# EXPERIMENT 2: LORENTZ FORCE

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LAB SECTION: THURSDAY 2PM

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# Worksheet

1. Measurement of $e/m$
   (a) When the beam of electrons is bent into a circle, the magnetic field $B$ points outwards towards us. The beam is moving in a counterclockwise circular motion. The charge of the electrons is negative. According to the right-hand rule, the magnetic field must point out to propel the electrons counterclockwise.
   (b) When magnetizing current is held constant and Accelerating Voltage is increased, the Radius of the circular electron beam increases, and vice versa. When Accelerating Voltage is held constant and Magnetizing Current is increased, the Radius decreases, and vice versa.
   (c) When the magnetic field is not perpendicular to the electron velocity, the shape of the beam becomes helical; the end of the electron beam is offset inwards or outwards from the beginning, creating a velocity component parallel to the magnetic field. If the electrons have both a parallel and a perpendicular velocity $\mathbf{v} = \mathbf{v}_{||} + \mathbf{v}_{\perp}$, the magnetic part of the Lorentz force equation $\boldsymbol{F}_e = -e(\mathbf{v} \times \mathbf{B})$ is altered. When $\mathbf{v} = \mathbf{v}_{||}$ (no parallel component) the cross product simply becomes $-\mathrm{v} \cdot \mathrm{B}$. When $\mathbf{v} = \mathbf{v}_{||} + \mathbf{v}_{\perp}$ (with parallel component) the cross product becomes $-\mathrm{v}_{\perp} \cdot \mathrm{B}$ which is less in magnitude. So, the overall magnetic force $\boldsymbol{F}_e$ decreases in magnitude with the introduction of a velocity component parallel to $\mathbf{B}$.
   (d) Starting from Equation 2.2 in the Manual,

$$evB = \frac{mv^2}{R} \tag{1}$$

The energy of the electron $eV$ can be written as:

$$eV = \frac{1}{2}mv^2 \text{ or } v = (\frac{2eV}{m})^{1/2} \tag{2}$$

Eliminating a velocity from both sides of Equation 2.2 and plugging in Equation 1:

$$eB = \frac{m}{R}(\frac{2eV}{m})^{1/2} \tag{3}$$

Lastly, solving for $e/m$ results in Equation 2.3:

$$\frac{e}{m} = \frac{2V}{B^2R^2} \tag{4}$$

2. Diodes
   (a) The transformer secondary signal was 33.4±0.5 V while the voltage rating was 12 V. The RMS Voltage of this signal was 11.7±0.5 V. It appears that the Voltage Rating corresponds more to the RMS Voltage than to the transformer secondary signal.
   (b) The DC Level (RMS Voltage) for the full-wave rectifier DC Power Supply was 11.3±0.5 V while the magnitude of the ripple was about 0.32±0.05 V.

# Introduction

The purpose of this lab was to determine the charge to mass ratio of the electron $e/m$ using two different methods and compare them to each other and to the experimentally accepted value. To do this, we mimicked J.J. Thompson's experiment using a Cathode Ray Tube (CRT) to shoot a beam of electrons in a circular path. When a Helmholtz Coil is set up above the beam in the CRT, a magnetic field B is induced in the CRT coming outwards towards us with magnitude:

$$B = \frac{8\mu_0 IN}{5\sqrt{5}R_c} \tag{1}$$

Where $I$ is the current running through the coil, $N = 130.00 \pm 0.01$ is the number of turns in the coil, $\mu_0 = 4\pi \cdot 10^{-7}$ Tm/A is the Vacuum Permeability, and $R_c = 0.150 \pm 0.005$m is the coil radius. The magnetic field causes the electron beam to travel in a counterclockwise circular pattern according to the magnetic portion of the Lorentz Force Equation:

$$|\boldsymbol{F}_e| = |-e(\mathbf{v} \times \mathbf{B})| = evB \tag{2}$$

Force is always perpendicular to velocity, hence circular motion. Since motion is circular, $|\boldsymbol{F}_e|$ can be equated to a centripetal force:

$$F_c = \frac{mv^2}{R} \tag{3}$$

Where $m$ is the electron's mass, $v$ is its velocity, and $R$ is the Radius of the circular beam.

$$evB = \frac{mv^2}{R} \tag{4}$$

The CRT has a Kent MODEL TG-13 base, which is connected to a 1601 Regulated Power Supply and a PASCO SF-9585A high voltage power supply to run a current $I$ through the Helmholtz coils and an Accelerating Voltage $V$ to the electrons, respectively. Thus, the Kinetic Energy of the electrons can be written as:

$$E = eV = \frac{1}{2}mv^2 \tag{5}$$

From these Equations, we derive Equation 2.3 from the Lab Manual (see Worksheet above)

$$\frac{e}{m} = \frac{2V}{B^2R^2} \tag{6}$$

Using the CRT base to vary the current and accelerating voltage, the diameter of the beam is measured, and from this the radius $R$ is determined. Equation (6) is the first method we used to determine the charge to mass ratio. Equation (6) can be rewritten as:

$$V = \frac{e}{m}\left(\frac{B^2R^2}{2}\right) \tag{7}$$

Which takes the form $y = mx + b$ where $b = 0$ in theory. The second method of determining charge to mass ratio is to run a linear regression on $V$ as a function of $B^2R^2/2$ using different $V$, $B$, and $R$ values and calculate the slope of the best-fit line to this data. This value will represent the charge to mass ratio $e/m$.

## Experimental Description and Results

To measure the value of $e/m$, several different variables needed to be collected. These included: Accelerating Voltages $V_{accel}$ (will be referred to as simply $V$), Magnetizing Currents $I_{coils}$ (will be referred to simply as $I$), and electron beam radii $R$. For the experimental setup, the HELMHOLTZ COILS input on the CRT base was connected to the 1601 Regulated Power Supply and the Multimeter in Series, and the HEATERS input on the CRT base was connected to the PASCO SF-9585A high voltage power supply. By tuning the 500V ADJUST knob and the CURRENT ADJ knob on the CRT base, various combinations of $V$ and $I$ could be attained. In total, we collected nine different combinations of data for these variables and measured the corresponding Diameter of the electron beam using a meterstick. Halving the Diameter gave the Beam Radii $R$. Table 1 has all recorded data. In addition to these collected variables, nine values for the magnetic field $B$ were calculated using Equation (1) (See Equation 2.4 in Manual). Two methods were used to calculate $e/m$: 1) Simply using Equation (6) (See Equation 2.3 in Manual) and 2) running a linear regression of $V_{accel}$ as a function of $B^2R^2/2$ (See Figure 1). See Figure 1 and Analysis Section for further details.

| Accelerating Voltage (V) | Magnetizing Current (A) | Electron Beam Radius (m) | e/m values (C/kg) |
|---|---|---|---|
| 158±1 | 1.795±0.005 | 0.0175±0.0005 | $(5.3\pm0.6)\times10^{11}$ |
| 108±1 | 1.792±0.005 | 0.0168±0.0005 | $(3.9\pm0.5)\times10^{11}$ |
| 195±1 | 1.792±0.005 | 0.0215±0.0005 | $(4.3\pm0.4)\times10^{11}$ |
| 195±1 | 1.016±0.005 | 0.0368±0.0005 | $(4.6\pm0.3)\times10^{11}$ |
| 195±1 | 1.398±0.005 | 0.0276±0.0005 | $(4.3\pm0.3)\times10^{11}$ |
| 127±1 | 1.399±0.005 | 0.0233±0.0005 | $(4.0\pm0.3)\times10^{11}$ |
| 103±1 | 1.003±0.005 | 0.0311±0.0005 | $(3.5\pm0.2)\times10^{11}$ |
| 152±1 | 1.400±0.005 | 0.0255±0.0005 | $(3.9\pm0.3)\times10^{11}$ |
| 171±1 | 1.796±0.005 | 0.0193±0.0005 | $(4.7\pm0.5)\times10^{11}$ |

**Table 1: Accelerating Voltage, Magnetizing Current, and Resulting Electron Beam Radius and Charge to Mass Ratio Values for nine different combinations of Voltage and Current.** On the CRT base, Accelerating Voltage Values were varied in the range 100-200V, while Current Values were varied in the range 1-2A. Nine unique combinations were made and recorded, and the Diameter of the circular beam was measured, divided by 2, and the resulting radius was recorded for each case. After calculating the Magnetic Fields, the charge to mass ratio $e/m$ was determined with Equation (6). This process represents method 1. An average value for $e/m$ is taken in the Analysis Section. Note: All uncertainties explained/derived in Analysis.

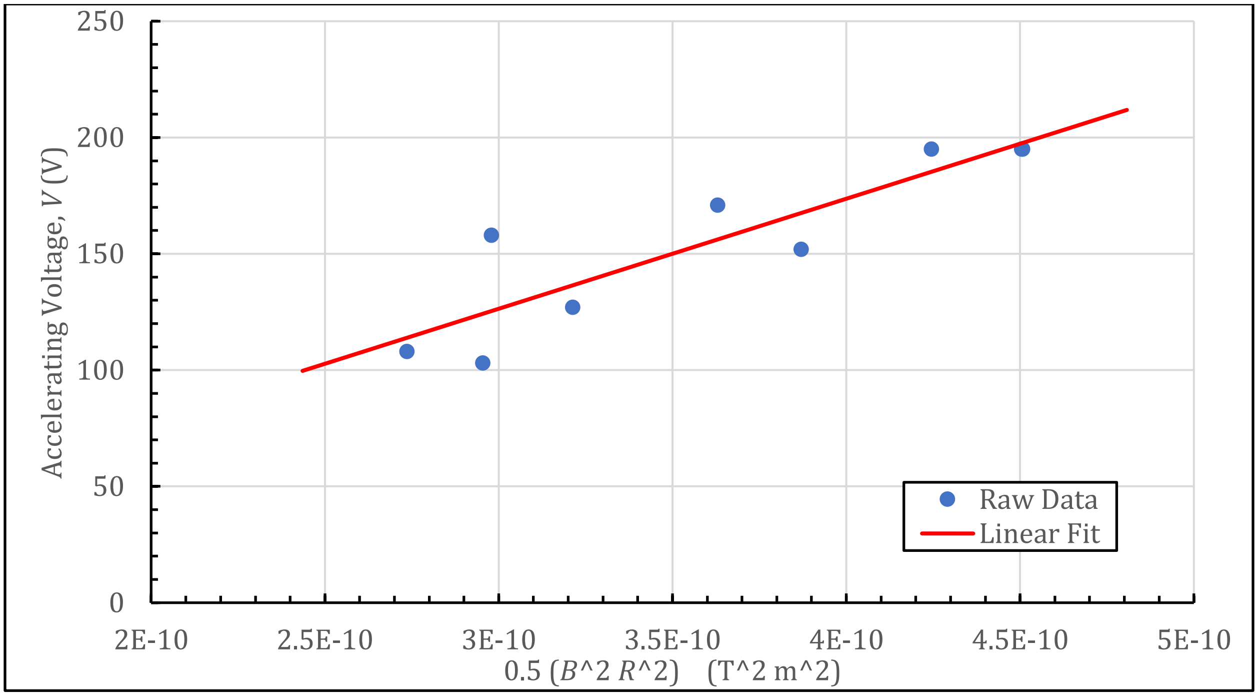


**Figure 1: Determination of Charge to Mass Ratio of the Electron using Linear Regression.** The nine different data configurations from Table 1 were used to calculate nine different values for Accelerating Voltage, $V$ and the value $0.5B^2R^2$. The plot is the Voltage $V$ as a function of $0.5B^2R^2$ for the various combinations of Voltage and Current. The blue circles are the raw data and the red line is a linear fit taking the form $V = a(0.5B^2R^2) + b$ where $a = (4.7 \pm 0.9) \times 10^{11}$ C/kg and $b = (-2 \pm 3) \times 10^1$ V. The slope $a$ represents the charge to mass ratio, therefore $a = e/m$.

## Analysis

The following are the values for the charge to mass ratio $e/m$ for an electron that were determined using the two different methods, along with the calculations required to determine them.

**Method 1:**

| Trial | e/m Value (C/kg) |
|---|---|
| 1 | $(5.3\pm0.6) \times 10^{11}$ |
| 2 | $(3.9\pm0.5) \times 10^{11}$ |
| 3 | $(4.3\pm0.4) \times 10^{11}$ |
| 4 | $(4.6\pm0.3) \times 10^{11}$ |
| 5 | $(4.3\pm0.3) \times 10^{11}$ |
| 6 | $(4.0\pm0.3) \times 10^{11}$ |
| 7 | $(3.5\pm0.2) \times 10^{11}$ |
| 8 | $(3.9\pm0.3) \times 10^{11}$ |
| 9 | $(4.7\pm0.5) \times 10^{11}$ |

**Table 2: Charge to Mass Ratio Values from Table 1 for Nine Trials.** This data was directly taken from the $e/m$ values in Table 1. The following derivations/calculations explain how the individual uncertainties were determined, along with an overall best value for $e/m$ with its uncertainty.

The nine different values for $e/m$ in Table 2 (calculated using Equation 6) were averaged to determine $e/m_{\text{best}}$. The error in $e/m$, or $\delta(e/m)$, was calculating using $\delta\left(\frac{e}{m}\right) = \frac{\sigma_{e/m}}{\sqrt{N}}$ where $\sigma_{e/m}$ is the standard deviation of the nine values and $N = 9$ is the total number of values.

$$\frac{e}{m} = (4.3 \pm 0.2) \times 10^{11}\ \text{C/kg}$$

The propagation for the error in each individual $e/m$ value in Table 2 is as follows. Starting with Equation 6, we can rewrite it as $\frac{e}{m} = 2(\frac{V}{xy})$ where $x = B^2$ and $y = R^2$. Therefore, the uncertainty is

$$\delta\left(\frac{e}{m}\right) = 2 \cdot \left(\frac{e}{m}\right)_{\text{best}} \sqrt{\left(\frac{\delta V}{V_{\text{best}}}\right)^2 + \left(\frac{\delta x}{x_{\text{best}}}\right)^2 + \left(\frac{\delta y}{y_{\text{best}}}\right)^2} \tag{8}$$

It is easy to see that $\frac{\delta x}{x_{\text{best}}} = 2\frac{\delta B}{B_{\text{best}}}$ and $\frac{\delta y}{y_{\text{best}}} = 2\frac{\delta R}{R_{\text{best}}}$. To calculate $\frac{\delta B}{B_{\text{best}}}$, we need Equation (1)

$$\frac{\delta B}{B_{best}} = \frac{8\mu_0}{5\sqrt{5}} \sqrt{\left(\frac{\delta I}{I_{\text{best}}}\right)^2 + \left(\frac{\delta N}{N_{\text{best}}}\right)^2 + \left(\frac{\delta R_c}{R_{c_{\text{best}}}}\right)^2} \tag{9}$$

Putting it all together, we get

$$\delta\left(\frac{e}{m}\right) = 2 \cdot \left(\frac{e}{m}\right)_{\text{best}} \sqrt{\left(\frac{\delta V}{V_{\text{best}}}\right)^2 + \left(\frac{16\mu_0}{5\sqrt{5}}\right)^2 \cdot \left[\left(\frac{\delta I}{I_{\text{best}}}\right)^2 + \left(\frac{\delta N}{N_{\text{best}}}\right)^2 + \left(\frac{\delta R_c}{R_{c_{\text{best}}}}\right)^2\right] + 4 \cdot \left(\frac{\delta R}{R_{\text{best}}}\right)^2} \tag{10}$$

$\delta V = \pm 1$ V, $\delta I = \pm 0.005$ A, $\delta N = \pm 0.01$ Turns, $\delta R_c = \pm 0.005$ m, and $\delta R = \pm 0.0005$ m.

$\delta V$ and $\delta I$ were determined by inspection on the equipment, $\delta N$ and $\delta R_c$ were given, and $\delta R$ is the error in the meterstick. Equations used to propagate this uncertainty were Equations A12,22,24,25.

**Method 2:**

The Linear Regression performed on $V$ as a function of $0.5B^2R^2$ represented in Figure 1 produced a slope $a = (4.7 \pm 0.9) \times 10^{11}$ C/kg and an intercept $b = (-2 \pm 3) \times 10^1$ V. It is important to note that the intercept *is* consistent with zero, i.e. the uncertainty overlaps the value with zero. This should be the case, for if $B = 0$ or $R = 0$, then there would be no Accelerating Voltage $V$. As noted in the introduction, the slope $a$ of the best fit line represents the charge to mass ratio. Therefore:

$$\frac{e}{m} = (4.7 \pm 0.9) \times 10^{11}\ \text{C/kg}$$

**Comparison**

Both values for $e/m$ using Methods 1 and 2 are not consistent with the accepted (theoretical) value for the charge to mass ratio for the electron: $1.759 \times 10^{11}$ C/kg. However, when compared to each other, their uncertainties do overlap, and the values are quite similar. Below are calculations for percent error between Methods 1 or 2 and the theoretical value.

$$\% \text{ error} = \left|\frac{\text{Experimental} - \text{Theoretical}}{\text{Theoretical}}\right| \times 100\%$$

Method 1: $\%\ \text{error} = (140 \pm 10)\ \%$

Method 2: $\%\ \text{error} = (170 \pm 50)\ \%$

These percent error values are extremely high (reasons in conclusion). Nevertheless, Method 1 seems to be more accurate than method 2.

## Conclusion

The goal of this experiment was to obtain a value for the charge to mass ratio of the electron using a Cathode Ray Tube to shoot a beam of electrons in a circular path under the influence of a uniform Magnetic Field. The value of $e/m$ was gathered using two different methods: directly using Equation (6), and through a linear regression. Not only did we want to obtain the values, but we wanted to show that they were similar to each other and to the theoretical value. Unfortunately, neither of the values calculated proved to be consistent with the theoretical value of $1.759 \times 10^{11}$ C/kg. Based on the percent errors, the value of $e/m$ was determined more accurately through Method 1 ($140 \pm 10\%$ as opposed to $170 \pm 50\%$). On the other hand, the results of Method 1 ($e/m = (4.3 \pm 0.2) \times 10^{11}$ C/kg) and the results of Method 2 ($e/m = (4.7 \pm 0.9) \times 10^{11}$ C/kg) are consistent with one another. These results imply that the calculations and analysis on the data were satisfactory, but there must have been random/systematic error when collecting the data to account for the large percent errors. For example, when measuring the Diameter of the electron beam, it was difficult to obtain an accurate reading with the meterstick. The CRT is spherical; therefore, the alignment and positioning were awkward and most likely inconsistent from measurement to measurement. This represents systematic error in the data for the Radius, which has a major impact on both values for $e/m$. Other Errors include random error in all of the equipment used, which is most probably miniscule compared to the error in Diameter measurement.

References

[1] Campbell, W.C. *et al*. Physics 4BL: Mechanics Lab Manual (ver. August 31, 2018). (University of California Los Angeles, Los Angeles, California).