EXPERIMENT 2: MEASUREMENT OF g

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WORKSHEET

2 DERIVATIONS

Before we derive equation 2.1,[1] let's define the variables not seen in that equation.
v_1 is the velocity of the ball at photogate 1 and v_2 is the velocity of the ball at photogate 2

We will be using a kinematic equation to describe the ball during time interval T_1 and $T_1 + T_2$. Here are the equations we begin with:

For the interval T_1 : $$d = v_1 T_1 + \frac{1}{2} g {T_1}^2 \quad (1)$$

For the interval $T_1 + T_2$: $$d + D = v_1(T_1 + T_2) + \frac{1}{2} g (T_1 + T_2)^2 \quad (2)$$

Solving (1) for v_1 and substituting into (2):

$$v_1 = \frac{d}{T_1} - \frac{gT_1}{2} \longrightarrow d + D = \left(\frac{d}{T_1} - \frac{gT_1}{2}\right)(T_1 + T_2) + \frac{1}{2} g (T_1 + T_2)^2 \quad (3)$$

(3) now can be simplified and solved for g in terms of measured variables:

$$g = \frac{2\left(D - d\left(\frac{T_2}{T_1}\right)\right)}{T_2(T_1+T_2)} \longrightarrow g = \frac{2}{T_1+T_2}\left(\frac{D}{T_2} - \frac{d}{T_1}\right)$$

3 PLOTS

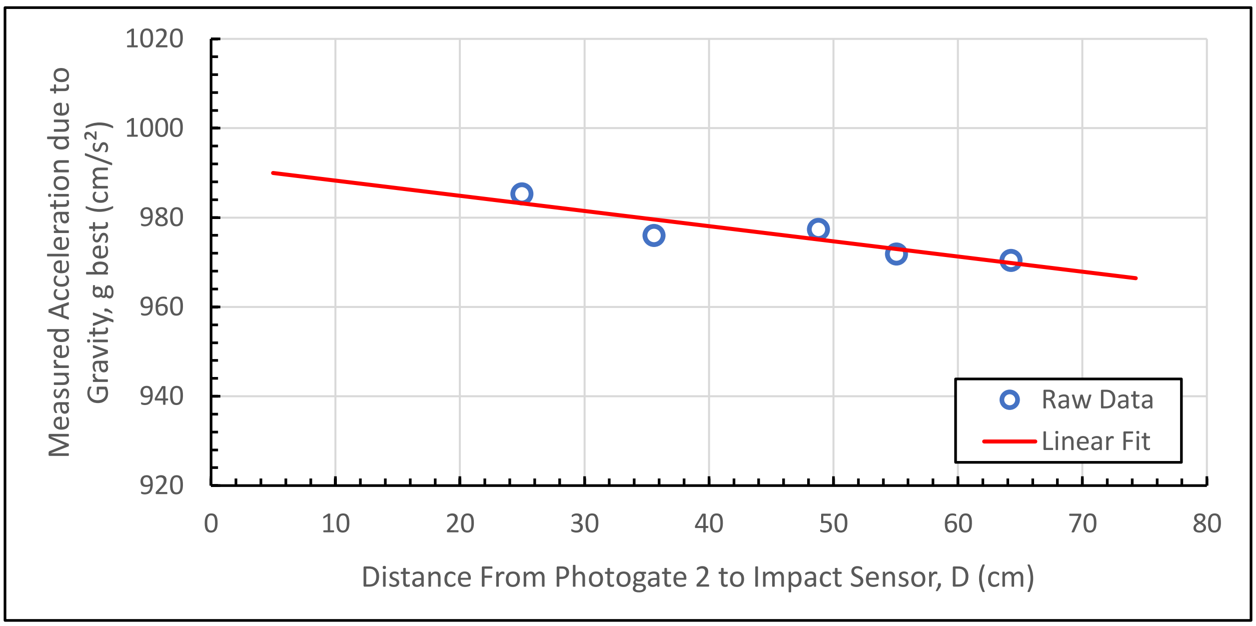

Figure 1: The Apparent Dependence of g on D. The blue points are the raw data, and the red line has the equation $g = aD + b$ where $a = (-0.34 \pm 0.09)$ s^{-2} and $b = (992 \pm 4)$ m/s^2. The data fail to show an independence of g on D, because the slope is not consistent with zero.

Based on common knowledge of gravity, we should not expect g to depend upon D. However, the data in Figure 1 shows a linear fit between g and D. One way to rule out a linear dependence would be to show that the slope of this linear fit is consistent with zero. Unfortunately, the slope in Figure 1 is not consistent with zero (-0.34 ± 0.09 does not include zero) and we cannot rule out a linear dependence with this data.

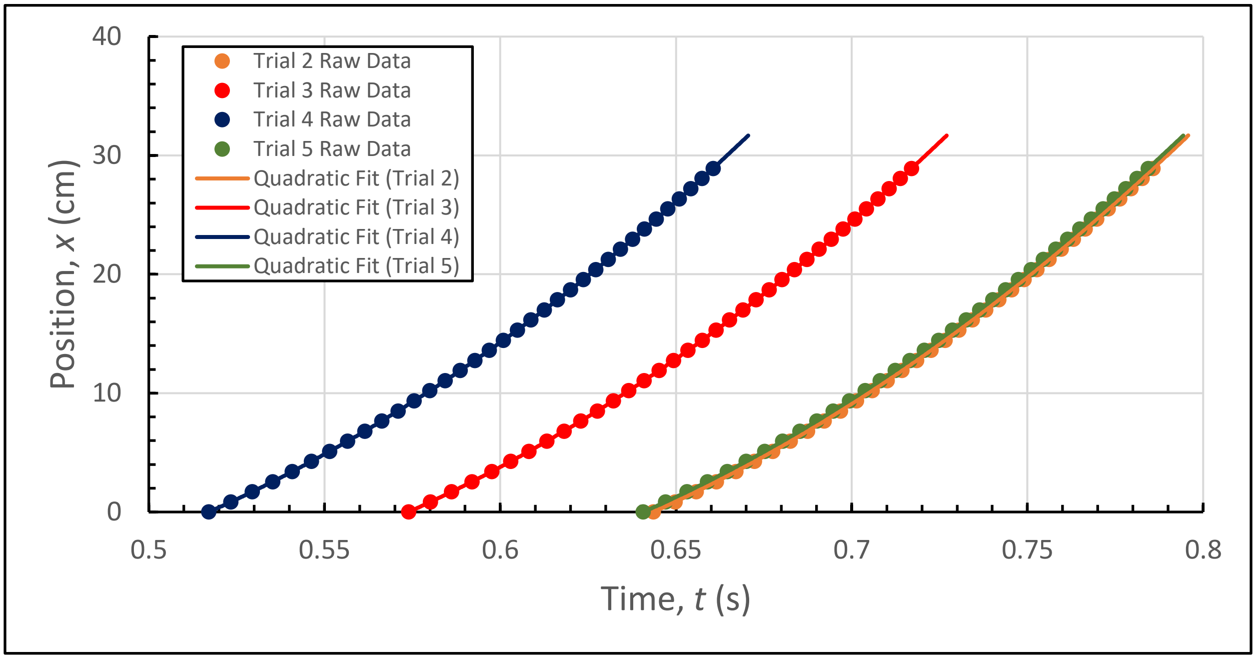

Figure 2: Position as a Function of Time for the Photogate Comb After Release for Several Different Trials. Each color represents a different trial, however each trial was performed under identical conditions. Each curve is a quadratic fit with equation $x(t) = at^2 + bt + c$. The parameters for each trial are as follows:
Trial 2: $a = (491.4 \pm 0.1)$ cm/s^2, $b = (-499.2 \pm 0.2)$ cm/s, and $c = (117.71 \pm 0.07)$ cm
Trial 3: $a = (487.9 \pm 0.4)$ cm/s^2, $b = (-427.9 \pm 0.5)$ cm/s, and $c = (84.9 \pm 0.2)$ cm
Trial 4: $a = (487.8 \pm 0.2)$ cm/s^2, $b = (-372.9 \pm 0.3)$ cm/s, and $c = (62.42 \pm 0.08)$ cm
Trial 5: $a = (490.8 \pm 0.2)$ cm/s^2, $b = (-498.2 \pm 0.3)$ cm/s, and $c = (117.71 \pm 0.09)$ cm

4 DATA TABLES

TRIAL	PHOTOGATE SPACING d (cm)	GAP TO IMPACT SENSOR D (cm)	MEASURED ACCELERATION g (cm/s^2)
1	8.32	48.82	977 ± 2
2	8.32	24.98	985 ± 2
3	8.32	35.59	976 ± 2
4	8.32	64.29	970 ± 2
5	8.32	55.08	972 ± 3

Table 1: Ball Drop Results from Different Heights, D. d is the distance between photogates, held constant. D is the distance between the second photogate and the impact sensor, varied. All g values calculated using equation 2.1[1]. Note, all uncertainties for d and D are δd and $\delta D = \pm\, 0.05$ cm.

TRIAL	CALCULATED ACCELERATION g (cm/s^2)
1	980.8 ± 0.1
2	982.9 ± 0.1
3	975.9 ± 0.4
4	975.5 ± 0.2
5	981.5 ± 0.2

Table 2: Photogate Comb Method Results for five trials. All values of g calculated by differentiating the position equations found in Figure 2 and 3. All uncertainties in g are strictly statistical uncertainties (no systematic) and come from regression analysis.

For the ball drop method, δg is composed of both statistical and systematic uncertainties because they both have significant values that cannot be ignored; they are within one order of magnitude of each other (more than 10 times the other). For the Photogate Comb Method, the statistical uncertainty dominates, and the systematic uncertainty can be ignored.

5 CONCLUSION

The two methods used to calculate g, the ball drop method and the photogate comb method, each produced values for g with different precision, accuracy, and uncertainty. Both methods have uncertainties δg, composed of statistical/systematic uncertainty. Statistical uncertainty comes from either equation ii.13[1] (ball method) or regression analysis (comb method). Systematic uncertainty is calculated by taking half the difference in the maximum and minimum values possible for g given the parameters. For the ball drop method, both systematic and statistical uncertainty contribute to δg. Statistical uncertainty was larger than systematic, but not large enough to ignore either uncertainty. For the comb method, δg has only statistical uncertainty, because the systematic uncertainty was significantly less in comparison. The average value of g for each method measures accuracy compared to the accepted value for g, 9.7955 ± 0.0003 m/s^2. $\bar{g}_{ball} = 9.76$ m/s^2 and $\bar{g}_{comb} = 9.793$ m/s^2. Both values are close, but the comb method proves to be more accurate. To analyze precision, we can look at the digits of uncertainty for each method. When using cm/s^2, the ball drop method only has uncertainty to the nearest 1cm/s^2 while for the comb we have to the nearest 0.1cm/s^2. In addition, the standard deviation of the set of data for g for the comb method was much smaller than that of the ball drop. Therefore, the Photogate Comb Method was both more accurate and precise in estimating the value for g.

6 EXTRA CREDIT

Trial (string)	Acceleration, g (cm/s^2)	Trial (glasses)	Acceleration, g (cm/s^2)
1	977.3 ± 0.2	1	954.8 ± 0.9
2	984.9 ± 0.2	2	989.1 ± 0.2
3	994.2 ± 0.1	3	980.4 ± 0.2
Average	985.5	Average	974.8

Table 3: Photogate comb Method Using String vs. Glasses Case, Three Trials Each.

Two new methods of dropping the comb were using a string and using a glasses case. To determine which method is more accurate or precise, I took the average values of g from three separate trials of each method and found their uncertainties through regression analysis. Table 3 summarizes the results. Using the same reasoning as described in the conclusion, the string method was more accurate and precise.

PRESENTATION MINI-REPORT

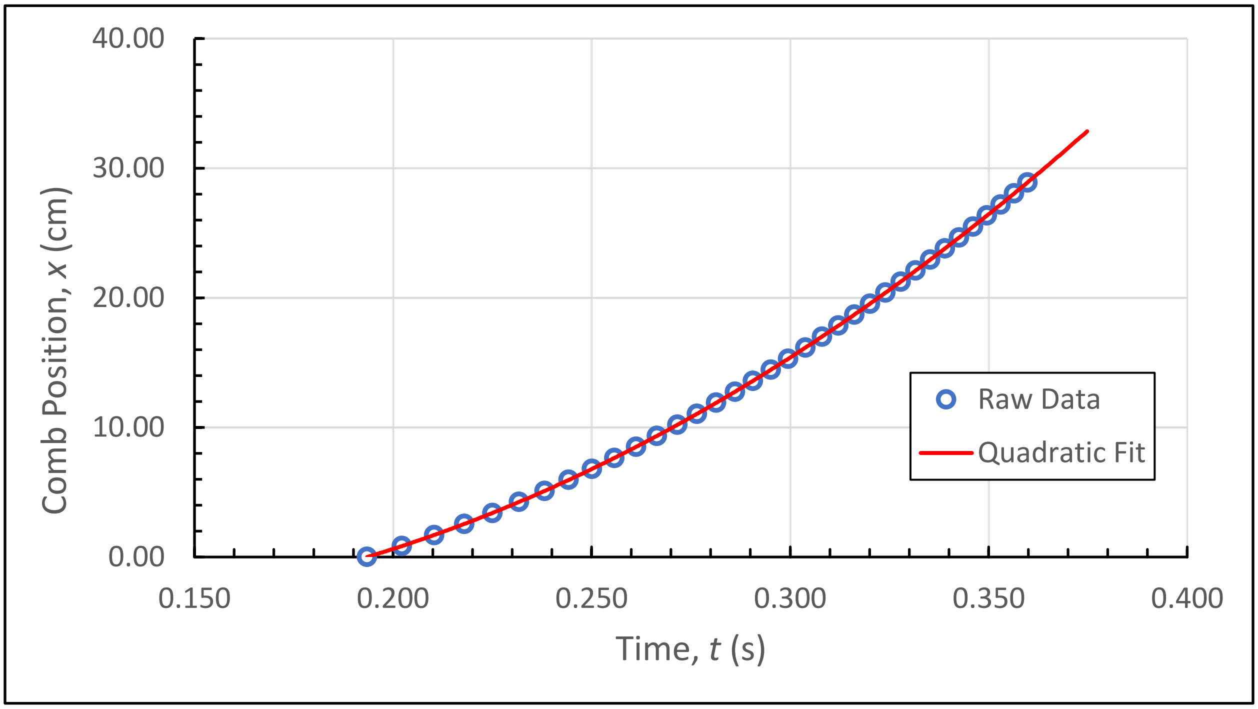

Figure 3: Position as a Function of Time for the Photogate Comb After Release. The time values, t, of the blue data points represent each instant the sensor was blocked by the front edge of a slot in the comb. The position values, x, of the blue data points track the comb's position as it travels through the sensor. The red curve is the quadratic relationship between position and time of the comb and has the equation $x(t) = at^2 + bt + c$ with parameters $a = (490.4 \pm 0.1)\ \mathrm{cm/s^2}$, $b = (-97.56 \pm 0.07)\ \mathrm{cm/s}$, and $c = (0.52 \pm 0.01)$ cm. The data and resulting curve show a non-linear, quadratic relationship between distance and time as the force of gravity acts on the comb.

To estimate the value of g, two methods were used in the experiment. The results of the second method (the photogate comb method) are shown in Figure 3 above. The comb is a long thin piece of metal with 35 evenly sized and spaced slots. When dropped vertically through a photogate sensor, time data is recorded. Each time the front edge of a slot in the comb blocks the sensor, a time value is recorded. This time corresponds to a certain position value x of the comb, all of which have values of $n\lambda$, or multiples of the distance from the back edge of one slot to the front edge of another ($\lambda = 0.85 \pm 0.05$ cm). The equation *x(t)* in Figure 3 is a position function of the comb. The acceleration of the comb (which we assume to be constant and equal to g) is determined by differentiating the position function twice, which in fact produces a constant. In this case, we get $a = g = 980.8 \pm 0.1\ \mathrm{cm/s^2}$. This value is very close to accepted value of g and is fairly precise. The uncertainty is dominated by statistical uncertainty (gathered form regression analysis).

Caption: 108 Paragraph: 192

References

[1] Campbell, W.C. et al. Physics 4AL: Mechanics Lab Manual (ver. April 3, 2017). (University of California Los Angeles, Los Angeles, California).