EXPERIMENT 3: AC CIRCUITS

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LAB SECTION: THURSDAY 2PM

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Worksheet

1. Equation 3.16 in the manual relates the Gain of the RLC circuit to the Resistance R, inductance L, capacitance C, and frequency ω:
$$G(\omega) = \frac{R}{R + i\left(\omega L - \frac{1}{\omega C}\right)} \tag{1}$$
Gain is maximized at 1, or when $\omega L - \frac{1}{\omega C} = 0$. Thus, the resonant frequency is:
$$\omega_{\text{res}} = \frac{1}{\sqrt{LC}} \tag{2}$$
Since the resonant frequency in this setup is out of range (too high), we need to increase the denominator of Equation 2 to decrease the resonant frequency to put it in our range. The most obvious answer would be to use an inductor/capacitor with a higher inductance/capacitance. Since we are already using the maximum for both, we could add inductors and capacitors into the circuit to increase the overall inductance and capacitance. Inductors add like resistors, so we should add inductors in series to the circuit. Capacitors add opposite to resistors, so we should add capacitors in parallel to the circuit. This would have the overall effect of decreasing the resonant frequency to put it in the BODE analyzers frequency range of 20 Hz to 20 kHz.
2. The resonant frequency (in Hertz) is defined as:
$$f_{\text{res}} = \frac{1}{2\pi\sqrt{LC}} \tag{3}$$
If we have a range of resonant frequencies of 88 MHz to 108 MHz and Inductance $L = 1$ mH, then we can solve Equation 3 for C to determine the minimum and maximum Capacitance and thus the range of Capacitance needed to receive the frequencies in the given range.
$$C = \frac{1}{L}\left(\frac{1}{2\pi f_{\text{res}}}\right)^2 \tag{4}$$
When $f_{\text{res}} = 88$ MHz, $C = 3.27 \cdot 10^{-15}$ F and when $f_{\text{res}} = 108\text{MHz}, = 2.17 \cdot 10^{-15}$ F. The range of Capacitance is $(2.17 \cdot 10^{-15} - 3.27 \cdot 10^{-15})$ F.
3. We are interested in receiving a photometer output frequency of 3 Hz, which is very low compared to the residual noise coming from the outlet. Therefore, we want low frequencies to pass through and cause the higher frequencies to attenuate. Thus, a low-pass filter should be used. The cutoff frequency ω_{c} and f_{c} are modeled as:
$$\omega_{\text{c}} = (RC)^{-1} \tag{5}$$
$$f_{\text{c}} = (2\pi RC)^{-1} \tag{6}$$
Capacitance is then:
$$C = (2\pi R f_{\text{c}})^{-1} \tag{7}$$
If $R = 1\ \text{k}\Omega$ and the frequency we are concerned with is 3 Hz, then we should use a Capacitor with Capacitance slightly lower than $C = 53.1\ \mu\text{F}$ so that the cutoff frequency is slightly higher than 3 Hz.
If we have a noisy signal where we are only interested in a frequency of 1 GHz, we should pass the frequencies first through a low-pass filter to set the upper bound at 1.2GHz, and then send the remaining frequencies through a high-pass filter to set the lower bound at 0.8GHz. These values are the cutoff frequencies for the filters. For the low-pass, $f_{\text{c}} = 1.2$ GHz when component values $R = 1000\ \Omega$ and $C = 1.33 \cdot 10^{-13}$ F. For the high-pass, $f_{\text{c}} = 0.8$ GHz when component values $R = 1000\ \Omega$ and $C = 1.99 \cdot 10^{-13}$ F.

Introduction

The purpose of this experiment was to construct three functional AC Circuits: RC, RL, and RLC. For the RC and RL circuits, the main goal was to determine the transient time τ using myDAQ to construct a graph for the RC circuit and using the oscilloscope to estimate it for the RL circuit.

The equations for transient time for an RC and RL circuit respectively are:

$$\tau = RC \tag{1}$$

$$\tau = L/R \tag{2}$$

For the RLC Circuit, the main goal was to determine the resonant frequency f_{res} with two methods: (1) using the oscilloscope to estimate the frequency and (2) using the BODE Analyzer to create a plot to find the resonant frequency. These values for f_{res} are then compared to the theoretical value of resonant frequency calculated using:

$$f_{\text{res}} = \frac{1}{2\pi\sqrt{LC}} \tag{3}$$

The other goal of the RLC circuit was to calculate the Q-factor using Equation 3.18 in the manual:

$$Q = \frac{f_{\text{res}}}{f_2 - f_1} \tag{4}$$

See Analysis for further details.

The RC circuit was constructed by connecting a $1000 \pm 50\ \Omega$ Resistor and a $1.0 \pm 0.1\ \mu\text{F}$ Capacitor in series while using an AC Power Source provided by a Rigol DG1022 Function/Arbitrary Waveform Generator. Using myDAQ, the Input Waveform V_b and Voltage Across the Capacitor $V(t)$ were acquired over time t. Through manipulation of these variables (See Analysis), a linear graph was constructed from which τ was determined. Note that the Function Generator supplied Power at 500.000 ± 0.001 Hz.

The RL circuit was constructed by connecting the same Resistor and a large coil of copper wire (1500 turns) in series with power supplied from the same AC source. Instead of myDAQ, the Voltage drop across the Resistor was acquired using Channel 1 of the Oscilloscope. Transient time τ was determined from analysis on oscilloscope (See Analysis). The Inductance L is then found using Equation 2. Note that the Function Generator supplied Power at 500.000 ± 0.001 Hz.

The RLC circuit was constructed similarly using the same Resistor, the same Capacitor, and the same coil used in the RL circuit (1500 turns) connected in series with power supplied from the same AC source. For the first method, Channel 1 on the oscilloscope measured the Voltage drop across the resistor (V_{out}) while Channel 2 measured the function generator signal. The waveform generator's driving frequency was then adjusted to observe the maximum possible V_{out} on the scope. This frequency was recorded as the resonant frequency for Method 1. For the second method, we used the BODE Analyzer software to plot the Gain versus frequency. The resonant frequency is the frequency at which gain is maximized.

Analysis

RC Circuit

The Voltage across a Capacitor can be modeled by Equation 3.3 in the Lab Manual:

$$V(t) = V_b\left(1 - e^{-t/RC}\right) \qquad (5)$$

Where V_b is the constant input waveform Voltage. We determined this value to be 2.93 ± 0.05 V (maximum on myDAQ). Equation 5 can be rearranged and *Linearized* to take the more convenient form:

$$\ln\left(\frac{V_b - V(t)}{V_b}\right) = -\frac{t}{RC} \qquad (6)$$

which takes the functional form of a line $y = mx + b$ with slope $m = -1/RC$ and intercept $b = 0$. A plot of this function is shown below as Figure 1.

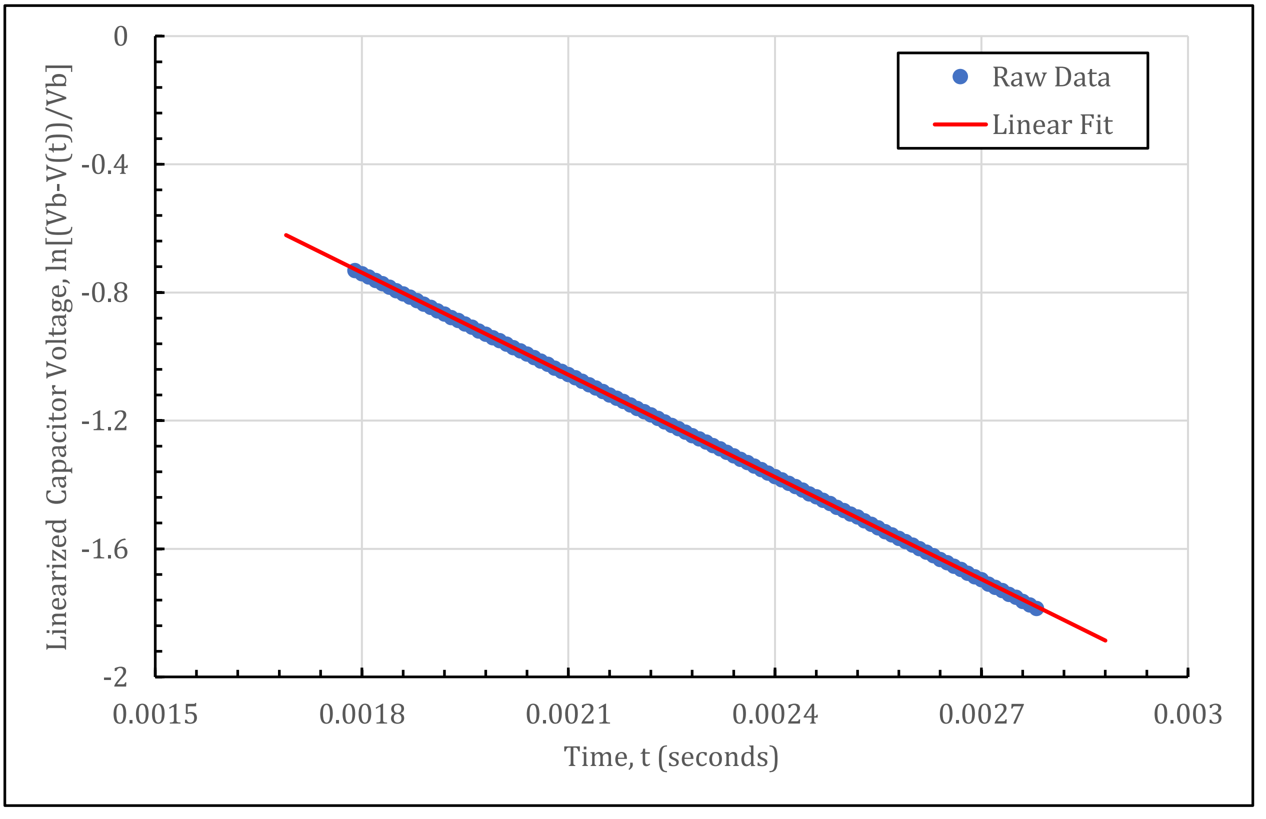

Figure 1: Determination of transient time from Linearized RC Transient Data. For the RLC circuit, input Voltage V_b and the Voltage across the Capacitor $V(t)$ were recorded over time using myDAQ. The data points above are from a single transient time period. The plot is the left side of Equation 6 as a function time. The blue circles are the raw data and the red line is the linear fit taking the form $\ln([V_b - V(t)]/V_b) = at + b$, where $a = (-1063.4 \pm 0.8)$ Hz and $b = (1.176 \pm 0.002)$. From Equation 6, it is apparent that the slope $a = -1/RC$.

The transient time for an RC circuit is given by Equation 1. Therefore, since the slope $a = -1/RC$, the transient time is:

$$\tau = -\frac{1}{a} = -(a)^{-1} \tag{7}$$

To reiterate, the functional form of the linearized transient state data is a line with slope $-1/RC$. Using Equation 7, the experimental value for transient time for the RC circuit is

$$\tau = 0.9404 \pm 0.0007 \text{ ms}$$

Propagation of uncertainty for this transient time was derived using Equation A.25 in the Manual:

$$\delta\tau = \tau_{\text{best}}|-1|\frac{\delta a}{a_{\text{best}}} \tag{8}$$

The theoretical transient time for the RC circuit should be $\tau = RC$ (Equation 1). With the Resistance and Capacitance given,

$$\tau = 1.0 \pm 0.1 \text{ ms}$$

The uncertainty for this theoretical transient time (using Equation A.24) is:

$$\delta\tau = \tau_{\text{best}}\sqrt{\left(\frac{\delta R}{R_{\text{best}}}\right)^2 + \left(\frac{\delta C}{C_{\text{best}}}\right)^2} \tag{9}$$

To compare measured and theoretical transient times, we calculated the percent error:

$$\%\ error = \left|\frac{Experimental - Theoretical}{Theoretical}\right| \times 100\% \tag{10}$$

$\%\ error = (6.0 \pm 0.5)\ \%$. The values are quite similar as the percent error is considerably low.

RL Circuit

The transient time for an RL circuit can be defined as the time it takes the inductor's current to reach 63.2% of the maximum current V_b/R. Since we used an oscilloscope to measure Voltage (and not current), the transient time can also be calculated as the time it takes the Voltage across the Resistor to reach 63.2% of V_b. This holds because of Ohm's Law. The peak-to-peak value of the Voltage across the Resistor was measured as 1.92$\pm$0.05 V, which is the value for V_b. 63.2% of V_b is 1.21$\pm$0.03 V. Since the Voltage at time $t = 0$ seconds was 0.96$\pm$0.05 V (and not zero), the transient time was the time at which the Voltage across the Resistor was 0.96$\pm$0.05 V plus 63.2% of V_b. This value is 2.16$\pm$0.06 V. The corresponding transient time was estimated to be:

$$\tau = 40.0 \pm 0.5\ \mu\text{s}$$

The error was approximated from the cursor tool on the oscilloscope. To determine a value for the inductance L, we can use $L = R\tau$ since we know $R = 1000 \pm 50\ \Omega$. Therefore, the inductance (whose value will be used later to calculate theoretical resonant frequency for the RLC circuit) is:

$$L = 40 \pm 2 \text{ mH}$$

The uncertainty propagation for inductance was derived using Equation A.24 in the Manual:

$$\delta L = L_{\text{best}} \sqrt{\left(\frac{\delta R}{R_{\text{best}}}\right)^2 + \left(\frac{\delta \tau}{\tau_{\text{best}}}\right)^2} \tag{11}$$

RLC Circuit

Method 1: Oscilloscope

The Waveform Drive Frequency that produced the maximum V_{out} was:

$$f_{\text{res},1} = 830 \pm 50 \text{ Hz}$$

Method 2: BODE Analyzer

The BODE Analyzer plotted the magnitude of the Voltage response gain $|V_{out}/V_{in}|$ as a function of logarithmic frequency. V_{out} is the Voltage across the Resistor while V_{in} is the Voltage across the entire circuit. Figure 2 is a recreation of the BODE Analyzer plot.

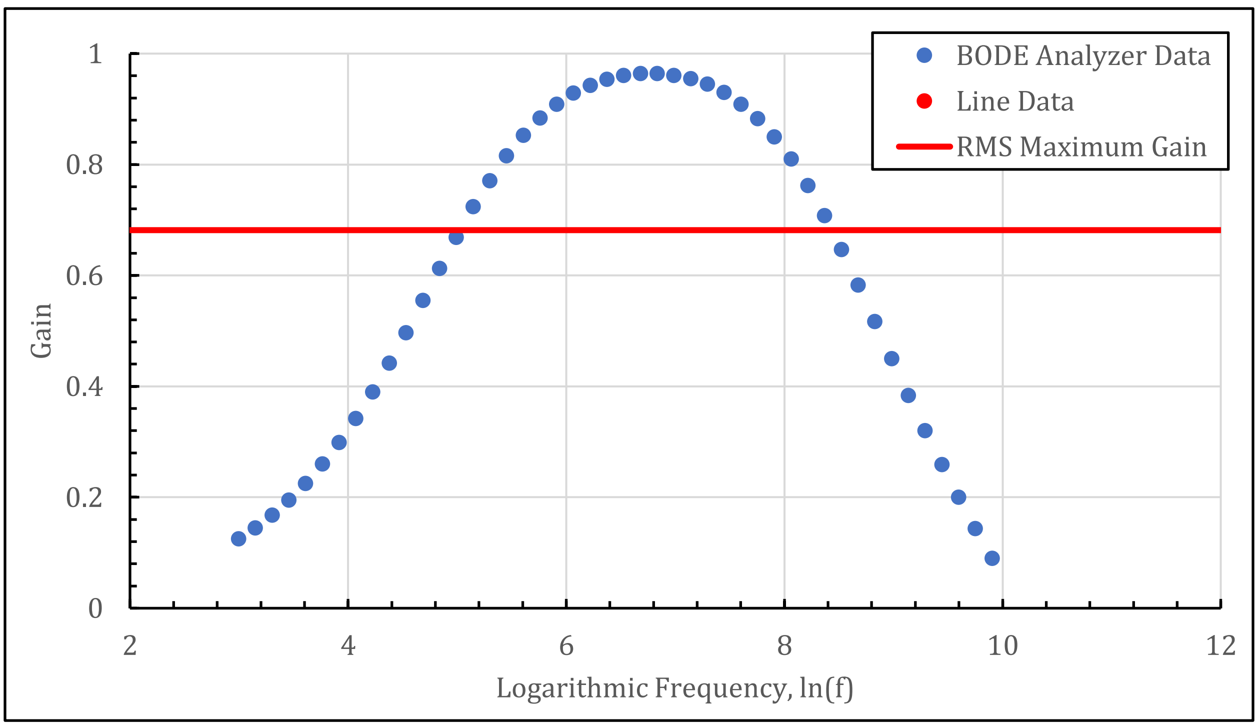

Figure 2: Determination of Resonant Frequency and Q-factor from BODE Analyzer Plot. Data for the Gain and Frequency (in Hertz) were copied from the BODE Analyzer Plot. The plot is Gain as a function of logarithmic Frequency. The blue points are the raw data for different frequencies. The red line represents the value at which the gain is $1/\sqrt{2}$ of its maximum value. Maximum Gain was estimated to be 0.964 ± 0.001 and $1/\sqrt{2} \cdot \text{Max Gain}$ is 0.6817 ± 0.0007. The resonant frequency is the frequency at which Gain is maximized. From the data, $f_{\text{res},2} = 820 \pm 50$ Hz. Uncertainty propagated from estimation using data and plot.

The functional form of the plot in Figure 2 appears to be Gaussian-like because the x-axis is logarithmic frequency as opposed to simply frequency.

Theoretical Resonant Frequency for the RLC circuit can be determined using Equation 3 since we know the Capacitance and Inductance in the circuit:

$$f_{\text{res,t}} = 800 \pm 40 \text{ Hz}$$

The uncertainty propagation for theoretical resonant frequency was derived using Equation A.24 and A.25 in the Manual. If we rewrite Equation 3 as $f_{\text{res}} = A(x)^{-1/2}$ where $A = 1/2\pi$ and $x = LC$, then:

$$\frac{\delta f_{\text{res}}}{f_{\text{res,best}}} = \left|-\frac{1}{2}\right| \frac{\delta x}{x_{\text{best}}} \tag{12}$$

$$\frac{\delta x}{x_{\text{best}}} = \sqrt{\left(\frac{\delta L}{L_{\text{best}}}\right)^2 + \left(\frac{\delta C}{C_{\text{best}}}\right)^2} \tag{13}$$

$$\delta f_{\text{res}} = f_{\text{res,best}} \left|-\frac{1}{2}\right| \sqrt{\left(\frac{\delta L}{L_{\text{best}}}\right)^2 + \left(\frac{\delta C}{C_{\text{best}}}\right)^2} \tag{14}$$

The resonant frequencies of the two methods are very similar with identical uncertainties. To determine which method is more accurate, we calculated percent errors (using Equation 10) between the measured and theoretical values:

Method 1: $\%\ error = (3.8 \pm 0.9)\ \%$

Method 2: $\%\ error = (2.5 \pm 0.8)\ \%$

Based on Percent Errors, method 2 appears to be the more accurate method, which makes sense because there was more analysis done with technology in determining the resonant frequency compared to method 1, which relied on some estimations by eye.

The Q-Factor for this RLC circuit is defined in Equation 4. The variables f_1 and f_2 are the frequencies at which the value for Gain in Figure 2 is equal to $1/\sqrt{2}$ times the Maximum Gain. This value, 0.6817 ± 0.0007, is identifiable as the red line on Figure 2. Using the raw data and the curve of blue points to estimate the values of these frequencies:

$$f_1 = 150 \pm 10 \text{ Hz and } f_2 = 4650 \pm 10 \text{ Hz}$$

Using Method 2's resonant frequency value to calculate the Q-factor, we obtained:

$$Q = 0.18 \pm 0.01$$

The uncertainty propagation for Q-factor was derived using Equation A.23 and A.24 in the Manual:

$$\delta Q = Q_{\text{best}} \sqrt{\left(\frac{\delta f_{\text{res}}}{f_{\text{res,best}}}\right)^2 + \left[\frac{(\delta f_2)^2 + (\delta f_1)^2}{\left(f_{2,\text{best}} - f_{1,\text{best}}\right)^2}\right]} \tag{15}$$

References

[1] Campbell, W.C. *et al*. Physics 4BL: Mechanics Lab Manual (ver. August 31, 2018). (University of California Los Angeles, Los Angeles, California).