

Mixed effects models 2

“Space is the place” - *Sun Ra*

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Motivation

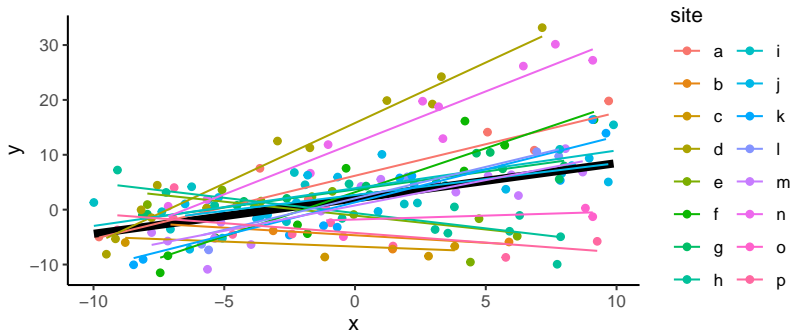
- How do I check if model results are valid?
 - Residual checks
 - Hypothesis testing
- What if my response variable is non-normal?
 - Generalized linear mixed models (GLMMs)
- Sampling over time or space
 - “Continuous” random effects
- Christmas-themed exercise!

Mixed effect model example

Let's go back to our earlier example:

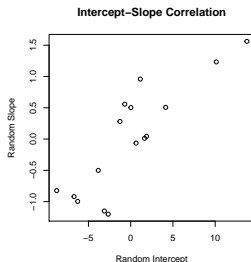
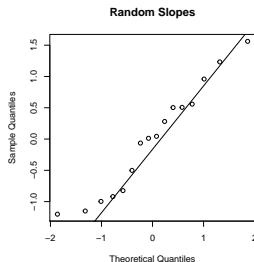
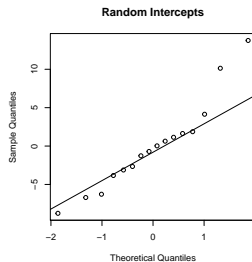
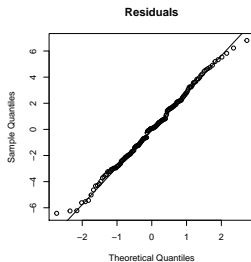
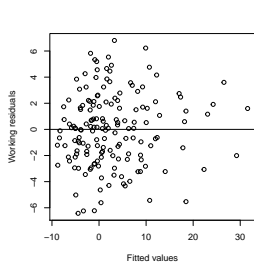
```
lmer(y ~ x + (x|site), data = dat)
```

- We're interested in predicting y using x (fixed effects)
- Data was collected at a number of *sites*, which may affect y
- Effect of each site is normally distributed (random intercept)
- Effect of site on slope of x is normally distributed (random slope)



Validation

- Similar to linear models, but we *also* check whether the random intercepts are normally distributed



Hypothesis testing

Is this fixed effect important? (e.g. ANOVA)

- Use likelihood-based test via drop1 (likelihood ratio test, AIC)
- Be careful to fit model with REML = FALSE!

```
lmm1 <- update(lmm1,REML=FALSE) #Refit model using ML rather than REML  
drop1(lmm1,test='Chisq') #x has a strong effect
```

```
## Single term deletions
```

```
##
```

```
## Model:
```

```
## y ~ x + (x | site)
```

```
##          npar      AIC      LRT  Pr(Chi)
```

```
## <none>      913.08
```

```
## x          1 917.85 6.7697 0.009272 **
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Hypothesis testing (cont.)

How do I know this effect is different from x ? - Use Wald Z-test (2-sided p-value from Z-test)

```
##           x
## 0.004659069
```

- `glht` from `library(multcomp)` works with `lmer` models if you are comparing between coefficients (e.g. “Is treatment A different from B and C?”)

What if my response variable is non-normal?

- Linear model (LM)

$$\hat{y} = X\beta$$

$$y \sim \text{Normal}(\hat{y}, \sigma)$$

- Linear mixed effects model (LMM)

$$\hat{y} = X\beta + U\zeta$$

$$y \sim \text{Normal}(\hat{y}, \sigma)$$

$$\zeta \sim \text{Normal}(0, \sigma_{\text{site}})$$

- Generalized linear model (GLM)

$$\text{logit}(\hat{\phi}) = X\beta$$

$$y \sim \text{Binomial}(\hat{\phi})$$

- Generalized linear mixed effects model (GLMM)

$$\text{logit}(\hat{\phi}) = X\beta + U\zeta$$

$$y \sim \text{Binomial}(\hat{\phi})$$

$$\zeta \sim \text{Normal}(0, \sigma_{\text{site}})$$

How do I fit GLMMs?

- `glmer` and `glmer.nb` from `library(lme4)` work for Binomial, Poisson, and Negative Binomial data

```
library(lme4)
glmm1 <- glmer.nb(y2~x+(x|site),data=dat) #Negative binomial GLMM
summary(glmm1) #glmer.nb takes a LONG time to run
```

- `glmmTMB` from `library(glmmTMB)` works for those above, *plus* a bunch of others
 - e.g. Zero-inflation, Beta-binomial, Spatial Models

```
library(glmmTMB)
glmm2 <- glmmTMB(y2~x+(x|site),data=dat,family=nbinom2())
summary(glmm2) #Similar results, but quicker
```


Fitting GLMMs - glmer.nb

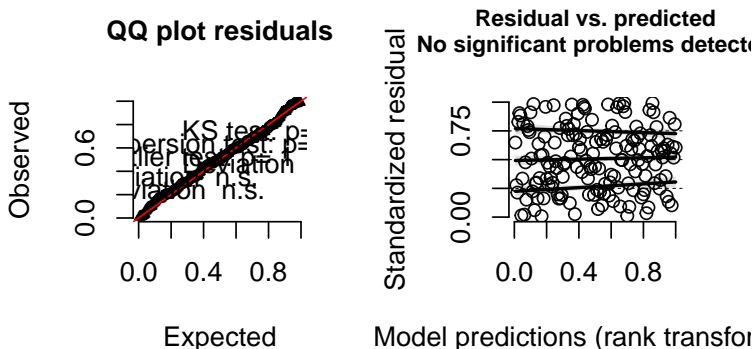
Fitting GLMMs - glmmTMB

```
## Family: nbinom2 ( log )
## Formula:          y2 ~ x + (x | site)
## Data: dat
##
##      AIC      BIC   logLik deviance df.resid
##    627.8    646.2   -307.9    615.8      154
##
## Random effects:
##
## Conditional model:
##   Groups Name      Variance Std.Dev. Corr
##   site  (Intercept) 1.43543  1.1981
##         x           0.02892  0.1701   0.92
## Number of obs: 160, groups: site, 16
##
## Overdispersion parameter for nbinom2 family (): 5.12
##
## Conditional model:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.34132    0.32172   1.061   0.2887
## x            0.11026    0.04697   2.348   0.0189 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual checks on glmmTMB models

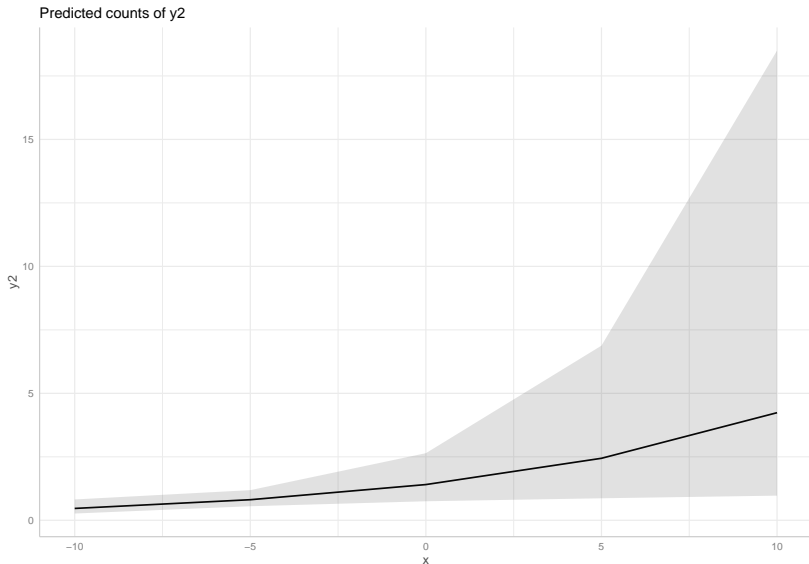
- Extract residuals and make your own plots, or use `simulateResiduals` from `library(DHARMa)` (see [here](#))
- DHARMa also has useful functions for checking overdispersion and zero-inflation (found [here](#))

DHARMa residual diagnostics



Partial residual plots for glmmTMB models

- `ggpredict()` from `library(ggeffects)` works with `glmmTMB` models



Spatial and Temporal Random Effects

"My data were sampled over time or over a geographic area (or both). Can I just use day or site as a random effect?"

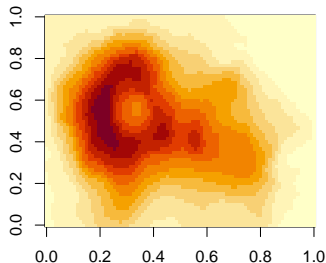
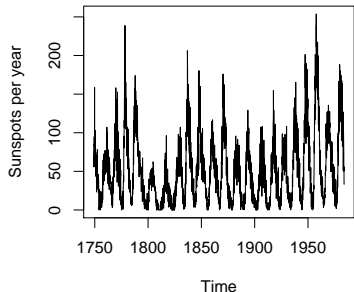
- Short answer: "Yes"
- Long answer: You might be able to do better, because of the **1st Law of Geography**:

"... everything is related to everything else, but near things are more related than distant things." Waldo Tobler

- If you have spatial or temporal information, this can help R to estimate random effects more accurately
 - Can improve prediction accuracy (smaller p-values)
 - Can give you hints about the underlying causal mechanisms

Temporal or Spatial Data

- Correlation is often present in temporal data or spatial data; causes may be unknown or “uninteresting”
- Usually we are interested in accounting for these patterns, in order to better estimate the “interesting” patterns on top of them



Covariance

- Normal distributions¹ don't just have a single σ , but a matrix of values
- If our data y are *independent*, then it looks like this:

$$y \sim \text{Normal}(\textcolor{brown}{M}, \textcolor{red}{\Sigma})$$

$$\textcolor{brown}{M} = [\mu_1, \mu_2, \mu_3]$$

$$\textcolor{red}{\Sigma} = \begin{bmatrix} \textcolor{red}{\sigma}^2 & 0 & 0 \\ 0 & \textcolor{red}{\sigma}^2 & 0 \\ 0 & 0 & \textcolor{red}{\sigma}^2 \end{bmatrix}$$

- Zeros mean “ μ_1 , μ_2 , & μ_3 aren't related to each other”
- Diagonal elements = *variance*, off-diagonal = *covariance*

¹Multivariate Normal

Covariance and Correlation

In real life, things may not be independent from each other. For example:

- $\sigma = 2$ (variance = $\sigma^2 = 4$)
- μ_1 and μ_2 are strongly correlated ($r=0.7$), but μ_3 is not related to anything ($r=0$). Shown here as a *correlation matrix* (R):

$$R = \begin{bmatrix} 1 & 0.7 & 0 \\ 0.7 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- When multiplied by the variance, this becomes the *covariance matrix* (Σ)

$$\Sigma = \begin{bmatrix} \sigma^2 \times 1 & \sigma^2 \times 0.7 & \sigma^2 \times 0 \\ \sigma^2 \times 0.7 & \sigma^2 \times 1 & \sigma^2 \times 0 \\ \sigma^2 \times 0 & \sigma^2 \times 0 & \sigma^2 \times 1 \end{bmatrix} = \begin{bmatrix} 4 & 2.8 & 0 \\ 2.8 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Gaussian Process Modelling

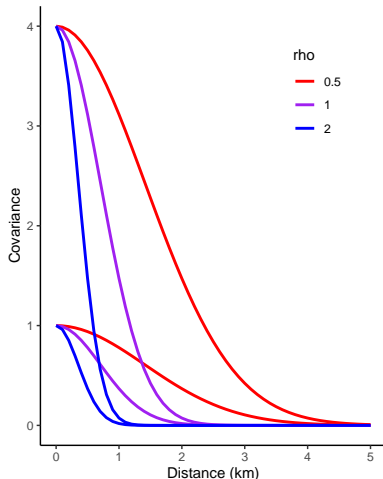
- We can model covariance between things as a function of *distance*, either in time or space
- Squared-exponential is fairly common²:

Σ = covariance

Σ = variance \times correlation

$$\Sigma = \sigma^2 \times e^{-\rho^2 \text{Dist}^2}$$

- Instead of finding a single σ value, R now looks for σ (maximum covariance) and ρ (decay with distance)



²Also common: AR-1 (temporal processes), Matérn (spatial processes)

Spatial random effects

- Say that we collected data at 16 sites, and we're interested in the effect of y on x
- Let's first fit a model with a random intercept for site

If we plot the intercepts for each site, we see that they are clustered:

