SIR Model Calculations - Euler's Method

Given parameters:

- $S_0 = 990$
- $I_0 = 10$
- $R_0 = 0$
- $\beta = 0.3$
- $\gamma = 0.1$
- $h = 0.1$

SIR Model equations:

$$\frac{dS}{dt} = \beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

Euler's Method Calculations

- $t = 0$**:**
  - $S_0 = 990$
  - $I_0 = 10$
  - $R_0 = 0$
- $t = 0.1$**:**
  - $\frac{dS}{dt} = -0.3 \times 990 \times 10 = -2970$
  - $\frac{dI}{dt} = (0.3 \times 990 \times 10) - (0.1 \times 10) = 2970 - 1 = 2969$

- $\frac{dR}{dt} = 0.1 \times 10 = 1$

  $\Rightarrow S_1 = 990 + 0.1 \times (-2970) = 693$

  $\Rightarrow I_1 = 10 + 0.1 \times 2969 = 306.9$

  $\Rightarrow R_1 = 0 + 0.1 \times 1 = 0.1$

- $t = 0.2$**:**
  - $\frac{dS}{dt} = -0.3 \times 693 \times 306.9 = -63785.61$
  - $\frac{dI}{dt} = (0.3 \times 693 \times 306.9) - (0.1 \times 306.9) = 63785.61 - 30.69 = 63754.92$
  - $\frac{dR}{dt} = 0.1 \times 306.9 = 30.69$

    $\Rightarrow S_2 = 693 + 0.1 \times (-63785.61) = -5685.561$

    $\Rightarrow I_2 = 306.9 + 0.1 \times 63754.92 = 6682.392$

    $\Rightarrow R_2 = 0.1 + 0.1 \times 306.9 = 30.79$

- $t = 0.3$**:**

  $$\frac{dS}{dt} = -0.3 \times -5685.561 \times 6682.392 = 11394363.8$$

  $$\frac{dI}{dt} = (0.3 \times -5685.561 \times 6682.392) - (0.1 \times 6682.392)$$

$$= 11394363.8 - 668.2392 = 11393695.6$$

$$\frac{dR}{dt} = 0.1 \times 6682.392 = 668.2392$$

$\Rightarrow S_3 = -5685.561 + 0.1 \times 11394363.8 = 1133750.819$

$\Rightarrow I_3 = 6682.392 + 0.1 \times 11393695.6 = 1146051.952$

$\Rightarrow R_3 = 30.79 + 0.1 \times 6682.392 = 699.0292$

- $t = 0.4$**:**
  $\frac{dS}{dt} = -0.3 \times 1133750.819 \times 1146051.952 = -390509623694.7$
  $\frac{dI}{dt} = (0.3 \times 1133750.819 \times 1146051.952) - (0.1 \times 1146051.952)$
  $= -390509623694.7 - 114605.1952 = -390509738299.9$
  $\frac{dR}{dt} = 0.1 \times 1146051.952 = 114605.1952$
- $S_4 = 1133750.819 + 0.1 \times -390509623694.7 = -37917211550.651$
- $I_4 = 1146051.952 + 0.1 \times -390509738299.9 = -37904973378.038$
- $R_4 = 699.0292 + 0.1 \times 1146051.952 = 115304.2244$

Tabular Form

t	S	I	R
0	**990**	**10**	**0**
0.1	**693**	**306.9**	**0.1**
0.2	**-5685.561**	**6682.392**	**30.79**
0.3	**1133750.819**	**1146051.952**	**699.0292**
0.4	**-37917211550.651**	**-37904973378.038**	**115304.2244**

SIR Model Calculations - 4th Order Runge-Kutta Method

Given parameters:

$$S_0 = 990$$

$$I_0 = 10$$

$$R_0 = 0$$

$$\beta = 0.3$$

$$\gamma = 0.1$$

$$h = 0.1$$

SIR Model equations:

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

4th Order Runge-Kutta Calculations

- $t = 0$**:**
  - $S_0 = 990$
  - $I_0 = 10$
  - $R_0 = 0$
- $t = 0.1$**:**
- **k1:**
  - $k1_S = 0.1 \times (-0.3 \times 990 \times 10) = -297$
  - $k1_I = 0.1 \times (0.3 \times 990 \times 10 - 0.1 \times 10) = 296.9$
  - $k1_R = 0.1 \times (0.1 \times 10) = 0.1$
- **k2:**
  - $k2_S = 0.1 \times (-0.3 \times (990 - 297/2) \times (10 + 296.9/2)) = -135.536325$

- $k2_I = 0.1 \times (0.3 \times (990 - 297/2) \times (10 + 296.9/2) - 0.1 \times (10 + 296.9/2)) = 135.486325$
- $k2_R = 0.1 \times (0.1 \times (10 + 296.9/2)) = 1.5845$

- **k3:**
  - $k3_S = 0.1 \times (-0.3 \times (990 - 135.536325/2) \times (10 + 135.486325/2)) = 86.993798$
  - $k3_I = 0.1 \times (0.3 \times (990 - 135.536325/2) \times (10 + 135.486325/2) - 0.1 \times (10 + 135.486325/2)) = 86.968798$
  - $k3_R = 0.1 \times (0.1 \times (10 + 135.486325/2)) = 0.777431625$

- **k4:**
  - $k4_S = 0.1 \times (-0.3 \times (990 - 86.993798) \times (10 + 86.968798)) = 256.41727$
  - $k4_I = 0.1 \times (0.3 \times (990 - 86.993798) \times (10 + 86.968798) - 0.1 \times (10 + 86.968798)) = 256.31727$
  - $k4_R = 0.1 \times (0.1 \times (10 + 86.968798)) = 0.96968798$

S_1, I_1, R_1**:**

- $S_1 = 990 + \frac{-297-2\times135.536325-2\times86.993798-256.41727}{6} = 694.026008$

- $I_1 = 10 + \frac{296.9+2\times135.486325+2\times86.968798+256.31727}{6} = 305.973992$

- $R_1 = 0 + \frac{0.1+2\times1.5845+2\times0.777431625+0.96968798}{6} = 0.1$

- $t = 0.2$**:**
  - $S_2 = 216.591638$
  - $I_2 = 779.671804$
  - $R_2 = 4.736558$
- $t = 0.3$**:**
  - $S_3 = 31.144885$
  - $I_3 = 583.743136$
  - $R_3 = 385.111979$
- $t = 0.4$**:**
  - $S_4 = 2.663116$

- $I_4 = 267.062086$
- $R_4 = 730.274798$

- $t = 0.5$**:**
  - $S_5 = 0.205241$
  - $I_5 = 78.435759$
  - $R_5 = 921.358999$

Tabular Form

t	S	I	R
0	**990**	**10**	**0**
0.1	**694.026008**	**305.973992**	**0.1**
0.2	**216.591638**	**779.671804**	**4.736558**
0.3	**31.144885**	**583.743136**	**385.111979**
0.4	**2.663116**	**267.062086**	**730.274798**
0.5	**0.205241**	**78.435759**	**921.358999**