EXPERIMENT 6: GEOMETRIC OPTICS

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LAB SECTION: THURSDAY 2PM

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Worksheet

1. Fiber Optic Cables
   a. The critical angle β at the fiber-cladding interface occurs when $n_f \sin(\beta) = n_{clad}$. Therefore $\beta = 1.20$ rads.
   b. Light will experience total internal reflection at this interface if $\beta > 1.20$ rads. The angles α and β are complementary (add up to $\pi/2$). Thus, the range of refracted angles α that will cause TIR is $0 < \alpha \leq 0.367$ rads.
   c. The angles θ and α are related through Snell's Law: $n_{air} \sin(\theta) = n_f \sin(\alpha)$. Using the range for α and solving for θ, the result is $0 < \theta \leq 0.569$ rads.
   d. If the light undergoes total internal reflection at the first fiber-cladding interface, then the light will continue to travel through the fiber part of the cable as long as the cable is straight. If there are bends and turns, then the light may exit the fiber.
2. The index of refraction for the prism can be calculated using Snell's Law:

$$n_{air} \sin(\theta_i) = n_{prism} \sin(\theta_t) \tag{1}$$

The value for $n_{air} = 1$, and the measured values for the incident and transmitted angles were $\theta_i = 0.7318 \pm 0.0009$ rads and $\theta_t = 0.5042 \pm 0.0009$ rads. The uncertainty is:

$$\delta n_{prism} = n_{prism} \cdot \sqrt{\left(\frac{\delta\theta_i \cos(\theta_i)}{\sin(\theta_i)}\right)^2 + \left(\frac{\delta\theta_t \cos(\theta_t)}{\sin(\theta_t)}\right)^2} \tag{2}$$

The answer is then $n_{prism} = 1.383 \pm 0.003$. Our experimental value for $\theta_c = 0.7388 \pm 0.0009$ rads. We can calculate θ_c using Snell's Law using the back interface and the recently calculated value for n_{prism}: $\theta_c = \arcsin(\frac{n_{air}}{n_{prism}})$. We get $\theta_c = 0.8081 \pm 0.0007$ rads. Uncertainty in this critical angle is:

$$\delta\theta_c = n_{air} \cdot \frac{\delta n_{prism}}{n_{prism}^2} \cdot \frac{1}{\sqrt{1 - \left(\frac{n_{air}}{n_{prism}}\right)^2}} \tag{3}$$

The percent error in these critical angles is calculated using:

$$\%\ error = \left|\frac{Experimental - Theoretical}{Theoretical}\right| \times 100\% \tag{4}$$

This results in a percent error of $8.6 \pm 0.8\%$ which is not incredibly large. However, our critical angles do not completely corroborate because the uncertainties do not allow the values to overlap.

3. The equation for addition of thin lens focal lengths with finite separation is:

$$\frac{1}{f_{total}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{e}{f_1 f_2} \tag{5}$$

Solving for e, we get:

$$e = f_2 + f_1 - \frac{f_1 f_2}{f_{total}} \tag{6}$$

Then:
$$\lim_{f_{total}\to\infty} e = f_2 + f_1 \tag{7}$$

As $f_{total} \to \infty$, the two initial parallel lines run through both lenses and end up parallel again once they pass through the last lens. They neither converge or diverge when $f_{total} \to \infty$. See Figure 1 Below.

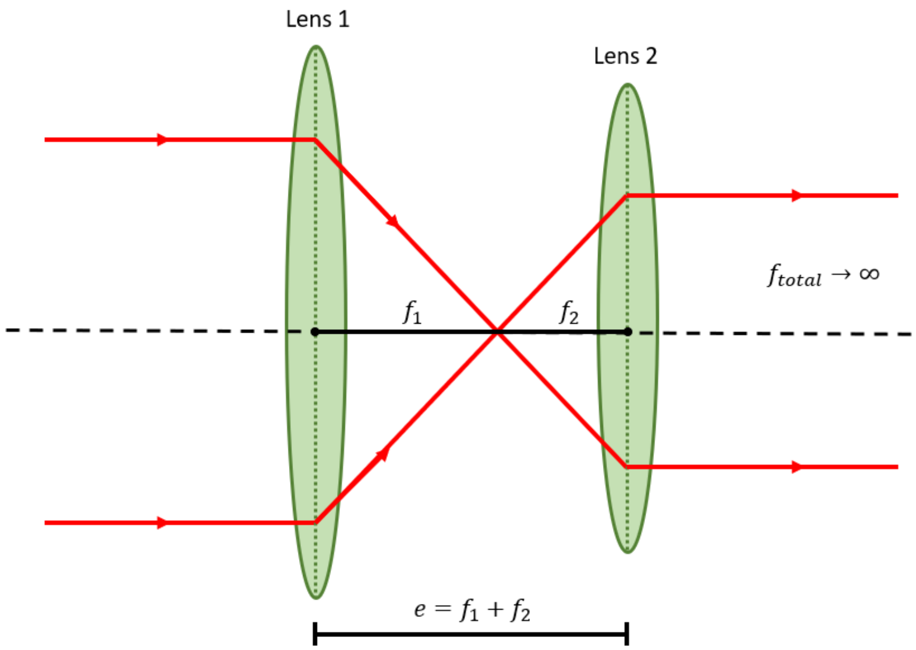

Figure 1: Resulting Beams when $f_{total} \to \infty$. As f_{total} gets larger and larger, the separation distance e approaches the value $f_1 + f_2$ and the exiting beams on the right of Lens 2 diverge more and more until they become parallel. The incoming parallel beams to the left of Lens 1 flip sides but still end up parallel once they travel through both lenses.

Presentation Report

Introduction

This lab was divided into three independent experiments, each exploring phenomena related to the geometric optics of lenses. We observed properties of three different thick lenses, observed properties of thin lenses with different diameters (along with a ball lens), and explored various image formations from different positions of a thin converging lens.

For the first part, the main purpose was to determine the focal lengths for three separate thick lenses: biconvex, biconcave, and planoconvex. To do this, we used a ray box to pass multiple light rays through each lens and measure the distance at which they converged. Another goal of this part was to observe the phenomenon of spherical aberration using the biconvex lens, and to both observe and measure the (de)magnification factor for a lens combination composed of one biconcave and one planoconvex lens. The Magnification Factor is given by:

$$M \equiv \frac{\text{final separation}}{\text{initial separation}} \tag{1}$$

Where "separation" refers to the distance between the parallel light rays either before or after magnification. Spherical Aberration is the phenomenon by which all light rays passing through a lens do not converge at a single point. Rather, there are *multiple* focal points observed.

For the second part, the main purpose was again to determine the focal lengths of various lenses, but also to calculate the dioptric power P of a single lens and a combination of two similar lenses and compare experimental and theoretical results. Dioptric Power can be determined from the focal length f using the Equation:

$$P = \frac{1}{f} \tag{2}$$

For a combination of two lenses, the total Dioptric Power is simply the sum of the Dioptric Powers for the individual lenses. Instead of thick lenses, we used thin lenses with different diameters in addition to a ball lens. To determine the focal length, we used a laser beam instead of a ray box. The laser beam was reflected off a beam splitter to produce parallel rays to pass through each lens. Using a screen on the other end, the focal length represents the distance between the screen and lens at which the two laser beams converge at the same point. For a ball lens, the theoretical focal length from the center of the sphere is given by the Equation:

$$f = \frac{nD}{4(n-1)} \tag{3}$$

Where n is the refractive index of the ball lens and D is its diameter. In our experiment, we measured the focal length from the surface of the lens f'. Thus, another goal of this part was to compare theoretical and measured focal lengths f for the ball lens.

For the last part, the purpose was to test/verify the relationship between the distance between an object and a thin lens o, the distance between the thin lens and the resulting image i, and the focal length f. This relationship is captured in Equation 3 below:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \tag{4}$$

We did so by taking several measurements for different object distances o and recording the resulting image distance i. Thus, we could compare the focal length gathered from the lens in part 2 to the focal length calculated using Equation 3 and the data from this part. For these same trials, we also measured the image size I to compare with the object size O to verify that the image is a magnification or demagnification based on object distance o.

Experimental Results

Part 1: Thick Lenses

The focal lengths of the three different thick lenses were determined using a ray box to pass multiple rays of light through each lens and observe at what position these rays converge. The baffle for the ray box was adjusted such that three rays can pass through. The ray box was placed on a blank sheet of white paper. The first lens observed was the biconvex lens. We placed the lens so that the center ray hit the center of the lens, and the center line of the lens was perpendicular to the incoming light rays. Then, we traced the lens and the outgoing rays and measured the distance from the center line of the lens to the position at which the three traced rays converged using a ruler. This distance is the focal length of the lens. The procedure was repeated for the planoconvex lens, which was placed such that its planar face was closest to the incoming rays. For the biconcave lens, the rays did not converge on the outgoing side of the lens. Instead, after moving the lens further from the incoming rays, we traced the outline of the lens and the outgoing, diverging rays, and then connected the traced rays at a position behind the lens. The focal length is still the distance between the center line of the lens and the position of convergence of the traced rays, except the reported value should be negative. Table 1 below summarizes the measurements.

Lens Type	Focal Length, f (cm)
Biconvex	5.98±0.05
Biconcave	-4.06±0.05
Planoconvex	8.83±0.05

Table 1: Measured Focal Lengths for Three Different Thick Lenses. The first column is the type of lens and the second is the respective measured focal length. Note that the distance measured for the Biconcave lens was 4.06±0.05, but since the outgoing rays diverged, the focal length is reported as negative.

To identify spherical aberration, we used the biconvex lens and the ray box with the baffle set to allow five separate rays through. The result: two different positions of convergence. The outermost two rays converged to a position closer to the lens than that of the three center rays. We measured the two different focal lengths. See Analysis for further details/calculations on spherical aberration.

We observed 1D Magnification and Demagnification using a combination of lenses: one biconcave and one planoconvex. For magnification, the biconcave lens was placed near the ray box (which was set to let three rays out) to divert the rays. Then, the planoconvex lens was placed aside the biconcave. The result: the light rays were again parallel except spaced further apart (magnified). The outgoing parallel rays form the ray box and outgoing parallel rays from the planoconvex lens were traced, and the separation distance (distance from the top ray to the bottom ray) was measured and recorded for both sets. The procedure was repeated for demagnification, in which case the lenses were placed in reverse order. See Analysis for calculations of (de)magnification factors for these lens combinations.

Part 2: Thin Lens Properties

The experimental setup used to determine the focal lengths of various lens is as follows. A laser, beam splitter, lens holder, and moveable screen were all set up atop a magnetic bench with a built-in ruler. The laser was placed on the left side and directed perpendicular to the ruler on the magnetic bench. The beam splitter was placed at ~45-degree angle such that the laser beam was immediately split and directed towards the lens inside the lens holder and screen. The lens holder was adjusted so two distinct points could be detected on the screen (placed to the right of the lens). See Figure 1 below.

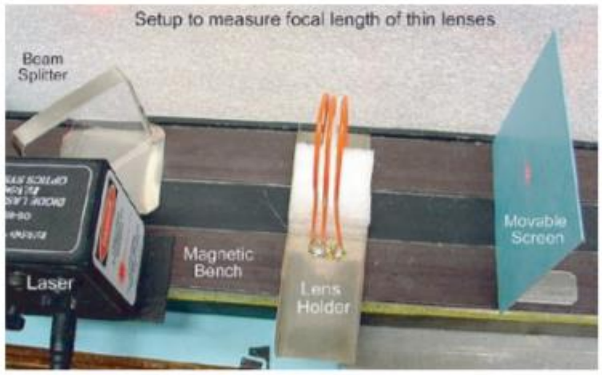

Figure 1: Experimental Setup for Determining Focal Lengths of Thin Lenses. A single beam is emitted by the laser. The beam splitter splits the beam into two parallel beams with a separation of about a centimeter. The two beams travel through the lens held in place by the lens holder and are seen on the screen as two distinct points. The moveable screen is adjustable.

To determine focal length, the screen was placed far away from the lens and brought closer until the two points converged. The distance between the center of the lens and the face of the screen was recorded using the metal ruler. Then, the screen was moved closer until the points noticeably diverged, and this distance was recorded. These two measurements were used to approximate error. The distance at which the points purely overlapped (somewhere between the previous two positions) was found and recorded. Thus, the focal length is this third measurement. The different lenses experimented with were: (1) a thin lens with diameter equal to 3.5 cm, (2) a second thin lens with same diameter, (3) a combination of the previous two lenses, (4) a thin lens with diameter equal to 2 cm, (5) a ball lens with diameter equal to 25.4 ± 0.5 mm. For the combination of lenses (3), the lenses were placed adjacent to one another in the lens holder and the distance between the center of the two lenses and the screen was measured as the focal length. For lens (4), the lens was already secured with a vertically twisted wire, so the lens holder was unnecessary. For lens (5), the distance between the edge of the sphere *nearest* to the screen and the screen was recorded for the focal length. See Table 2 below for results.

Lens Number	Lens Type	Focal Length, f (cm)
(1)	Thin Lens with diameter 3.5cm	15.4±0.3
(2)	Thin Lens with diameter 3.5cm	10.5±0.6
(3)	Combination of Above Lenses (1) & (2)	6.1±0.2
(4)	Thin Lens with diameter 2cm	16.0±0.7
(5)	Ball Lens with diameter 2.54 ± 0.05cm	0.47±0.05

Table 2: Focal Length Measurements for the 5 Different Lenses Used. The first column is the Lens number corresponding to the discussion above. Note that there are no uncertainties for the diameters for the first, second, and fourth lenses because we did not actually measure them, and their values are not used in any calculation. The uncertainties in focal lengths will be discussed/propagated in the analysis.

Part 3: Image Formation with Lenses

The experimental setup used to investigate image formation with thin lenses is as follows: A PASCO Scientific Diode Laser (operating on 5 Volts) was placed on the left side of the Magnetic Bench. An iris with film grating (acting as the "object") was placed a short distance to the right of the diode laser. Following that was the 2cm diameter thin lens and the moveable screen. See Figure 2 below.

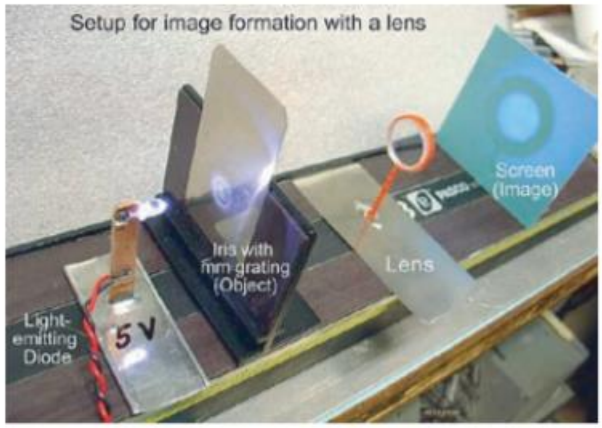

Figure 2: Experimental Setup for Investigating Image Formation With a Thin Lens. The light emitting diode illuminates the object (a grid) through a circular slit to projected through the lens onto the screen to form an image. The distance between the lend and object is o and the distance between the screen (image) and lens is i. The 2cm diameter lens used was the same lens used in Part 2, with focal length $f = 16.0 \pm 0.7$ cm.

To verify Equation 3, we varied and measured the object distance o and measured the resulting image distance i with the ruler for various values of o such that $o > f$, ensuring some values of o in the interval $f < o < 2f$ and some values $o > 2f$. The screen was moved to the position at which the image appeared most sharp/clearly. The Object and Image sizes can be represented equivalently by the grating spacing of the grid (the height of a single box). The Object grating spacing is reported in the manual to be 1.0 ± 0.5 mm. The Image Grating Spacing was determined by taking a measurement of the height of a single box for all trials. See Table 3 below for results.

Trial	Object Distance, o (cm)	Image Distance, i (cm)	Image Grating Spacing, I (cm)
1	24.09$\pm$0.05	46.98$\pm$0.05	0.19$\pm$0.05
2	33.40$\pm$0.05	30.84$\pm$0.05	0.09$\pm$0.05
3	41.43$\pm$0.05	26.56$\pm$0.05	0.07$\pm$0.05
4	27.49$\pm$0.05	38.14$\pm$0.05	0.14$\pm$0.05
5	31.48$\pm$0.05	32.32$\pm$0.05	0.11$\pm$0.05

Table 3: Resulting Image Distances and Image Grating Spacing for Five Unique Object Distances. The object and image distances and resulting image grating spacings were measured and recorded for five different positions of the lens from the object. Once the lens was placed, the screen was adjusted to the position at which the image was sharpest. All uncertainties are ± 0.05 cm because we used the same ruler for every measurement.

Analysis

Part 1: Thick Lenses

There were no calculations necessary to determine the focal lengths of the three different thick lenses. See Table 1 for those values. However, some analysis is necessary to determine the degree of spherical aberration on the biconvex lens and magnification and demagnification factors of the biconcave/planoconvex lens combination.

Spherical Aberration occurred on the biconvex lens when five light rays were passed through it. The central three rays converged to a longer focal point $f_c = 6.02 \pm 0.05$ cm while the outer two rays converged to a shorter focal point $f_o = 5.36 \pm 0.05$ cm. The distance from the lens to the midpoint of these two focal points to the lens was measured to be $f_m = 5.69 \pm 0.05$ cm. The percent difference between the central ray focal point and the midpoint focal point is calculated by:

$$\%\ difference = \left|\frac{f_m - f_c}{f_c}\right| \times 100\% \tag{5}$$

This results in a percent difference of $5.5 \pm 0.2\%$, which is a measure of the degree of spherical aberration that occurs with this specific lens.

Magnification occurred when the biconcave lens diverted the rays first and then the planoconvex lens made them parallel once again. The separation distance between the magnified top and bottom rays was measured to be 3.64 ± 0.05 cm and the separation distance between the initial rays from the ray box was measured to be 1.82 ± 0.05 cm. Thus, using Equation 1, the magnification factor is:

$$M_1 = 2.00 \pm 0.06$$

Demagnification occurred when the lenses were reversed. The final separation distance was measured to be 0.96 ± 0.05 cm with the same initial separation distance of 1.82 ± 0.05 cm. Again, using Equation 1, the demagnification factor is:

$$M_2 = 0.53 \pm 0.02$$

Note that the magnification and demagnification factors of the same combination of lenses should be the reciprocals of one another. Our measurements are consistent with this fact. The uncertainty propagation for the M is as follows: If we let the final and initial separation distance be F and I respectively where δF and δI are both ± 0.05 cm:

$$\delta M = M_{best} \cdot \sqrt{\left(\frac{\delta F}{F_{best}}\right)^2 + \left(\frac{\delta I}{I_{best}}\right)^2} \tag{6}$$

Part 2: Thin Lens Properties

The uncertainties in the focal lengths given in Table 2 were estimated by halving the difference in the distances between the lens and when the points began to converge and between the lens and when the points began to diverge.

The Dioptric Power of a lens is given by Equation 2: the inverse of focal length. If two lenses are used, the theoretical total Dioptric Power should be:

$$P_{tot} = P_1 + P_2 \tag{7}$$

The experimentally determined dioptric powers for the first and second 3.5cm diameter lenses respectively are $P_1 = 0.065 \pm 0.001$ cm-1 and $P_2 = 0.095 \pm 0.005$ cm-1. The experimentally determined dioptric power for the combination of those two lenses is $P_{tot} = 0.164 \pm 0.005$ cm-1. The theoretical total dioptric power is simply the sum of the two individual powers P_1 and P_2 determined experimentally. This results in $P_{tot} = 0.160 \pm 0.005$ cm-1.

The uncertainty propagation for the experimentally determined powers is:

$$\delta P = P_{best}|-1|\frac{\delta f}{f} \tag{8}$$

And for the theoretically determined total power is:

$$\delta P_{tot} = \sqrt{(\delta P_1)^2 + (\delta P_2)^2} \tag{9}$$

The percent error in the theoretical and experimental total dioptric powers can be calculated using:

$$\%\, error = \left|\frac{Experimental - Theoretical}{Theoretical}\right| \times 100\% \tag{10}$$

% error $= 2.5 \pm 0.6$ %. The percent error is low and the uncertainties for P_{tot} overlap meaning that the experimental and theoretical values are consistent.

The theoretical focal length for a ball lens measured from the center of the ball is given by Equation 3. The refractive index of the ball lens is given as $n \approx 1.5$ (no uncertainty) while the diameter is given as $D = 2.54 \pm 0.05$ cm. To determine the experimental focal length, we added the radius of the ball $D/2$ to the focal length from the edge of the ball $f' = 0.47 \pm 0.05$ cm given in the last row of table 2. In Equation form:

$$f = f' + D/2 \tag{11}$$

The theoretical focal length from the center of the ball is $f = 1.91 \pm 0.04$ cm while the experimental focal length is $f = 1.74 \pm 0.06$ cm. Uncertainty propagations for theoretical and experimental, respectively are:

$$\delta f = \frac{n}{4(n-1)} \cdot \delta D \tag{12}$$

$$\delta f = \sqrt{(\delta f')^2 + (\delta D/2)^2} \tag{13}$$

Using Equation 10, the percent error is $8.9 \pm 0.7\%$. This error is neither large nor small in magnitude.

Part 3: Image Formation with Lenses

Equation 4 can be verified by comparing the theoretical and experimental image distances for the 2cm lens. The theoretical values for i can be determined by solving Equation 4 for i. The experimental values for i are the image distances measured for the five different trials (see Table 3).

Trial	Theoretical Image Distance (cm)	Experimental Image Distance (cm)	Percent Error (%)
1	48 ± 6	46.98 ± 0.05	1.4 ± 0.2
2	31 ± 3	30.84 ± 0.05	0.41 ± 0.04
3	26 ± 2	26.56 ± 0.05	1.9 ± 0.1
4	38 ± 4	38.14 ± 0.05	0.37 ± 0.04
5	33 ± 3	32.32 ± 0.05	0.67 ± 0.06

Table 4: Verification of Equation 4 by Comparison of Theoretical and Experimental Image Distances for a 2cm Lens. The theoretical image distances come from Equation 4 using the corresponding object distances in Table 3 and the value for the focal length of 16.0±0.7 cm from Part 2. The Experimental image distances are the measured values for i (see Table 3 Column 3). The percent error was calculated using Equation 10.

The uncertainty propagation for Theoretical Image Distance is:

$$\delta i = (i_{best})^2 \cdot \sqrt{\left(\frac{\delta o}{o^2}\right)^2 + \left(\frac{\delta f}{f^2}\right)^2} \tag{14}$$

Considering that the percent errors are all very low for each trial and the uncertainties for Image Distances overlap, Equation 4 is verified with this procedure.

The Image and Object sizes, or equivalently, grating spaces, I and O respectively are related by:

$$\frac{I}{O} = \frac{i}{o} = \frac{f}{o - f} \tag{15}$$

Magnification occurs when $I > O$ or when $f < o < 2f$. Demagnification occurs when $I < O$ or when $o > 2f$. The Object and Image grating spaces are known for each trial; thus, we could verify if there is magnification or demagnification by checking if both conditions for the ratio I/O and the interval for o are satisfied. O is given in the manual as 1.0 ± 0.5 mm. The focal length for the 2cm lens was measured to be $f = 16.0 \pm 0.7$ cm. Therefore $2f = 32 \pm 1$ cm. Therefore if $I > 1.0 \pm 0.5$ mm and 16.0 ± 0.7 cm $< o < 32 \pm 1$ cm, the magnification property is verified. Table 5 summarizes the verification of magnification/demagnification using this method.

Trial	Is $f < o < 2f$	Is $I > O$	Magnification/ Demagnification	Property Verified
1	Yes	Yes	Magnification	Yes
2	No	No	Demagnification	Yes
3	No	No	Demagnification	Yes
4	Yes	Yes	Magnification	Yes
5	Yes	Yes	Magnification	Yes

Table 5: Verification of Image Magnification Property Due to a Thin Lens. Column 2 determines whether the values for o in Table 3 Column 2 are between f and $2f$. Column 3 determines whether the Image Grating Spacing is greater than the Object Grating Spacing. If these 2 columns match, then the property is verified (Column 5). Column 4 determines whether the object position causes magnification or demagnification. Note that if o does not fall within the specific range, it is assumed that $o > 2f$ because there was no data taken for values of $o < f$.

For us to be able to see a clear image on the screen, the image must be *real* as opposed to *virtual*. Real images will occur when $o > f$. All five trials satisfied this condition. For a thin converging lens like the one used in this experiment, a real image infers an inverted image. Therefore, all the images in trials 1-5 should have been inverted, and they were.

Conclusion

The main purpose of this lab was to investigate the various properties of different types of lenses. These types included biconcave, biconvex, and planoconvex thick lenses, different sized thin lenses, and a ball lens. For the thick lenses, the goal was to determine all focal lengths and observe and quantify spherical aberration on the biconvex lens and magnification/demagnification on the combination of biconcave and planoconvex lenses. The percent difference in focal lengths due to spherical aberration was $5.5 \pm 0.2\%$ which allowed us to conclude that the degree of spherical aberration is considerably noticeable for this lens. The magnification and demagnification factors for the biconcave and planoconvex lenses were 2.00 ± 0.06 and 0.53 ± 0.02 respectively. Since the same lenses were used for both instances, these values should be the inverse of one another. With the uncertainties, the values do overlap with this property. For the thin lenses, the goal was to determine focal lengths and dioptric powers for the different thin and ball lenses. For the larger diameter lenses, the focal lengths and powers were determined individually and when placed together. Thus, an experimental and theoretical value for P_{tot} was determined. The percent error between these two values was $2.5 \pm 0.6\%$. Because the uncertainties allow the two values to overlap and the percent error was low, our experimental results are consistent with the relevant Equations. For the spherical lens, the uncertainties in focal lengths for theoretical and experimental values did not allow the them to overlap and the percent error of $8.9 \pm 0.7\%$ allowed us to conclude that our experimental results did not agree with what was predicted. This may have been due to the difficulty in taking a measurement for the focal length because of how close it was to the lens. To improve this part, we could use a ball lens with a larger focal length to get a more accurate reading. For part 3, the results were very conclusive. For the five trials of different object distances, Equation 4 was verified for each of them because the theoretical and experimental image distances overlapped with uncertainties and had very low percent errors. We were also able to verify the properties of magnification for thin converging lenses, because the object distance and Image grating spacing agreed for all trials.

References

[1] Campbell, W.C. *et al*. Physics 4BL: Mechanics Lab Manual (ver. August 31, 2018). (University of California Los Angeles, Los Angeles, California).