EXPERIMENT 7: DIFFRACTION AND INTERFERENCE

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Worksheet

1. If the slit spacing d increases, then the diffraction peaks must get closer together according to the equation:

$$x = \frac{m\lambda D}{d} \tag{1}$$

   x is the distance between peaks, so when the denominator d increases, x decreases.

2. Single slit interference is responsible for the shape of the intensity curve's envelope while double slit interference is responsible for the oscillations within the envelope.

Presentation Report

Introduction

This lab was divided into three separate experiments each having to do with the diffraction of light. When a beam of light (i.e. laser light) travels through a single slit, a combination of two slits (double slit), or even a grating with multiple slits, constructive and destructive interference of light waves occur resulting in the diffraction of light. When light passes through the slit(s), its intensity varies as a function of the distance from the center of the slit(s) at some given distance away from the slit(s) In the case of a single slit, the position of Intensity *minima* is given by:

$$b\sin(\theta) = m\lambda \tag{1}$$

Where b is the slit width, θ is the angle between the center of the slit and the minimum position, m is an integer corresponding to position of minima with respect to the center, and λ is the wavelength of the light. If the slit is spaced a distance D from the screen on which Intensity is being measured, then:

$$b = \frac{m\lambda D}{x_{min}} \tag{2}$$

Where x is the separation distance between minima. This Equation holds if we assume the small angle approximation $\sin\theta \approx \tan\theta$ and $\tan\theta = x/D$. If a double slit is used, Equation 2 still holds for the slit widths. However, the distance between slits, the slit spacing d, is also an important quantity. Using trigonometry, the position of Intensity *maxima* is given by:

$$d\sin(\theta) = m\lambda \tag{3}$$

Using the same small angle approximation, the slit spacing can be represented by:

$$d = \frac{m\lambda D}{x_{max}} \tag{4}$$

Except this time the separation distance x is the distance between maxima. For the first part of the experiment, the objective was to verify Equations 2 and 4 using single and double slits with known parameters (b & d) and comparing the results, and then later to determine these same parameters for an unknown double slit. This was achieved through measuring Intensity as a function of a position for a known wavelength of light traveling through the slits and analyzing the results.

In the case of multiple slits, i.e. a diffraction grating, the position of maxima is given by:

$$d\sin(\theta) = n\lambda \tag{5}$$

Where n is different than m used in previous Equations. Typically, diffraction gratings have a specific number of lines per meter N which is given by the reciprocal of slit spacings d. Thus,

$$N = \frac{n}{d} = \frac{\sin\theta}{\lambda} \tag{6}$$

The purpose of this next part was to verify the diffraction grating spacing N by running laser light through a diffraction grating of known N and measuring the angles of maxima. In addition, another goal was to determine the bandwidth of visible light by running a beam of white light through the same grating. The purpose of the final part of the experiment was to determine the width of a strand of hair by analyzing light's diffraction pattern as a result of running a laser beam past the stretched hair. The thickness of the hair is analogous to the slit width of a single slit.

Experimental Results

Part 1: Single and Double Slit Diffraction Patterns

This part of the experiment used a laser, known single slit with parameter $b = 0.04$ mm, known double slit with parameters $b = 0.04$ mm and $d = 0.125$ mm, unknown double slit, photodetector (photometer) with linear translator, magnetic bench, and myDAQ software. The photodetector's output voltage (measures light intensity) was connected to channel 1 of myDAQ. The light sensor is able to move horizontally by a rod which is connected to a built-in potentiometer. The output voltage of this potentiometer was connected to channel 0 of myDAQ while an input of about 5.00 ± 0.01 Volts was supplied to its input via a DC power supply. The laser was positioned on the left of the bench with the photodetector on the right. Before using any diffracting elements, the laser light was centered directly on the photodetector's sensor set to a distance of 1.00 ± 0.05 cm by the rod. With the data acquisition program set to collect intensity voltage as a function of position voltage at a rate of 100 points per second, the rod was rotated at a rate of approximately 3 seconds per turn changing the position of the sensor. Once the position reads about 4.00 ± 0.05 cm on the photodetector, data collection was stopped. The resulting laser beam intensity profile was used to calibrate the potentiometer output voltages to photometer displacements. These displacements are directly correlated to the position on the diffraction pattern. The known single slit diffracting element ($b = 0.04$ mm) was placed between the laser and detector at a distance of $D = 40.00 \pm 0.05$ cm to the left of the detector and aligned so the beam was diffraction pattern was clearly centered on the sensor. The same procedure for data acquisition was carried out. Refer to Figure 1 for the diffraction pattern with calibrated position data. The experiment was repeated except the single slit element was replaced with the known double slit ($b = 0.04$ mm and $d = 0.125$ mm) at the same position D. Refer to Figure 2 for the diffraction pattern. Lastly, the experiment was repeated with the unknown double slit replacing the known double slit. The parameters b and d are to be determined later in analysis. Refer to Figure 3 for the diffraction pattern.

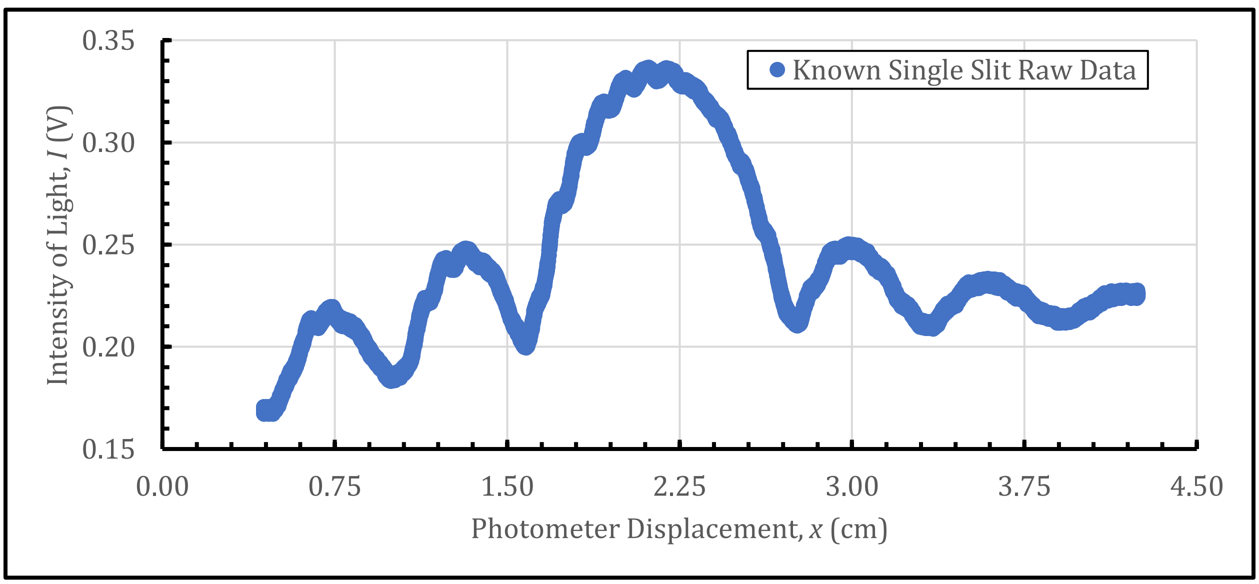

Figure 1: Diffraction Pattern for Beam Diffracted Through Single Slit with Width 0.04 mm. The Output Voltage from the Photodetector as a function of the Photometer's displacement using the rod for light diffracted through the known single slit is plotted above. Note that the y-axis is effectively the Light Intensity of the diffracted light. Potentiometer voltages were converted to displacements. The general shape of the plot is expected for that of single slit diffraction, ignoring the slight perturbations within the peaks.

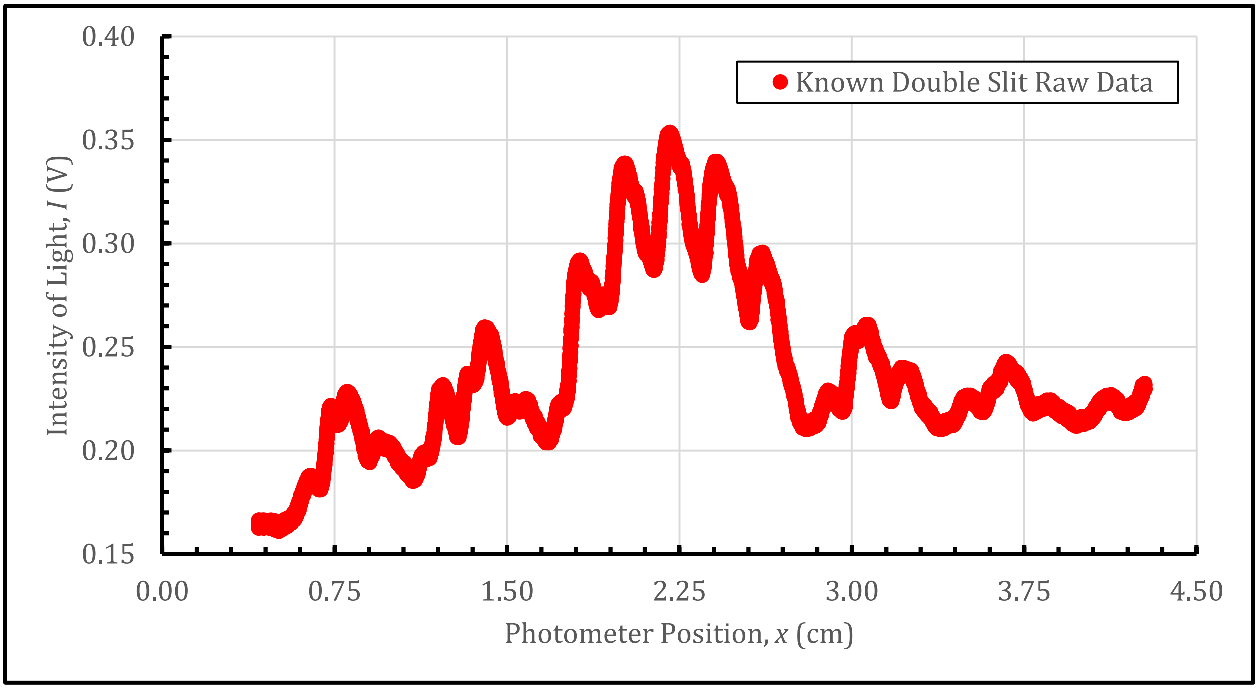

Figure 2: Diffraction Pattern for Beam Diffracted Through Double Slit with Width 0.04 mm and Spacing 0.125 mm. The Intensity Output Voltage from the Photodetector as a function of the Photometer's displacement using the rod for light diffracted through the known double slit is plotted above. Potentiometer voltages were converted to displacements. Note that the y-axis is effectively the Light Intensity of the diffracted light. The general shape of the plot is expected for that of double slit diffraction; there are oscillations (maxima and minima) in intensity within an "envelope." This envelope pattern mirrors the single slit pattern and can be seen encompassing the overall shape of the plot.

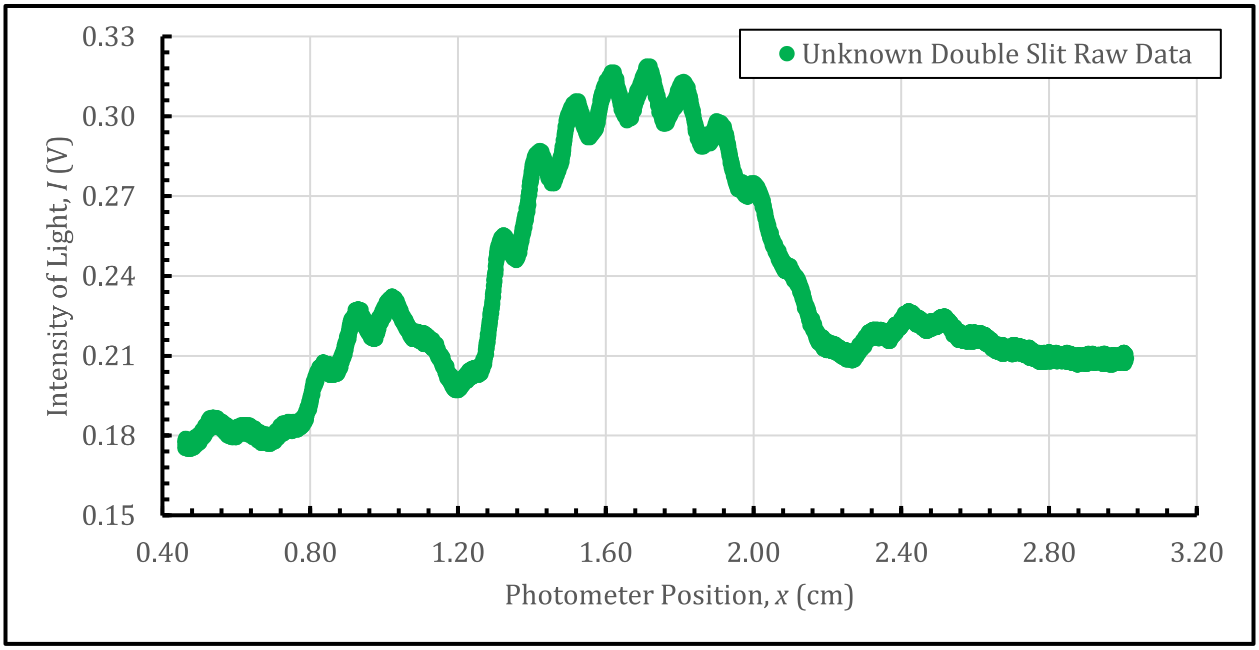

Figure 3: Diffraction Pattern for Beam Diffracted Through Double Slit with Unknown Parameters. The Intensity Output Voltage from the Photodetector as a function of the Photometer's displacement using the rod for light diffracted through the unknown double slit is plotted above. Potentiometer voltages were converted to displacements. Once again, the general shape of the plot is expected for that of double slit diffraction. Note that the y-axis is effectively the Light Intensity of the diffracted light. The envelope pattern is apparent as well, but the oscillations within the envelope are less magnified.

Part 2: Diffraction Grating

To observe the diffraction spectrum created by a diffraction grating, the laser beam was directed through a beam expander, which transmitted the light through the diffraction grating with known diffraction spacing of $N = 600$ lines/mm. The grating diffracts the beams in different directions (angles). The several beams were observed by aligning a piece of white paper with a printed protractor up to the grating and positioning the origin of the protractor where the beams emerge. See Figure 4. The angles between maxima (center beam and two beams to the left and right) were $\theta_{laser} = \pm 22.5 \pm 0.5°$.

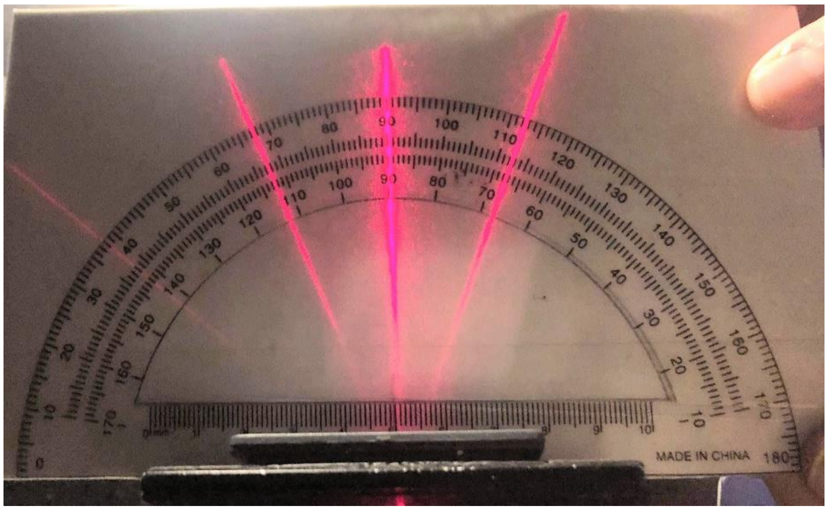

Figure 4: Diffraction Spectrum for Laser Beam transmitted through Diffraction Grating. The laser beam was sent through a beam expander and then though the diffraction grating. The resulting beams are illuminated on the protractor. The magnitude of the angle between the center beam and the right beam is the same as that of the center beam and left beam.

To observe the dispersion of white light, a single beam of white light was emitted from the ray box and sent directly through the diffraction grating with the protractor positioned similarly (Figure 5). The diffraction angles for red, green, and blue light are given in Table 1.

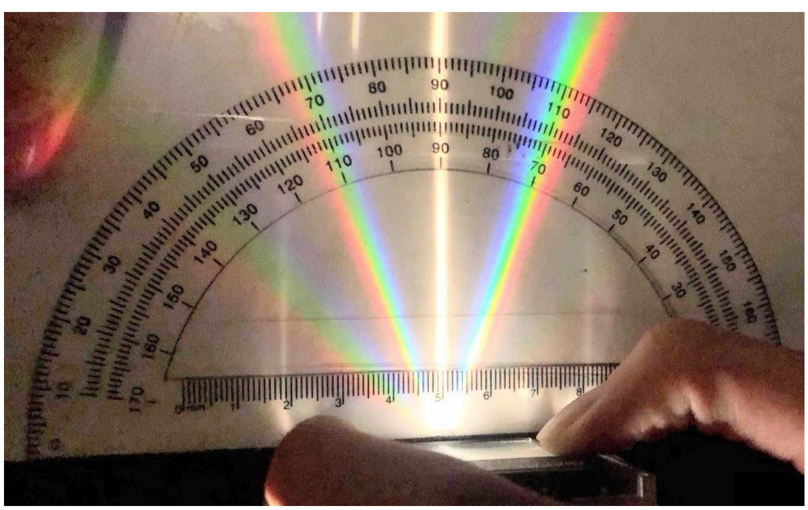

Figure 5: Dispersion of White Light by a Diffraction Grating. After traveling through the diffraction grating, white light is dispersed into a spectrum at an angle relative to the beams original path. From the image, red light is deflected more than green and blue light on both sides.

Color	Angle, θ (degrees)	Angle Range (degrees)
RED	23 ± 1	22.0 ± 0.5 – 24.0 ± 0.5
GREEN	19 ± 1	18.0 ± 0.5 – 20.0 ± 0.5
BLUE	16 ± 1	15.0 ± 0.5 – 17.0 ± 0.5

Table 1: Diffraction Angles for Dispersed White Light as a Spectrum. The angles were recorded from the center white beam to the right (most clear) spectrum in Figure 5. The uncertainties in angle are a result of the range of angles. The uncertainty in the bounds of the angle range comes from the protractor measurements. For each color, the angle ranged about ±1 degree from the center of the color. See Analysis for calculations of bandwidth/wavelengths.

Part 3: Diffraction of Light Through Hair

To determine the thickness of hair, we placed a single strand of hair vertically upwards along the magnetic slide support using the magnetic strips to hold it down. The hair was placed a distance of $L = 100.00 \pm 0.05$ cm from the left of the screen. The laser was pointed directly at the hair and the diffraction pattern appeared on the white paper. The points at which Intensity was maximized (brightest spots) were located on the paper by marking them with a pencil. The diffraction pattern looked similar to that of the single slit. There was clearly a well-defined envelope but there were no apparent oscillations in Intensity within the peaks. After marking positions of maxima, the measured distance x_{max} between adjacent maxima was 0.45 ± 0.05 cm (measured with ruler).

Analysis

Before any analysis is done using the plots, it is important to note how the position data on the x-axes were calibrated. From the beam profile described in the first part of experimental results, the channel 0 voltage data determined at photometer displacement $x = 1.00, 2.50,$ and 4.00 ± 0.05 cm were recorded. A linear plot of the photometer displacement as a function of this channel 0 voltage for these three points was created, and the slope of the bet fit line was the conversion factor from voltage to photometer displacement. Its value was 1.00 ± 0.02 cm/V. The uncertainty was determined from linear regression. Before plotting, all channel 0 data were multiplied by the conversion factor to covert from Volts to centimeters.

Part 1: Single and Double Slit Diffraction Patterns

The first step in analyzing the diffraction patterns is to compare the theoretical known parameters of the single and double slits to the parameters obtained from the plots and Equations 2 and 4. To compare, the percent difference between experimental and theoretical parameters is determined using:

$$\%\,error = \left|\frac{Experimental - Theoretical}{Theoretical}\right| \times 100\% \tag{7}$$

For the known single slit, the only parameter that needed to be calculated was the slit width b using Equation 2. The values for λ and D are already known. If we use adjacent minima, then the value for m is 1, no uncertainty. From Figure 1, the two adjacent minima nearest and to the right of the center peak are separated by a distance $x_{min} = 0.6 \pm 0.1$ cm. The uncertainty in x_{min} was determined from the difficulties in discerning true minima/maxima values from the Figures. Using $\lambda = 670$ nm (no uncertainty reported) and $D = 40.00 \pm 0.05$ cm, the slit width is:

$$b = 0.045 \pm 0.008 \text{ mm}$$

The theoretical slit width for the single slit is 0.04 mm (no uncertainty reported). This gives a percent error of 12.5 ± 0.5 %. This error is relatively high, however, the uncertainty in the experimental value for b allows it to overlap with the theoretical value.

For the known double slit, the parameters that needed to be calculated were the slit widths b and the slit spacings d using Equations 2 and 4 respectively. Using adjacent minima from the envelope in Figure 3 (the two minima to the right of the central peak), the separation distance is $x_{min} = 0.6 \pm 0.1$ cm. Using the same λ and D, the slit widths for the double slit are:

$$b = 0.045 \pm 0.008 \text{ mm}$$

The uncertainty in slit width is propagated with:

$$\delta b = b_{best}\sqrt{\left(\frac{\delta D}{D_{best}}\right)^2 + \left(\frac{\delta x}{x_{best}}\right)^2} \tag{8}$$

The theoretical slit width for the single slit is 0.04 mm. This again gives a percent error of 12.5 ± 0.5 %. Similarly, this is a high percent error but the values uncertainty is large enough to overlap with the theoretical. To determine the slit spacing d, multiple separation distances were determined from the 5 different maxima within the center envelope peak in Figure 3. Since the distances are all between adjacent maxima, $m = 1$ for all calculations in Equation 4. Table 2 below summarizes the analysis.

Maxima Used	Separation Distance, x_{max} (cm)	Slit Spacing, d (mm)
$0 \rightarrow 1$	0.20 ± 0.05	0.13 ± 0.03
$0 \rightarrow -1$	0.20 ± 0.05	0.14 ± 0.03
$1 \rightarrow 2$	0.20 ± 0.05	0.14 ± 0.03
$-1 \rightarrow -2$	0.20 ± 0.05	0.14 ± 0.03

Table 2: Determining Slit Spacing from Separation Distance Between Adjacent Maxima for KNOWN Double Slit. The left column is identifying which maxima in Figure 3 are being used to measure the separation distance. '0' corresponds to the center peak, '1' to the peak to the right, '-1' to the peak to the left, etc. The right column is the experimental slit spacing, determined from Equation 4. An average (best) value is given below with uncertainty. The center column is the distance between adjacent maxima. Its uncertainty is lower than that of adjacent minima because the peaks are sharper and more easily discernible from the plots.

The average of the four slit spacings in Table 2 produces a best value for d.

$$d = 0.14 \pm 0.03 \text{ mm}$$

The uncertainty in the best/average value for d is propagated from Equation 4 rather than using a statistical uncertainty because it is insignificant

$$\delta d = d_{best}\sqrt{\left(\frac{\delta D}{D_{best}}\right)^2 + \left(\frac{\delta x}{x_{best}}\right)^2} \tag{9}$$

The theoretical slit width for the double slit is 0.04 mm while the theoretical slit spacing is 0.125 mm. This gives a percent error of 12.0 ± 0.9 %. Once again, the uncertainty is large enough for the experimentally calculated value for d to overlap with its known theoretical value.

It is important to note that the first minimum in both the known single and double slit are similar due to the single slit envelope within the double slit plot. This is why both values for b are identical.

The exact same method was used to determine b and d for the unknown double slit. The same uncertainty propagations were applied. The only difference is that there are no theoretical parameters to compare to. However, it was determined before experimentation that the slit width b for the unknown double slit was noticeably larger than that of the known single and double slits. Using Figure 3, the separation distance between adjacent minima was $x_{min} = 0.51 \pm 0.1$ cm. The adjacent minima were taken to the left of the central peak as opposed to the right. This distance produces a slit width of:

$$b = 0.05 \pm 0.01 \text{ mm}$$

Uncertainty propagated using Equation 8. This slit width is noticeably larger than the width for the known slits, $b = 0.04$ mm. A similar table was constructed to determine d. See Table 3.

Maxima Used	Separation Distance, x_{max} (cm)	Slit Spacing, d (mm)
$0 \rightarrow 1$	0.10 ± 0.05	0.3 ± 0.1
$0 \rightarrow -1$	0.10 ± 0.05	0.3 ± 0.1
$1 \rightarrow 2$	0.09 ± 0.05	0.3 ± 0.1
$-1 \rightarrow -2$	0.10 ± 0.05	0.3 ± 0.1

Table 3: Determining Slit Spacing from Separation Distance Between Adjacent Maxima for UNKNOWN Double Slit. The same method was used as described for the known slit above and in Table 2. The distances between maxima for the unknown double slit are noticeably smaller, contributing to a larger slit spacing. The uncertainty in separation distances are equal to those in Table 2.

The average of the four slit spacings in Table 3 produces a best value for d.

$$d = 0.3 \pm 0.1 \text{ mm}$$

The uncertainty was propagated using Equation 9 rather than using statistical uncertainty.

Part 2: Diffraction Grating

The diffraction patterns for double slits and diffraction gratings are similar in overall shape. The maxima are produced at the same angles, but for the diffraction grating, the intensity is much sharper at maxima and it is much darker in between maxima.

For the laser beam going through the beam expander and diffraction grating, we reported the angle that produced this *first* ($n = 1$) sharp intensity. This angle $\theta = 22.5 \pm 0.5°$ corresponds to the angle in Equations 5 and 6. Therefore, if we want to compare the values for the diffraction spacing N, all that is needed to do is:

$$N = \frac{\sin\theta}{\lambda} \tag{10}$$

Since the wavelength of the laser beam is still $\lambda = 670$ nm with no uncertainty, the experimental value for the diffraction spacing is:

$$N = 570 \pm 10 \text{ lines/mm}$$

The uncertainty is propagated as:

$$\delta N = \frac{\delta\theta \cos\theta}{\lambda} \tag{11}$$

Note that the uncertainty in θ needed to be converted to radians before determining δN. The theoretically reported value for N is 600 lines/mm with no uncertainty. The percent error between experimental and theoretical (using Equation 7) is thus $5 \pm 1\,\%$. The values do not overlap considering uncertainty but the magnitude of the percent error is satisfyingly low.

For the beam of white light going straight through the grating, the white light was dispersed into a color spectrum at angles according to Table 1. The angles were measured to the first intensity spike, therefore again, $n = 1$. Experimentally determined wavelengths can be determined by solving Equation 10 for λ and using the theoretical value for N. To determine the bandwidth frequency for visible light, we converted the maximum and minimum wavelengths of light (corresponding to the upper bound of red and lower bound of blue light, respectively) to frequencies using:

$$f = \frac{c}{\lambda} \tag{12}$$

Where $c = 3 \times 10^8$ m/s is the speed of light. Table 4 summarizes the wavelengths and corresponding frequencies for the three colors analyzed within the spectrum.

Color	Wavelength, λ (nm)	Frequency, f (THz)
RED	650 ± 30	460 ± 20
GREEN	540 ± 30	550 ± 30
BLUE	460 ± 30	650 ± 40

Table 4: Experimentally Determined Wavelengths and Frequencies for Red, Green, and Blue Visible Light. Wavelengths were determined using the best angles in Table 1 and Equation 10 with $N = 600$ lines/mm. Frequencies were determined using Equation 12. The values fall within the generally accepted ranges for these colors. Note that the frequencies in the table for red and blue light do not indicate the upper and lower bounds of bandwidth (see below). Uncertainties derived below.

$$\delta\lambda = \frac{\delta\theta \cos\theta}{N} \tag{13}$$

$$\delta f = f_{best}|-1|\frac{\delta\lambda}{\lambda_{best}} \tag{14}$$

To determine the bandwidth for visible light, we determined the frequencies for the *most* red light using the largest angle in its range from table 1 ($24.0 \pm 0.5°$) and for the *most* blue light using the smallest angle in its range from table 1 ($15.0 \pm 0.5°$). Thus, the bandwidth ranges from:

$$(440 \pm 10) - (700 \pm 20) \text{ THz}$$

Part 3: Diffraction of Light Through Hair

The thickness of the strand of hair is analogous to the slit width b of a single slit. Thus, with the measured separation distance between maxima, we can determine the thickness l of the hair by substituting l for b and L for D in Equation 2, which will read:

$$l = \frac{m\lambda L}{x_{max}} \tag{15}$$

Since we are still measuring adjacent maxima, $m = 1$. Wavelength is again 670 nm for the laser beam. $L = 100.00 \pm 0.05$ cm is the distance between the hair and the diffraction pattern. The separation distance x_{max} as opposed to x_{min} is used. x_{max} was recorded in experimental results ($= 0.45 \pm 0.05$cm). The uncertainty is thus:

$$\delta l = l_{best}\sqrt{\left(\frac{\delta L}{L_{best}}\right)^2 + \left(\frac{\delta x}{x_{best}}\right)^2} \tag{16}$$

The thickness of the hair was determined to be:

$$l = 0.15 \pm 0.02 \text{ mm}$$

This value seems reasonably small enough to be the thickness of a single strand of hair.

Conclusion

The overall goal of this lab was to observe the different diffraction properties of light using various tools. We were able to verify the validity of Equations 2 and 4 by comparing the experimental and theoretical slit widths and spacings for known single and double slits. For the known single slit, the experimentally determined slit width was $b = 0.045 \pm 0.008$ mm which, with its uncertainty, overlaps with the given value for this width with a 12.5 ± 0.5 % error. As for the experimental slit width of the known double slit, the value came out to be exactly the same as that for the single slit with the same percent error. For the slit spacing, we determined $d = 0.14 \pm 0.03$ mm corresponding to a percent error of 12.0 ± 0.9 %. Although the percent errors indicate these experimentally determined values are not completely accurate, the uncertainties overlap with the known parameters, which allowed us to verify the validity of Equations 2 and 4.

Since Equations 2 and 4 held under the given experimental conditions, they could be used to find the slit width and spacing of an unknown double slit. These values were $b = 0.05 \pm 0.01$ mm and $d = 0.3 \pm 0.1$ mm. There were no values to directly compare these to, however it was known the slit width for this unknown double slit was larger than the known by inspection. The results agreed with this inspection.

Any considerable errors from this portion of the lab most likely arose from the perturbations in light intensity being read by the photometer or the difficulties in determining positions of minima/maxima on the plots. These errors could be avoided by conducting the experiment in a more light-deprived environment.

The next goal was to verify the diffraction spacing N of a known diffraction grating with $N = 600$ lines/mm. After directly measuring the angles between adjacent intensity maxima, the experimental value for N was calculated to be 570 ± 10 lines/mm, giving a percent error of 5 ± 1 %. This value appears both accurate and precise; the percent error is low and the uncertainty is small compared to N. But the uncertainty does not overlap with the theoretical value. This could be attributed to the error arising from the non-permanent positioning of the protractor. The protractor was held by hand up against the diffraction grating and needed to be held perfectly horizontal. Thus, its position was most likely not perfect for measuring the beam angles. Despite this, we concluded that the experiment did in fact verify the diffraction spacing of the grating. A possible way to eliminate this error would be to fix the protractor at the ideal horizontal position to better measure the angles.

For the dispersion of white light through the diffraction grating, the goal was to obtain experimental values for wavelengths and frequencies of red, green, and blue visible light, and to calculate a bandwidth (range of frequencies) for the entire spectrum of visible light. The wavelengths and frequencies found in Table 4 fall within the range of generally accepted wavelengths and frequencies for the respective colors. The bandwidth was calculated to be $(440 \pm 10) - (700 \pm 20)$ THz for visible light, which makes sense logically.

The goal of the final part of the experiment was to simply determine the thickness (width) of a single strand of hair. Using the same procedure used to determine the slit width for a single slit (while replacing appropriate values), the thickness of the hair was determined to be $l = 0.15 \pm 0.02$ mm. This value seems to be reasonable, however there is no value to truly compare it to because the thickness of hair varies. It seems small enough to be an accurate value for the true thickness, and the small uncertainty points to preciseness of the measurements. To produce more accurate results, the positions of maxima could have been recorded and measured using the photodetector instead of eyeballing with a pencil and ruler.

References

[1] Campbell, W.C. *et al*. Physics 4BL: Mechanics Lab Manual (ver. August 31, 2018). (University of California Los Angeles, Los Angeles, California).