EXPERIMENT 5: SPEED OF SOUND AND LIGHT

SAMUEL ELLISON – UID # 204977052

LAB PERFORMED ON 2/14/2019

LAB SECTION: THURSDAY 2PM

TA NAME: RHYAN GHOSH

LAB PARTNERS: ERIC WONG AND MELVIN PRASETYO

Worksheet

1.

(a) The expression for the speed of light c in terms of time cursor measurements t_1 and t_2 and length of the beam path Δx is:

$$c = \frac{x_2 - x_1}{t_2 - t_1} \tag{1}$$

Where x_2 and x_1 are the two measurements for the final and initial positions of the beam of light, and t_2 and t_1 are the final and initial cursor positions of the time measurements on the oscilloscope.

(b) Using Equations A.23 and A.24 in the lab manual, an equation for the uncertainty in the speed of light can be determined.

$$\delta c = c_{best} \cdot \sqrt{2 \cdot \left[\left(\frac{\delta x}{\Delta x}\right)^2 + \left(\frac{\delta t}{\Delta t}\right)^2\right]} \tag{2}$$

Therefore $\delta c = \pm(6 \times 10^6)$ m/s

(c) If beam path length has infinite precision, $\delta x = \pm 0$ cm. Therefore Equation 2 produces $\delta c = \pm(6 \times 10^6)$ m/s, which is still the same.

(d) If the scope's time cursors are infinitely precise, $\delta t = \pm 0$ ns. Therefore Equation 2 produces $\delta c = \pm(2 \times 10^5)$ m/s, which is significantly lower.

(e) The time cursor uncertainty is the main contributor because when taken out, the uncertainty in c drops significantly.

2. For our experiment, the equation we used to calculate the speed of light was:

$$c = \frac{4d}{t_{far} - t_{near}} = \frac{4d}{\Delta t} \tag{3}$$

Where d is one fourth of the total distance the beam of light travels and $t_{far} - t_{near}$ is the total time Δt that the beam travels. Therefore, the uncertainty in c is:

$$\delta c = c_{best} \cdot \sqrt{\left(\frac{\delta d}{d}\right)^2 + 2 \cdot \left(\frac{\delta t}{\Delta t}\right)^2} \tag{4}$$

The measured variables were $d = 308.1 \pm 0.1$ cm, $t_{far} = 40 \pm 3$ ns and $t_{near} = 2 \pm 3$ ns. The result is $c = (3.2 \pm 0.4) \times 10^8$ m/s. The defined speed of light in vacuo does fall within the uncertainties of this experimental value for the speed of light.

3. A non-dispersive dispersion relation might take the form:

$$\omega = vk \tag{5}$$

Having group velocity of a wave packet:

$$v_g \equiv \frac{\partial w}{\partial k} = v \tag{6}$$

A dispersive dispersion relation might take the form:

$$\omega = v\sqrt{k} \tag{7}$$

Having group velocity of a wave packet:

$$v_g \equiv \frac{\partial w}{\partial k} = \frac{v}{2\sqrt{k}} \tag{8}$$

Presentation Report

Introduction

The overall purpose of the lab was to determine values for the speed of sound v in air and compare them to the theoretically accepted value. There were two separate experiments to determine two different values for v.

For the first sound experiment, *traveling* sound waves were produced and analyzed to determine the phase velocity of sound. Since this value v is constant, we generated traveling sound waves with various frequencies f using a speaker. The wavelength λ, wavevector k, and angular frequency ω could be calculated from analysis. The phase velocity of sound is related to angular frequency and wavevector if the medium is non-dispersive via:

$$\omega = v_p k \tag{1}$$

Thus, another goal was to determine the dispersion relation for sound waves in the medium of air. For different frequencies and wavelengths come different values for ω and k, thus the speed of sound was determined using a linear regression for Equation 1. Angular frequency is related to frequency and wavevector (spatial frequency) is related to wavelength via:

$$\omega = 2\pi f \tag{2}$$

$$k = \frac{2\pi}{\lambda} \tag{3}$$

For the second sound experiment, *standing* sound waves were produced and analyzed to determine v. Sound waves were produced by a speaker and reflected backwards, resulting in interference. Instead of varying the frequency of the waves, we kept f constant and measured the average wavelength of the standing waves as the reflecting surface moved further from the speaker. The speed of sound is then simply:

$$v = f\lambda \tag{4}$$

The theoretically accepted value for the speed of sound is 343 m/s in air. Apart from determining the speed of sound using the two methods, the goal was to see which method was more precise and which was more accurate, and to see whether the calculated values agree with the accepted value within the experimental uncertainty.

Experimental Results

Part 1: Traveling Sounds Waves

To produce traveling sound waves, we emitted sound from a stationary loudspeaker which was connected to a linear track slider with a ruler on it. This sound was detected with a microphone at the same height as the loudspeaker and was connected to a linear track slider with a ruler on it. The frequencies of the sound waves were produced by connecting the loudspeaker to the Rigol Function Generator. The Oscilloscope was connected to the microphone in one channel and the Rigol Function Generator in another to show the Function Generator wave and the sound wave picked up by the microphone. The generator's wave was very clean with little noise while the microphone's wave still appeared sinusoidal but had significant noise. To determine the wavelength of the sound waves, the peaks of both waves were lined up by shifting the microphone via the slider, and the measurement on the ruler was recorded. Then, the slider was moved so that the wave peaks again lined up after a sound wave phase shift of 2π. The ruler measurement was recorded again. Thus, the difference in these ruler recordings is the wavelength of the traveling sound wave. This procedure was conducted for frequencies of 4.00000 ± 0.00001 kHz, 6.00000 ± 0.00001 kHz, 8.00000 ± 0.00001 kHz, 10.00000 ± 0.00001 kHz, and 12.00000 ± 0.00001 kHz. Angular frequencies ω and Spatial frequencies k were determined from these experimental results and plotted below as a dispersion relation for traveling sound waves.

The slope of the plot a in Figure 1 represents the value determined for the phase velocity of sound (Equation 1). See Analysis for further details.

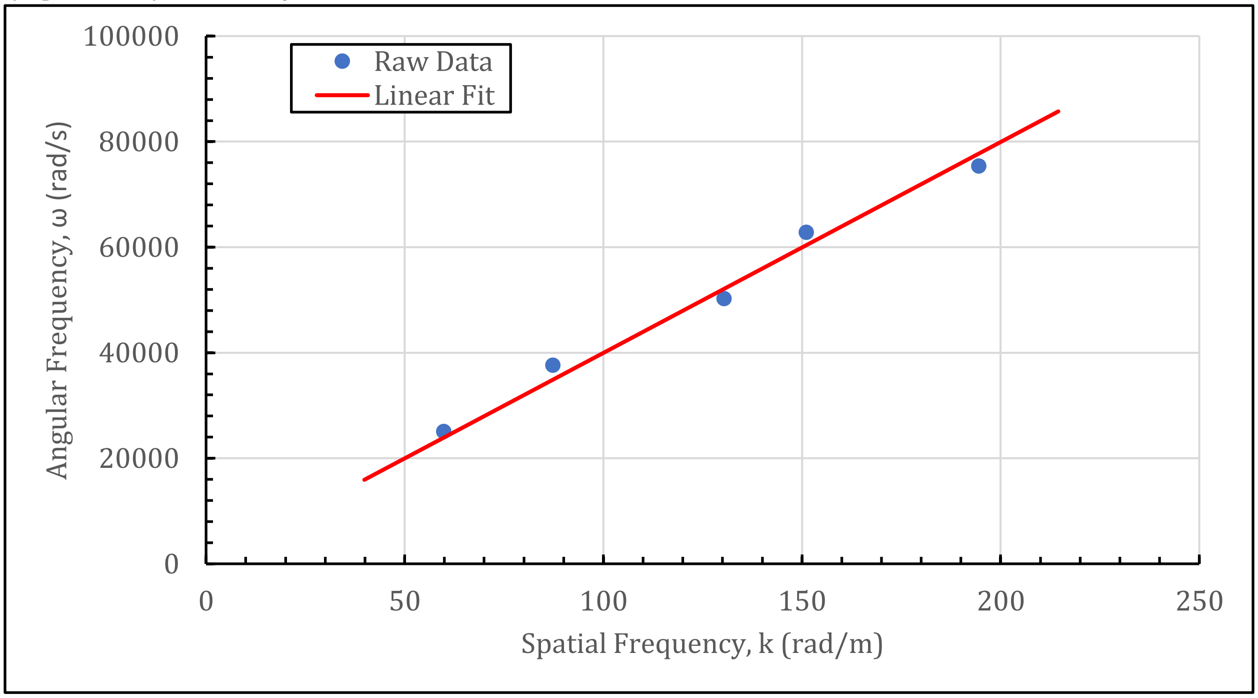

Figure 1: Dispersion Relation for Traveling Sound Waves. Frequencies f and wavelengths λ were converted to Angular Frequencies ω (y-axis) and Spatial Frequencies k (x-axis), respectively using Equations 2 and 3 for five different frequencies of sound (see above). The blue points are the raw data, and the red line is the linear fit with intercept set to zero. The line takes the form $\omega = ak$ where $a = 400 \pm 8$ m/s with correlation coefficient $r = 0.999$. The intercept is required to be zero because a spatial frequency of zero or an angular frequency of zero are both indicative of no sound wave being produced. When one is zero, the other must also be zero.

Part 2: Standing Sound Waves

Sound waves were produced and detected using the same loudspeaker and microphone as in part 1, except now a reflecting surface (whose position could be varied) was used to reflect the emitted waves back towards the source, resulting in interference and thus standing sound waves. To determine the speed of sound for these waves, all we needed were values for frequency and wavelength. The reflecting surface had a built-in potentiometer to record its position electrically. Before producing sound waves, the potentiometer was calibrated. After connecting the potentiometer's input to a DC Power Supply at 1.993 ± 0.001 Volts and output to a multimeter to read its voltage output, the reflecting surface was moved in increments of 5.00 ± 0.05 cm using a ruler and the voltage was recorded. Distance was plotted as a function of Output Voltage (Figure 2) to determine the calibration factor to convert the output voltages from the potentiometer into position values for the reflecting surface.

The slope of the trendline a in Figure 2 represents the calibration factor C used to convert Potentiometer Output Voltages into Reflecting Surface Positions, d.

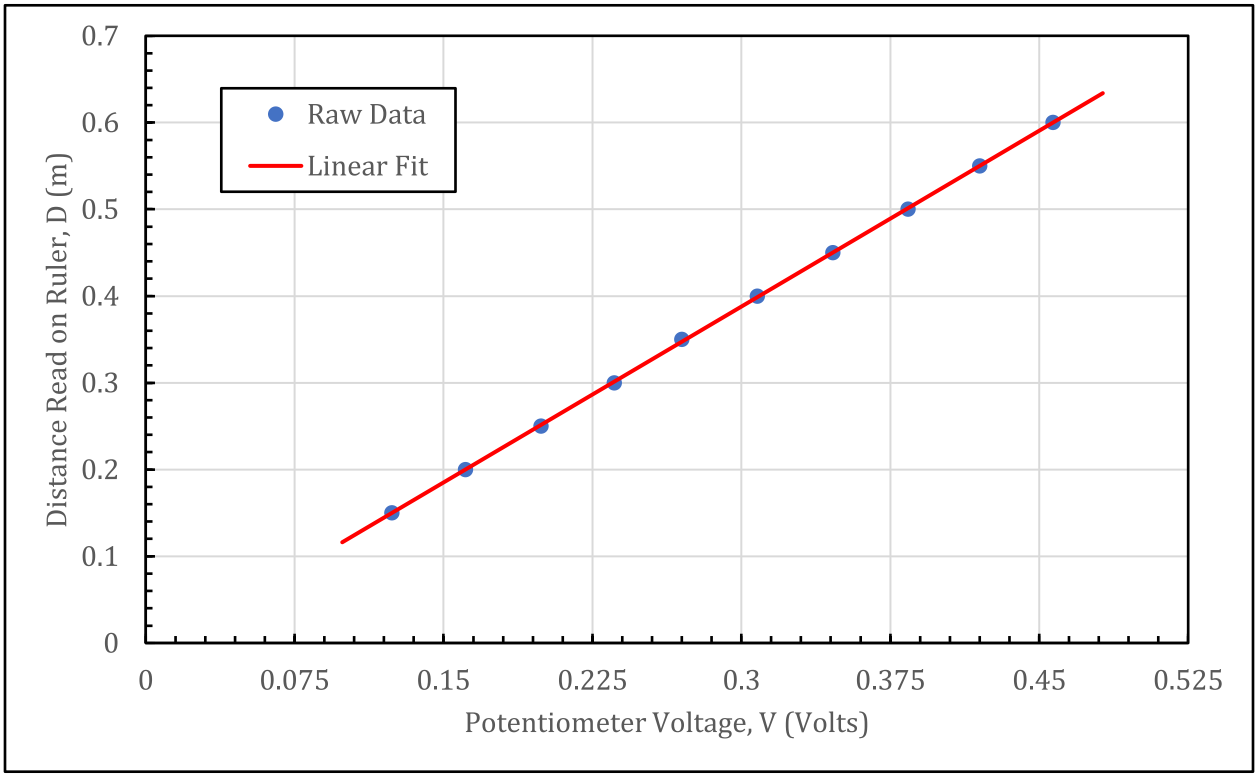

Figure 2: Potentiometer Calibration. Potentiometer Output Voltage was read off the Digital Mulitmeter every 5.00 ± 0.05 cm the refelcting surface was moved for 10 different distances. The Distance read off the ruler as a function of Potentiometer Output Voltage was plotted. The blue points are the raw data and the red line is the linear fit taking the form $D = aV + b$ where $a = 1.352 \pm 0.004$ m/V and $b = -0.018 \pm 0.001$ m and having a correlation coefficient $r \approx 1$. The slope a is the calibration factor C. Therefore, $C = 1.352 \pm 0.004$ m/V. Note that the intercept is not required to be consistent with zero.

Now the standing waves were produced and analyzed. The Rigol Function Generator was set to produce waves with frequency equal to 6.00000 ± 0.00001 kHz. The potentiometer's output voltage was connected to Channel 0 of myDAQ, while the microphone's detection signal (DC) was connected to Channel 1 of myDAQ. The DC power supply still provided power at same Voltage. The program on the computer was set to collect data continuously at a rate of one point every 0.1 seconds and plot the microphone signal as a function of the output voltage of the potentiometer (reflecting surface position after calibration) as the reflecting surface was slowly moved further from the stationary speaker and microphone at a constant rate. The data was collected for several other frequencies, but the only one analyzed was that of 6.00000 ± 0.00001 kHz.

From analysis of the data gathered for this part of the experiment, five different wavelengths were determined from Figure 3 using the 6 positions of maxima.

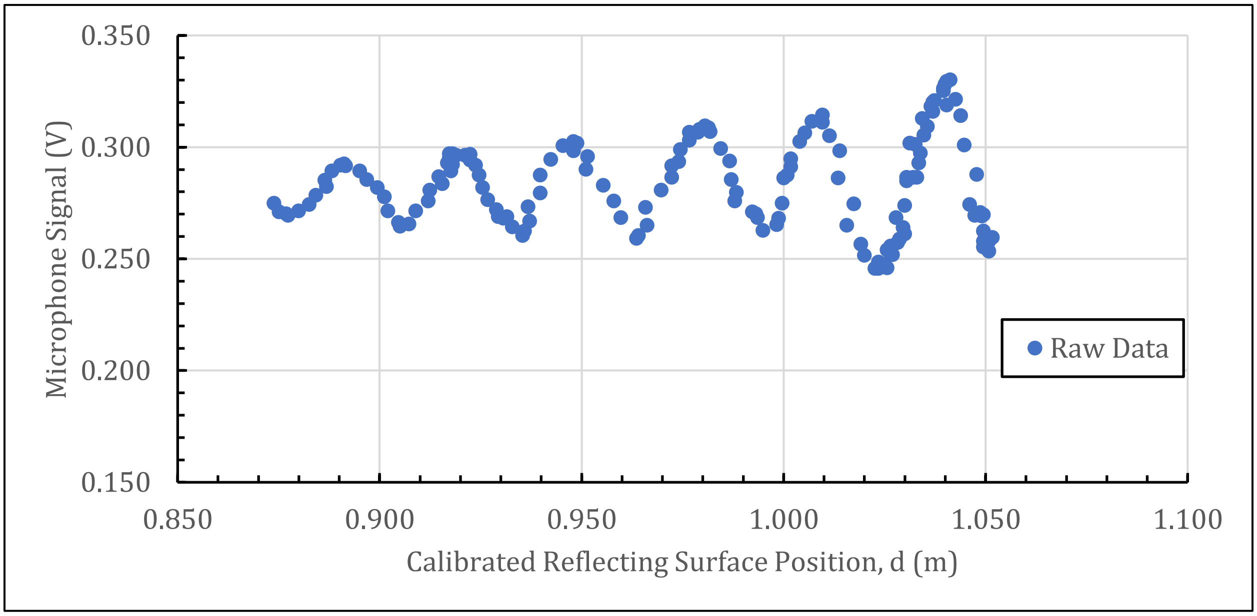

Figure 3: Standing Wave Microphone Signal for Sound produced at 6.00000 ± 0.00001 kHz. Microphone Signal Data and Potentiometer Output Voltage Data were gathered from myDAQ. The Potentiometer Output Voltages were multiplied by $C = 1.352 \pm 0.004$ m/V to give d, the Calibrated Reflecting Surface Position. The plot is Microphone Signal as a function of Reflecting Surface Position. The blue points are the raw data. The plot takes the form if a sinusoidal wave. The important features of the plot are the values for the position d at which The Microphone Signal is maximized. Distances between maxima are the antinodal spacings. The values for d are used to determine wavelength, see below.

The values for wavelengths (calculation details in analysis) are summarized in Table 1 below.

Wavelength Number	Wavelength, λ (m)
1	0.063 ± 0.009
2	0.058 ± 0.008
3	0.065 ± 0.008
4	0.058 ± 0.008
5	0.056 ± 0.007

Table 1: Wavelengths for Standing Waves Determined from Figure 3. Five different values for the wavelength of the standing sound wave produced with a frequency of 6.00000 ± 0.00001 kHz. The values are determined from the peak to peak distances of Figure 3 (See Analysis) and uncertainties are propagated in the analysis. An average value of wavelength is used to determine the speed of sound later.

Analysis

Part 1:

Figure 1 is a plot with trendline resembling Equation 1. Therefore, the slope of this trendline a corresponds with the phase velocity of sound v_p. The correlation coefficient $r = 0.999$ ensures this relationship is in fact linear, meaning Equation 1 holds for sound waves traveling through an air medium. Thus, air is a non-dispersive medium. For traveling waves, we determined:

$$v_p = 400 \pm 8 \text{ m/s}$$

Uncertainty from regression is the same for both the slope and the velocity. The percent error for the phase velocity of traveling waves and the theoretically accepted value for the speed of sound can be determined using:

$$\%\, error = \left|\frac{Experimental - Theoretical}{Theoretical}\right| \times 100\% \tag{5}$$

% error (traveling waves) $= 17 \pm 2$ %

Part 2:

Determining the speed of sound from the traveling sound waves was much more cumbersome. The wavelength was determined from Figure 3, a plot of the Microphone Signal Output as a function of the Calibrated Position of the Reflecting Surface. The position was determined using the calibration factor C from Figure 2. The Potentiometer Output Voltage data collected by myDAQ was multiplied by C, with the result being the x-axis in Figure 3.

The value for the wavelength of the traveling sound waves is double the distance between the positions of maxima (or minima) on Figure 3. This factor of 2 in Equation 6 is because in a standing wave, the node spacing (or equivalently, antinode spacing, which is peak to peak distance) is one half of the wavelength. There are six maxima in the figure, therefore there are five different differences between maxima, meaning five different wavelength calculations. Hence the five values in Table 1. If we call the position of one maximum d_1 and another maximum d_2 (where $d_2 > d_1$), then:

$$\lambda = 2(d_2 - d_1) \tag{6}$$

In which any value for d is:

$$d = CV \tag{7}$$

Where C is the calibration factor and V is the potentiometer Output Voltage. Thus, the uncertainty propagation for the wavelength values in Table 1 for Standing Waves is:

$$\delta\lambda = 2 \cdot \sqrt{(d_2)^2 \cdot \left[\left(\frac{\delta C}{C}\right)^2 + \left(\frac{\delta V}{V_2}\right)^2\right] + (d_1)^2 \cdot \left[\left(\frac{\delta C}{C}\right)^2 + \left(\frac{\delta V}{V_1}\right)^2\right]} \tag{8}$$

The values for δC and δV respectively are ± 0.004 m/V (from Figure 2) and ± 0.0000001 V (from myDAQ data). The values for the positions d were determined by simply observing the graph and finding the corresponding data points to each maximum.

The average value for the wavelength of the standing sound wave (of the five wavelengths in Table 1) with uncertainty is:

$$\lambda_{avg} = 0.060 \pm 0.002 \text{ m}$$

The statistical uncertainty is determined using Equation A.14 in the Manual:

$$\delta\lambda_{avg} = \frac{\sigma}{\sqrt{N}} \tag{9}$$

Where σ is the standard deviation of the sample of wavelengths and N is the number of values (5). Since the frequency f was held constant, the speed of sound for the standing waves was calculated using Equation 4, where $f = 6000.00 \pm 0.01$ Hz and $\lambda = \lambda_{avg}$. The result is:

$$v = 360 \pm 10 \text{ m/s}$$

The uncertainty in this speed was propagated with:

$$\delta v = v_{best} \cdot \sqrt{\left(\frac{\delta f}{f_{best}}\right)^2 + \left(\frac{\delta\lambda_{avg}}{\lambda_{avg}}\right)^2} \tag{10}$$

The percent error for the standing wave speed (using Equation 5) is:

% error (standing waves) $= 5 \pm 3$ %

Conclusion

The main purpose of this lab was to experimentally determine the speed of sound using two different methods: one via traveling waves and the other via standing waves, and to compare these experimental values with the theoretically accepted value for the speed of sound, $v = 343$ m/s. Within the traveling waves portion, another goal was to determine the dispersion relation for sound waves propagating through air using Equation 1. If angular frequency is in fact a linear function of wavevector for sound in air, then air is a non-dispersive medium for sound.

For the traveling waves, the experimentally determined value for the speed of sound (or equivalently, the phase velocity of sound in air) was $v_p = 400 \pm 8$ m/s. Since the plot of ω as a function of k in Figure 1 showed a conclusive linear relationship ($r = 0.999$), air was determined to be a non-dispersive medium for sound. For standing waves, the experimentally determined value for the speed of sound was $v = 360 \pm 10$ m/s.

To determine which method for measuring the speed of sound was more accurate, we looked at the percent errors. To determine which method had better resolution (more precise), we looked at the relative uncertainties δv. Method 1 using traveling waves was more precise (± 8 m/s $< \pm 10$ m/s) mainly because we solely used a regression to determine the uncertainty in speed. Method 2 using standing waves was more accurate (5 ± 3 % $< 17 \pm 2$ %) mainly because we used the myDAQ to gather data as opposed to measuring the wavelength using a ruler, therefore there was less margin for errors. Overall, the accepted value for the speed of sound, 343 m/s, is not within the uncertainty range of either of the experimentally determined speeds. This can be attributed to errors, such as background noise interfering with the sound waves before being detected by the microphone (random error) and error involved with measuring the wavelength of the wave by eye with a ruler. An alternative way to perform the experiment to prevent these errors would be to perform the lab in an isolated location with little background noise interference, and to use more finely tuned equipment to read distances for calculating the wavelengths.

References

[1] Campbell, W.C. *et al*. Physics 4BL: Mechanics Lab Manual (ver. August 31, 2018). (University of California Los Angeles, Los Angeles, California).