University of California at Merced
CSE 107 Introduction to Digital Image Processing
Midterm
Spring 2022

April 11, 2022
Exam start: 1:30pm
Exam End: 2:45pm

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

NAME: ______SOLUTION__________________________

This exam is open notes and open book. You can use your electronic devices to reference your electronic notes, a PDF of the text, the lecture notes, and the homework solutions. Please silence cell phones. Answer as many questions as possible. Partial credit will be given where appropriate. Be sure to clearly indicate your final answer for each question. Also, be sure to state any assumptions that you are making in your answers.

The exam consists of 9 pages including this one.

You have 75 minutes in which to complete the exam. Good luck!

Problem	Possible	Score
0	1	
1	7	
2	5	
3	6	
4	4	
5	10	
Total	33	

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Problem 0 (1 point):

Write your name at the top of each page of this exam.

Problem 1 (7 points): True/false and multiple choice.

Circle T or F:

a) In the retina, rods are less sensitive to color than cones. T / F

b) Perceived brightness by the human visual system is purely a function of local intensities. T / F

c) Using root-mean-square-error (RMSE) to compare pairs of images always agrees with how humans perceive the differences. T / F

d) Histogram equalization always increases the contrast in an image. T / F

Circle one of A-D:

e) The rapid eye movements that move your focus of attention around in a scene are called:

A. Ballistics
B. Saccades
C. Translationals
D. Realignments

f) Generally, what does the standard deviation tell you about a grayscale image:

A. How much the image differs from its inverse (negative).
B. Whether the image was taken in portrait or landscape mode.
C. Its contrast.
D. The minimum and maximum intensity values.

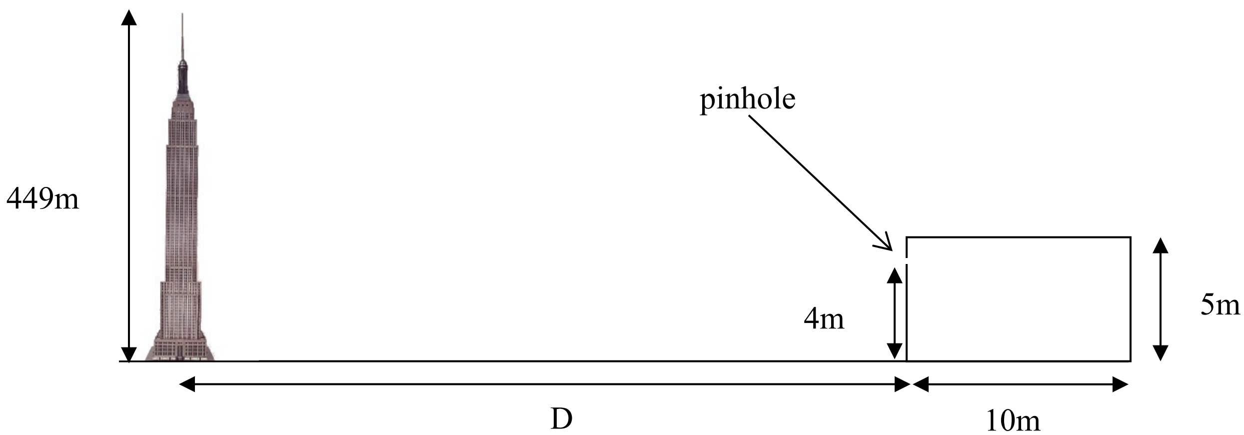
g) Which of the following CANNOT be modeled as an affine transformation? Assume a planar object is being imaged.

A. Rotating the camera from landscape to portrait view.
B. Moving the object closer to the camera along the optical axis.
C. Moving the camera perpendicular to the optical axis.
D. Tilting the object away from the camera.

Problem 2 (5 points): Pinhole camera geometry.

Even before images were able to be fixed on light-sensitive media, cameras were used to view scenes. A *camera obscura* was a room which was turned into a camera by making a small hole in one wall. This pinhole-like lens would then form (an upside down) image on the opposite wall of the scene outside the room.

Suppose you needed to design a camera obscura for viewing the Empire State Building. Assume your room is a fixed size: the room is square measuring 10m along each dimension. The walls of the room are 5m tall. The hole in the wall that will face the Empire State Building is 4m off the ground. The Empire State Building is 449m tall. See the diagram below.



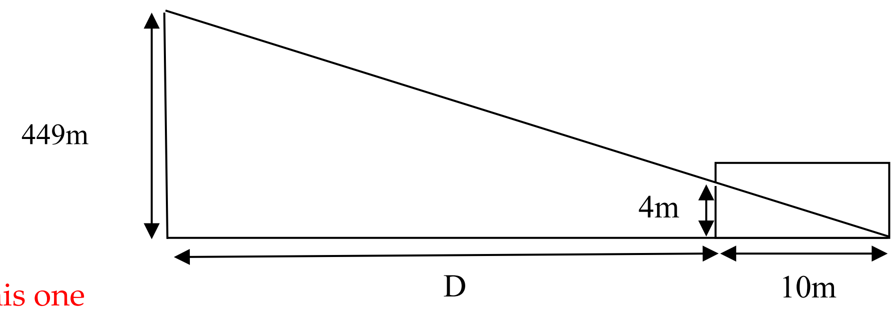
What is the **closest** that your camera obscura can be to the Empire State Building and **still have the building's image fit on the wall opposite the hole**? That is, find the minimum value of D in meters so that the tip of the building is projected on the wall and not the floor, and the base of the building is projected on the wall and not the ceiling. The only free parameter you have is the distance the room is from the building—you cannot change the dimensions of the room nor the height of the hole. (Make sure you show your work so you can get partial credit. The next page is blank for you to continue work on this problem.)

Solution: Note that the distance at which the top of the building is not projected onto the floor is greater than the distance at which the base is not projected onto the floor. See the figure on the next page.

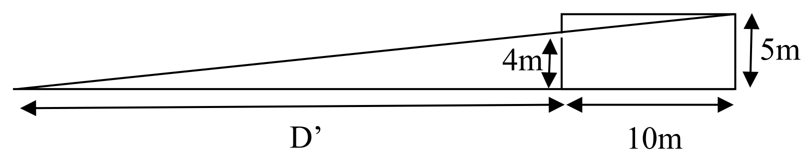
Problem 2 (continued)

Prove that the function that computes the minimum value of intensities in an image is non-linear.

That is, we are interested in this configuration



and not this one



since D' << D.

By the similar triangles in the figure at the top:

$$\frac{D + 10m}{449m} = \frac{10m}{4m}$$

Solving for D:

$$D = \left(\frac{10m}{4m}\right) 449m - 10m = \boxed{1112.5m}$$

Problem 3 (6 points): Nonlinearity.

Prove that the function that computes the minimum value of intensities in an image is non-linear.

Let h be the function which computes the minimum value of intensities in an image.

To show the function h is nonlinear, we just need to find any pair of images $f_1(x,y)$ and $f_2(x,y)$ and any two scalar constants a_1 and a_2 such that

$$h[a_1 f_1(x,y) + a_2 f_2(x,y)] \neq a_1 h[f_1(x,y)] + a_2 h[f_2(x,y)]$$

Suppose we pick

$$f_1 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 10 \\ \hline\end{array} \quad f_2 = \begin{array}{|c|c|}\hline 15 & 1 \\ \hline 2 & 10 \\ \hline\end{array} \quad a_1=1 \quad a_2=1$$

Then

$$h[a_1 f_1(x,y) + a_2 f_2(x,y)] = h[\begin{array}{|c|c|}\hline 16 & 3 \\ \hline 5 & 20 \\ \hline\end{array}] = 3$$

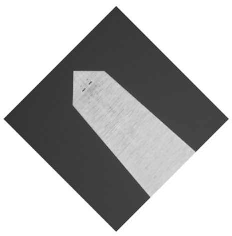
But

$$a_1 h[f_1(x,y)] + a_2 h[f_2(x,y)] = h[\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 10 \\ \hline\end{array}] + h[\begin{array}{|c|c|}\hline 15 & 1 \\ \hline 2 & 10 \\ \hline\end{array}] = 1+1 = 2$$

(To prove non-linearity, just need show the linearity equation does not hold for some specific choice of image pair and scalar pair.)

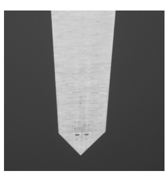
Problem 4 (4 points): Geometric transformations.

An “original” version of an image is shown above. Indicate which affine transformation matrix from the right column below will create each of the transformed images in the left column when applied to the original image. (Write A-D in the underlined spaces.) (Assume that the center of the image is at location (0,0) so that translations are not necessary.)



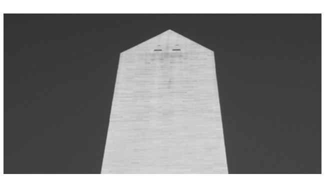
Matrix: _D_

A: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



Matrix: _C_

B: $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



Matrix: _A_

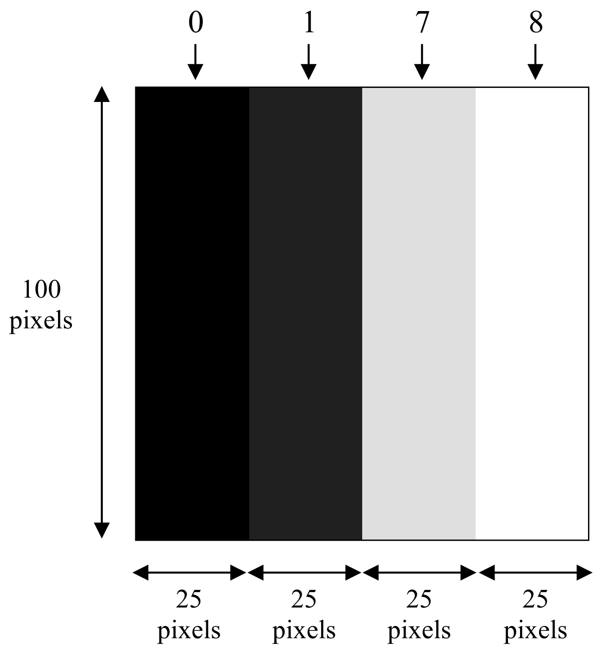
C: $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix: _B_

D: $\begin{bmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

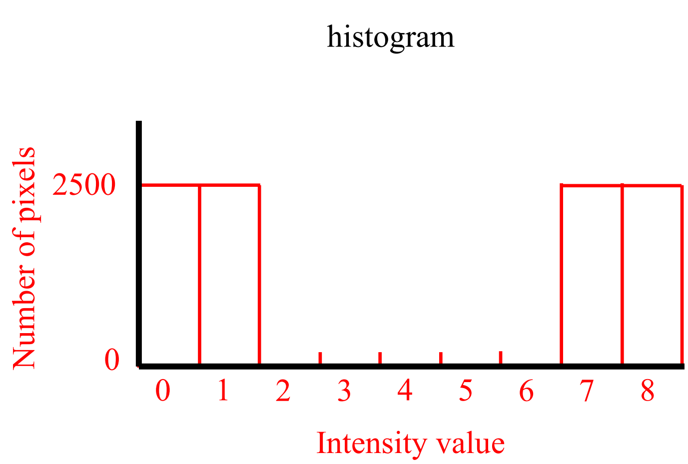
Problem 5 (10 points): Histogram equalization.

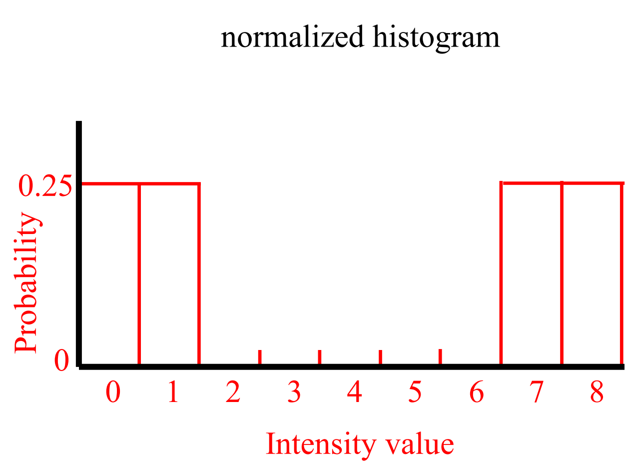
Consider the 100x100 pixel image below which can take values 0-8 (L=9):



The vertical strips are 25 pixels wide and have values 0, 1, 7, and 8.

(a) Plot the histogram and normalized histogram of this image on the graphs below. Make sure to label the axes.





Problem 5 (continued)

(b) Compute the probabilities $p_r(r_k)$ of the different image levels for the image:

r_k	$p_r(r_k)$
$r_0 = 0$	0.25
$r_1 = 1$	0.25
$r_2 = 2$	0
$r_3 = 3$	0
$r_4 = 4$	0
$r_5 = 5$	0
$r_6 = 6$	0
$r_7 = 7$	0.25
$r_8 = 8$	0.25

(c) Compute the mean and variance of the image.

mean = ___4____ variance = __12.5__

(If you have trouble computing the variance without a calculator, you can leave the result in equation form below for full credit.)

$$mean = \sum_{i=0}^{8} xp(x) = (0)(0.25) + (1)(0.25) + (7)(0.25) + (8)(0.25)$$
$$= \frac{0}{4} + \frac{1}{4} + \frac{7}{4} + \frac{8}{4} = 4$$

$$variance = \sum_{i=0}^{8} (x - mean)^2 p(x)$$
$$= (0-4)^2(0.25) + (1-4)^2(0.25) + (7-4)^2(0.25) + (8-4)^2(0.25)$$
$$= \frac{16}{4} + \frac{9}{4} + \frac{9}{4} + \frac{16}{4} = \frac{50}{4} = 12.5$$

Problem 5 (continued)

(d) Now, you will perform histogram equalization on the image.

Using the intensity transformation $s_k=T(r_k)$ that corresponds to histogram equalization, compute the values in the equalized image:

$$s_k = T(r_k) = (L-1)\sum_{i=0}^{k} p(r_i)$$

$$S_0 = (8)(0.25) = 2$$
$$S_1 = (8)[(0.25) + (0.25)] = 4$$
$$S_2 = (8)[(0.25) + (0.25)] = 4$$
$$S_3 = (8)[(0.25) + (0.25)] = 4$$
$$S_4 = (8)[(0.25) + (0.25)] = 4$$
$$S_5 = (8)[(0.25) + (0.25)] = 4$$
$$S_6 = (8)[(0.25) + (0.25)] = 4$$
$$S_7 = (8)[(0.25) + (0.25) + (0.25)] = 6$$
$$S_8 = (8)(1) = 8$$

s_k
$s_0 = 2$
$s_1 = 4$
$s_2 = 4$
$s_3 = 4$
$s_4 = 4$
$s_5 = 4$
$s_6 = 4$
$s_7 = 6$
$s_8 = 8$

(e) Finally, sketch the histogram equalized **IMAGE** below, indicating the pixel values.

2	4	6	8