Spatiotemporal models

"Space is the place" - Sun Ra

Samuel Robinson, Ph.D.

Oct 27, 2023

Outline

- Spatial and temporal data
  - Some basic GIS (`sf`)
- How to think about space and time
  - Plotting
  - Variograms
  - “Continuous” random effects
  - Kernels and
- Some common modeling approaches
  - GLS (covariance)
  - Basis functions (GAMs)

Spaaaaace

Some common problems

- My data were sampled over time or space. I'm not really interested in time or space *per se*, so can I just ignore them and run my models?
- I am actually interested in how something changes over time or space. Can I just use day or location (lat/lon) as another term in my model?
- My supervisor told me to look for something called autocorrelation, and it sounds scary

A common approach: random effects

"Can I just use day or site as a random effect?"

- Short answer: "Yes"
- Long answer: You might be able to do better, because of the **1st Law of Geography**:

"*. . . everything is related to everything else, but near things are more related than distant things.*" Waldo Tobler

- If you have spatial or temporal information, this can help R to estimate random effects more accurately
  - Can improve prediction accuracy (smaller p-values)
  - Can give you hints about the underlying causal mechanisms

Part 1: Time and Space in R

How R deals with time

- Dealing with time in R is somewhat annoying, but not complicated
- Common methods: `as.Date` (days), `as.POSIXlt` (date + time)
- Both require a date/time format: see `?strptime` for examples
- You can transform to specific formats (e.g. day of year) using `format`
- `difftime` is useful for getting differences in time points

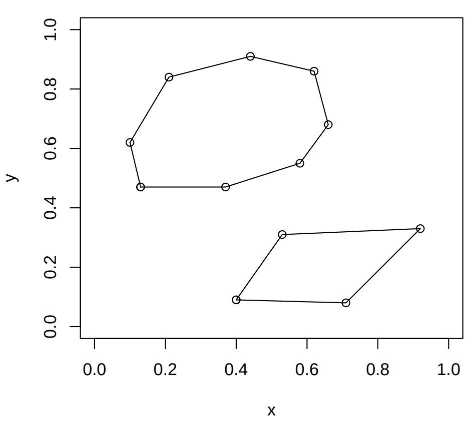
```
##    x         d1         d2
## 1  5 2010-05-06 2010-06-13
## 2 10 2021-11-14 2022-10-14
```

```
dExamp %>%
  mutate(across(c(d1,d2),~as.Date(.x,format='%Y-%m-
  mutate(doy=format(d1,format='%j')) %>%
  mutate(dChange=difftime(d2,d1,units='days'))
```

```
##    x         d1         d2 doy  dChange
## 1  5 2010-05-06 2010-06-13 126  38 days
## 2 10 2021-11-14 2022-10-14 318 334 days
```

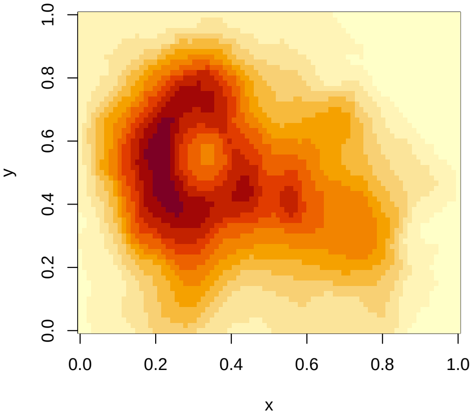
Two main types of spatial data

Vector data: points, lines, and polygons



Common R packages: `sf`, `sp`, `gstat`, `spdep`

Raster data: cells



Common R packages: `stars`, `terra`

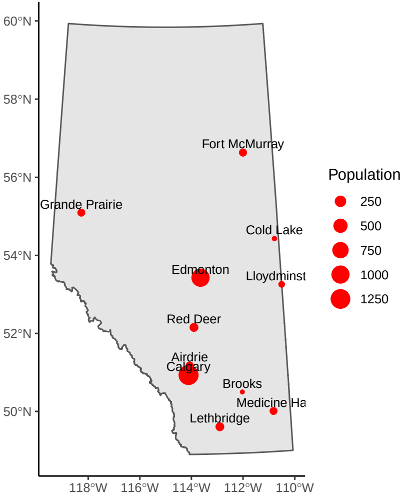
R as a GIS

- A **Geographic Information System** (GIS) is a system for organizing, analyzing, and displaying spatial information
- Common platforms and tools: ArcGIS, QGIS, PostGIS, Python
- A number of R packages are specifically written for dealing with GIS data, usually specific to raster or vector formats
- Ecologists mostly deal with vector data (site locations, boundary polygons) but raster data is sometimes used (NDVI, land cover classes)
- I'll show you a couple practical tips for using the `sf` package (see here also), but there are many other packages out there If you're dealing with large amounts of

  spatial data *I would encourage you to take a formal GIS course*, as there is a LOT to learn!

Common tasks: making maps

- Vector data are often encoded as *shapefiles* (set of several files)
- Point data can also be read in as *csv* files, which need to be turned into an `sf` object
- Be careful: shapefiles can be very large, which can easily crash R

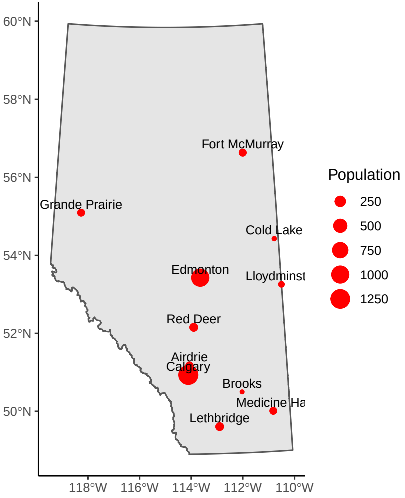


Common tasks: making maps (cont.)

```r
#Reads AB boundary shapefile
abBound <- read_sf('./shapefiles/AB_only.shp')

#Reads city csv
abCities <- read.csv('./shapefiles/abCities.csv') %>%
  st_as_sf(coords = c('lon','lat'),crs=4326) #Converts
#NOTE: crs 4326 is common lat/lon format

#Make map
ggplot()+
  #Add boundary
  geom_sf(data=abBound)+
  #Add cities
  geom_sf(data=abCities,aes(size=pop),col='red')+
  #Add labels
  geom_sf_text(data=abCities,aes(label=name),
               size=3,nudge_y=25000)+
  labs(x=NULL,y=NULL,size="Population")
```

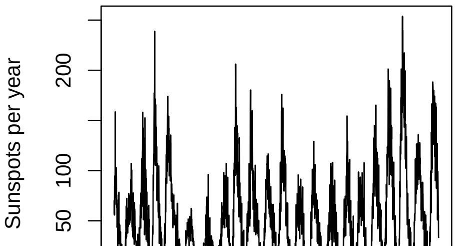


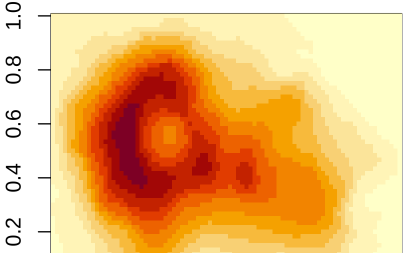
Common tasks: reprojection

Part 2: Spatiotemporal modeling

Temporal or Spatial Data

- Last week we talked about *cross*-correlation (i.e. correlation between columns of data); this week we're mostly talking about *auto*-correlation (i.e. correlation between individual data points in a single column)
- Correlation is often present in temporal data or spatial data; causes may be unknown or "uninteresting"
- Usually we are interested in accounting for these patterns, in order to better estimate the "interesting" patterns on top of them





Covariance

- Normal distributions[1] don't just have a single σ, but a matrix of values
- If our data y are *independent*, then it looks like this:

$$y \sim Normal(M, \Sigma)$$

$$M = [\mu_1, \mu_2, \mu_3]$$

$$\Sigma = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

- Zeros mean "μ_1, μ_2, & μ_3 aren't related to each other"
- Diagonal elements = *variance*, off-diagonal = *covariance*

[1] Multivariate Normal

Covariance and Correlation

In real life, things may not be independent from each other. For example:

- $\sigma = 2$ (variance $= \sigma^2 = 4$)
- μ_1 and μ_2 are strongly correlated (r=0.7), but μ_3 is not related to anything (r=0). Shown here as a *correlation matrix* (R):

$$R = \begin{bmatrix} 1 & 0.7 & 0 \\ 0.7 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- When multiplied by the variance, this becomes the *covariance matrix* (Σ)

$$\Sigma = \begin{bmatrix} \sigma^2 \times 1 & \sigma^2 \times 0.7 & \sigma^2 \times 0 \\ \sigma^2 \times 0.7 & \sigma^2 \times 1 & \sigma^2 \times 0 \\ \sigma^2 \times 0 & \sigma^2 \times 0 & \sigma^2 \times 1 \end{bmatrix} = \begin{bmatrix} 4 & 2.8 & 0 \\ 2.8 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Gaussian Process Modelling

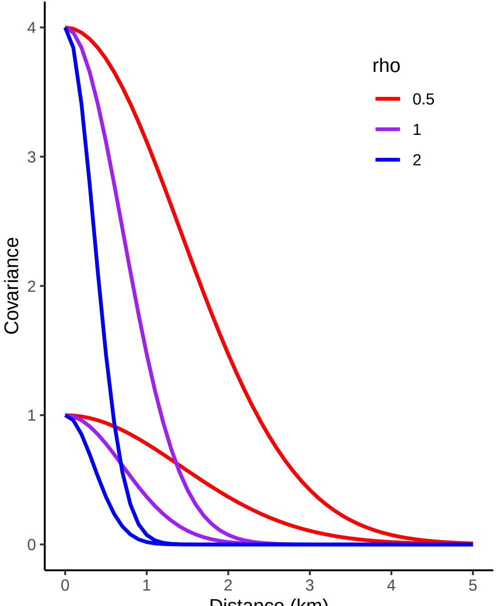
- We can model covariance between things as a function of *distance*, either in time or space
- Squared-exponential is fairly common[2]:

$$\Sigma = covariance$$

$$\Sigma = variance \times correlation$$

$$\Sigma = \sigma^2 \times e^{-\rho^2 Dist^2}$$

- Instead of finding a single σ value, R now looks for σ (maximum covariance) and ρ (decay with distance)

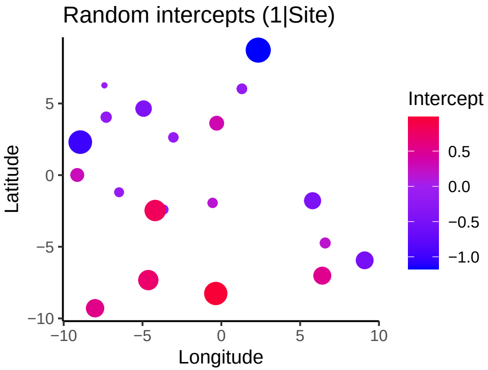


Spatial random effects

- Say that we collected data at 16 sites, and we're interested in the effect of y on x
- Let's first fit a model with a random intercept for site

```
#Same syntax as lmer models:
lmm2 <- glmmTMB(y~x+(1|site),data=dat2)
```

- If we plot the intercepts for each site, we see that they are clustered:



Spatial random effects (cont.)

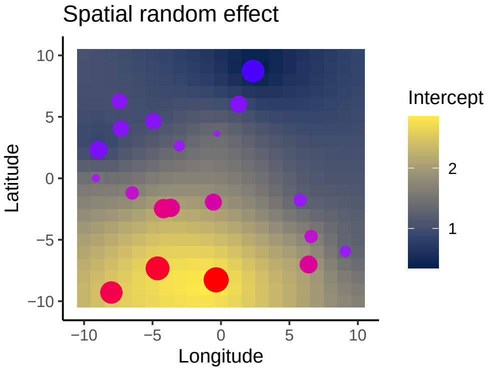
- Re-fit model with a spatial (exponential) random effect

```
#Coordinates
dat2$coords <- numFactor(dat2$lon,dat2$lat)

#Group factor (only 1 here)
dat2$group <- factor(rep(1,nrow(dat2)))

#Fit model with spatial random effect
lmm3 <- glmmTMB(y~x+exp(coords+0|group),data=dat2)
```

- Clustering effect modeled as a spatial random effect



Challenge

Problem: hard for large datasets

Solution: basis function

A challenger approaches

- Ho ho ho! Merry Christmas! In order to maximize the number of presents that you get from Santa Claus, you've decided to apply an analytic approach, and have collected data across Alberta on *number of Christmas presents received*
- You've also collected data on things that might influence Saint Nick's generosity (*naughtiness*, *presence of milk and cookies*, *chimney width*)
- Fit a GLMM to the present data, one using spatial random intercepts, and one using "regular" random intercepts
- Which type of snack should you leave out for Santa? Which area might you consider moving to??

Two-column slide