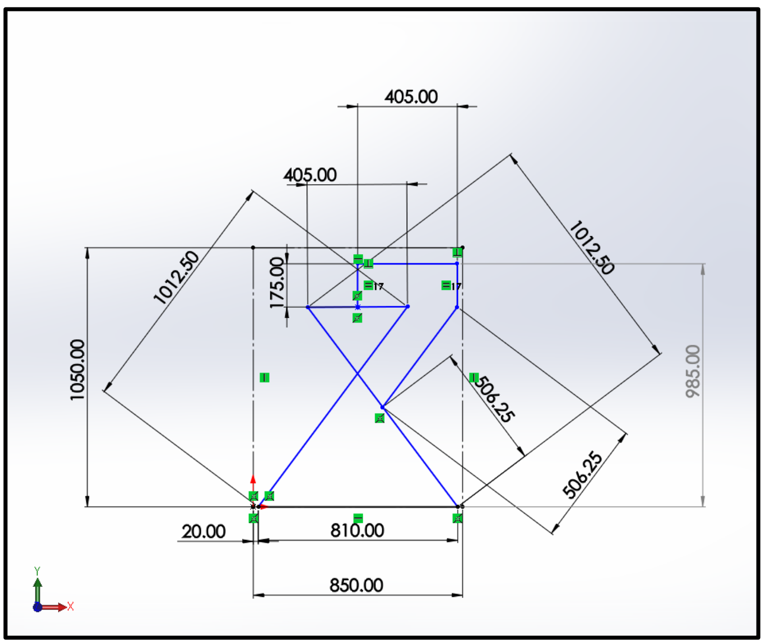


Figure 1: *One Dimensional Sketch of Mechanism. All dimensions are in millimeters and were made into 3D linkages to the right. Note that link 5 bisects link 4 and is half its length, and that the length of the platform is the same as link 3.*

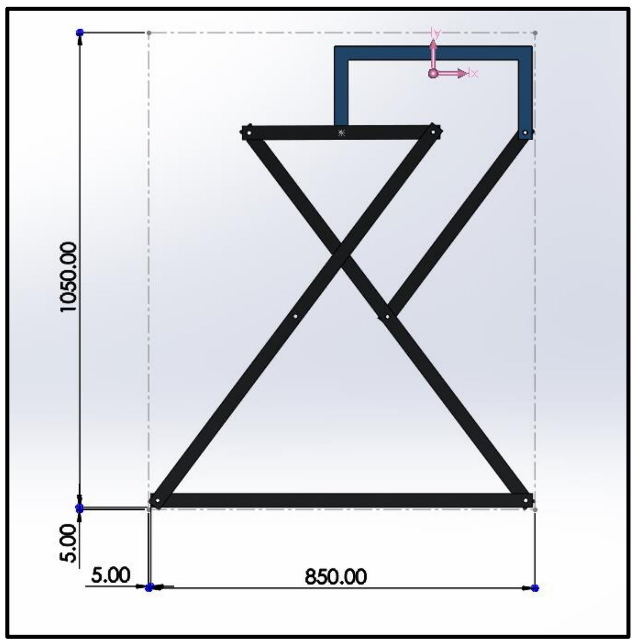


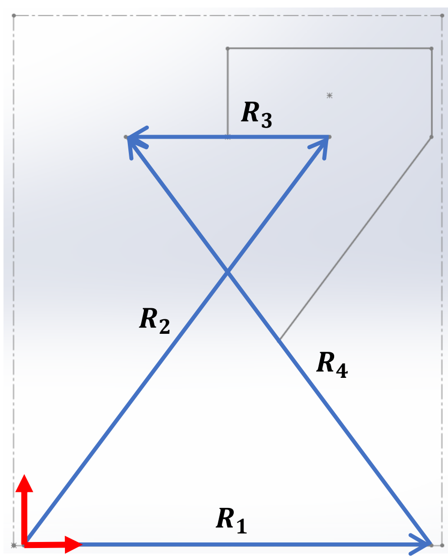
Figure 2: *Planar View of 3D model of Mechanism. The Center of Mass of the Tabletop is shown as the pink origin. The rectangular outline encloses the entire mechanism for this position. Each link has a thickness and width of 30mm.*

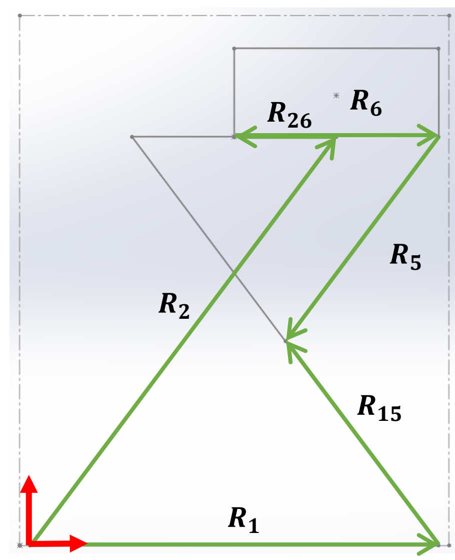
(2) The range of the input angle $[\theta_{2,min}, \theta_{2,max}]$ was determined through trial and error. The angle was varied within the sketch (Figure 1) until the tabletop began to displace vertically/angularly. The range was entered into MATLAB to check the displacements to see if requirements were met (i.e. vertical displacement < 4mm, horizontal displacement > 850 mm). An ideal (but not unique) range was found to be:

$$\theta_2 = [36.87°, 96°]$$

$\theta_{2,min}$ corresponds to the position at which the Chebyshev coupler and opposite leg are both oriented vertically, i.e. $\theta_{2,min} = \cos^{-1}\left(\frac{r_1}{r_2}\right) = \cos^{-1}\left(\frac{4}{5}\right)$

The entire Mechanism can be described with these two vector loops shown below:





$$\boldsymbol{R_2 + R_3 - R_4 - R_1 = 0} \qquad \boldsymbol{R_2 + R_{26} + R_6 + R_5 - R_{15} - R_1 = 0}$$

$\boldsymbol{r_1}$	$\boldsymbol{r_2}$	$\boldsymbol{r_3}$	$\boldsymbol{r_4}$	$\boldsymbol{r_5}$	$\boldsymbol{r_6}$	$\boldsymbol{r_{26}}$	$\boldsymbol{r_{15}}$
810mm	1012.5mm	405mm	1012.5mm	506.25mm	405mm	C_1	C_2
$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	$\boldsymbol{\theta_4}$	$\boldsymbol{\theta_5}$	$\boldsymbol{\theta_6}$	$\boldsymbol{\theta_{26}}$	$\boldsymbol{\theta_{15}}$
0°	Input	Unknown	Unknown	Unknown	Unknown	C_3	C_4

Where the Constraints are:

C_1: $r_{26} = 0.5r_3$ C_2: $r_{15} = 0.5r_{15}$ C_3: $\theta_{26} = \theta_3$ C_4: $\theta_{15} = \theta_4$

Thus, a system of four equations and four unknowns was constructed and solved using MATLAB for every position of the input angle over its interval. Two separate Newton's method iterations are used; the first uses the first vector loop to determine θ_3 and θ_4, and the second uses these values and determines θ_5 and θ_6. However, this is not sufficient to determine the displacement of the platform's center of mass.

Once the position of the platform's center of mass was determined, a new vector with constant properties was used to find its location from the endpoint of R26. See the appendix for a more detailed computation of the center of mass. The plots below illustrate the platform's movements.

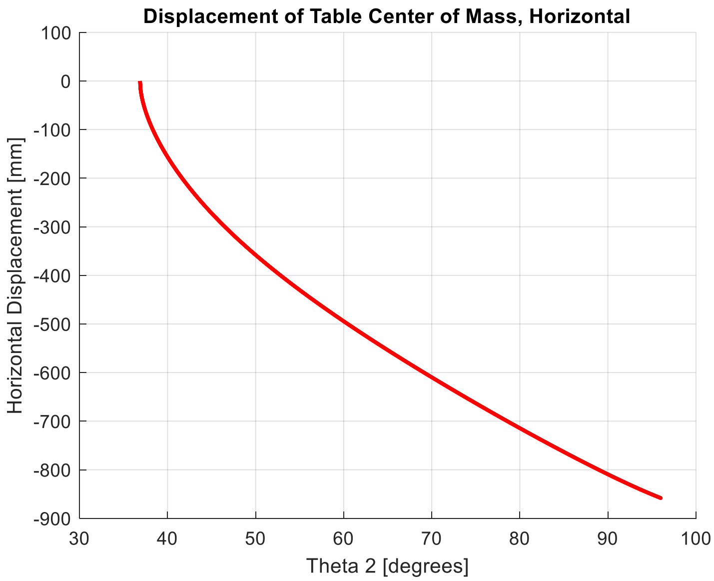


Figure 3: *Horizontal Displacement of the Platform's Center of Mass over one cycle of the input angle. The total displacement condition is met:*

$$858.089mm > 850mm$$

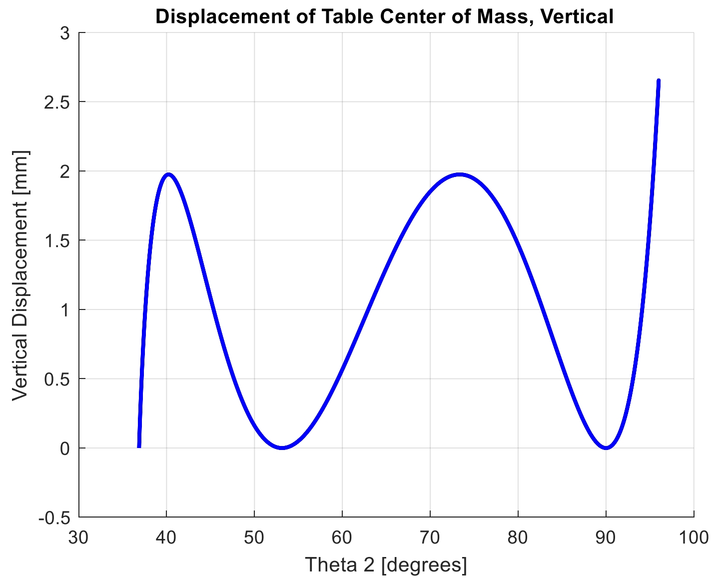


Figure 4: *Vertical Displacement of the Platform's Center of Mass over one cycle of the input angle. The total displacement condition is met:*

$$2.655mm < 4mm$$

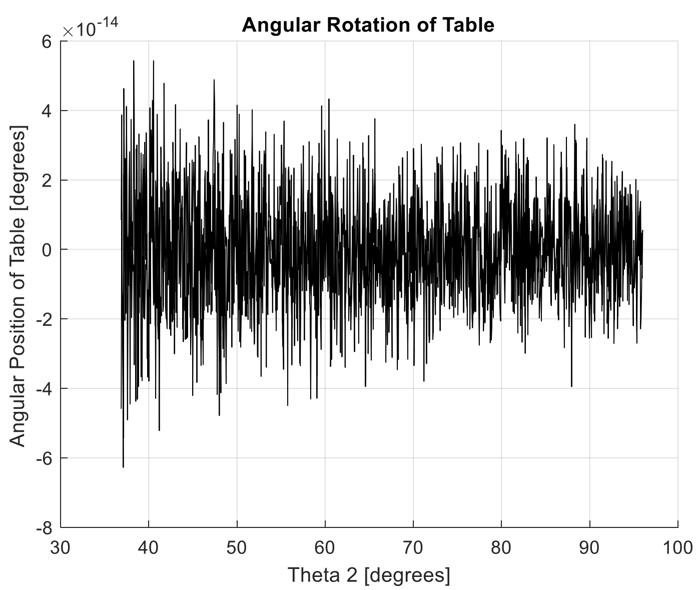


Figure 5: *Angular Displacement of the Platform's Center of Mass over one cycle of the input angle. The total displacement condition is met:*

$$1.171e\text{-}13 < 1°$$

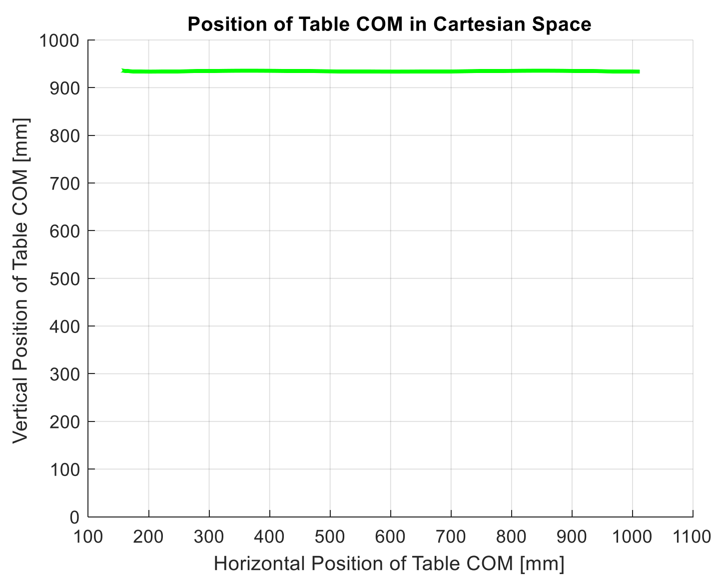


Figure 6: *Visualization of the overall position of the platform COM in space. The axes are scaled to emphasize that the vertical displacement is seemingly negligible.*

The motor drives the angular position, velocity, and acceleration of link 2 over time according to:

$$\theta_2(t) = \frac{\theta_{2,max} - \theta_{2,min}}{2}\sin(wt) + \frac{\theta_{2,max} + \theta_{2,min}}{2}, t \in \left[0, \frac{6\pi}{w}\right] sec$$

$$\frac{d\theta_2}{dt} = \omega_2(t) = w\frac{\theta_{2,max} - \theta_{2,min}}{2}\cos(wt)$$

$$\frac{d^2\theta_2}{dt^2} = \alpha_2(t) = -w^2\frac{\theta_{2,max} - \theta_{2,min}}{2}\sin(wt)$$

Where $w = 1°/\mathrm{s} = \pi/180$ rad/s. The interval is 3 full cycles, or $t \in [0,1080]$ sec. To determine the Torque applied to link 2 as a function of time, the Power Equation was used:

$$P(t) = \tau(t)\omega_2(t) = \frac{dT}{dt} + \frac{dU}{dt}$$

$$\frac{dT}{dt} = \sum_{j=2}^{6} A_j(t)\omega_2(t)\alpha_2(t) + \sum_{j=2}^{6} B_j(t)\omega_2(t)^3$$

$$A_j(t) = m_j\left(x_{Gj}'^2 + y_{Gj}'^2\right) + I_{Gj}\theta_j'^2$$

$$B_j(t) = m_j\left(x_{Gj}'x_{Gj}'' + y_{Gj}'y_{Gj}''\right) + I_{Gj}\theta_j'\theta_j''$$

$$\frac{dU}{dt} = \sum_{j=2}^{6} m_j g y_{Gj}'(t)\omega_2(t)$$

$$m_j = \rho_A V_j$$

$$I_{Gj} = \frac{m_j}{12}(W^2 + r_j^2)$$

The four unknown output angles from above were recalculated using Newton-Raphson for the time dependent input angle. All kinematic coefficients (linear and angular) were evaluated again using Newton-Raphson on the first and second derivatives of the vector loops. These coefficients are all time dependent according to the input angle. The density of the link material (Aluminum) is $\rho_A =$ 2698.9 kg/m³, and the width of each link is $W = 30$mm.

Putting everything together:

$$\tau(t) = \sum_{j=2}^{6} A_j(t)\alpha_2(t) + \sum_{j=2}^{6} B_j(t)\omega_2(t)^2 + \sum_{j=2}^{6} m_j g y_{Gj}'(t)$$

The MATLAB program generated the following plot for input Torque of the motor as a function of time over 3 full cycles of the input angle.

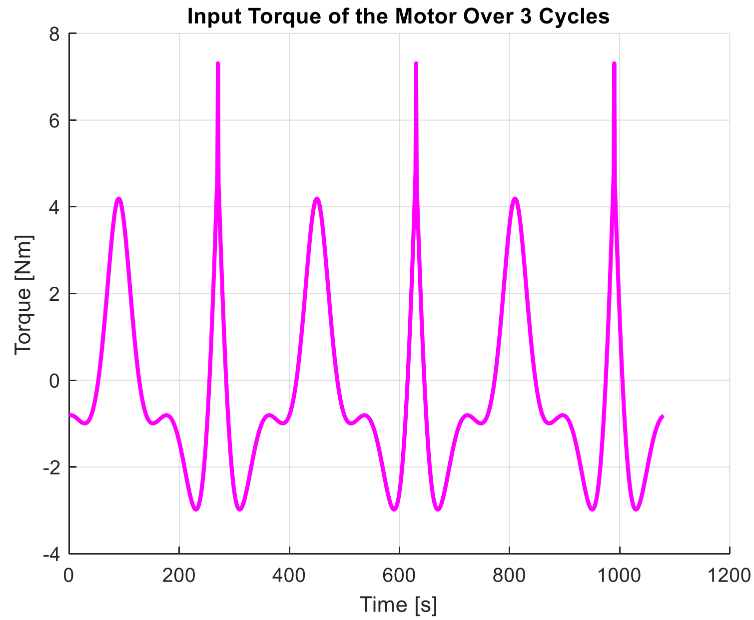


Figure 7: *Input Torque produced by Motor as a function of time. The torque appears to oscillate periodically with many local maxima and minima. The global maximum is 7.3097 Nm and global minimum is -2.9836 Nm. Therefore, the input Torque must change direction over time. The sharp peaks at the global maximum occur regularly over the three cycles, which corresponds to the time at which the mechanism changes its movement direction. The time between these peaks is equal to one full cycle.*

The table below summarizes the results of the simulation:

	Maximum Horizontal Displacement	**Maximum Vertical Displacement**	**Maximum Angular Displacement**
Value	858.089mm	2.655mm	1.171E-13°
Requirement	$x > 850$mm	$y < 4$mm	$\theta < 1°$
Requirement Met?	Yes	Yes	Yes

Table 1: Design Requirements and the simulation results. All conditions specified have been met.

Appendix 1: Center of Mass Calculation

The platform takes the following two-dimensional shape:

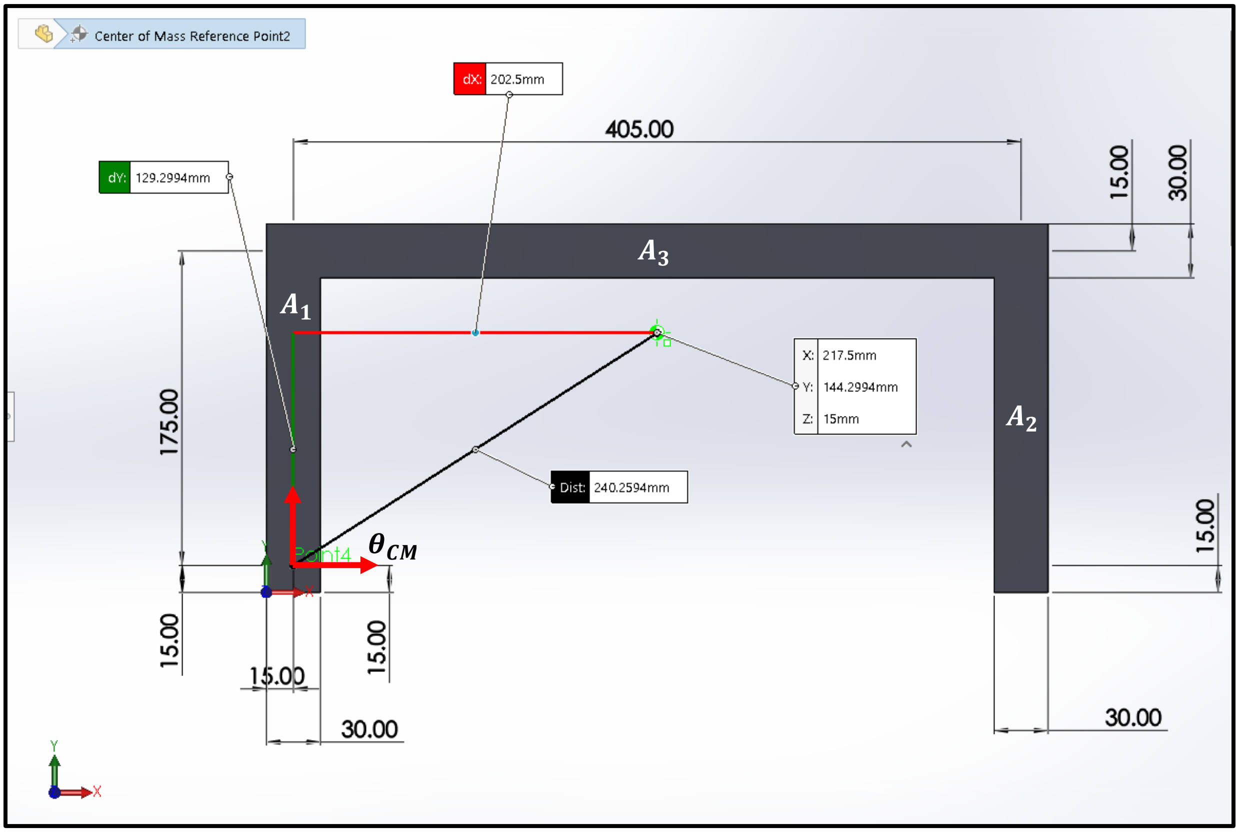


Figure 8: Platform Dimensions for Center of Mass Calculation. Note that the shape has been broken down into three separate rectangles, and the Origin is placed at the joint connecting the platform to the coupler. The COM calculation below is consistent with SolidWorks calculation. The joint holes are omitted because the MATLAB simulation is 1 Dimensional and therefore has no holes at joints. In 3D, these holes would have an effect on COM.

To calculate the Center of Mass, the platform was decomposed into three rectangles, each with their own centers. The following relation describes the positions of the COM (relative to the origin) of the entire platform with respect to the parameters of the rectangles:

$$x_G = \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3}{A_1 + A_2 + A_3} = \frac{(6150)(0) + (6150)(405) + (11250)(202.5)}{6150 + 6150 + 11250} = 202.5mm$$

$$y_G = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3}{A_1 + A_2 + A_3} = \frac{(6150)(87.5) + (6150)(87.5) + (11250)(175)}{6150 + 6150 + 11250} = 129.3mm$$

To determine the positions of the center of mass over time with respect to the actual origin (placed at the lower joint of link 2), we need a vector ***D*** to describe its location since the platform is changing angles. The overall center of mass position is therefore:

$$x_{G6}(t) = r_2\cos(\theta_2(t)) + r_{26}\cos(\theta_3(t)) + D\cos\left(\theta_6(t) + \theta_{CM}\right)$$

$$y_{G6}(t) = r_2\sin(\theta_2(t)) + r_{26}\sin(\theta_3(t)) + D\sin\left(\theta_6(t) + \theta_{CM}\right)$$

Where $D = \sqrt{x_G^2 + y_G^2} = 240.26mm$ and $\theta_{CM} = \tan^{-1}\frac{y_G}{x_G} = 32.56°$