Samuel Fafel - CSCE420 - Homework 3

1. **Translate the following sentences into First-Order Logic.**
    a. bowling balls are sporting equipment:
    $\forall x(BowlingBall(x) \rightarrow SportingEquipment(x))$

    b. horses are faster than frogs:
    $\forall x(Horse(x) \rightarrow (Speed(x) > \forall y(Frog(y) \rightarrow Speed(y))))$

    c. all domesticated horses have an owner:
    $\forall x(DomesticatedHorse(x) \rightarrow \exists y(Owner(y) \land Owns(y,x)))$

    d. the rider of a horse can be different than the owner:
    $\exists x \exists y \exists z(Horse(x) \land Rider(y) \land Owner(z) \land Rides(y,x) \land Owns(z,x) \land \neg(y = z))$

    e. a finger is any digit on a hand other than the thumb:
    $\forall x(Digit(x) \land OnHand(x) \land \neg Thumb(x) \rightarrow Finger(x))$

    f. an isosceles triangle is defined as a polygon with 3 edges connected at 3 vertices, where 2 (but not 3) edges have the same length:
    $\forall x(Polygon(x) \land Edges(x) = 3 \land Vertices(x) = 3 \land \exists y \exists z (Edge(y) \land Edge(z) \land EqualLength(y,z) \land PartOf(y,x) \land PartOf(z,x) \land \neg\forall w(Edge(w) \land PartOf(w,x) \rightarrow EqualLength(y,w))) \rightarrow IsoscelesTriangle(x))$

2. **Convert the following first-order logic sentence into CNF:**
$\forall x\ person(x) \land [\exists z(petOf(x,z)) \land \forall y(petOf(x,y) \rightarrow dog(y))] \rightarrow doglover(x)$

    1. Eliminate Implications
    2. Move NOTs inward
    3. Standardize Variables
    4. Skolemize
    5. Drop Universal Qualifiers
    6. Distribute $\land$ over $\lor$

$[\neg person(x) \lor \neg petOf(x,f(x)) \lor \neg petOf(x,g(x)) \lor dog(g(x)) \lor doglover(x)]$

f(x) denotes a potential pet of x
g(x) denotes a potential dog of x

3. Determine whether or not the pairs of predicates are unifiable.

a. owes(owner(X), citibank, cost(X))   owes(owner(ferrari), Z, cost(Y))
   u = {X/ferrari, Z/citibank, Y/X}
   unified: owes(owner(ferrari), citibank, cost(ferrari)

b. gives(bill, jerry, book21)   gives(X,brother(X),Z)
   u = {X/bill, Z/book21}
   unified: gives(bill, brother(bill), book21)

c. opened(X, result(open(X), s0)))   opened(toolbox, Z)
   These are not unifiable because result(open(X), s0) is a function that depends on variable X while Z is the second predicate is a single variable. There is no way to unify these without a condition that would make the result(open(X), s0) equivalent to Z. Specifically, the first predicate's second argument is a function, while the second predicate's second argument is a variable.

4. Consider the following situation:

Marcus is a Pompeian.
All Pompeians are Romans.
Caesar is a ruler.
All Romans are either loyal to Caesar or hate Caesar (but not both).
Everyone is loyal to someone.
People only try to assassinate rulers they are not loyal to.
Marcus tries to assassinate Caesar.

a. Translate these sentences to First-Order Logic.

1. Pompeian(Marcus)
2. $\forall$x(Pompeian(x) $\rightarrow$ Roman(x))
3. Ruler(Caesar)
4. $\forall$x(Roman(x) $\rightarrow$ Loyal(x, Caesar) $\oplus$ Hate(x, Caesar))
5. $\forall$x$\exists$y(Loyal(x,y))
6. $\forall$x$\forall$y((Ruler(y) $\wedge$ Assassinate(x,y)) $\rightarrow$ $\neg$Loyal(x,y))
7. Assassinate(Marcus, Caesar)

b. Prove that Marcus hates Caesar using Natural Deduction. Label all derived sentences with the ROI and which prior sentences and unifier were used.

- Marcus is a Roman (from sentence 1 "Marcus is a Pompeian" and sentence 2 "All Pompeians are Romans").
    - ROI: Universal Instantiation (UI) from sentence 2, then Modus Ponens (MP) with sentence 1 as the particular instance.
    - Derived from sentences: 1, 2.
- Marcus is not loyal to Caesar (from sentence 7 "Marcus tries to assassinate Caesar" and sentence 6 "People only try to assassinate rulers they are not loyal to").
    - ROI: Universal Instantiation (UI) from sentence 6, then Modus Ponens (MP) with sentence 7.
    - Derived from sentences: 6, 7.
- Marcus either hates Caesar or is loyal to Caesar (from sentence 4 "All Romans are either loyal to Caesar or hate Caesar" and the conclusion from step 1 that Marcus is a Roman).
    - ROI: Universal Instantiation (UI) from sentence 4, then Modus Ponens (MP) with the conclusion from step 1.
    - Derived from sentences: 1, 2, 4.
- Marcus hates Caesar (from the conclusion in step 3 and the conclusion in step 2).
    - ROI: Disjunctive Syllogism (DS) with step 3's disjunction and step 2's negation.
    - Derived from sentences: 1, 2, 4, 6, 7.

c. Convert all the sentences into CNF

1. Pompeian(Marcus)
2. (¬Pompeian(x) ∨ Roman(x))
3. Ruler(Caesar)
4. ((¬Roman(x) ∨ Loyal(x, Caesar)) ∧ (¬Roman(x) ∨ ¬Hate(x, Caesar)) ∧ (¬Roman(x) ∨ Hate(x, Caesar)) ∧ (¬Roman(x) ∨ ¬Loyal(x, Caesar)))
5. (Loyal(x, f(x)))
6. (¬Ruler(f(x)) ∨ ¬Assassinate(x, f(x)) ∨ ¬Loyal(x, f(x)))
7. Assassinate(Marcus, Caesar)

d. Prove that Marcus hates Caesar using Resolution Refutation.

Roman(Marcus) : (1) and (2), u = {x/Marcus}
Loyal(Marcus, Caesar) : Roman(Marcus) and (4), u = {x/Marcus}
¬Hate(Marcus, Caesar) : Roman(Marcus) and (4), u = {x/Marcus}
¬Loyal(Marcus, Caesar) : (6) and (7), u = {x/Marcus, f(x) = Caesar}
Hate(Marcus, Caesar) : ¬Loyal(Marcus, Caesar) and (4), u = {x/Marcus} (Contradiction!)

5. Write a KB in First-Order Logic with rules/axioms for…

a. **Map-Coloring**

color(c) #Where 'c' is a color
state(s) # Where 's' is a state
colored(s, c) # State 's' is colored with color 'c'
adjacent(s_1, s_2) # State 's_1' is adjacent to state 's_2'

$\forall s \exists c(\text{colored}(s, c))$ # Every state must have a color
$\forall s \forall c_1 \forall c_2(\text{colored}(s,c_1) \land \text{colored}(s,c_2) \rightarrow c_1 = c_2)$ # State must have only one color
$\forall s_1 \forall s_2 \forall c(\text{adjacent}(s_1, s_2) \rightarrow (\text{colored}(s_1, c) \rightarrow \neg\text{colored}(s_2, c)))$ # No adjacent colors

b. **Sammy's Sport Shop**

obs(x, q) # Observed Box 'x' to have a ball of color 'q'
label(x, q) # Box 'x' is labeled with color 'q'
cont(x, q) # Box 'x' contains balls of color 'q'

$\forall x \exists q(\text{cont}(x, q))$ # Each box contains something
$\forall x_1 \forall x_2 \forall q\ (x_1 \neq x_2 \rightarrow \text{cont}(x1, q) \rightarrow \neg\text{cont}(x_2, q))$ # No boxes with same contents
$\forall x \forall q(\text{label}(x, q) \rightarrow \neg\text{cont}(x, q))$ # Every label is incorrect
$\forall x \forall q_1 \forall q_2\ (\text{cont}(x, q_1) \land \text{cont}(x, q_2) \rightarrow q_1 = q_2)$ # Box must have only one type of content

c. **Wumpus World**

adjacent(x, y, p, q) # defines room at (x, y) is adjacent to (p, q)
stench(x, y) # means there is a stench in room (x, y)
breezy(x, y) # means there is a breeze in room (x, y)
wumpus(p, q) # means the Wumpus is in room (p, q)
pit(p, q) # means there is a pit in room (p, q)
safe(x, y) # means room (x, y) is safe

$\forall x \forall y \forall p \forall q(\text{adjacent}(x, y, p, q) \rightarrow (\text{stench}(x, y) \leftrightarrow \text{wumpus}(p, q)))$
All rooms adjacent to wumpus have stench
$\forall x \forall y \forall p \forall q(\text{adjacent}(x, y, p, q) \rightarrow (\text{breezy}(x, y) \leftrightarrow \text{pit}(p, q)))$
All rooms adjacent to pit have breeze
$\forall x \forall y(\text{safe}(x, y) \leftrightarrow \neg\text{wumpus}(x, y) \land \neg\text{pit}(x, y))$ # Safe if no wumpus and no pit

d. **4-Queens**

queen(r, c) #means there is a queen in row 'r' and column 'c'

$\forall r \forall c(\text{queen}(r, c) \rightarrow \neg(\exists c_2\ (c_2 \neq c \land \text{queen}(r, c_2))))$ # No queen in same column
$\forall r \forall c(\text{queen}(r, c) \rightarrow \neg(\exists r_2\ (r_2 \neq r \land \text{queen}(r_2, c))))$ # No queen in same row
$\forall r \forall c(\text{queen}(r, c) \rightarrow \neg(\exists r_2\ \exists c_2\ (r_2 \neq r \land c_2 \neq c \land |r - r_2| = |c - c_2| \land \text{queen}(r_2, c_2))))$

No queen in same diagonal