Agent-Based Computational Modeling and International Relations Theory: *Quo Vadis?*

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Summary

Agent-based computational modeling (ABM, for short) is a formal and supplementary methodological approach used in international relations (IR) theory and research, based on the general ABM paradigm and computational methodology as applied to IR phenomena. ABM of such phenomena varies according to three fundamental dimensions: scale of *organization*—spanning foreign policy, international relations, regional systems, and global politics—as well as by *geospatial* and *temporal* scales. ABM is part of the broader complexity science paradigm, although ABMs can also be applied without complexity concepts. There have been scores of peer-reviewed publications using ABM to develop IR theory in recent years, based on earlier pioneering work in computational IR that originated in the 1960s that was pre-agent based. Main areas of theory and research using ABM in IR theory include dynamics of polity formation (*politogenesis*), foreign policy decision making, conflict dynamics, transnational terrorism, and environment impacts such as climate change. Enduring challenges for ABM in IR theory include learning the applicable ABM methodology itself, publishing sufficiently complete models, accumulation of knowledge, evolving new standards and methodology, and the special demands of interdisciplinary research, among others. Besides further development of main themes identified thus far, future research directions include ABM applied to IR in political interaction domains of space and cyber; new integrated models of IR dynamics across domains of land, sea, air, space, and cyber; and world order and long-range models.

Keywords: international relations, IR theory, agent-based modeling ABM, computational modeling, social science methodology, formal models in social science, computer models, object-oriented modeling, empirical international relations theory

Subjects: Qualitative Political Methodology, Quantitative Political Methodology, World Politics

Introduction

What Is an ABM?

An agent-based computational model in international relations (IR) is an object-oriented simulation model of a system of political entities that interact on foreign policy issues on an organizational scale or "level of analysis" (country, relational, regional, or global). Accordingly, the landscape of theory and research on agent-based computational IR varies in terms of such components (specific systems, issues, and scales) and their combinations across different ABMs. In order to assess and understand ABMs in IR it is worthwhile to take a closer look at each dimension and its combinations, which defines the structure and state of the field.

Two preliminary terminological clarifications are in order. First, the acronym "ABM" is used to denote (1) an agent-based computational *model*, as well as (2) agent-based *modeling* as a specific methodology that is used in many domains of science (social, natural, and engineering), with meaning usually clarified by context. Second, the term "computational" is always implied in reference to ABM, because there are many agent-based models that are not necessarily computational, such as game-theoretic models, Richardsonian conflict models, and others where actors are the fundamental units of analysis.

The methodology of ABM in IR theory and research is summarized in Figure 1. The process begins by selecting a part of the IR real-world universe to study (explanandum, in the lower center). In a way that parallels other formal approaches (e.g., mathematical modeling), an abstraction selects a set of concepts, entities, research questions, and other modeling elements to create a formal model. In this case the formalization uses a computational structure, such as provided by a preexisting toolkit (e.g., MASON, NetLogo, Repast, or other) or by native code in some object-oriented programming (OOP) language (see Section "Object Oriented Modeling (OOM) and Object Oriented Programming (OOP)," below). Once the agent-based simulation model is created (explanans, in the upper right side of the cycle), it must then be subject to verification and validation tests with empirical data. A sufficiently verified and validated model is considered ready for analysis of scenarios, "what if" research questions, and other scientific uses.

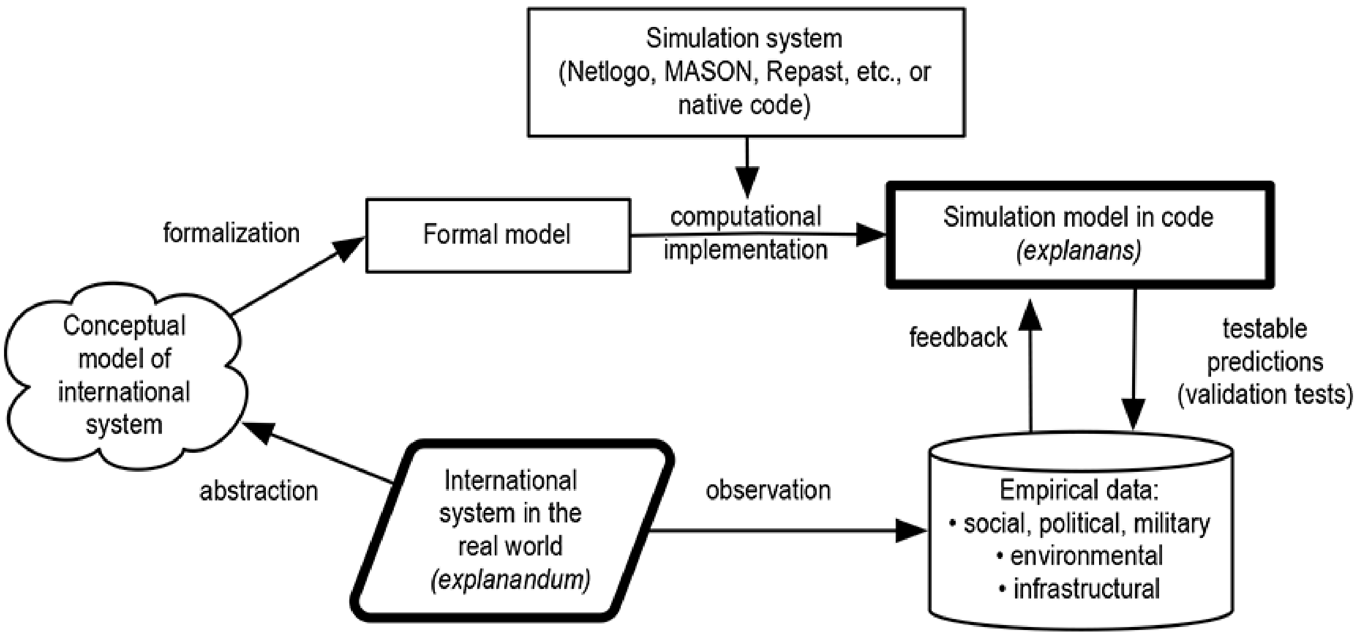

Figure 1. Formal methodology of agent-based modeling applied to international relations theory.

Note: Adapted from Cioffi-Revilla (2017, ch. 8).

Complexity of IR and ABMs

First, international relations phenomena are complex, in a strict scientific sense, because they generally satisfy the characteristics of complex phenomena (Byrne & Callaghan, 2014; Cioffi-Revilla, 2017, ch. 1, 7; Conte et al., 2012; Epstein, 1999; Page, 2015; Paravantis, 2016):

1. *Cardinality*. The number of agents or entities in the international system, as well as in IR ABMs is large. For example, in every epoch the international system is composed of multiple actors, even during bipolar epochs.
2. *Diversity*. In addition to being large, entities are diverse in nature, including state and non-state actors, relations, and dynamics.
3. *Dimensionality*. The state of each entity in an international system is determined by many variables and parameters, so IR phenomena and their ABMs have what is known in systems terminology as "high dimensionality." Note that dimensionality, cardinality, and diversity represent separate aspects of a complex system, not the same dimension of complexity. Examples include goals, capabilities, vulnerabilities, allies, dependencies, and infrastructure, and relevant geospatial features, among others.
4. *Connectivity*. Entities and dynamics in IR phenomena and in ABMs are linked or interact through interaction network structures that connect agents and entities. Organizations, both internal and external (e.g., alliances) to each actor, provide typical examples of connectivity structures.
5. *Nonlinearity*. Patterns of interactions in IR and in ABMs are generally nonlinear, as opposed to linear dependencies in simple systems. This property is primarily responsible for generating emergent phenomena at macro-levels above the individual entities, such as emergent relational, regional, systemic, or global phenomena (polarization, warfare, alliances, international regimes and organizations). For example, perceived security threats are a nonlinear function of objective threat conditions (a phenomenon known in psychology as the Weber-Fechner law); and compound events in deterrence are exponentially related to the number of deterrence requirements (Wohlstetter, 1959, 1968); among other known patterns, so foreign decisions are the result of anything but linear thinking or simple processes (see, e.g., Cioffi, 1989, 2017, ch. 6; Zinnes & Gillespie, 1976; Zinnes & Muncaster, 1987; Zinnes et al., 1978).

Multiple Modeling Scales

The scale of IR ABMs is at least three-dimensional, according to organization, space, and time.

1. *Organizational scale*. An IR ABM may range from single-actor decision making, to group decision making (e.g., bureaucratic systems or processes), or be national, regional, or global in organizational scope.
2. *Spatial scale*. The geographic scope of an ABM, when specified, determines its spatial scale. While some models include two countries, others include several or the entire international system.
3. *Temporal scale*. IR ABMs operate on a time-scale of that range from tens of milliseconds (human decision making) to millennia (rise and fall of civilizations). ABMs of the international system of states are typically calibrated on a scale of years and run through several centuries.

Methodologically, therefore, every IR ABM can be situated in a three-dimensional space consisting of the above dimensions. From simplest to most complex, in relative terms, IR ABMs range from the most individualistic (e.g., a single agent decisionmaker), referencing a single actor (e.g., within a foreign policy bureaucracy or agency), with a short temporal range (e.g., a single decision in time); to the most comprehensive political collective (global system), in all continents and space where political agents interact (governmental and non-governmental), from the origin of politics in complex societies to the present (the so-called long-range consisting of the past 10,000 years). The organizational (O), geospatial (G), and temporal (T) scales of a political ABM specify its "OGT scale," which is more accurate than simply referring to a two-dimensional spatio-temporal scale. The OGT scales generate a three-dimesnional space where each model is situated and may be compared to others.

Object-Oriented Modeling (OOM) and Programming (OOP)

While IR ABMs differ in terms of complexity, OGT scales, and other specific characteristics, they all have the object-oriented modeling (OOM) paradigm of computational social science in common. The defining feature of OOM is the focus on political entities, which comprise ("encapsulate" in proper OOM terminology) variables and dynamics. By contrast, other modeling methodologies, such as statistics and equation-based mathematical models in social science focus primarily on variables, leaving actors implicit. Two examples of OOM theory in IR are deterrence theory and balance of power theory, because the units of analysis in each case consist of countries (strategic actors) which, in turn, consist of capabilities and other features. In other words, strategic capabilities are conceptualized as constituent attributes of actors, not as separate or isolated variables. By contrast, a Richardsonian arms race model is more directly focused on variables such as levels of armaments or hostility, rather than being primarily focused on the actors themselves.

A significant methodological implication and theoretical advantage of the object orientation of ABM is that it enables the researcher to include essential but often heterogeneous features of IR within a single integrated model, such as the environment of the international system being modeled, capabilities, belief systems, norms, energy and information flows, hierarchies and networks of couplings and embeddedness, systems-of-systems, and similar real-world features and phenomena. Modeling such features is often impossible or intractable through other approaches, such as statistical or traditional mathematical models.

Computational IR

The ABM approach to IR theory is a specific modeling methodology and scientific tradition within the broader field of computational social science, which comprises several other modeling traditions, such as complexity theory, network science, microsimulation, and system dynamics, among others (see Cioffi-Revilla, 2017; Gilbert & Troitzsch, 2005; Taber & Timpone, 1996). Hybrid ABMs are those that combine agent-based computational components with other nonagent formal structures, such as system dynamics (e.g., Boyle et al., 2006) or evolutionary learning (Cioffi, De Jong, & Bassett, 2012). Hybrid ABMs have grown in quantity and quality as researchers have gained familiarity with computational modeling methods to fulfill theoretical needs.

Why ABMs Support Interdisciplinary Research Necessary for Understanding Politics

Finally, as a field of computational social science, ABM research in fields such as IR is subject to the same opportunities, limitations, and other operating aspects as agent-based modeling research in other fields in the natural, social, and engineering sciences where computational simulation modeling is used. A major property of ABM is its powerful capacity for interdisciplinary integration across subject matter necessary for IR theory. For example, ABM can integrate political with social, cultural, economic, geographic and other aspects necessary to create viable regional or global models of the international system (Cioffi, 2016). This is also the reason why much research on social ABMs is presented at conferences in many fields, including natural science and engineering sciences conferences, not just in social science. Purely mathematical, noncomputational models are more limited in this regard, given the complexity of interdisciplinary models (Bonabeau, 2002).

Chronological Overview of ABM in IR

This article focuses on peer-reviewed IR ABM research published in recent years, as well as on earlier models that somehow have escaped the attention of other reviewers but hold high significance. Peer-reviewed publications include traditional journals and academic presses, as well as PhD dissertations, refereed conference papers, and post-conference proceedings, among others. The total population of such channels has grown significantly over time, as progress and interest in the field has grown.

Table 1 contains the results of surveying the large field of ABMs in IR using the three dimensions of IR in terms of organizational, geospatial, and temporal scales. Column 1 provides a name and number that identifies each ABM. Some modelers name their models (e.g., GeoSim, MayaSim, AfriLand), while others do not, so a name is assigned to models that appear to lack a name. The practice of naming models is to be encouraged, for it facilitates referencing and omits having to always name the authors or make an arbitrary selection of authors. Column 2 provides a brief description of the referent system in the empirical world (i.e., the system being modeled) and the phenomenon or theme being investigated within the system. Column 3 specifies the approximate degree of empirical fidelity of the ABM, with low meaning highly abstract or theoretical, high meaning that a large amount and variety of empirical data or validated mechanisms were used to build the model, and medium being somewhere in between. Clearly, values of empirical calibration or fidelity are somewhat subjective and intended only as coarse measures so as to categorize ABMs on a simple scale of abstraction. Column 4 specifies the programing language or toolkit used to implement the model in code. Finally, Column 5 provides references to one or more publications where the model is documented. Each model is listed in chronological order of the year in which it was first published.

Table 1. Agent-Based Computational Models in International Relations by Date of Publication

Model number and name	Referent system; research themes	Abstract scale	Source code	Publications
		1977		
1. Machiavelli in Machina	International state system; conflict	Medium	FORTRAN	Bremer & Mihalka, 1977
		1980		
2.Computational 2-Person Iterated Prisoner’s Dilemma	Multi-actor system; emergence of cooperation	Low		Axelrod, 1980a, 1980b, 1981, 1984, 1986, 1987; Axelrod & Dion, 1988; Wu & Axelrod, 1995
		1986		
3. Norms Game	Multi-actor system; norm emergence and stability	Low		Axelrod, 1986, 1987
		1990		
4. Realpolitik Among Hexagons	International state system	Medium		Cusack & Stoll, 1990
		1992		
5. Concurrent Interstate Conflict	International system; onset of wars, alliances	Medium	Lisp	Duffy, 1992, 1993
		1993		

Model number and name	Referent system; research themes	Abstract scale	Source code	Publications
6. Landscape Theory	International state system; alliance formation	Medium	Pascal	Axelrod & Bennett, 1993; Axelrod, 1997 in R; Bennett, 2000 in Pascal
		1994		
7. EOS	Neolithic polities system	Low		Doran et al., 1994
		1995		
8. Tribute Model	International state system	Low	Pascal	Axelrod, 1995
		1997		
9. ISAAC	Landscape of combat; emergent phenomena of military combat	Medium	ANSI C and EINSTein (C++)	Ilachinski, 1997, 2004, 2005, 2012
10. GeoSim	Balance of power system; territorial change	Medium	Repast	Cederman, 1997, 2001, 2002, 2003; Cederman et al., 2010, 2011; Cioffi & Gotts, 2003; Johnson et al., 2011
		2000		
11. Two-Level National Security Management	International state system with domestic opposition actors; maintaining security and democracy	Medium	Pascal	Simon & Starr, 2000

Model number and name	Referent system; research themes	Abstract scale	Source code	Publications
		2002		
12. Economic Geography, Trade and War	International state system; onset of war	Medium	Gauss	Bearce & Fisher, 2002
13. Civil Violence	Generic agent society; onset of civil violence warfare	Low	Sugarscape	Epstein, 2002
14. War and Trade	International state system; emergence of war	Medium	C++	Min, 2002; Min et al., 2004
15. Client States	International system; formation of U.S. client states	Medium	Repast	Sylvan & Majeski, 2002, 2003
		2003		
16. Radical Islamist Terrorism TAP	Middle East and North Africa (MENA) region; onset of terrorism	High	Java	MacKerrow, 2003
17. Historical Dynamics	System of polities; rise and fall of polities	Low	APL	Turchin, 2003
		2004		
18. Etruscan Politogenesis	Regional Etruscan polity system; emergence of politogenesis	High	NetLogo?	Cecconi et al., 2004, 2015

Model number and name	Referent system; research themes	Abstract scale	Source code	Publications
19. Emergent Polarity	International system; emergence of polarity	Low	Pascal	Cederman, 2004
20. Asymmetric Power	International system with asymmetric power; emergence of cooperation	Low	C+	Majeski, 2004, 2005
21. Silk Road Simulation	Network of silk road polities; emergence and self-organization	Medium	Java	Malkov, 2004, 2006
22. Conflict Wargame	International system; strategic-operational effects of conflict	High	Aide de Camp 2 (AIDE2)	Selke, 2004
		2005		
23. Bronze Age Mesopotamia	Mesopotamian polities system; emergence of sociopolitical complexity	High	ENKIMDU	Christiansen & Altaweel, 2005, 2006; Wilkinson et al., 2007
24. International Norms	International system; emergence of international norms	Medium		Hoffmann, 2005, 2006, 2008
25. TNet-CTNet	Complex adaptive system of terrorist and CT networks; emergent evolving networks	Medium-High	SOTCAC in C++	Ilachinski, 2005, 2012
		2007		
26. Small-Scale Chiefdoms	Small-scale agent society; politogenesis	Medium	C++	Alden & Choi, 2007

Model number and name	Referent system; research themes	Abstract scale	Source code	Publications
27. Titicaca Warfare	Titicaca regional polities system; conflict	Medium	NetLogo	Griffin & Stanish, 2007; Stanish & Levine, 2011
		2008		
28. REsCAPE	Society with ethnic groups; onset of conflict and warfare	Framework	Java	Bhavnani, 2008; Bhavnani & Miodownik, 2009
29. Hierarchies	Inner Asia regional system; conflict among polities	High	MASON	Cioffi et al., 2011; Cioffi, Honeychurch, & Rogers, 2015; Rogers, 2017
		2009		
30. AfriLand	Regional international system; political stability, transnational conflict	Medium	MASON	Cioffi & Rouleau, 2009a, 2009b
		2010		
31. Cycling Polities	System of chiefdoms; cycling over time	Medium	Matlab	Gavrilets et al., 2010; Turchin et al., 2013
		2011		
32. Peruvian Politogenesis	Aspero polity and Norte Chico region, Supe River Valley, Peru; polity formation	High	NetLogo	Auble, 2010; Auble & Cioffi, 2013; Cioffi-Revilla, 2017, pp. 335–341
33. SOC and Polity Cycling	Population landscape; emergence of polities	Medium	NetLogo	Griffin, 2011

Model number and name	Referent system; research themes	Abstract scale	Source code	Publications
34. NormSim	International system; emergence of international norms and institutions	Medium	MASON	Rouleau, 2011
		2012		
35. RiftLand	East African regional international system; disasters, crises, conflict	High	MASON and GeoMASON	Cioffi et al., 2012; Kennedy et al., 2010, 2012; Sutherland, 2012
36. SW Puebloan	Puebloan polities system; collective goods	High		Kohler et al., 2012a, 2012b
37. Identity Conflicts	Interstate system with identities; emergence of conflict	Medium	NetLogo	Gartzke & Weisiger, 2013
		2013		
38. Canonical Polity Cycling	Politogenesis; emergence and cycling of first complex polities	Medium	NetLogo	Dover, 2013; Dover & Cioffi, 2015
39. MayaSim	Maya polities system; polity dynamics	High	NetLogo	Heckbert, 2013
40. Spatial Tribute Model	International state system	Low	MASON	Masad, 2013
41. Baghdad Sunni-Shiite War	Baghdad, Iraq; warfare between Sunnis and Shiites	High		Weidmann & Salehyan, 2013

Model number and name	Referent system; research themes	Abstract scale	Source code	Publications
		2014		
42. Jerusalem Segregation and Violence	Jerusalem, Israel; segregation and emergence of urban violence	High	Repast and Java	Bhavnani et al., 2014
		2015		
43. Northern Jazirah Politogenesis	Regional Northern Jazirah polities system, Iraq; emergence of polity settlement hierarchies	High	Repast	Altaweel, 2015
44. NorthLands	Boreal and Arctic regions; climate change and sociopolitical adaptation	High	MASON and GeoMASON	Cioffi et al., 2015, 2016
45. Paths to Great Power War	International multipolar state system; onset of great power war	Medium	NetLogo	Luteijn, 2015
		2016		
46.ZambeziLand	Zambezi River, Southern Africa; polity formation	Medium	Python	Bogle & Cioffi, 2016
47. CLM-2016	Pacific region international system; deterrence, proliferation nukes	High	Construct	Carley et al., 2016
48. Masad, 2016	International system; conflicts and crises	Medium	Python	Masad, 2016

Model number and name	Referent system; research themes	Abstract scale	Source code	Publications
49. Maidan	Contemporary Ukrainian society; collective action vs. Russian annexation	Low	NetLogo	Pugacheva, 2016, pp. 354–365
50. Referenda v. Propaganda	Global system; top-down and bottom-up institutional effects	Low	Java	Ulloa et al., 2016
		2017		
51. RealLand	International state system; onset of war, alliances, territorial dynamics	High	NetLogo	Selke, 2017; Cioffi-Revilla, 2017

Note: Self-organized terrorist-counterterrorist adaptive coevolution.

Source: Prepared by the author.

The reader will note that numerous international conflict models are included in this survey of ABM in IR theory and research. However, not all forms of conflict or warfare are included, given the international orientation of this survey. For example, ABMs of purely internal or civil violence (e.g., Bahvnani & Choi, 2012; Bennett, 2008; Cioffi & Rouleau, 2010; Collins et al., 2013; Goh et al., 2006; Harrison, 2016; Keller et al., 2010; Kuznar, Sedlmeyer, & Kreft, 2008; Makowski & Rubin, 2013; McFarlane, 2016; Weidmann, 2016) are omitted, as they more properly belong to the broader realm of politics or political science, such as comparative politics, political sociology, or political anthropology. On the other hand, ABMs of transnational conflict that spills over more than a single country or polity are included (e.g., Axelrod, 1997; Bhavnani et al., 2014; Ilachinski, 2012; MacKerrow, 2003, Pugacheva, 2016; Ulloa et al., 2016).

ABMs of polity formation (e.g., Axelrod, 1995; Bogle & Cioffi, 2016; Cederman, 1997; Doran et al., 1994; Masad, 2013) are included within the scope of this survey, given the fundamental nature of polities as building blocks of international systems. Recall that polities vary according to organizational scale, so instances include chiefdoms, states, empires, and the global polity system in recent centuries.

Areas of Theory and Research

Table 1 would have been a lot smaller if the survey had been confined to models of the contemporary international system. However, such a restriction would have greatly diminished the value of ABM. Accordingly, this assessment includes ABMs of ancient and current international systems, small and large units, and brief and long epochs of time. ABM methodology is viable across the full range of all three "OGT" dimensions mentioned earlier: organizational, geospatial, and temporal.

Given the several dozen ABMs identified in Table 1, it seems useful to highlight several areas of IR theory and research that form clusters of scientific inquiry. The many ABMs that populate Table 1 represent a great variety of models contributed by computationally oriented political scientists, archaeologists, and computer scientists, to name some the main disciplines involved in this interdisciplinary effort.

IR ABMs identified in Table 1 also vary in terms of (1) system complexity, (2) substantive phenomena (e.g., norms, warfare, trade, alliances, and combinations thereof), and (3) OGT scales (organizational, geospatial, and temporal dimensions). At one end of the spectrum are simple models focused on a single theme involving one or few actors within a relatively short time span (e.g., models 3, 7, 8, 13, and 17). At the other end of the spectrum are models composed of numerous heterogeneous actors in dynamic interaction over many themes or issues during a long period of history (e.g., 16, 23, 29, 35, and 41).

The main themes investigated by the ABMs in Table 1 have been numerous, diverse, and include the following: polity formation, foreign policy decision making, conflict and polity dynamics, transnational terrorism, politics of international trade, international cooperation, alliances, norms, and effects of climate change. Each of these themes represents a cluster of research questions.

Polity formation at all levels or scales of organization is a clear theme that has emerged over the past decades. The phenomenon of polity formation is also known as *politogenesis* (Cioffi-Revilla, 2017, chs. 5, 7), a term originally introduced by Russian computational social scientists (Bondarenko & Korotayev, 2011; Grinin, 2009; Grinin & Korotayev, 2009). Polity formation refers to the first (i.e., earliest) emergence of chiefdoms, states, and empires in human societies in various parts of the ancient world, from a long-range historical perspective (Auble, 2010; Auble et al., 2013; Bogle & Cioffi, 2016; Cecconi et al., 2015; Cioffi-Revilla, 2017, chapter 5). Polity formation also refers to the formation of a generic polity or political system, in terms of norms, hierarchies, or control mechanisms, regardless of time or space, from a more abstract or theoretical perspective (Alden & Choi, 2007; Axelrod, 1995; Cederman, 1997; Dover, 2013; Dover & Cioffi, 2015; Gavrilets, Anderson, & Turchin, 2010). In addition, besides instances consisting of state and nonstate polities, polities can also form in network-oriented or transnational forms, as in the case of terrorist organizations with putative or ideologically claimed control or governance over a set of individuals or community. ABMs of polity formation are and will remain essential in IR theory and research, because polities of all types or organizational forms are fundamental to international relations (Cioffi-Revilla, 2017, chs. 5–7).

Foreign policy decision making has been another IR ABM cluster, based on earlier work centered on developing a deeper understanding of how leaders and foreign policy elites make decisions in the international system (Simon & Starr, 2000; Sylvan & Majeski, 2002, 2003; Taber & Timpone, 1996). ABMs provide an ideal modeling framework for implementing theories of decision making, given their ability to render an ecology of relevant actors connected through various information and influence and control channels. Although some foreign policy ABMs have been informed by specific historical actors and circumstances, others are more theoretical or abstract, as was the case in earlier categories and in several of those identified below. A specifically valuable feature of ABMs in this area is their ability to implement algorithms that accurately reflect known practices and procedures in foreign policy and national security bureaucracies, in both formal and informal channels. Hence, this is an area where the information-processing paradigm of computational social science plays a central role.

Conflict, warfare, and polity dynamics has been a major focus of IR ABM theory and research. Although the majority of models in this area references modern or contemporary international systems (e.g., Alt et al., 2009; Bahvnani et al., 2014; Bennett, 2008; Cederman, 2003, 2008), several models already exist of earlier, formative epochs (Doran & Palmer, 1995). Most models in this cluster consist of statelike agents in interaction with one another. The environment in which they interact varies widely across models, ranging from highly abstract tilelike topologies to high-resolution landscapes consisting of terrain, hydrology, climate, and infrastructure (e.g., Kennedy et al., 2012). Regardless of their level of realism or data-intensive detail, all models in this category thus far consist mainly of land and limited air space. This category of models represents the closest we have so far in terms of a computational science of IR history, including its origins thousands of years ago (Cioffi-Revilla, 2017, ch. 5). As mentioned in section "Directions for the Future," the ABMs in this survey do not include the full environment of the contemporary international system, including land, sea, air, space, and cyber (LSASC) domains.

Although most of the ABMs in this cluster have been state-based, some have been network-oriented and have used clever diffusion mechanisms to represent dynamics (Malkov, 2004, 2006).

Transnational terrorism constitutes another cluster of IR ABMs (e.g., Bhavnani et al., 2014; Ilachinski, 2005, 2012; MacKerrow, 2003). (Note that this category in Table 1 excludes models involving strictly domestic terrorism, which is not an IR area of research.) ABMs in this cluster represent a smaller number than those ABMs that have focused on domestic violence and single-country terrorism. There is some underrepresentation in this category, however, because some of the ABMs that have been created in this area have remained unpublished (in the so-called gray literature) or are actually classified as secret by government agencies, for understandable reasons. There is still an abundance of well-published models in this cluster, which is enough to represent a vibrant and active research area. This is an area where comparative computational analysis (so-called model-to-model or M2M comparative research) could be fruitful, particularly when using aspects of complexity in terrorism: cardinality, dimensionality, and other components discussed earlier in Section "Introduction".

Cooperation was among the first clusters to form, based on computational modeling of IPD (Iterated Prisoners' Dilemma) games (Axelrod, 1984). This research program inspired a generation of researchers in computational social science, well beyond IR theory. In IR itself, ABMs of international cooperation have also inspired numerous new insights into processes ranging from arms control to trade, the latter constituting an area that could be considered its own cluster (Min, 2002; Min et al., 2004), as well as norms (Axelrod, 1986; Bhavnani, 2006; Hoffmann, 2005; Rouleau, 2011) and alliances (Axelrod & Bennett, 1993; Bennett, 2000; Gartzke & Weisiger, 2013). Indeed, cooperation represents a superclass of IR ABMs, much like conflict, so separate clusters on aspects of cooperation are clearly discernable in Table 1.

Climate change has generated a cluster of IR ABMs in the early 21st century (e.g., Cioffi et al., 2015), thanks to several key developments. Besides the necessary interdisciplinary cooperation to undertake this category of computational models (and multiyear funding), Simons's theoretical triad of human, artificial, and natural (HAN) systems (specifically, the coupled, adaptive, evolving dynamics; Simon, 1996), and the increasing availability of remote sensing data as well as detailed climate data with sufficient resolution, has enabled a new class of ABMs that would have been unthinkable just a few years earlier. In addition, powerful platforms or toolkits, such as MASON and Repast, have made possible scientific work of a kind that was simply not feasible before such systems existed. The idea that climate change can affect IR phenomena predates ABMs in this area, but it was not until the first computational simulation models were created that such effects could be demonstrated using coupled human-artificial-natural systems in specific geographic regions of the world (e.g., Cioffi et al., 2015). Such recent models will likely grow and improve to include phenomena linked to climate change, such as human migrations, infrastructure risks, and sociopolitical change.

Enduring Challenges

Most of the previous advances were achieved in spite of numerous, enduring challenges. The following are some of the major ones. Some can be remedied or mitigated; others are harder to overcome or need to be factored in, or managed as best as possible when conducting ABM research in IR. Additional challenges will no doubt arise when pursuing future research directions in Section "Directions for the Future". Learning, publication, duplication, and achieving measurable accumulation of scientific knowledge will continue to pose enduring challenges.

Learning ABM in IR

Computational modeling is not part of the standard training of IR researchers, nor is mathematical modeling for that matter. Neither form of formal model (mathematical or computational) is part of the graduate curriculum. Exposure to ABMs in IR (or other areas of social science) at the undergraduate level is rare. Unlike students in other areas of science, exposed to mathematical and computational methods of scientific inquiry since high school, few social scientists experience similar exposure. Today, basic training in multivariate statistical analysis remains the core methodological toolset, just as it was twenty years ago.

The advent of effective and free teaching tools, such as NetLogo, is changing the educational landscape in ways that favor much earlier exposure to scientific ideas in IR. Many NetLogo models can be built and used by secondary or high school students to learn about social science, including history. This includes interdisciplinary fields such as IR, even if computational social science is not (yet) taught at pre-university levels. In addition, entertainment computational simulation games, such as *Civilization* and *World of Warcraft* (Bainbridge, 2010; van Creveld, 2013, ch. 6), among others, increase exposure to computational simulation modeling of international relations.

In the 1960s the teaching of physical and biological sciences in the United States was in disarray, leaving much to be desired—especially in light of Sputnik and other scientific accomplishments by Soviet scientists. As a response, the U.S. National Science Foundation instituted the Physical Sciences Study Committee and its biological counterpart. These efforts created new, excellent teaching materials, including innovative textbooks (PSSC, 1960) that instructed a new generation of physical and biological scientists. Perhaps comparable efforts are necessary today to advance computational social science and ABM approaches across the full spectrum of the social sciences, including IR.

Publishing an ABM

Political ABMs are difficult to describe in short publications (i.e., papers, chapters), and sometimes even in books. The main exceptions are doctoral dissertations (and master's theses), which require extensive description and documentation of methods and modeling details. The need to use online "Supplemental Information" mitigates but does not entirely eliminate this challenge. Use of online depositories is valuable but does not solve all problems. Published code may still be difficult or impossible to comprehend, and therefore useless, unless it is written according to high standards. And the long-term sustainability of digital archives poses its own challenges.

The fundamental reason why ABMs are challenging to describe (with sufficient information so as to enable reproducibility) is that the referent system, as well as the simulation system, is complex, in the strict sense explained in Section "Introduction": most IR ABMs are composed of numerous, diverse, and interconnected entities whose state is determined by many variables and parameters linked via nonlinear dependencies. Processes of adaptation and evolution make description additionally demanding. Mathematical, graphical, algorithmic, and other types of descriptions are all necessary, not optional. By contrast, most statistical models are relatively easier to describe, even when they have nonlinear, time-dependent, or probabilistic components. Complexity in ABMs also raises the issue of Occam's Razor: the enduring balance between abstract simplicity and realistic representation (Edmonds & Moss, 2005).

Existing protocols for describing ABMs in general, or for use in other fields (e.g., the ODD protocol used in "individual-based" models in ecology; Railsback & Grimm, 2012; Object Design and Description), are only somewhat useful for social and IR ABMs. This is because extant protocols such as ODD were created for biological entities in mind, not for modeling entities such as people, belief systems, decision making, human groups, norms, networks, and political institutions. Accordingly, ABM description protocols created in other disciplines have an inherent tendency to fall short of what is needed to describe IR ABMs with sufficient information to enable reproducibility.

We need new protocols, perhaps through extensions of extant frameworks, such as the now popular ODD, augmented by proper human and social components, as well as specific details on complexity features. For instance, additional information on agent decision-making features, networks structures, evolutionary characteristics, among others commonly found in IR ABMs, would improve IR ABM descriptions. Another strategy for improvement could be the use of formal systems protocols, such as SysML (Systems Modeling Language; Delligatti, 2014; Friedenthal et al., 2015). Even without these more specialized systems protocols, more extensive use of UML (Unified Modeling Language) class, sequence, and state diagrams would go a long way toward improving the current situation (Cioffi-Revilla, 2017, pp. 53–73).

Avoiding Reinventing the Wheel

Although the number of existing IR ABMs may seem relatively large at fifty or so, a surprisingly small number of them appear to be based on a thorough assessment of prior work. This challenge seems sufficiently present (and corrosive of scientific practice) to warrant highlighting. In fact, some publications after the year 2000 lack references to prior literature, as if IR ABM were being invented *ex nihilo*. Few publications seem founded on thorough familiarity and understanding of prior models focused on the same or similar phenomena and can claim to advance knowledge through a systematic strategy of model development that builds on earlier research done by others.

Ideally, an ABM should be grounded and build on prior models in the same or neighboring themes, unless a demonstrably superior fresh start is being proposed on the basis of a thorough and well-documented assessment of prior work. The risk is otherwise high of reinventing the wheel, and not even realizing that similar or even scientifically superior wheels have already been created.

Accumulation of Knowledge

A related (and arguably greater) risk in the use of ABM in IR, when models are poorly or loosely grounded in earlier work, consists of failing to accumulate knowledge. All too often an ABM model fails to reflect greater scientific information and understanding of real world dynamics than its predecessors. This occurs for a variety of reasons beyond failure to check prior literature. For example, a modeler selects a language that is incapable of implementing desirable features found in earlier models, representing "a step backward," or a more limited single-processor version is implemented and claimed as novel when earlier distributed versions accomplished more.

Ideally, every ABM on a given theme or cluster of research questions should provide a net advance in terms of scientific knowledge. This should be demonstrated explicitly, not simply assumed or alluded to in implicit arguments. As is the case with mathematical models, the power of computational models (agent-based or otherwise) lies in their ability to build on previous models and advance the frontiers of knowledge, such that each generation of IR scientists literally knows more about international relations phenomena than any previous generation.

Evolving Standards and Methodology

As in all fields of contemporary science, computational theory and methodology in IR is a work-in-progress: an evolving enterprise involving a community of practitioners. A major challenge involves keeping up with the latest ideas in areas of computational concepts, theories, models, programming languages, data structures, algorithms, and technologies that support their implementation. This is particularly challenging, because relevant developments originate from within the IR research community itself, as well as from the broader computational sciences community.

For example, several years ago the introduction of GPUs (graphics processing units) seemed to provide a revolutionary solution to overcoming problems of computational speed in large-scale multiagent systems. This idea seemed especially appealing for solving research problems in computational IR requiring large numbers of agents to be included in a given model. As a result, practitioners who were interested in these developments had to meet the challenges of learning the technology, new programming environment, and other aspects in order to exploit the technology. Another example is the case of quantum computing, now looming in the horizon of computational modeling. These technological examples are typical of computational science, which is more independent on technical advances than mathematical modeling is as a less dynamic tool set or methodology.

Journals and reviewers in general bear special responsibility for being familiar with high standards and implementing them. This will remain a demanding challenge for the foreseeable future. Unlike mathematical models, where standards have existed for several centuries and the formal structures evolve more slowly, ABMs in computational science are younger and more rapidly evolving.

Interdisciplinary Research

The interdisciplinary nature of agent-based computational modeling poses advantages and disadvantages. Much of the content of this entry is based on the positive advantages provided by these new approaches. However, the disadvantages or risks of interdisciplinary research cannot be ignored, because this research creates many challenges as well as opportunities. First, interdisciplinary work requires paying attention to that undertaken in other disciplines. In turn, this necessitates gaining extensive, deep familiarity with the relevant literature, including journals, proceedings, and other sources of scientific information in other disciplines. Attending scientific meetings of disciplines far from one’s own original field is another requirement.

The challenges imposed by the rigors of interdisciplinary research go a long way in explaining why IR ABMs so often seem to lack sufficient grounding in relevant disciplines, or why communication across disciplines remains so challenging. Disciplines are also protectors of scientific “turf,” and incursions that are often perceived as uninformed or lacking in expertise are not only unwelcome—they are also rejected outright, as when significant contributions are ignored by discipline-based turf protectors. In IR ABMs this is a challenging problem that can be solved only by increasing and deepening education, scientific exchange, rigorous communication, and sustained collaboration across disciplines that for too long have been accustomed to “protecting their territorial integrity” often at the price of ignorance and lack of scientific progress. ABMs provide a scientific venue for overcoming such collective action problems.

Directions for the Future

This assessment of ABM in IR theory and research suggests a number of directions for future research. First, further development of all the main topics identified in Section “Areas of Theory and Research” is necessary and will likely continue, since ABM methodology and its broader complexity science has been far from being fully utilized. For example, few IR ABMs make extensive use of insights and understanding provided by complexity science, such as mathematical properties of results from simulations (e.g., Cederman, 2003; Cioffi, Honeychurch, & Rogers, 2015; Ilachinski, 2012). The amount of complexity science effective used by IR ABMs is still scant compared to its potential for scientific progress.

Second, from a purely methodological perspective, the research community will likely develop increasing interest in technological aspects of ABM approaches to IR theory and research. These would include the use of big data in phases of model development (from motivation to analysis), further use of GIS (geographic information systems) to support geospatial and environmental modeling when needed, new visualization facilities, advances in evolutionary computation, and parallel computing, among other possibilities. The latter has numerous aspects and architectures that are relevant to ABM in IR theory and research, such as for modeling cognitive structures that support richer representations of decision making, parallel distribution of interaction networks among agents, implications of Amdahls' Law for gaining speed up with parallelized computations (Cioffi, 2014), and ways of overcoming challenges with model components that are computationally intensive, among others.

Third, from a more substantive perspective, a set of four additional themes readily suggests themselves, based on the assessment already provided in section "Areas of Theory and Research."

Outer Space and Cyberspace

The contemporary international system is now as dependent on space-based (earth-orbiting technologies and similar systems) and cyber systems (the Internet and World Wide Web as information space) and processes as any other major aspect of civilization: that is, "space" and "cyber," for short. Indeed, our society has become dependent on space and cyber for maintaining and enhancing our quality of life, as well as its fundamental viability. Accordingly, international relations today—from foreign policy decision making within the bureaucracies and executives of foreign ministries all the way to global strategic levels—critically depend on space and cyber systems. Leaders and publics from virtually all countries and regions of the world alike rely increasingly on cyber and space for a multitude of interactions and transactions on multiple scales, from local to global.

ABMs are fully capable of including space and cyber entities within their ontology. Such entities are human and technological, natural, and artificial, and consistent with concepts discussed earlier in Section "Introduction". For example, consider the space domain. The satellite fleets of major powers in the contemporary international system—United States, EU, Russia, China, India, Brazil—as well as federated space-based assets, are a major component of national security capabilities. They are composed of people, organizational entities, norms, physical assets, and extensive and complex infrastructure systems. Both governmental and civilian entities are related through a network of dependencies.

In cyber, the ontology is not less complex, albeit quite different in terms of composition and topology. The World Wide Web, the physical Internet, and numerous critical cyber networks all provide essential information-processing and other forms of support to governmental and nongovernmental interactions in the international system. Governance of the Internet may not be governmentally controlled, but it still represents an issue of fundamental national security for all actors in the international system, especially the great powers. Cybersecurity dilemmas are no longer a matter of science fiction. The complexity of cybersecurity satisfies all essential features explained in Section "Introduction": cardinality (large number of entities), dimensionality (many variables determine the state of entities), diversity (many different types of entities), connectivity (network structure of the cyberworld), and nonlinearity (emergent phenomena generated by interacting components).

Both space and cyber components of the international system are not only complex; they are also adaptive and evolutionary. This is mostly due to the advanced technological nature of cyber and space domains, as opposed to earlier and more traditional domains of the international system in terms of land (the oldest), sea, and air. Adaptation and evolution are distinctive features of advanced technologies, such as space systems and cyber systems. As complex adaptive system components of the international system, systems of space- and cyber-based components obey a variety of properties and governing principles, such as fitness functions of various kinds (e.g., single, multiple, and dynamic) that determine which systems are selected, maintained, and improved, and which are discarded from the technological landscape. For example, today leaders of most developed countries rely on technologically sophisticated systems of space-based communications supported by a constellation of satellite networks to exchange information on policy issues ranging from national security to economic development and global governance. All such systems are adapted and evolved from earlier generations of legacy systems. Whereas earlier land- and sea-based systems were subject to local weather, today's space-based and cyber systems are vulnerable to space weather conditions that affect the integrity and viability of orbiting communications infrastructure. Such features and phenomena represent a challenge for improving ABMs in IR in areas such as information warfare, hybrid conflicts, and global public opinion dynamics, among others.

Integrated Models

Many IR ABMs today still lack proper integration. The current international system consists of land, sea, air, space, and cyber components. By contrast, most IR ABMs in Table 1 are limited to land alone, as if the world were still confined to conventional balance of power and ground-based interactions and strategies exclusively based on terrestrial combat. Clearly, a new generation of integrated ABMs is needed for representing the contemporary international system in modeling accurate ways to address research questions and phenomena resulting from the land-sea-air-space-cyber (LSASC) integration.

Besides integration of LSASC components and environments, integration also means composing IR ABMs in such a way that they include human, artificial, and natural (HAN) environments. This requires integration of all three components of the HAN triad (e.g., Cioffi, 2016; Kennedy et al., 2014), not just one or two at a time (e.g., Cederman & Girardin, 2007a, 2007b; Liu et al., 2007; Ostrom, 2009). No country exists independently of the land, sea, and air it is situated in, so IR theory and research stand to gain from inclusion of natural environments. HAN components are another way of viewing LSASC components and both serve the purpose of verifying completeness in IR ABMs.

From the substantive perspective of IR issues (transnational terrorism, national security, migrations, pandemics, alliances, foreign aid, arms control, nonproliferation), greater integration is also needed across issues, because polities in the international system concurrently deal with issues (i.e., in active parallel modes), not in sequential or serial processes ("one-at-a-time"). Concurrency in managing national security and policy issues on multiple spatial and temporal scales is a hallmark of governance in every polity. This is true at scales or levels from domestic, to national, to international, to global.

Properly designed ABMs are fully capable of handling the complexities of LSASC, HAN, and multi-issue integration. In addition, features of the international system include continuities and discontinuities, or continuous and discrete data, respectively. For example, both MASON and Repast have extensive discrete and continuous facilities for representing and processing information, including 3D, for data structures and complicated event scheduling, besides an increasing number of code libraries. The international system itself is a spherical topology of adjacent and remote actors, as an interaction network, not a flat 2D world as in a paper map, so different time zones and other real-world features affect daily interactions. While interactions today occur on a 24/7/365 basis, such a system requires both continuous and discrete data structures and algorithms for scientific modeling and simulation. Efforts in this direction will almost always require distributed architectures, which is an area where much exists already and can be leveraged. Also, the availability of such distributed systems will grow in the future.

World Order

The structure and dynamics of world order are fundamental and perennial themes in IR theory, but one that is only indirectly addressed by models in Table 1. This is arguably true on both regional and global scales, since world order is akin to a fractal or multiscale property: in politics, some system or pattern of structured order always exists at domestic, national, bilateral, multilateral, regional and global levels. The classical theory of world order, based on balance of power among the main actors or great powers, includes dynamic processes such as arms races and security alliances that explain war and peace.

In the early 21st century the only ABMs that are related to world order are mainly those that examine norms (e.g., Axelrod, 1986; Degterev, 2016 Hoffmann, 2008; Rouleau, 2011), not institutions such as specific historical alliances and forms of international governance. Although norms are significant for understanding world order, they are insufficient, especially in the computational reconstruction of major processes such as the system of Westphalian alliances, the Concert of Europe, the League of Nations, and the United Nations system of institutions. According to IR theory, each of these historical systems emerged from the ashes and failure analysis of the previous system in preventing general war. Reforming the current UN system or designing and implementing a new system of world order capable of preventing general war is a scientific and policy task that can be supported by appropriate agent-based modeling of international order to test and refine plans for improving international peace and security.

ABMs of world order need to explain how peace and war occur in the contemporary international system, specifically in terms of the polities and dynamics operating today, with proper nouns, not just in terms of nameless actors that rise and fall as in the Chinese game of Go. Real world countries require realistic, high-resolution detailed representations in terms of LSASC and HAN components, otherwise they are not sufficiently explanatory and, therefore, fail as empirical theories aiming to explain how the real-world works. Such high-fidelity ABMs must be virtual worlds *in silico* capable of replicating the known historical record and explaining the present and providing insight about future history of the international system.

Such ABMs of world order are feasible and desirable, given proper scientific planning and implementation. While the earliest proposals in this direction date to the early 2000s, as of this writing no one has yet created a high-fidelity ABM of the international system on a global scale—comparable, for example, to current weather and climate (long-range) models. However, there is sufficient progress with regional models to support a more global effort in this direction. A high-fidelity ABM of the entire international system today would require interdisciplinary organization, deep expertise in computational modeling, and rigorous reliance on best practices concerning the entire lifecycle of ABM based on the MDIVVA process summarized earlier in section "Introduction."

Long-Range Models

No science is complete without explaining and understanding when, where, and how its phenomena began and how it evolved into its current form. All the IR ABMs discussed thus far may be designated as short-range in terms of temporal scale. Long-range IR theories are like theories in geology or cosmology: they explain the origin and total historical evolution of the international system, from the period of initial formation thousands of years ago to the present time. The international system has always been composed of state and non-state actors as the main two classes of entities. During its formative phase, the international system began with nonstate polities today called "chiefdoms" in social science. These early polity systems were frequently at war, and in some cases in constant warfare in the form of raiding and related violent interactions. Eventually some chiefdoms evolved into states, and later some states evolved into empires, forming the first inter-state international systems. Other forms on nonstate polities, such as terrorist actors and international organizations eventually appeared.

Long-range ABMs of IR are another future research direction. Above all, they should be capable of generating thousands of years of history, not just decades or centuries, because the long-range scale of the international system is empirically millenarian. Moreover, initial conditions for such long-range ABMs correspond to hunter-gatherer societies, because they generated the first chiefdom-level polities that constituted the earlier interpolity system. The formation of chiefdom-based interpolity systems occurred independently at least four times in different and far flung regions of the ancient world: the Near East or West Asia, *ca.* 10,000 years ago; the Far East or East Asia, *ca.* 8,000 years ago; the Andean and coastal region of Peru and northern Bolivia, *ca.* 5,000 years ago; and in Mesoamerica, *ca.* 4,000 years ago.

Moreover, long-range ABMs must be able to generate how, where, and when the four regional systems eventually fused into larger systems until the present state of globalization. For example, whereas the international systems of west and east Asia evolved independently from their period of initial formation up to *ca*. 1500 BC, and then began sporadic and gradual fusion, both New World systems rapidly fused with the European system 500 years ago through conquest. Just like 21st-century cosmologists have developed detailed models of the early and expanding universe, so someday IR scientists will have available long-range ABMs of the international system. Experience in modeling early polity systems will play a key role, along with new concepts and computational technologies.

Long-range ABMs of the international system will also be able to look into the future, into time-scales when new types of polities will form and evolve, including the first space- and cyber-based polities. Some of these polities will likely have recognizable forms (such as polities with familiar forms of governance institutions) while others will consist of polities purposively evolved for such future environments. If humanity is able to survive the current existential threat of weapons of mass destruction, new forms of polities will be required for spacefaring civilization. ABMs can provide insights and deeper understanding, especially when used in combination with other tools and paradigms available across the sciences.

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