Part3 Lab3

Saturday, December 4, 2021 11:27 PM

2-1

$y = x + \epsilon \quad \in R^p$

$E x = \mu \quad E(x-\mu)(x-\mu)^T = \Sigma$

$E \epsilon = 0 \quad E \epsilon \epsilon^T = \sigma^2 I_{p \times p}$

From noisy $y \to \hat{x}$

$$\min_{\hat{x}} E\left[\|\hat{x} - x\|^2 \mid y \right]$$

$$\hat{x} = E[x \mid y]$$

Alternatively, we added

$$\hat{x} = \mu + H(y - \mu)$$

H: $p \times p$ matrix

$$\min_{H \in R^{p \times p}} E\|x - (\mu + H(y-\mu))\|^2$$

$$H = \Sigma(\Sigma + \sigma^2 I)^{-1}$$

Step 1

$$J = E\|x - \mu - H(x - \mu + \epsilon)\|^2$$

$$= E\,Tr\left[(I-H)(x-\mu)(x-\mu)^T(I-H)^T + H\epsilon\epsilon^T H^T\right]$$

$$= Tr\left[(I-H)\Sigma(I-H)^T + \sigma^2 H H^T\right]$$

$$\|x\|^2 = x^T x = Tr[x^T x] = Tr(x x^T)$$

Step 2

$$\left(\frac{d}{dH} Tr(HB^T) = B\right) - 2(I-H)\Sigma + 2\sigma^2 H = 0$$

$$\Rightarrow \hat{x} = \mu + \Sigma(\Sigma + \sigma^2 I)^{-1}(y - \mu)$$

#