EXPERIMENT 4: MAGNETISM

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LAB SECTION: THURSDAY 2PM

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Worksheet

1. To determine the polarity of the Hall Probe, you could use a permanent magnet whose polarity (North and South Poles) are already known. Magnetic field lines on a permanent magnet are emitted from the North pole and loop around to the South pole. You can use this fact to determine the probe's polarity. If you hold the first side of the probe near the North pole of the magnet and the reading is positive, then this side of the probe should be the South pole, because the magnetic field lines travel from the North pole of the magnet and enter the South pole of the probe. Alternatively, if this first side produces a negative reading, then that side of the probe is the North pole.

2.

Material	Magnetism
Copper	Diamagnetic
Aluminum	Paramagnetic
Tantalum	Paramagnetic
Bismuth	Paramagnetic
Carbon	Diamagnetic
Iron	Diamagnetic
Nickel	Paramagnetic
Glass	Inconclusive
Rocks	Inconclusive

Table 1: Magnetism in Materials Using a Torsion Pendulum. A torsion pendulum apparatus was set up using a laser, a screen, a mirror, a magnet, and various materials. The laser was pointed directly at the mirror (which was connected to the magnet) and reflected towards the screen. When a material was brought near the magnet, a torque was applied to the mirror, which caused the laser's position on the screen to change. If the laser moved in a way indicating that the magnet and material attract, the material is paramagnetic. Conversely, If the laser moved in a way indicating that the magnet and material repel, the material is diamagnetic.

3. Since there is a gravitational force $F_g = Mg$ acting downward on $\mathbf{m}_1$ there must be an equal and upward force F_{B_2} acting on $\mathbf{m}_1$. In order for this force to act upward, the magnetic dipoles $\mathbf{m}_1$ and $\mathbf{m}_2$ must be antiparallel. If we set these forces equal, we have:

$$Mg = -\nabla U \tag{1}$$

From Equation 4.5 in the Manual, $U = -\boldsymbol{m}_1 \cdot \boldsymbol{B}_2$, so:

$$Mg = -\nabla(\boldsymbol{m}_1 \cdot \boldsymbol{B}_2) \tag{2}$$

Since we know $\boldsymbol{m}_1 = m_1\hat{\boldsymbol{k}}$ and $\boldsymbol{B}_2$ from Equation 4.3 in the Manual, we can take the dot their dot product, then the negative gradient of that result, and end up with the following:

$$Mg = \frac{3m_1^2\mu_0}{2\pi z_0^4} \tag{3}$$

Solving for z_0 gives:

$$z_0 = \left[\frac{3m_1^2\mu_0}{2\pi Mg}\right]^{1/4} \tag{4}$$

Introduction

There were four main independent experiments in this lab. One, we explored the dependence of radial distance on the magnetic field strength of a toroid. Two, we explored the dependence of radial distance on the magnetic field strength of a permanent magnet. Three, we explored the relationship amongst the distance between two permanent magnets and the Force between their dipoles. And lastly, we determined maximum induced voltage on solenoids with varying numbers of coil turns N, caused by running a current through a smaller, inner solenoid.

For the first part, the purpose was to show there exists a linear relationship between the magnetic field strength from a toroid B and the reciprocal of the radial distance from its center, and to determine a value for the permeability of free space μ_0 (and compare it to its theoretical value). To do this, we simply took measurements of the magnetic field strength at increasing distances, plotted the results, and examined if there was a linear relationship. From the plot (See Figure 1), the slope can be used to determine an experimental value for μ_0. Note that the equation for the magnetic field strength from a toroid is given by Equation 4.1 in the Manual:

$$B = \frac{\mu_0 I}{2\pi\rho} \tag{1}$$

Where I is the current running through the toroid and ρ is the radial distance from its center (r).

For the second part, the purpose was to show there exists a linear relationship between the magnetic field strength from a permanent magnet B and the cube of the reciprocal of the radial distance ($1/r^3$) from the magnet. This was achieved in a similar manner as for the first part.

For the third part, the purpose was to show there exists a linear relationship between the Force between two dipoles of permanent magnets F and the reciprocal of the distance between them to the fourth ($1/r^4$). This was accomplished using a scale to read the weight of a magnet and how it changes depending on its distance from the other magnet and plotting the results.

For the last part, the purpose was to construct a functional RL series circuit, where the inductor is composed of a large solenoid enclosing a smaller solenoid with and without a ferromagnetic core. Thus, when an alternating current is supplied to the circuit, a voltage is induced onto the larger solenoid. This induced voltage is given by Equation 4.7 in the Manual:

$$V_{ind} = -\frac{d\Phi}{dt} \tag{2}$$

Note that Φ is the magnetic flux through one loop of a solenoid. The goal was to obtain values for the maximum induced voltage V_{ind} on the large solenoids with different numbers of turns and compare this to the theoretical value found using:

$$V_{ind,max} = I_0 \mu_0 \omega A n N \tag{3}$$

Instead of comparing the voltages directly, we took theoretical and experimental ratios of these voltages and compared those. We also investigated the effect on the induced voltage by inserting a ferromagnetic core into the smaller solenoid by calculating the voltage's amplification factor (ratio).

Experimental Results

Part 1: Dependence of radial distance on the magnetic field strength of a toroid

To create a magnetic field using the toroid, we applied a Voltage using a Model 1601 Regulated DC Power Supply, which in turn applied a current through the wires. When applying a Voltage of 10.58 ± 0.02 V, the current was 0.96 ± 0.01 A (read from 8010A Digital Multimeter). The resulting magnetic field strength B was measured using a Gauss/Teslameter Model 5080. Its units are milliTesla (mT) with an uncertainty of ± 0.02mT. We clamped its probe to a Mascot slider with a ruler on it (error of ± 0.05cm) to accurately measure the distance from the center of the toroid. The variable r is the distance from the end of the probe to the center. Starting with the end of the probe positioned directly in the center (distance $r = 0.00 \pm 0.05$ cm), we took B measurements every 0.50 ± 0.05 cm increase until the end of the probe exited the outer radius of the toroid. The data relevant to the analysis includes the r and B values between the inner and outer radii of the toroid. This corresponded to radial distances from 2.50 ± 0.05 cm to 14.50 ± 0.50 cm. An important characteristic of the Teslameter is the angle at which the probe's plane is oriented. In order to maximize the magnetic flux through this plane, the probe plane is oriented parallel to the vertical wires of the toroid (thus the angle between the wires and the plane is 0 ± 1 degrees).

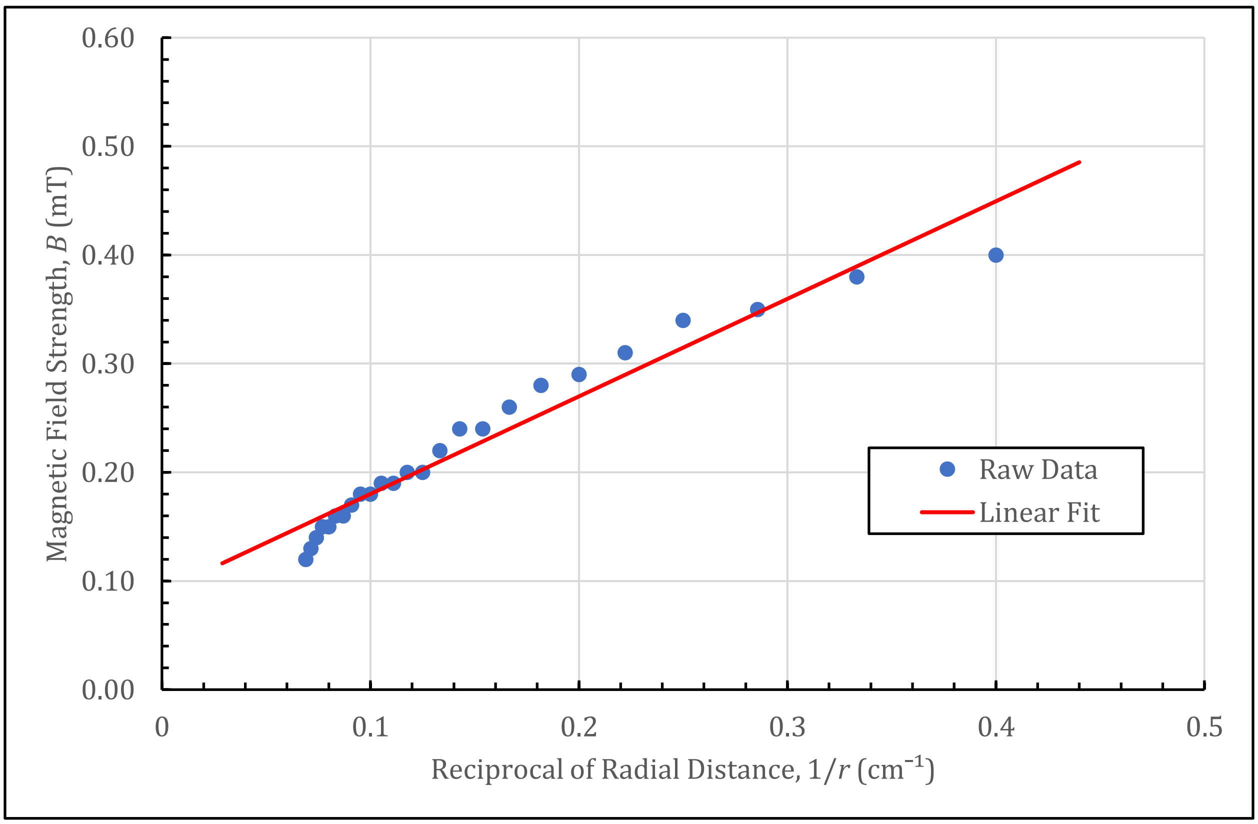

Figure 1: Linear Dependence of Magnetic Field Strength of a Toroid on Reciporical of Radial Distance. B values were collected every 0.50 ± 0.05cm within the range 2.50 ± 0.05 cm to 14.50 ± 0.05 cm. The blue points are the raw data and the red line is the linear fit with equation taking the form $B = a(1/r) + b$ where $a = 0.90 \pm 0.04$ cm mT and $b = 0.090 \pm 0.008$ mT and has correlation coefficient $r = 0.97$.

Part 2: Dependence of radial distance on the magnetic field strength of a permanent magnet

A permanent magnet naturally has a magnetic field stemming from its north pole to its south pole. The cylindrical magnet was mounted on a board for support and arranged such that the its flat circular faces are perpendicular to the table. Using the same Teslameter probe and slider with ruler technique, we obtained B values every 0.50 ± 0.05 cm beginning at 3.81 ± 0.05 cm (width of 2 fingers) from the magnet in the z-direction. The z-direction is the axis perpendicular to the flat faces of the magnet. The probe was oriented such that its flat face was always parallel to the magnets flat face. We stopped after 21 data points were collected.

Part 3: Relationship amongst the distance between two permanent magnets and the Force between their dipoles

Two permanent magnets, each with their own dipoles, exert a Force on one another. We placed one magnet flat and centered on an ACCULAB scale (zeroed the scale) and attached the other one to the vertically oriented slider in a way such that the magnets were colinear and the two nearest faces were of opposite polarity. Beginning at a separation distance between these two faces of 17.40 ± 0.05cm, we recorded the value on the scale in grams with an error of ± 0.5g every 0.50 ± 0.05cm until the distance was 5.50 ± 0.05cm.

Part 4: induced voltage on solenoids

An RL series circuit was created using a 10.0 ± 0.5 Ohm Resistor, an Inductor composed of a primary coil with $N' = 175.0 \pm 0.1$ turns within a Faraday coil with $N = 500.0, 1000.0, or\ 1500.0 \pm 0.1$ turns, and an Alternating Current supplied from the RIGOL Waveform Generator at a frequency of 1.00000 ± 0.00004 kHz. Using the oscilloscope, we measured the peak-to-peak Voltages across the resistor on Channel 1 and the inductor on Channel 2. The Voltage across the resistor is used to determine the current through the circuit I_0 (and thus the smaller solenoid) through Ohm's Law, and the Voltage across the inductor is the maximum induced voltage on the larger solenoid, V_{ind}. We did this for all three values of N for the larger solenoid both with and without a ferromagnetic core inserted inside the smaller solenoid. Thus, the Amplification Factor represents the ratio between V_{ind} values with and without the core for the same turns. All Voltages read off the oscilloscope have an error of ± 0.02 V. Table 1 summarizes this section.

Turns in Larger Solenoid, N	Maximum Induced Voltage, V_{ind} (V)	Amplification Factor	Current, I_0 (A)
500.0±0.1	0.44±0.02	2.5±0.1	0.090±0.005
1000.0±0.1	0.84±0.02	2.57±0.07	0.090±0.005
1500.0±0.1	1.22±0.02	2.64±0.05	0.090±0.005

Table 1: Results of RL circuit Data. For the three different N values, V_{ind} was obtained with the oscilloscope. The second column are V_{ind} values *without* the ferromagnetic core, while the third column is the Amplification Factor which is the ratio V_{ind}(with core) over V_{ind} (without core). It should be noted that the induced voltage is amplified consistently amongst large solenoids with different numbers of turns. The last column is the current running through the resistor and smaller solenoid, calculated as the Voltage drop across the resistor over the resistance. Note that it is constant for all trials, with and without the core.

Analysis

Part 1:

To prove linear dependence of the magnetic field strength B on the reciprocal of radial distance $1/r$ we can rewrite Equation 1 in a linear form:

$$B = \frac{\mu_0 I}{2\pi} x \tag{4}$$

Where $x = 1/r$. Thus, B as a function of x is linear. Figure 1 shows this linear dependence. To determine how well B and x are linearly related, we can use the correlation coefficient r. After performing a linear regression, we get $r = 0.97$. Since the number of measurements $N \geq 25$, a value of r with this magnitude surely constitutes a linearly relationship between the two variables, thus confirming that the magnetic field strength of a toroid varies linearly with $1/r$.

The best fit line modeled by $B = a(1/r) + b$ in Figure 1 mirrors Equation 4 above, where the slope $a = \frac{\mu_0 I}{2\pi}$ and $x = 1/r$. Thus, we can derive an expression to determine the permeability of free space μ_0:

$$\mu_0 = 2\pi\left(\frac{a}{I}\right) \tag{5}$$

Using $a = 0.90 \pm 0.04$ cm·mT from Figure 1 and $I = 0.96 \pm 0.01$ A from the Multimeter reading, we obtain:

$$\mu_0 = 6 \pm 2 \text{ cm} \cdot \text{mT} \cdot \text{A}^{-1} \quad \text{OR} \quad (6 \pm 2) \times 10^{-5} \text{ m} \cdot \text{T} \cdot \text{A}^{-1}$$

Uncertainty was propagated using Equation A.22 and A.24:

$$\delta\mu_0 = 2\pi \cdot \mu_0 \sqrt{\left(\frac{\delta a}{a_{best}}\right)^2 + \left(\frac{\delta I}{I_{best}}\right)^2} \tag{6}$$

The theoretical value for the permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$ m · T · A^{-1} *exactly*. To compare the experimental and theoretical values, the percent error was calculated using:

$$\%\, error = \left|\frac{Experimental - Theoretical}{Theoretical}\right| \times 100\% \tag{7}$$

% error = 4600 ± 100 %

This error is extremely large, meaning there is a huge difference in the experimentally calculated and theoretical value for the permeability of free space. Therefore, there must have been some errors while the experiment was conducted. See conclusion.

Part 2:

The magnetic field strength of a permanent magnet is proportional to $1/r^3$ where r is the radial distance from the magnet. After collecting data as described in part 2 of the experimental results, each distance r was inverted and cubed. A plot of B as a function $1/r^3$ was used to test if these variables are linearly related. See Figure 2 below.

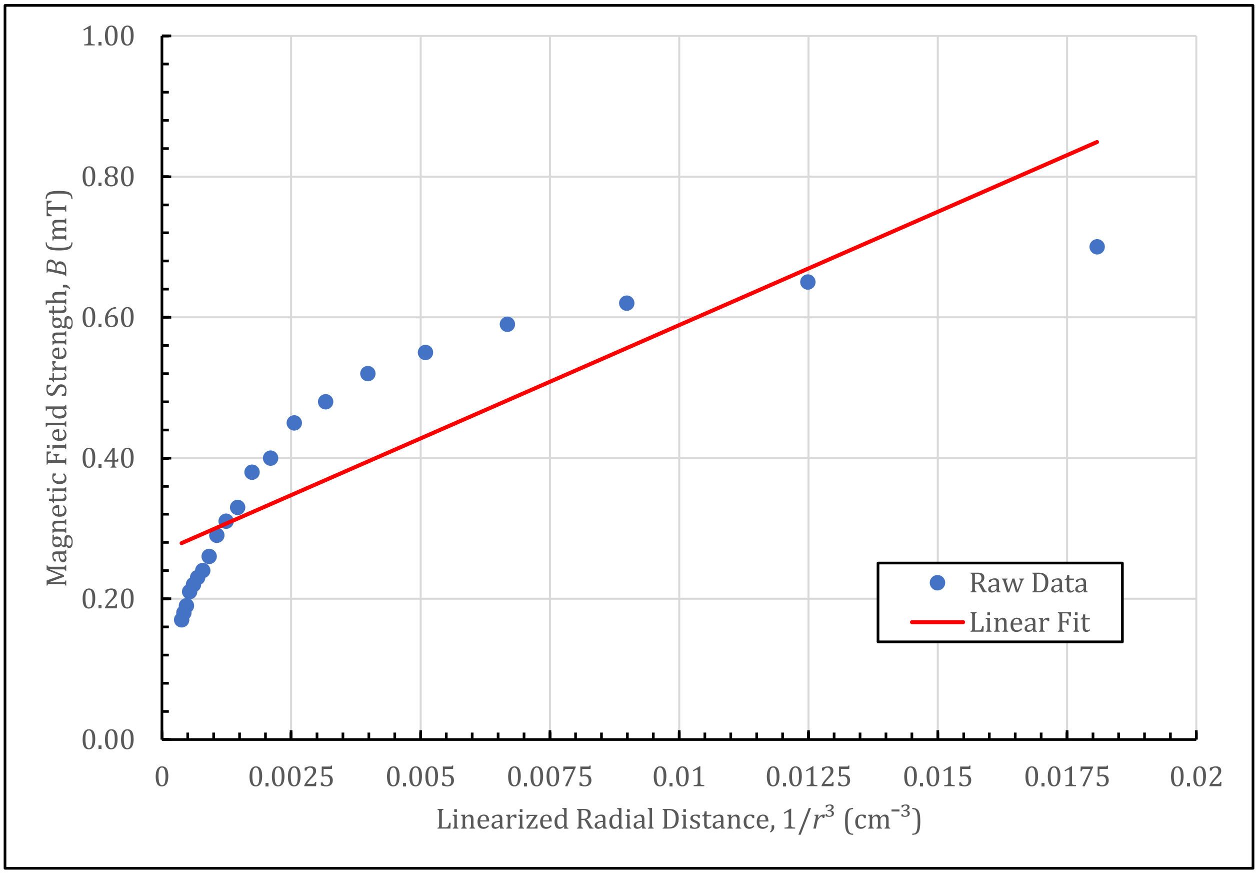

Figure 2: Linear Dependence of Magnetic Field Strength of a Permanent Magnet on Reciporical of Radial Distance Cubed. B values were collected every 0.50 ± 0.05cm within the range 3.81 ± 0.05cm to 13.81 ± 0.05cm. The blue points are the raw data and the red line is the linear fit with equation taking the form $B = a(1/r^3) + b$ where $a = 32 \pm 4$ $\mathrm{cm}^3 \cdot \mathrm{mT}$ and $b = 0.27 \pm 0.02$ mT and has correlation coefficient $r = 0.87$.

To determine how well B and $1/r^3$ are linearly related, we can again use the correlation coefficient r. After performing a linear regression, we get $r = 0.87$. Despite the fact that r is large enough to show linear relation, our number of measurements $N = 21$ is too low (less than 25) to prove a significant linear relationship between the two variables.

Part 3:

The force between dipoles is proportional to $1/r^4$ where r is the radial distance between poles of two equal permanent magnets. After collecting data as described in part 3 of the Experimental results, each distance r was inverted raised to the fourth. A plot of F as a function $1/r^4$ was used to test if these variables are linearly related. See Figure 3 below. The force F was obtained by converting the mass read on the scale to a Force by multiplying by $g = 9.81$ m/s^2 and dividing by 1000 to get the force in Newtons.

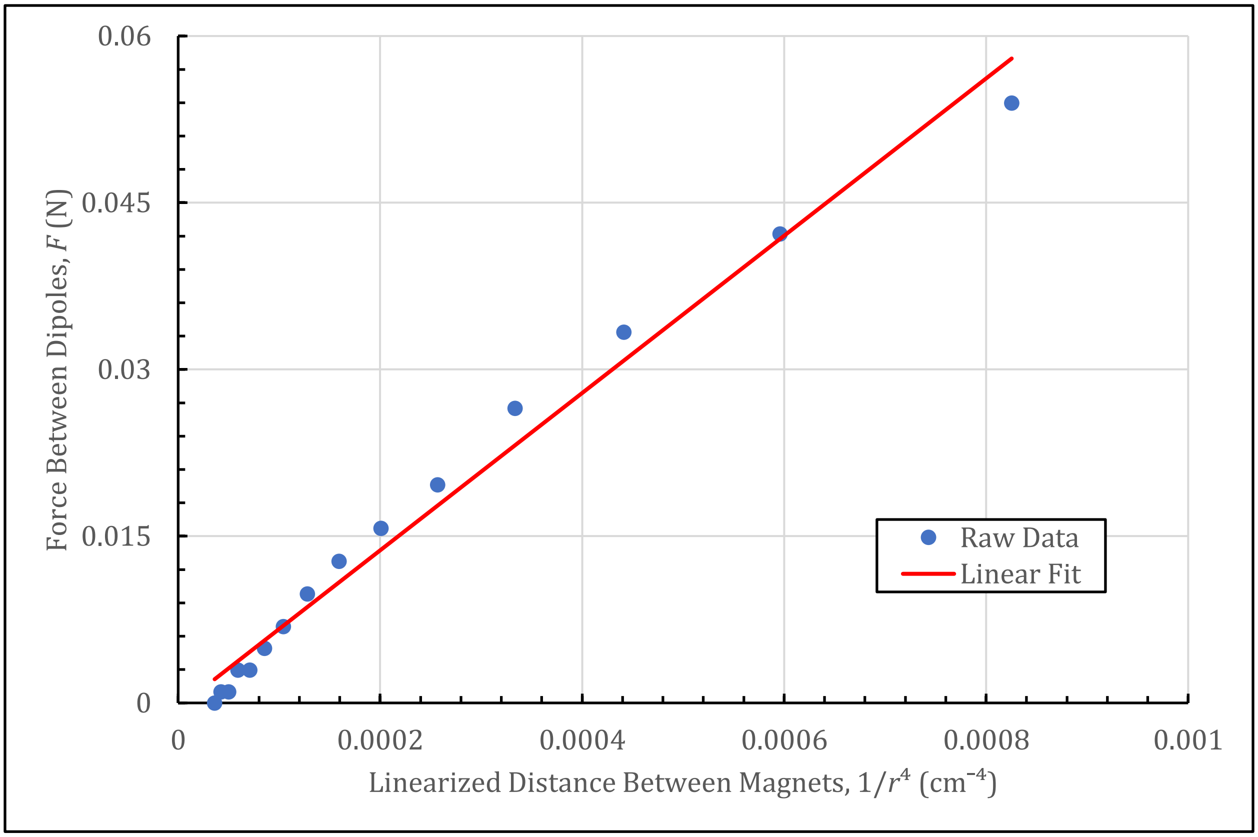

Figure 3: Linear Dependence of the Force Between Dipoles of two Permanent Magnet on Reciporical of Radial Distance to the fourth. The weight read by the scale was recorded every 0.50 ± 0.05cm beginning at a distance between magnets of 17.40 ± 0.05cm and decreasing to 5.50 ± 0.05cm. The scale readings were converted to Forces. The blue points are the raw data and the red line is the linear fit with equation taking the form $F = a(1/r^4) + b$ where $a = 71 \pm 2$ $\text{cm}^4 \cdot \text{N}$ and $b = -0.0004 \pm 0.0008$ N and has correlation coefficient $r = 0.99$.

To determine how well F and $1/r^4$ are linearly related, we can again use the correlation coefficient r. After performing a linear regression, we get $r = 0.99$. This r value is almost maximized, however once again our number of measurements $N = 15$ is too low (less than 25) to indicate a significant linear relationship between the two variables.

Part 4:

The Values Amplification Factor and Current Values from Table 1 were calculated as follows:

$$A.F. = \frac{V_{ind,with\ core}}{V_{ind,without\ core}} \tag{8}$$

$$I_0 = \frac{V_r}{R} \tag{9}$$

Where V_r is the Voltage across the resistor, which is always constant at 0.900 ± 0.002 V. The uncertainty propagations for these two variables are as follows:

$$\delta A.F. = A.F._{best}\sqrt{\left(\frac{\delta V_{ind,with\ core}}{V_{ind,with\ core}}\right)^2 + \left(\frac{\delta V_{ind,without\ core}}{V_{ind,without\ core}}\right)^2} \tag{10}$$

Where $\delta V_{ind,with\ core}$ and $\delta V_{ind,without\ core}$ are both ± 0.02 V.

$$\delta I_0 = I_0\sqrt{\left(\frac{\delta R}{R_{best}}\right)^2 + \left(\frac{\delta V_r}{V_{r,best}}\right)^2} \tag{11}$$

Where δV_r is also ± 0.02 V. These uncertainties use Equation A.25 in the manual.

To see how accurate the experimental V_{ind} values were, we calculated the ratio of V_{ind} for 1500.0 ± 0.1 turns to V_{ind} for 1000.0 ± 0.1 turns and V_{ind} for 500.0 ± 0.1 turns both for experimental and theoretical values for the induced voltage and compared using percent error. For the experimental ratios, we simply took the voltages gathered from the oscilloscope. For the theoretical ratios, we derived an equation using Equation 3 in the Introduction. Since we are taking ratios, the only variable in Equation 3 that changes is the number of turns in the larger solenoid N. Therefore, the theoretical ratio can be written as:

$$\frac{V_{ind,1500}}{V_{ind,\#}} = \frac{N_{1500}}{N_{\#}} \tag{12}$$

Where # denotes either 1000 or 500, depending on which ratio we are taking, and $N_{1500} = 1500.0 \pm 0.1$ turns. Table 2 below summarizes the results.

Ratio Type	Theoretical Value	Experimental Value	Percent Error (%)
$\frac{V_{ind,1500}}{V_{ind,1000}}$	1.5000±0.0002	1.45±0.04	3.3±0.3
$\frac{V_{ind,1500}}{V_{ind,500}}$	3.0000±0.0006	2.8±0.1	6.7±0.5

Table 2: Theoretical and Experimental Ratios of Maximum Induced Voltages and Their Comparison. The Maximum Induced Voltages on the larger solenoid as a result of the current running through the interior solenoid were recorded for different numbers of turns of coil on the larger solenoid, and the above ratios were taken as the experimental values. The theoretical ratios are simply the ratios of the turns (with uncertainty). Percent error values were calculated using Equation 7.

Since the percent errors between ratios are fairly low, therefore the results of the experiment match with the expected results. Any error could have resulted from magnetic fields from unwanted sources contributing to the induced voltages on the larger solenoid.

The uncertainty propagation for Theoretical Ratios is:

$$\delta\left(\frac{V_{ind,1500}}{V_{ind,\#}}\right) = \left(\frac{V_{ind,1500}}{V_{ind,\#}}\right)\sqrt{\left(\frac{\delta N_{1500}}{N_{1500}}\right)^2 + \left(\frac{\delta N_{\#}}{N_{\#}}\right)^2} \tag{13}$$

The uncertainty propagation for Experimental Ratios is:

$$\delta\left(\frac{V_{ind,1500}}{V_{ind,\#}}\right) = \left(\frac{V_{ind,1500}}{V_{ind,\#}}\right)\sqrt{\left(\frac{\delta V_{ind,1500}}{V_{ind,1500}}\right)^2 + \left(\frac{\delta V_{ind,\#}}{V_{ind,\#}}\right)^2} \tag{14}$$

Both uncertainties came from Equation A.24 in Manual. All uncertainties in Measured Induced Voltages are ± 0.02V and all uncertainties in turn numbers are ± 0.1turns.

The maximum induced Voltage on the larger solenoid depends on the number of turns N in that solenoid because of Equation 2. V_{ind} for just *one* loop depends on the change in magnetic flux over time through that one loop. But for many turns N, Equation 2 becomes:

$$V_{ind} = -N\frac{d\Phi}{dt} \tag{15}$$

If the change in flux over time is still the same for different numbers of turns, the induced voltage is increased proportionally to the number of turns in the solenoid.

Conclusion

The overall purpose of the lab was to investigate the dependence of magnetic field strength from a toroid and a permanent magnet on radial distance, to explore the relationship of a Force between dipoles and the distance between two permanent magnets, and to determine induced voltage ratios both theoretically and experimentally from an outer solenoid on an inductor in an RL series circuit. For part 1, the results of the experiment confirmed there is a linear relationship between the magnetic field of a toroid and the reciprocal of the radial distance from its center. The correlation coefficient $r = 0.97$ with $N = 25$ measurements indicates linear relation. However, the experimental results for the calculation of the permeability of free space did not align with expected theoretical value. The 4600 ± 100 percent error between experimental and theoretical values for μ_0 indicates that there was a large source of error in this part of the lab. This error may have been caused by interference from all the magnets and other toroids with their own independent magnetic fields affecting the measurements for just a single toroid. In order to eliminate this error, we would need to perform the lab in conditions with no other sources of magnetic field near the toroid. In addition, it was difficult to tell of the probe's plane was exactly 0 ± 1 degrees in comparison to the copper wires, because the plane was very thin and almost appeared to bend. To eliminate this error, we could use a larger, more accurate probe and measure the magnetic field to more significant figures if possible.

For part 2, the results of the linear regression indicated that there could possibly be a linear relationship between the magnetic field of a permanent magnet and $1/r^3$ because of the correlation coefficient $r = 0.87$. However, since not enough data points were collected, this linear relationship could not be completely confirmed by the experiment. In order to accurately demonstrate this linear relationship, more data would need to be taken.

For part 3, the results conclude similarly to part 2. A linear regression of the Force between dipoles and $1/r^4$ produced a very high correlation coefficient $r = 0.99$, but again since there were not enough data points, we could not conclude a linear relationship. Similarly, to demonstrate linear dependence of F on $1/r^4$, more data should be collected and analyzed.

For the last part, our results show that the addition of a ferromagnetic core to the inductor amplifies the maximum induced voltage on the outer solenoid by about the same amount for the three different inductors with larger solenoids of different turn amounts (See Table 1). The two theoretical and experimental ratios for maximum induced voltages without the ferromagnetic core were respectively similar. The first ratio had a percent error of 3.3 ± 0.3 % while the second had one of 6.7 ± 0.5 %. These are small percent errors, but any error could be attributed to the fact that there were other sources of magnetic fields besides the smaller interior solenoid that could have affected the induced voltage on the larger solenoid. Again, to eliminate this error, we would need to isolate the system so that there is no foreign magnetic field source influencing the induced voltage on the inductor.

References

[1] Campbell, W.C. *et al*. Physics 4BL: Mechanics Lab Manual (ver. August 31, 2018). (University of California Los Angeles, Los Angeles, California).