EXPERIMENT 3: CONSERVATION OF MECHANICAL ENERGY

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LAB SECTION: WEDNESDAY 2PM

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WORKSHEET

2 DISCUSSION

The glider was pulled in the positive x-direction to the right, released, and accelerated left through the photogate in the negative x-direction. The photogate was placed in a position in which the sensor was in between the 30th and 31st tooth *from the right* when facing the wall. In order to calculate potential and kinetic energy in the same positions, we determine potential and kinetic energy equations as functions of the same variable, $\bar{x}(i)$, the average position between two measured positions, x_i and x_{i+1}. Average position is defined in Equation 3.4[1]:

$$\bar{x}(i) \equiv \frac{1}{2}(x_{i+1} + x_i).$$

Since the equation for kinetic energy has a velocity term instead of a position term, we need to substitute for this velocity variable, shown in equation 3.3[1]:

$$v(\bar{x}(i)) = \frac{x_{i+1} - x_i}{t_{i+1} - t_i}.$$

Now we can substitute in for potential and kinetic energies using equations 3.1[1] and 3.2[1], and determine values for each energy at the same positions:

$$K(\bar{x}(i)) = \frac{1}{2}M\left(\frac{x_{i+1} - x_i}{t_{i+1} - t_i}\right)^2 \text{ and } U(\bar{x}(i)) = \frac{1}{2}k(\bar{x}(i))^2$$

The Kinetic Energy is stored within the movement in the glider while the Potential Energy is stored within the spring's compressions and extensions.

3 PLOTS AND TABLES

Mass of glider with photogate comb attached: $M = 224.30 \pm 0.05\text{g}$

Hanging Mass, *m* (kg)	Applied Force, *F* (N)	Displacement, *x* (m)
0.00260	0.0255	0.0056
0.01940	0.1901	0.0318
0.03440	0.3371	0.0556
0.10260	1.0055	0.1678
0.11940	1.1701	0.1943
0.13440	1.3171	0.2193

Table 1: Data used in Determining the spring constant *k*. The uncertainties for the Hanging mass, Applied Force, and Displacement are $\delta m = 0.00005$ kg, $\delta F = 0.0005$, and $\delta x = 0.0005$ m, respectively. Note that all masses were measured in grams and converted to kilograms, and all displacements measured in centimeters converted to meters. Applied Force is simply mg, hanging mass times acceleration due to gravity. See figure 1 below for a calculation of the spring constant k.

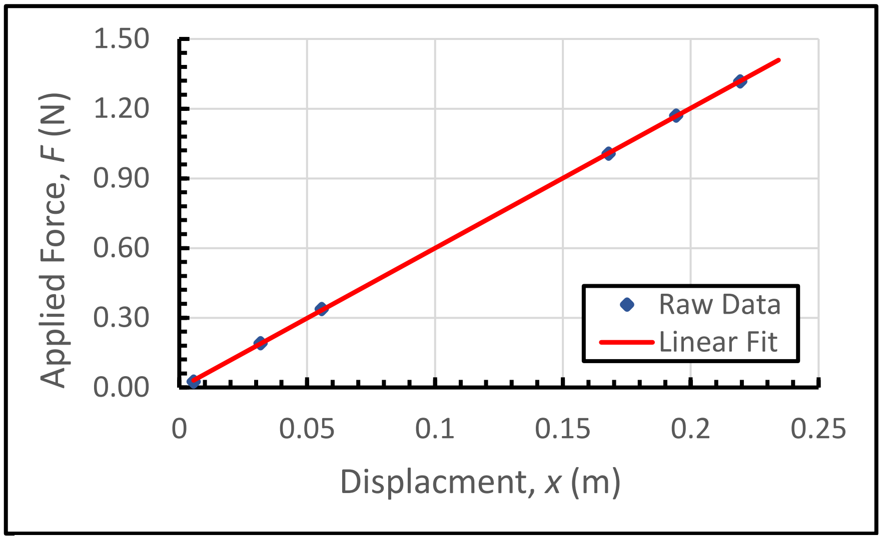

Figure 1: Determining the Spring Constant k from Hooke's Law. The blue points are the raw data and represent the force applied on the spring F by the hanging mass which displace the spring a certain distance x. The red line is the linear fit to the data and has the equation $F = ax + b$ where $a = (6.03 \pm 0.02)$ N/m and $b = (-0.003 \pm 0.003)$ m. Hooke's Law states that a spring force is proportional to its displacement: $F = kx$[2] where F is the spring force, k is the spring constant, and x is the spring's displacement. Therefore, the slope of the red linear fit $a = k$. So, the spring constant $k = (6.03 \pm 0.02)$ N/m.

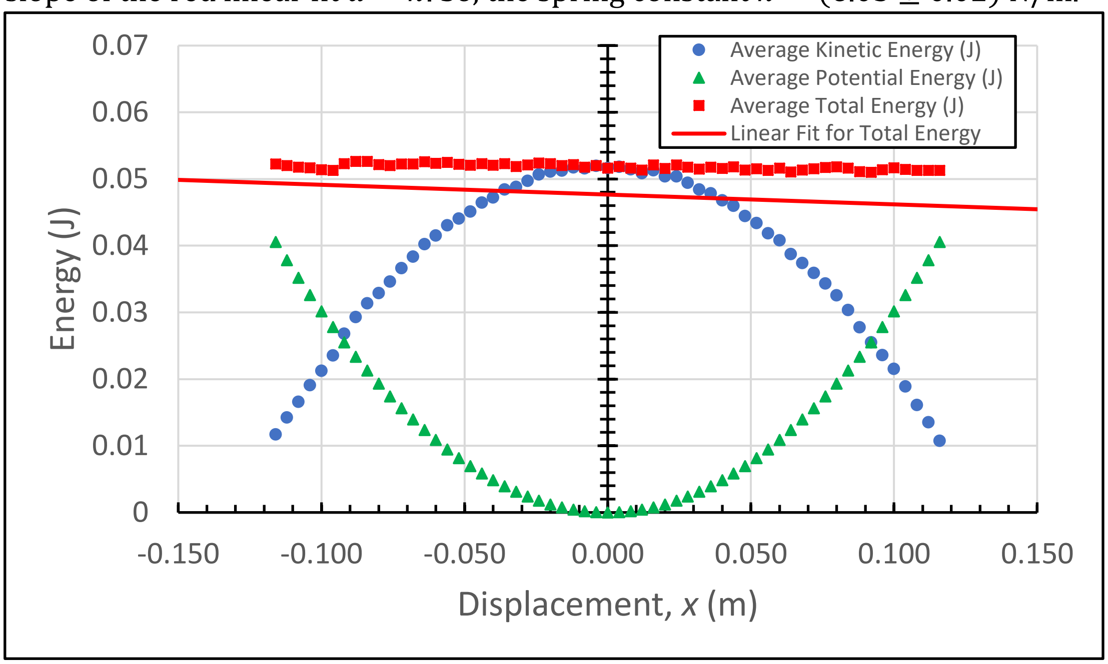

Figure 2: Conservation of Mechanical Energy. The blue circular points are the Average Kinetic Energies (K) at various glider displacements. The green triangular points are the Average Potential Energies (U) at these same glider displacements. The red square points are the Average Total Energies (E) and represent the sum of the Kinetic and Potential Energies at each glider displacement: $E = U + K$. The red line is the downward sloping linear fit to the Mechanical Energy and has the equation $E = ax + b$ where the slope $a = (-0.0041 \pm 0.0006)$ N and the intercept $b = (0.05181 \pm 0.00004)$ J. The slope of the line is negative, indicative of a dissipation in Total Energy due to a frictional force between the glider and the track. The magnitude of this force is representative of the slope a, both having units in Newtons.

The coefficient of friction μ can be determined using the equation $\boldsymbol{F} = \mu\boldsymbol{N}$,[2] where $\boldsymbol{F}$ is the frictional force and $\boldsymbol{N}$ is the Normal Force acting on the object undergoing friction (glider). Using Newton's Second Law, $\boldsymbol{N} = Mg$ where M is the mass of the glider and comb. The frictional force $\boldsymbol{F}$ comes from the magnitude of the slope a of the equation for the linear fit in Figure 2. Solving for μ gives $\mu = \frac{|a|}{Mg}$. Substituting values and using equation ii.23[1] to calculate the uncertainty, $\mu = (0.00173 \pm 0.00003)$. Note that μ has no units.

4 EXTRA CREDIT

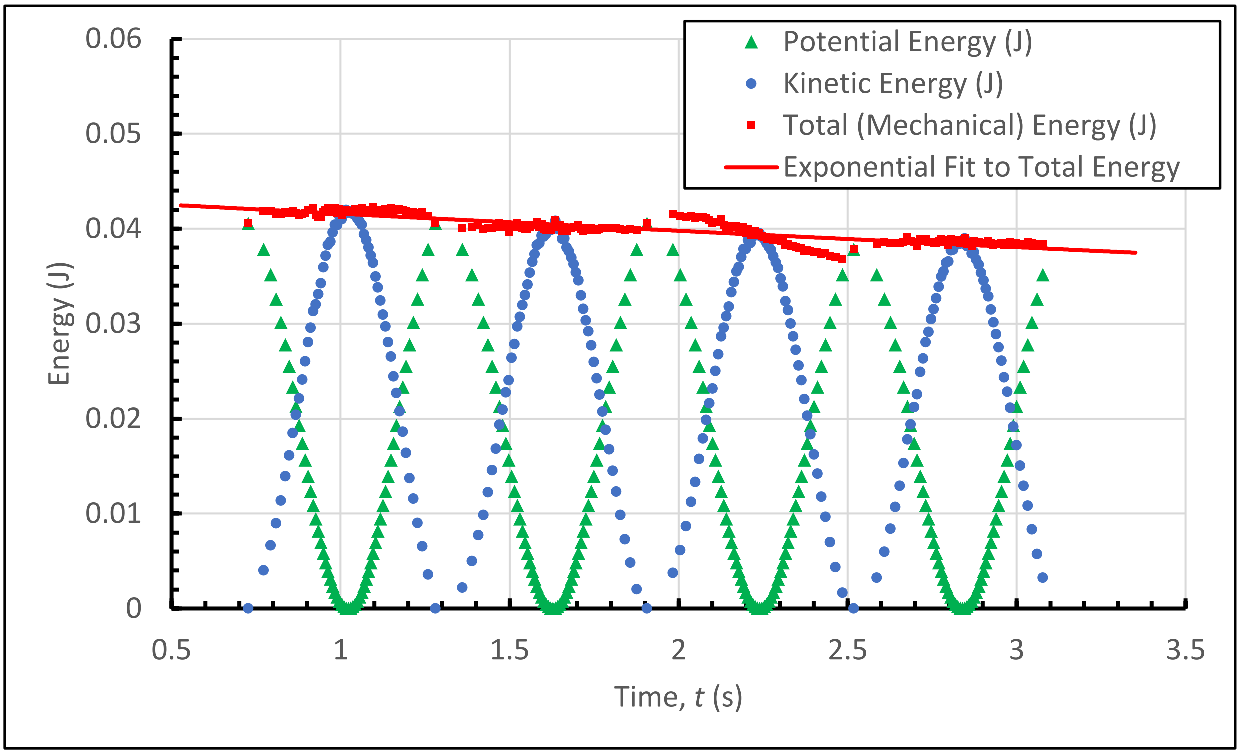

Figure 3: Conservation of Mechanical Energy Over Two Full Periods of Oscillation by the Glider Spring System. All data resemble the same variables as in Figure 2, except over two full oscillations of the glider-spring system. All Energies are plotted with respect to time t instead of displacement. The red curve is an exponential fit to the Total Energy with equation $E = \alpha e^{\beta t}$ where $\alpha = (0.044 \pm 0.003)$ J and $\beta = (-0.044 \pm 0.002)$ s^{-1}. An exponential regression was performed to determine uncertainties in α and β.

The oscillation amplitude A is related to the total energy of a system E by the equation $E = \frac{1}{2}kA^2$ [2] where k is the spring constant. If we want A to decrease by a factor of e, then we need E to decrease by a factor of e^2. So, $E_2 = \frac{E_1}{e^2}$. If we take the ratio of E_1 to E_2 using the exponential fit in Figure 3 and solve for $t_2 - t_1$, the time needed to cause such a decrease is $t = -\frac{2}{\beta}$ or $t = 45$ seconds. This value represents an estimation in the time it takes the glider's oscillation amplitude to decrease by a factor of *e*.

Analysis of a Glider-Spring System to Verify Conservation of Mechanical Energy

S. L. Ellison[1]

Abstract:

This report confirms the Conservation of Energy by demonstrating how the sum of the Kinetic and Potential Energies of a system remains relatively constant. The system analyzed is composed of a glider-spring mass traveling on a low-friction track. A photogate comb is attached above the glider with two similar springs on each end. The system is displaced from equilibrium, released, and set in motion. The glider's movement stores Kinetic Energy while the spring's extensions/compressions store Potential Energy. To calculate Energies, the spring constant, displacement, and velocity of the system are measured over time. A simple application of Hooke's Law is used to calculate the spring constant: the slope of a displacement versus Force linear fit. The displacement is determined by tracking the attached comb with a photogate sensor, from which velocity is derived. A plot of Kinetic, Potential, and Total Energies with respect to displacement show a mirrored relationship between Kinetic and Potential Energy. Total Energy remains relatively constant but decreases slightly and linearly over time. An analysis of the energy dissipation can reveal the frictional force acting on the glider, from which an accurate value for the coefficient of friction between the track and glider can be calculated.

Word Count: 199

[1] *Department of Mechanical Engineering University of California Los Angeles*

References

[1] Campbell, W.C. *et al*. Physics 4AL: Mechanics Lab Manual (ver. April 3, 2017). (University of California Los Angeles, Los Angeles, California).

[2] Young, H. D. *et al*. University Physics with Modern Physics. 143, 184, 442 (Pearson, 2016).