Report on Comparative Analysis of SIR Epidemic Model Using Euler and RK4 Methods (Without Normalization)

GROUP 3

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Introduction

This report analyzes the Susceptible-Infected-Recovered (SIR) model, a fundamental model for describing infectious disease spread. We use Euler and fourth-order Runge-Kutta (RK4) methods to investigate the impact of varying transmission (β) and recovery (γ) rates. We do not normalize the equations by the total population.

Model Formulation (Without Normalization)

The SIR model is defined by the following system of ODEs:

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

Where:

- $S(t)$ is the number of susceptible individuals at time t.
- $I(t)$ is the number of infected individuals at time t.
- $R(t)$ is the number of recovered individuals at time t.
- β is the transmission rate.
- γ is the recovery rate.

Numerical Methods

- **Euler Method:** A first-order numerical method.
- **RK4 Method:** A fourth-order Runge-Kutta method.

Implementation and Parameters

- Initial conditions: $S(0) = 990$, $I(0) = 10$, $R(0) = 0$.
- Time step (dt): 0.1
- Varying parameters:
  - β: 0.1, 0.2, 0.3, 0.4, 0.5.
  - γ: 0.05, 0.1, 0.15, 0.2, 0.25.

Results and Analysis

Varying Beta (β) with Gamma (γ) held constant at 0.1

Analysis:

- Higher β leads to a rapid decline in $S(t)$, a steep rise and peak in $I(t)$, and a faster increase in $R(t)$.
- The peak of $I(t)$ shifts earlier and higher with increasing β.
- The intersection of $S(t)$ and $R(t)$ occurs earlier with higher β.
- RK4 provides smoother, more accurate curves.

Varying Gamma (γ) with Beta (β) held constant at 0.3

Analysis:

- Higher γ leads to quicker recovery, resulting in a lower and earlier peak in $I(t)$.
- The peak of $I(t)$ lowers and advances with higher γ.
- The intersection of $S(t)$ and $R(t)$ occurs earlier with higher γ.
- RK4 maintains better accuracy.

Graphs:

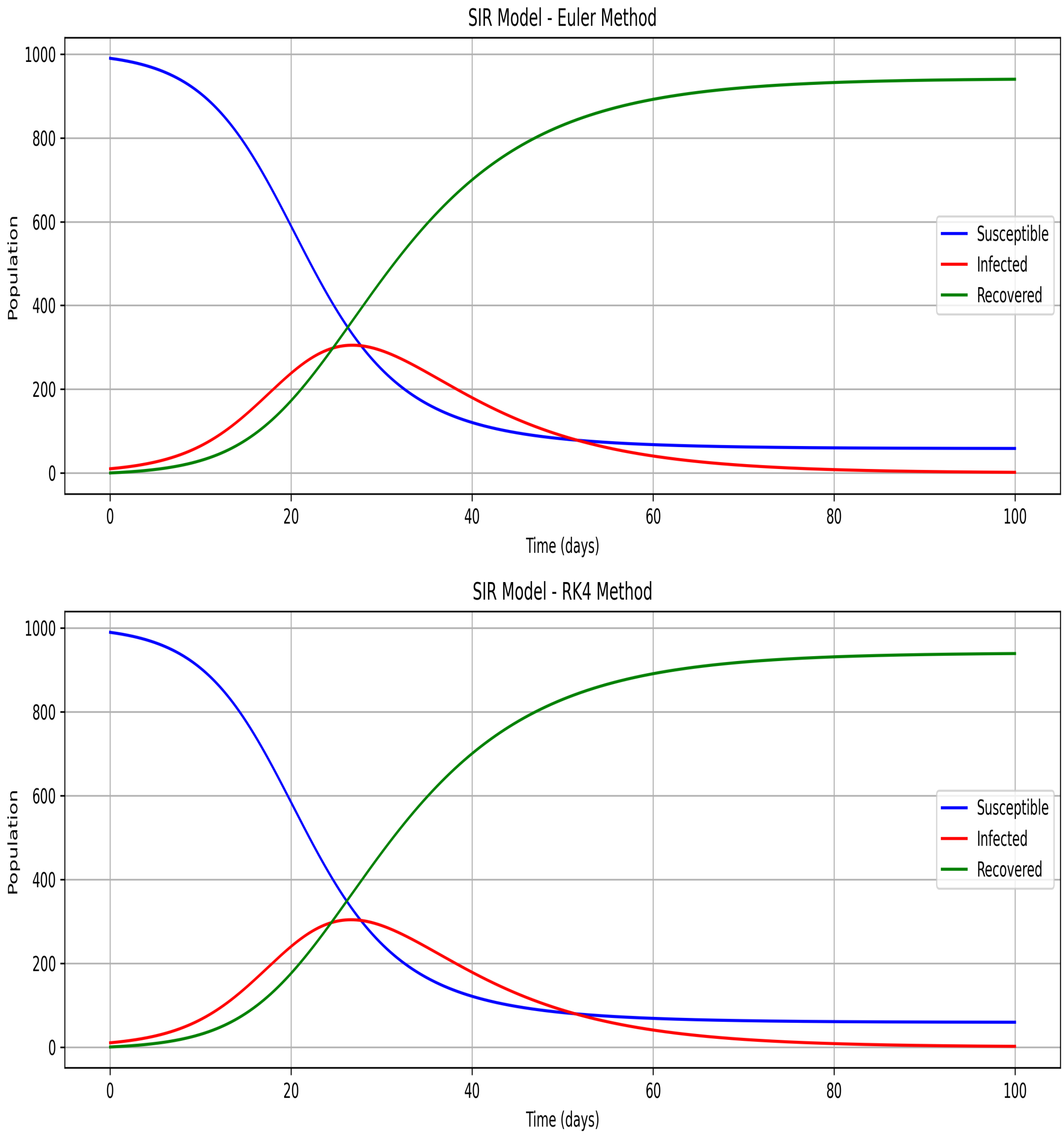

Figure 1: **Euler and R.K.4 methods on the S.I.R. model**

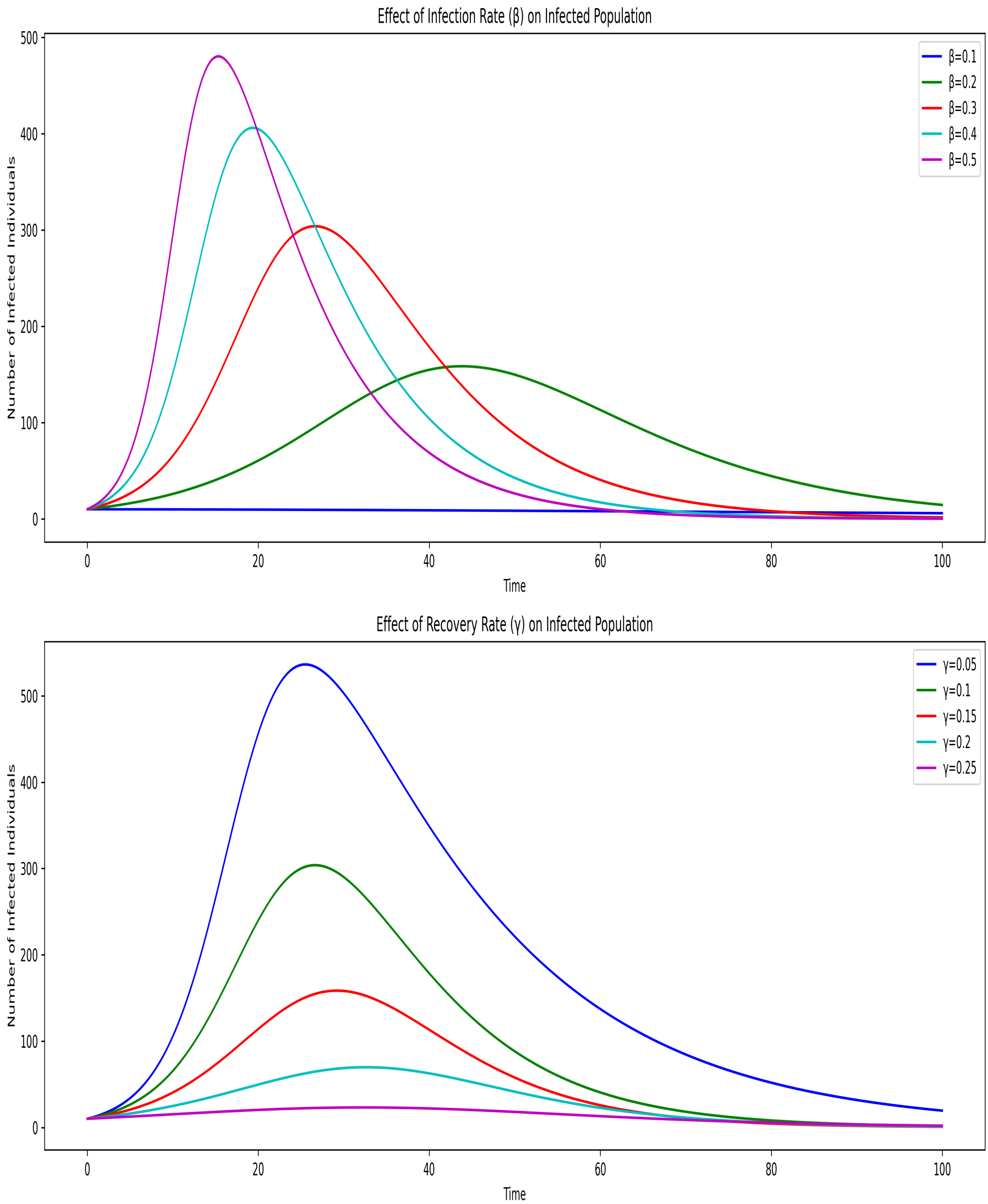

Figure 2: **Effects of varying β and γ on the Infected population**

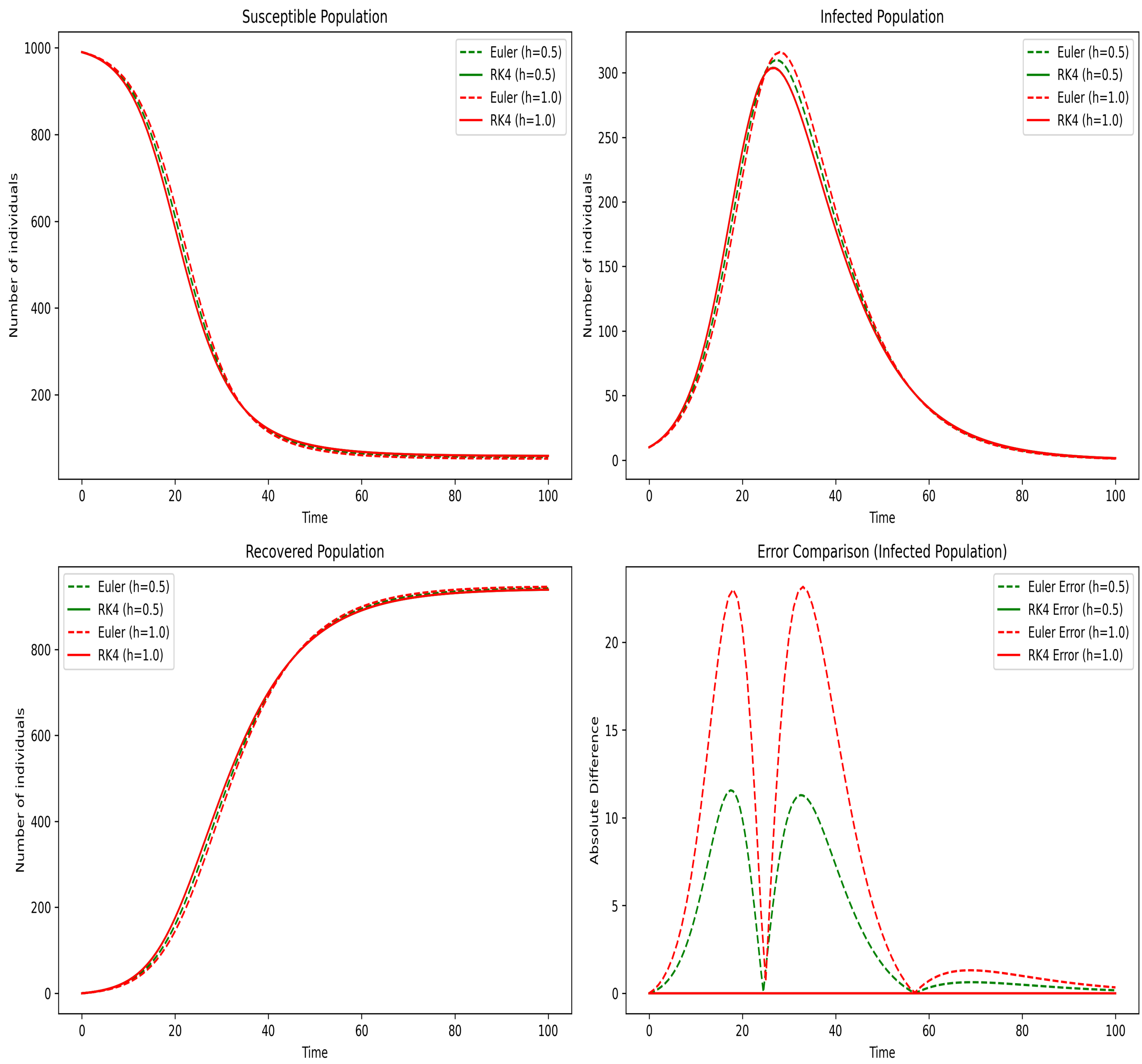

Figure 3: **Effects of varying stepsizes on the S.I.R. model**

Comparison of Euler and RK4 Methods

- **Accuracy:** RK4 is more accurate.
- **Stability:** RK4 is more stable.
- **Computational Cost:** RK4 is more computationally intensive.

Relation of Findings on the S.I.R. model to Covid-19

Infection rate β and its relation to Covid-19

- Higher β (transmission) $\rightarrow$ rapid COVID-19 spread.
- β influenced by viral mutations, social interactions.
- Public health measures aimed to reduce β. Lower beta values in the model reduce the peak of infection.

Recovery Rate γ and its relation to COVID-19

- Higher γ (recovery) $\rightarrow$ faster COVID-19 control.
- Vaccination, healthcare access increased γ.
- Larger gamma values in the model, reduce the peak of the infection.

Model Dynamics and COVID-19 Waves

- Infection peaks (I) reflect COVID-19 ”waves.”
- S/R intersections show shifts in population immunity.
- Real-world factors (variants, immunity) add complexity.

Numerical Methods and COVID-19

- Accurate simulations (RK4) vital for COVID-19 forecasting.
- Euler method is less accurate than RK4, especially with larger step sizes.

Overall Relation

- SIR model provides fundamental understanding of COVID-19 spread.
- β and γ relate to real-world pandemic factors.
- The model allows analysis of β and γ changes and their impact.

Discussion

By not normalizing, we observe the raw impact of β and γ on the counts of individuals.

References

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- EUROPEAN ACADEMIC RESEARCH Vol.XI,Issue 9/December 2023.
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- Smith, H. L., & Waltman, P. (1995). *The theory of the chemostat: dynamics of microbial competition.* Cambridge university press.

Conclusion

This report analyzes the SIR model without normalization, demonstrating the impact of β and γ. RK4 maintains superior accuracy and stability.