Pricing problems with Thompson Sampling

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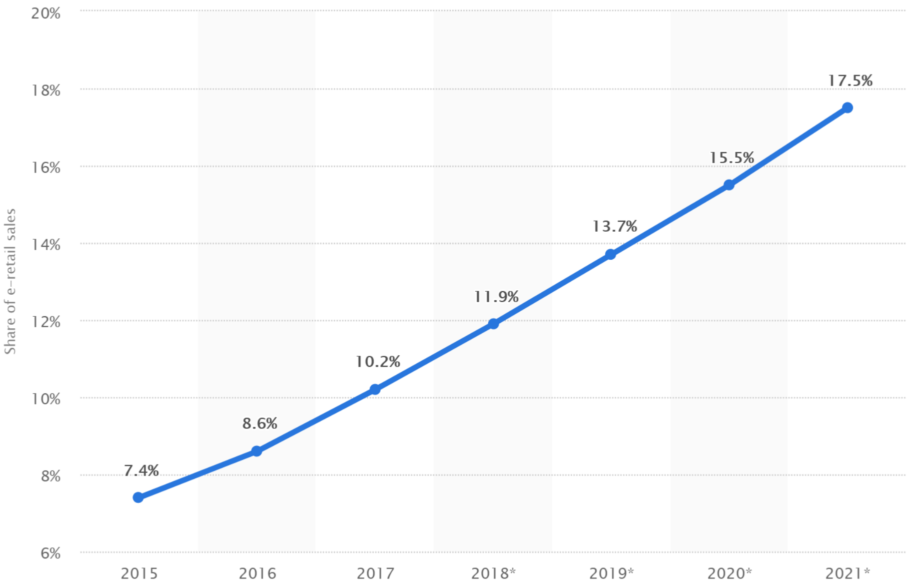
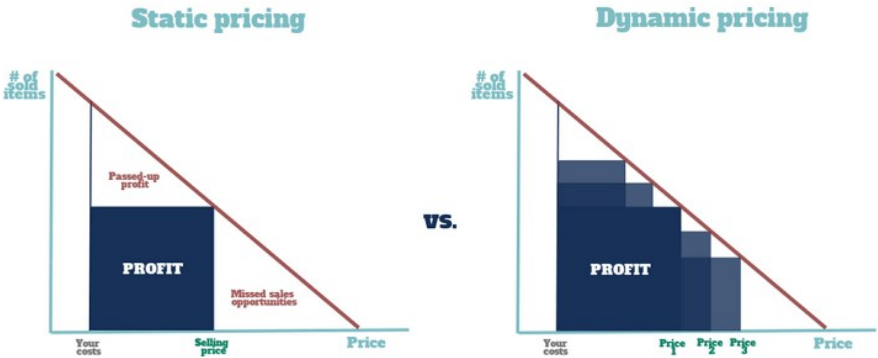
E-commerce trend



Dynamic Pricing



1. Maximise revenue
2. Attract more customers
3. Clear excess inventory

We can model dynamic pricing as sales promotions

The New York Times

H&M, a Fashion Giant, Has a Problem: $4.3 Billion in Unsold Clothes

Search

Bloomberg

A Power Plant Is Burning H&M Clothes Instead of Coal

The New York Times

Burberry to Stop Burning Clothing and Other Goods It Can't Sell

The New York Times

Back on Trend? H&M Makes AI, Loyalty Drive to Ride Fashion Cycle

By Reuters

April 17, 2019

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Why Thompson Sampling?

Good empirical performance

1. An Empirical Evaluation of Thompson Sampling (2011). Chapelle, Li

Already extended to revenue management problems

1. Online Network Revenue Management Using Thompson Sampling (2017). Ferreira, Simchi-Levi, Wang
2. Thompson Sampling for Complex Online Problems (2014). Gopalan, Mannor, Mansour.
3. Optimization of a SSP's Header Bidding Strategy using Thompson Sampling (2018). Jauvion, Grislain, Sielenou, Garivier, Gerchinovitz
4. Thompson Sampling for Dynamic Pricing (2018). Ganti, Sustik, Tran, Seaman

Classical Thompson Sampling

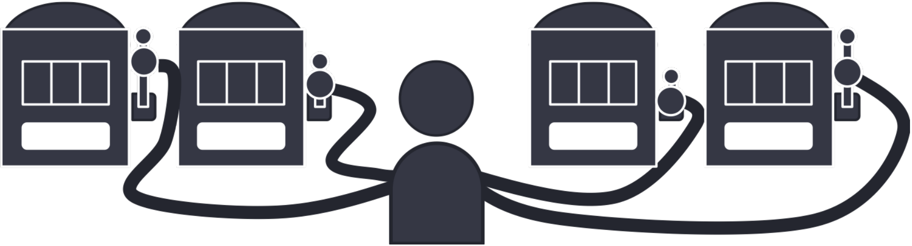

Figure: Multi-armed bandit problem

Key ideas of Thompson Sampling:

1. Probability matching
2. Bayesian updating

Pricing problem can be modelled as MAB problem.

MNL model

Goal: To learn the true demand under each price point

Data was created by:

$$X_{ik} = \frac{e^{V_i - P_{ik}}}{\sum_{j=0}^{N} e^{V_j - P_{jk}}}$$

1. Fixed V_i
2. Assumed true demand follows a Normal distribution
3. Created historical data for each of the 3 price points
4. Initialised prior distributions for each price point using MLE
5. Choose a price point and observe X_{ik}
6. Add observation to historical data and update distribution

MNL model

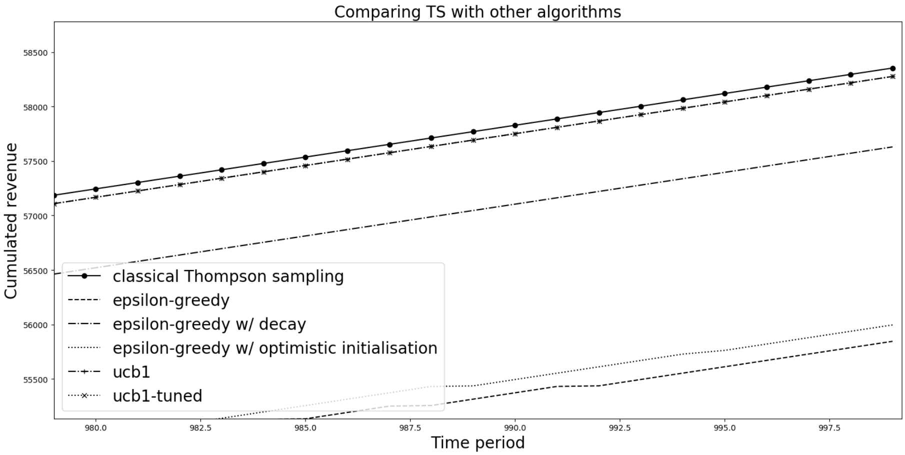

Figure: Comparing revenue between all algorithms (MNL model)

Constant price elasticity model

Goal: To learn the true demand under each price point

Data was created by:

$$d_i(p_i) = f_i \left(\frac{p_i}{p_{0,i}} \right)^{\gamma_{*,i}}$$

1. Fixed $\gamma_{*,i}, f_i, p_{0,i}$ as baseline demand using data from dataset
2. Assumed true demand follows a Normal distribution
3. Created historical data for each of the 3 price points
4. Initialised prior distributions for each price point using MLE
5. Choose a price point and observe $d_i(p_i)$
6. Add observation to historical data and update distribution

Constant price elasticity model

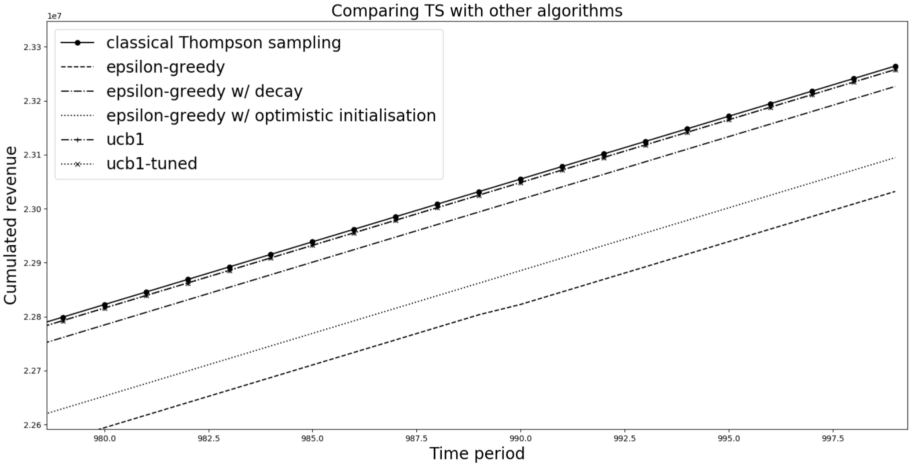

Figure: Comparing revenue between all algorithms (constant elasticity model)

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Adapted TS

Proposed by Ganti, Sustik, Tran, Seaman in Thompson Sampling for Dynamic Pricing (2018).

Goal: To learn elasticity γ_* instead of demand

1. Place a prior distribution on $\gamma_t \sim N(\mu, \Sigma)$
2. Sample from γ_t until all components are negative
3. Use a time series model to forecast demand $f_{i,t}$
4. Solve the optimisation problem to obtain new prices
   $p_t = \arg\max_p \sum_{i=1}^{N} \frac{p_i^2 f_{i,t}\gamma_{*,i}}{p_{i,t-1}} - p_i f_{i,t}\gamma_{*,i} + p_i f_{i,t}$
5. Apply prices p_t and observe reward
6. Update distribution of γ_t

Adapted TS

Goal: To learn elasticity γ_* instead of demand

Non-stationary demand model:

$$d_{i,t}(p_i) = f_{i,t}\left(\frac{p_i}{p_{i,t-1}}\right)^{\gamma_{*,i}}$$

1. Used auto regressive model $f_{i,t} = 0.05 + \sum_{\tau=t-1}^{0} 0.5^{t-\tau} d_{i,\tau} + \epsilon_t$
2. Fixed $\gamma_{*,i}, d_{i,0}$ as baseline demand using data from dataset
3. Random initialisation of parameters for prior distribution γ_t

Stationary demand model:

$$d_i(p_i) = f_i\left(\frac{p_i}{p_{0,i}}\right)^{\gamma_{*,i}}$$

1. Fixed $\gamma_{*,i}, f_i, p_{0,i}$ as baseline demand using data from dataset
2. Random initialisation of parameters for prior distribution γ_t

With inventory constraints

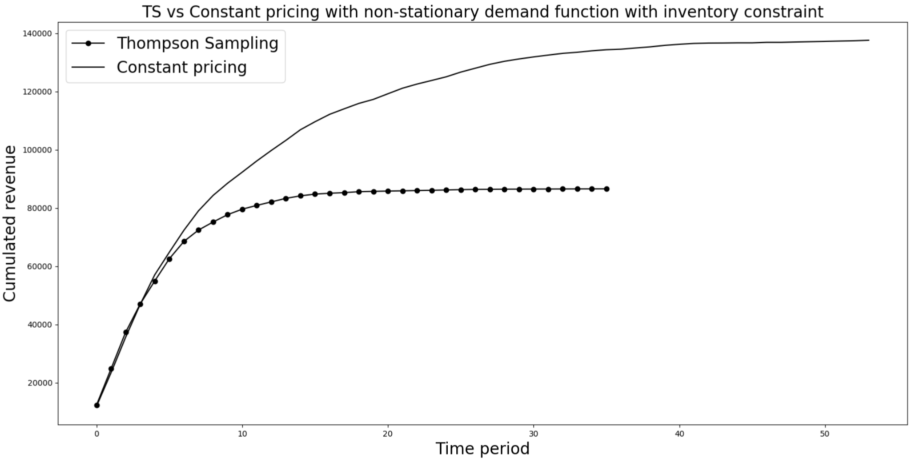

Figure: Cumulated revenue of TS and constant pricing with inventory constraints

Without inventory constraints

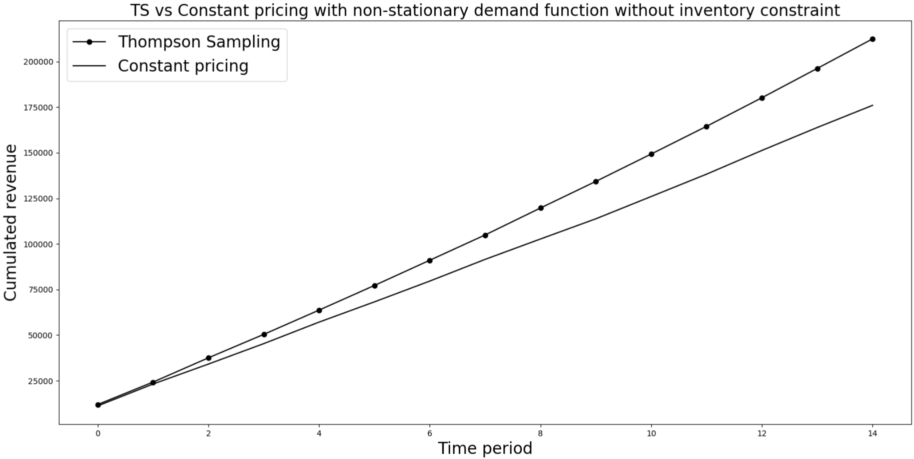

Figure: Cumulated revenue between TS and constant pricing without inventory constraints

Conclusion

- Thompson sampling may work just as well as if not better than commonly used algorithms if given good priors
- Dynamic Thompson sampling works best if inventory constraints are not a concern
- When Thompson sampling works well, it helps with maximising revenue and clearing inventory

References

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