Samuel Fafel - CSCE420 - Homework 2

1.

a. Implication Introduction

X	Y	Z	¬X	¬Z	¬X^Y^¬Z	X^Z → Y
0	0	0	1	1	0	1
0	0	1	1	0	0	1
0	1	0	1	1	1	1
0	1	1	1	0	0	1
1	0	0	0	1	0	1
1	0	1	0	0	0	0
1	1	0	0	1	0	1
1	1	1	0	0	0	1

A material condition (→) is true unless its antecedent is true and its consequent is false.

For the transformed rule X^Z → Y to be true, either X^Z is false, or both X^Z and Y are true.

We can see that when ¬X^Y^¬Z is True, X^Z → Y is also True, which suffices to prove that Implication Introduction is a sound rule of inference.

b. Conjunctive Rule Splitting

A	B	C	D	A^B	C^D	A^B→C^D	A^B→C
0	0	0	0	0	0	1	1
0	0	0	1	0	0	1	1
0	0	1	0	0	0	1	1
0	0	1	1	0	1	1	1
0	1	0	0	0	0	1	1
0	1	0	1	0	0	1	1
0	1	1	0	0	0	1	1
0	1	1	1	0	1	1	1
1	0	0	0	0	0	1	1
1	0	0	1	0	0	1	1
1	0	1	0	0	0	1	1
1	0	1	1	0	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	1
1	1	1	1	1	1	1	1

We can see that every row where (A^B→C^D) is True, A^B→C is True. There is no row where the premise is True and the conclusion is False. Therefore, whenever the premise is True, the conclusion is guaranteed to be True.

c. **Natural Deduction**

Assumption: $A \wedge B \rightarrow C \wedge D$
Show: $A \wedge B \rightarrow C$
Proof:

(1) $A \wedge B$ (Given)
(2) $C \wedge D$ (Modus Ponens from (1))
(3) C (Simplification from (2))
(4) $A \wedge B \rightarrow C$ (From (1) to (3), conditional proof)

Therefore, we've shown that if we assume $A \wedge B \rightarrow C \wedge D$, we can derive $A \wedge B \rightarrow C$ using Natural Deduction, confirming that $A \wedge B \rightarrow C \wedge D \models A \wedge B \rightarrow C$

d. **Resolution**

(1) $A \wedge B \rightarrow C \wedge D$ (Given)
(2) $\neg(A \wedge B) \vee (C \wedge D)$ (eliminate biconditionals and implications)
(3) $(\neg A \vee \neg B) \vee (C \wedge D)$ (DeMorgan's)
(4) $(\neg A \vee \neg B \vee C)$ and $(\neg A \vee \neg B \vee D)$

P1: $\neg A \vee \neg B \vee C$
P2: $\neg A \vee \neg B \vee D$

(1) $A \wedge B \rightarrow C$ (Given)
(2) $\neg(A \wedge B) \vee C$ (eliminate biconditionals and implications)
(3) $\neg A \vee \neg B \vee C$ (DeMorgan's)

Q: $\neg A \vee \neg B \vee C$

Negation of Q is:
Q': $A \wedge B \wedge \neg C$
Q'1: A
Q'2: B
Q'3: $\neg C$

From P1 and Q'1:
$\neg B \vee C$

From $\neg B \vee C$ and Q'2:
C

So we have C, which contradicts Q'3 ($\neg C$). Thus, by contradiction, the statement $A \wedge B \rightarrow C \wedge B \models A \wedge B \rightarrow C$ is proven by resolution.

2. Sammy's Sport Shop
   a. Propositional Knowledge Base (see sammy.kb)
   b. Prove that Box 2 must contain white balls **(C2W)** using Natural Deduction:
   Given: {O1Y, L1W, O2W, L2Y, O3Y, L3B}
      i. L1W → C1Y v C1B (Label is False)
      ii. O1Y → C1Y v C1B (Observation from Box 1 is Yellow)
         (i) + (ii) = C1Y v C1B (no new information)
      iii. L2Y → C2W v C2B
      iv. O2W → C2W v C2B
         (iii) + (iv) = C2W v C2B (no new information)
      v. L3B → C3Y v C3W
      vi. O3Y → C3Y v C3B
         (v) + (vi) = C3Y (Box 3 must contain only yellow)
      vii. C3Y → C1W v C1B v C2W v C2B
         (ii) + (vii) = C1B (Box 1 must contain both)
         (iv) + (vii) = C2W v C2B (no new information)
      viii. C1B → C2Y v C2W v C3Y v C3W
         (iv) + (viii) = C2W (Box 2 must contain only white)
      ix. C2W (By natural deduction)
   c. Conversion from KB to CNF (see sammy.cnf)
   d. Prove that Box 2 must contain white balls **(C2W)** using Resolution:
   Given: {O1Y, L1W, O2W, L2Y, O3Y, L3B}
   Add ~C2W and attempt to find a contradiction.
      i. L2Y and ~L2Y v C2W v C2B = C2W v C2B
      ii. O2W and ~O2W v C2W v C2B = C2W v C2B (no new information)
      iii. C2W v C2B and ~C2W = C2B
      iv. L3B and ~L3B v C3Y v C3W = C3Y v C3W
      v. O3Y and ~O3Y v C3Y v C3B = C3Y v C3B
      vi. from (iii) and (v), C2B and C3Y v C3B = C3Y
      vii. C3Y and C3Y v C3W = C3Y v C3W (no new information)
      viii. Considering the restraints, we know that ~(C1B ^ C2B). From (iii), we know C2B, so also ~C1B
      ix. L1W and ~L1W v C1Y v C1B = C1Y v C1B
      x. from (viii) ~C1B, so C1Y
      xi. from (x) and (iii) and (vi), C1Y and C2B and C3Y, meaning no box contains only white balls. This is a contradiction, therefore ~C2B is False, therefore C2B is True.

3. **Forward Chaining:**

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HaveMountainBike → HaveBike
AvisOpen → CarRentalOpen
CarRentalOpen → IsNotAHoliday
HaveMoney ^ CarRentalOpen → CanRentCar
CanRentCar → CanDriveToWork
CanDriveToWork → CanGetToWork
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4. **Backward Chaining:**

Trace	Comment
CanGetToWork	initialize with query
CanBikeToWork	pop CanGetToWork, replace with antecedent of (a)
HaveBike, WorkCloseToHome, Sunny	replace CanBikeToWork with antecedents of (d)
HaveMountainBike,WorkCloseToHome, Sunny	replace HaveMountainBike with antecedent of (e)
WorkCloseToHome, Sunny	pop HaveMountainBike (fact)
Sunny	pop WorkCloseToHome (fact)
[backtrack] CanDriveToWork	Sunny is not provable, therefore CanBikeToWork is not provable, so move to next rule for CanGetToWork (b)
OwnCar	replace CanDriveToWork with antecedent of (g)
[backtrack] CanRentCar	OwnCar is not provable, so move to next rule for CanDriveToWork (j)
HaveMoney, CarRentalOpen	replace CanRentCar with antecedents of (k)
CarRentalOpen	pop HaveMoney (fact)
HertzOpen	replace CarRentalOpen with antecedent of (l)
[backtrack] AvisOpen	HertzOpen not provable, so move to next rule for CarRentalOpen (m)
∅	pop AvisOpen (fact); stack becomes empty; return success!