Spatiotemporal models

“Space is the place” - Sun Ra

Samuel Robinson, Ph.D.

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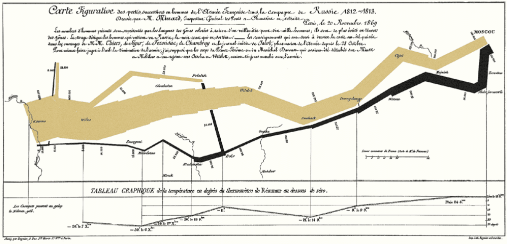
Outline

- Spatial and temporal data
  - Some basic GIS (`sf`)
- How to think about space and time
  - Plotting
  - Variograms
  - “Continuous” random effects
- Some common modeling approaches
  - GLS (covariance)
  - Basis functions (GAMs)



Some common problems

- My data were sampled over time or space. I'm not really interested in time or space *per se*, so can I just ignore them and run my models?
- I am actually interested in how something changes over time or space. Can I just use day or location (lat/lon) as another term in my model?
- My supervisor told me to look for something called autocorrelation, and it sounds scary

A common approach: random effects

"Can I just use day or site as a random effect?"

- Short answer: "Yes"
- Long answer: You might be able to do better, because of the **1st Law of Geography**:

"*. . . everything is related to everything else, but near things are more related than distant things.*" Waldo Tobler

- If you have spatial or temporal information, this can help R to estimate random effects more accurately
  - Can improve prediction accuracy (smaller p-values)
  - Can give you hints about the underlying causal mechanisms

Part 1: Time and Space in R

How R deals with time

- Dealing with time in R is somewhat annoying, but not complicated
- Common methods: `as.Date` (days), `as.POSIXlt` (date + time)
- Both require a date/time format: see `?strptime` for examples
- You can transform to specific formats (e.g. day of year) using `format`
- `difftime` is useful for getting differences in time points

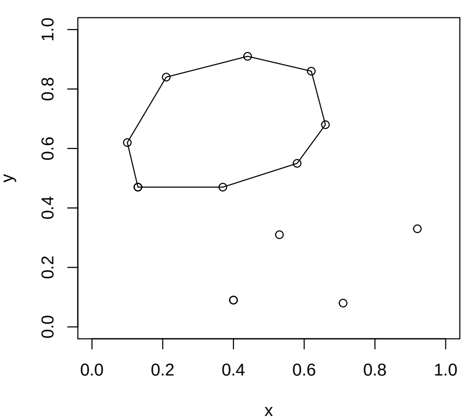
```
##    x         d1         d2
## 1  5 2010-05-06 2010-06-13
## 2 10 2021-11-14 2022-10-14
```

```
#Convert data to Date format
dateForm <- '%Y-%m-%d'
dExamp %>%
  mutate(across(c(d1,d2),
                ~as.Date(.x,format=dateForm))) %>%
  #Get day of year
  mutate(doy=format(d1,format='%j')) %>%
  #Get difference in time between d2 and d1
  mutate(dChange=difftime(d2,d1,units='days'))
```

```
##    x         d1         d2 doy  dChange
## 1  5 2010-05-06 2010-06-13 126  38 days
## 2 10 2021-11-14 2022-10-14 318 334 days
```

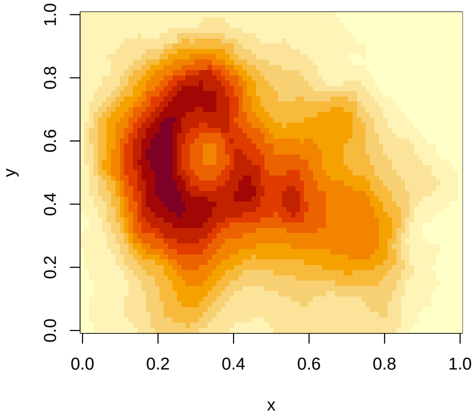
Two main types of spatial data

Vector data: points, lines, and polygons



R packages: `sf`, `sp`, `gstat`, `spdep`

Raster data: cells



R packages: `stars`, `terra`

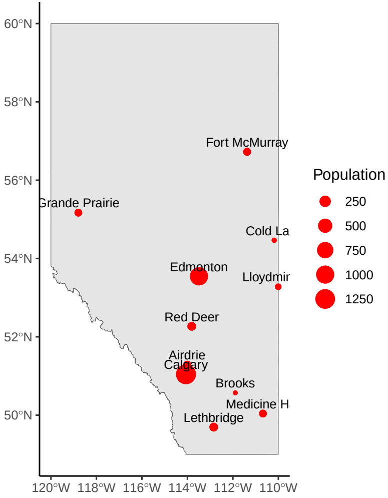
R as a GIS

- A **Geographic Information System** (GIS) is a system for organizing, analyzing, and displaying spatial information
- Common platforms and tools: ArcGIS, QGIS, PostGIS, Python
- A number of R packages are specifically written for dealing with GIS data, usually specific to raster or vector formats
- Ecologists mostly deal with vector data (site locations, boundary polygons) but raster data is sometimes used (NDVI, land cover classes)
- I'll show you a couple practical tips for using the `sf` package (see here also), but there are many other packages out there

If you're dealing with large amounts of spatial data *I would encourage you to take a formal GIS course*, as there is a LOT to learn!

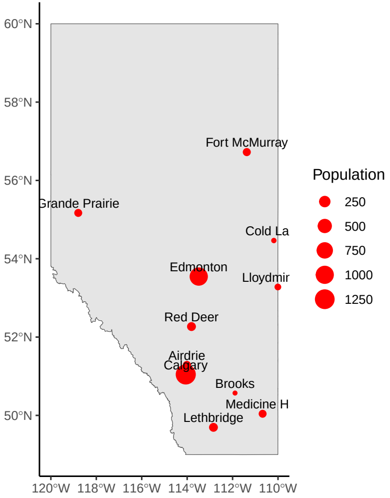
Common tasks: making maps

- Vector data are often encoded as *shapefiles* (set of several files)
- Point data can also be read in as *csv* files, which need to be turned into an `sf` object
- `sf` objects can be displayed in `ggplot` using `geom_sf`. Common aesthetics (colour, size) can be mapped onto the plot
    - Objects are layered on the map in order of coding
- Be careful: shapefiles can be very large, which can easily crash R!

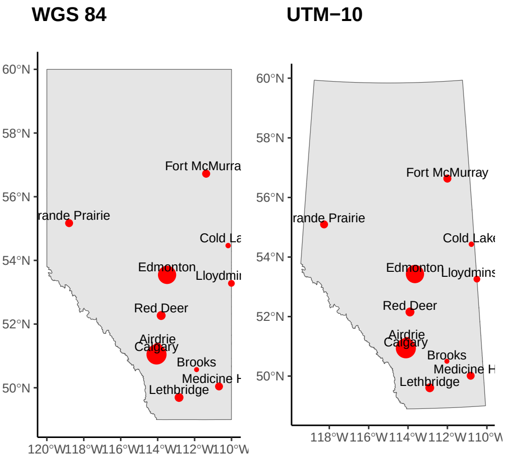


Common tasks: making maps (cont.)

```r
#Reads AB boundary shapefile
abBound <- read_sf('./shapefiles/AB_only.shp') %>%
  st_transform(4326)

#Reads city csv
csvPath <- './shapefiles/abCities.csv'
abCities <- read.csv(csvPath) %>%
  #Converts to sf
  st_as_sf(coords = c('lon','lat'),crs=4326)
#NOTE: crs 4326 is common lat/lon format

#Make map
(p1 <- ggplot()+
  #Add boundary
  geom_sf(data=abBound)+
  #Add cities
  geom_sf(data=abCities,aes(size=pop),col='red')+
  #Add labels
  geom_sf_text(data=abCities,aes(label=name),
  size=3,nudge_y=0.25)+
  labs(x=NULL,y=NULL,size="Population"))
```



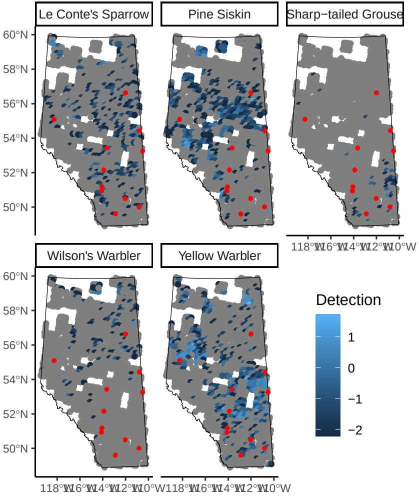
Common tasks: reprojection



- The world is not flat: all maps have to "bend" the data somehow. This is called the map **projection**
- Some map projections preserve *area*, others preserve *distance*. Degrees are not all the same distance apart!
- Usually we're interested in absolute distance between locations, so *Mercator* (UTM) is a good choice, but be careful which UTM zone you choose!
- `sf` uses `crs` codes: **4326** is for lat/lon (WGS 84), 3401 is an Alberta-specific UTM projection
- Many others are available

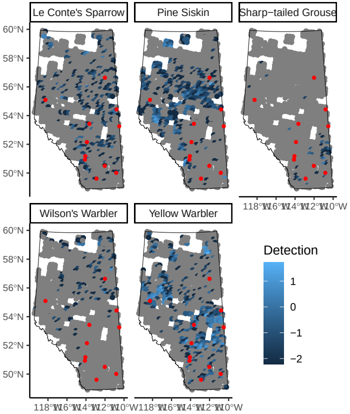
First challenge

- Make this map of bird counts from the ABMI dataset, with Alberta cities and provincial boundaries overlaid on top
- `medDetects` is the median detection rate at each site over several years (I used log(`medDetects`) for this map)



First challenge results

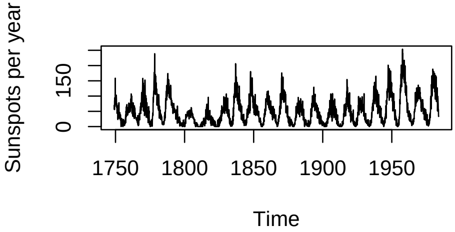
```r
abBound <- read_sf('./shapefiles/AB_only.shp') %>%
  st_transform(3401)
birdDat <- read.csv('./shapefiles/birdDat.csv') %>%
  st_as_sf(coords = c('lon','lat'),crs=4326) %>%
  st_transform(st_crs(abBound))
abCities <- read.csv('./shapefiles/abCities.csv') %>%
  st_as_sf(coords = c('lon','lat'),crs=4326) %>%
  st_transform(st_crs(abBound))

ggplot()+
  geom_sf(data=birdDat,aes(col=log(medDetects)))+
  geom_sf(data=abBound,fill=NA,col='black')+
  geom_sf(data=abCities,col='red',size=1)+
  facet_wrap(~Common.Name)+
  labs(col='Detection')+
  theme(axis.text = element_text(size=8),
        legend.position=c(0.85,0.25))
```

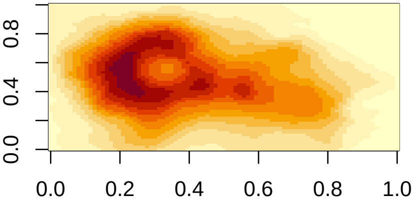


Part 2: Spatiotemporal modeling

Temporal or Spatial Data

- Correlation is often present in temporal data or spatial data; causes may be unknown or "uninteresting"
- Usually we are interested in accounting for these patterns, in order to better estimate the "interesting" patterns on top of them
- Last week we talked about *cross*-correlation (i.e. correlation between columns of data); this week we're talking about *auto*-correlation (i.e. correlation between individual data points in a single column)





Covariance

- Normal distributions[1] don't just have a single σ, but a matrix of values
- If our data y are *independent*, then it looks like this:

$$y \sim \mathit{Normal}(\hat{y}, \Sigma)$$

$$\hat{y} = [\mu_1, \mu_2, \mu_3]$$

$$\Sigma = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

- Zeros mean "μ_1, μ_2, & μ_3 aren't related to each other"
- Diagonal elements = *variance*, off-diagonal = *covariance*

[1] Multivariate Normal

Covariance and Correlation

In real life, things may not be independent from each other. For example:

- $\sigma = 2$ (variance $= \sigma^2 = 4$)
- μ_1 and μ_2 are strongly correlated (r=0.7), but μ_3 is not related to anything (r=0). Shown here as a *correlation matrix* (R):

$$R = \begin{bmatrix} 1 & 0.7 & 0 \\ 0.7 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- When multiplied by the variance, this becomes the *covariance matrix* (Σ)

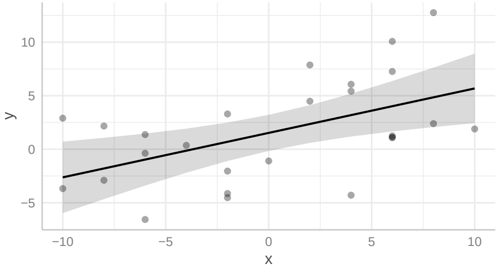
$$\Sigma = \begin{bmatrix} \sigma^2 \times 1 & \sigma^2 \times 0.7 & \sigma^2 \times 0 \\ \sigma^2 \times 0.7 & \sigma^2 \times 1 & \sigma^2 \times 0 \\ \sigma^2 \times 0 & \sigma^2 \times 0 & \sigma^2 \times 1 \end{bmatrix} = \begin{bmatrix} 4 & 2.8 & 0 \\ 2.8 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Let's start with an example

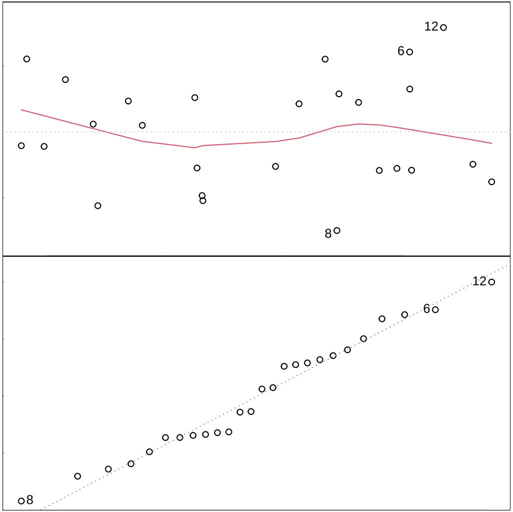
- Say we're fitting a simple linear regression on a dataset collected across space

```
##           y         x site         lat         lon
## 1  5.816386  3.396386    a -1.42694611 -1.43172453
## 2 -6.741759 -6.424300    b  0.08295583 -1.49349903
## 3 -4.210695 -2.177401    c  0.95595705 -3.91433926
## 4  1.732103  9.617379    d  2.10483276 -4.26778886
## 5  1.711212 -5.182335    e  0.17610414  0.04349044
```

Model: lm(y ~ x)

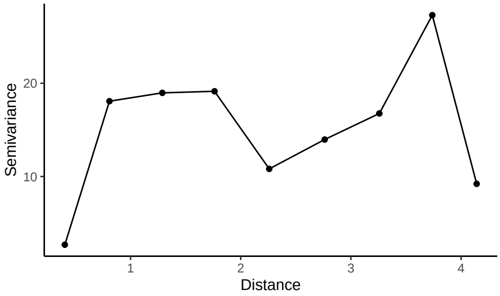


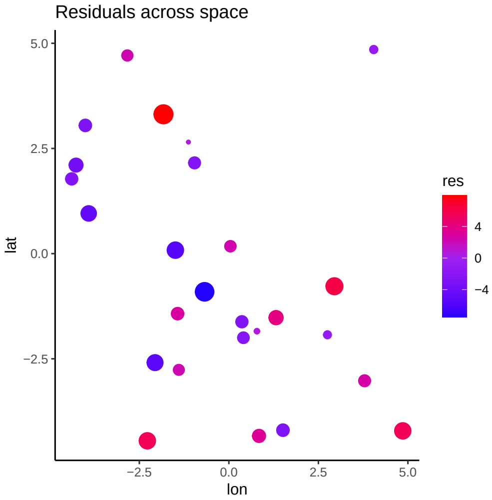
Things look mostly OK, right?



Spatial residual plot

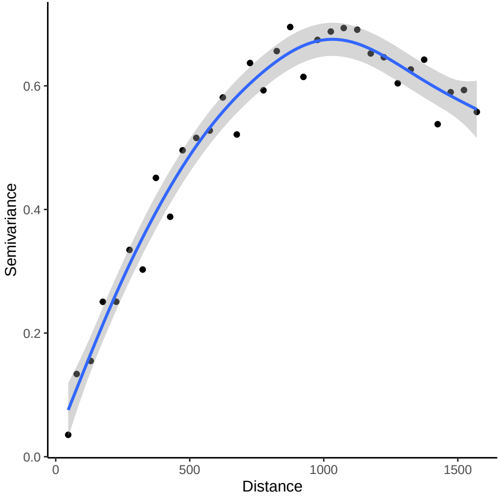
- Residuals are spatially *non-independent*!
- *Variograms* are a common tool to examine how variance changes with distance
- Uncorrelated spatial data will have a *flat* variogram (no change in semivariance with distance)





How do variograms work?

- Variograms compare the squared difference (variance) between values spaced at different distances
- If close values are similar, variance increases with increasing distance before leveling off
- Analysis of the specific shape of the curve is called *variography*, and is important for spatial modeling



Gaussian Process Modelling

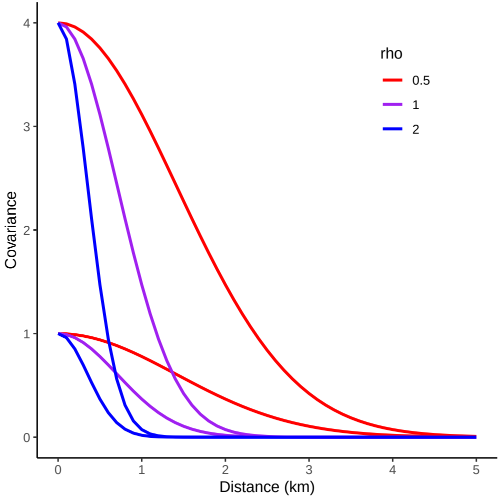
- We can model covariance between things as a function of *distance* and use it to predict what other points in between might be (a.k.a. Kriging)
- Squared-exponential is fairly common[2]:

$$\Sigma = covariance$$

$$\Sigma = variance \times correlation$$

$$\Sigma = \sigma^2 \times e^{-\rho^2 Dist^2}$$

- Instead of finding a single σ value, R now looks for σ (maximum covariance) and ρ (decay with distance)



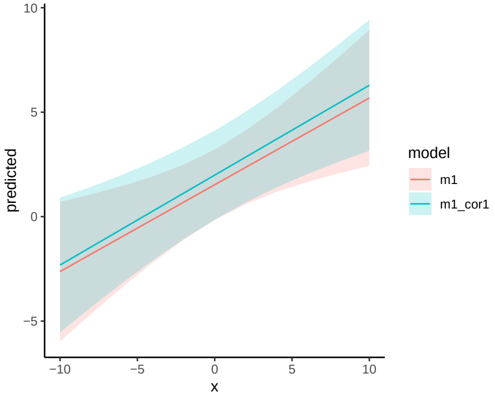
[2] Also common: AR-1 (temporal processes), Matérn (spatial processes)

How do we do this in R?

- For simpler models, you can use a generalized linear system (`gls`)
- Here I've used `corGaus` to specify a Gaussian (squared-exponential) kernel

```
corForm <- corGaus(form = ~ lat + lon)
m1_cor <- gls(y ~ x, correlation = corForm,
              dat = dat2_small)
```

```
## Generalized least squares fit by REML
##   Model: y ~ x
##   Data: dat2_small
##        AIC      BIC    logLik
##   152.1316 156.8439 -72.06582
##
## Correlation Structure: Gaussian spatial correlation
##  Formula: ~lat + lon
##  Parameter estimate(s):
##     range
## 0.8284849
##
## Coefficients:
##                 Value Std.Error  t-value p-value
## (Intercept) 1.9868593 1.0339275 1.921662  0.0666
## x           0.4302774 0.1141634 3.768962  0.0009
##
##  Correlation:
##   (Intr)
## x -0.027
##
```



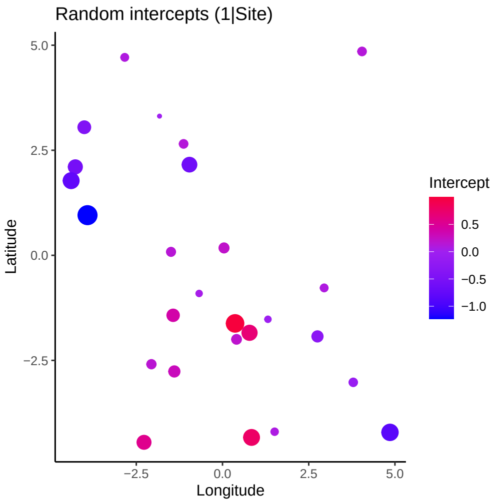
Not too bad in this case, but in general, high spatial correlation effectively means *your sample size is smaller*

Random effects: standard

- Say that we collected data at 16 sites, and we're interested in the effect of y on x
- Let's first fit a model with a random intercept for site

```
#Same syntax as lmer models:
lmm2 <- glmmTMB(y~x+(1|site),data=dat2)
```

- If we plot the intercepts for each site, we see that they are clustered

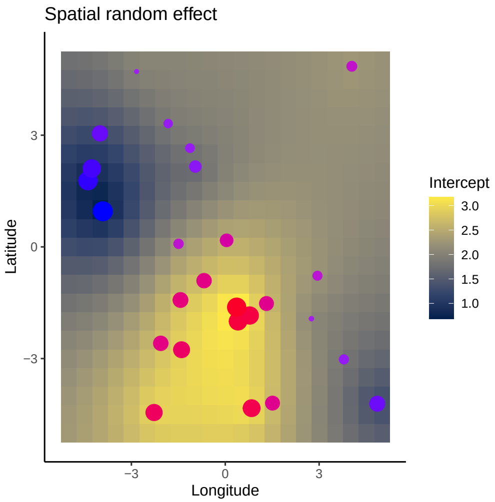


Random effects: spatial

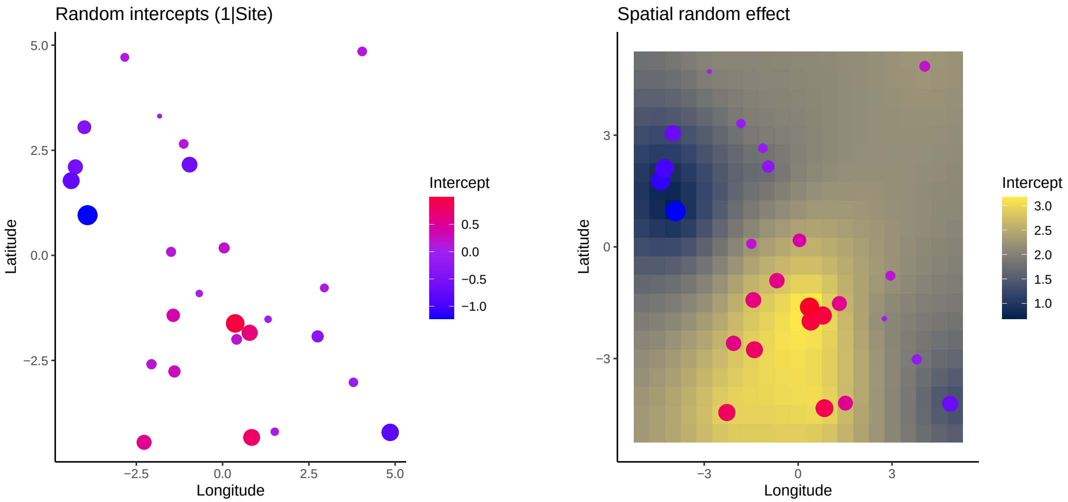
- Re-fit model with a spatial (exponential) random effect

```
#Coordinates
dat2$coords <- numFactor(dat2$lon,dat2$lat)

#Group factor (only 1 here)
dat2$group <- factor(rep(1,nrow(dat2)))

#Fit model with spatial random effect
lmm3 <- glmmTMB(y~x+exp(coords+0|group),data=dat2)
```

- The spatial random *field* is shown in the background, along with individual draws from the field (sites)

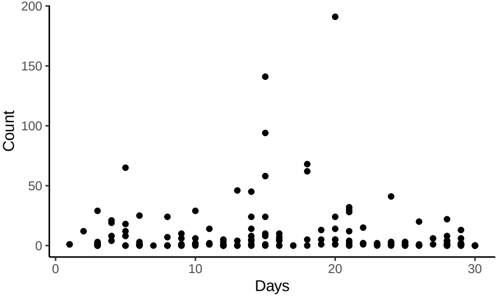


How do these differ?



Second challenge

```
##    count temperature site days
## 1      1    5.310173    i    0
## 2      1    7.442478    e    0
## 3     12   11.457067    m    1
## 4      3   18.164156    n    2
## 5      1    4.033639    a    2
## 6      2   17.967794    c    2
```

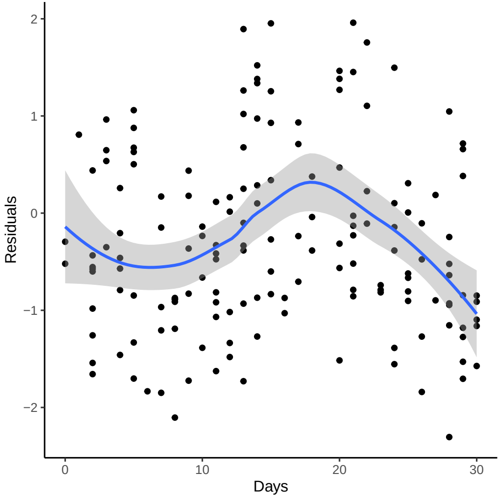


- Things can vary in *time* as well as space
- There is a dataset found here (`timeDat.csv`) with count data recorded at each location, along with temperature and a `day` variable
- How does count change with temperature? Is there `temporal` autocorrelation between the data?
- Fit a model that accounts for location, as well as any temporal autocorrelation

Second challenge results

```r
dat3 <- read.csv('./timeDat.csv') %>%
  mutate(site=factor(site))

#Naive random effect model
m2 <- glmmTMB(y~x+(1|site),
              data=dat3,
              family='nbinom2')

m2Res <- residuals(m2,'deviance')

dat3 %>% mutate(m2Res) %>%
  mutate(days=as.numeric(days)) %>%
  ggplot(aes(x=days,y=m2Res))+
  geom_point()+
  geom_smooth(method='loess',formula=y~x)+
  labs(x='Days',y='Residuals')
```

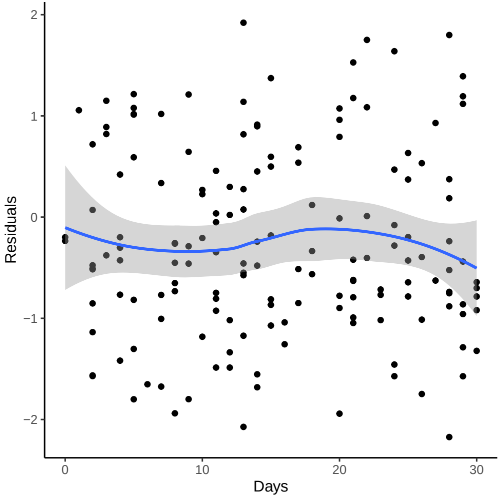
Looks like a fairly strong pattern in residuals



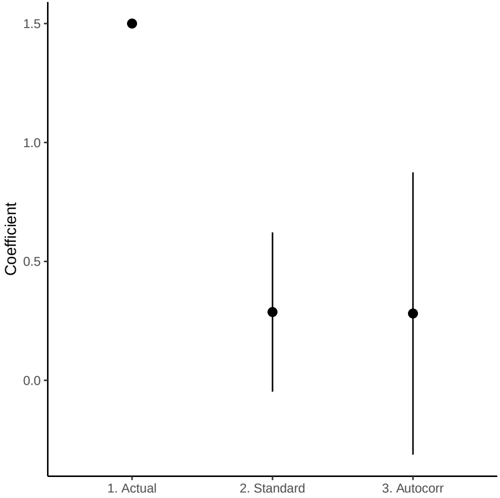
Second challenge results (cont)

```r
#Days as "numeric factor"
dat3$dayF <- numFactor(dat3$days)
dat3$group <- factor(rep(1,nrow(dat3)))

#Fit model with temporal random effect
m2_ac <- glmmTMB(y~x+(1|site)+
                exp(dayF+0|group),
              data=dat3,
              family='nbinom2')
m2acRes <- residuals(m2_ac,'deviance')

dat3 %>% mutate(m2acRes) %>%
  mutate(days=as.numeric(days)) %>%
  ggplot(aes(x=days,y=m2acRes))+
  geom_point()+
  geom_smooth(method='loess',formula=y~x)+
  labs(x='Days',y='Residuals')
```

Pattern has largely disappeared

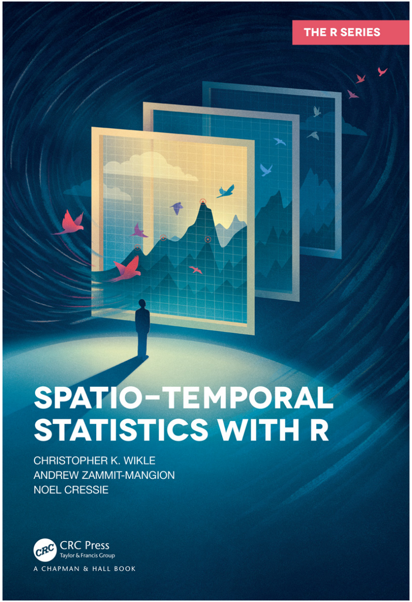


Second challenge results (cont. . .)



What if my data varies over space AND time?

- You can jointly estimate covariance functions for different types of distances, however. . .
- Make sure that these can actually be estimated! Did you actually gather data at all combinations of space and time?
- In the literature these can be *separable* or *non-separable* covariance functions (see here or here for more details)

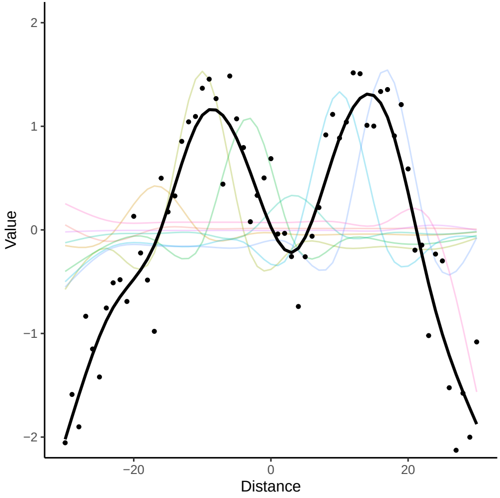


Problem: hard for large datasets

- You'll notice that I've only been doing this for correlated random intercepts
- OK because the covariance matrix for intercepts is small (26 x 26), but the covariance matrix for the entire dataset is large (260 x 260)
- If data points are all from different times, this can start using a lot of memory and can take a *long time*
- In general, *covariance-based methods* do not scale well to larger datasets, so we need a better way of doing things

Solution: basis functions

- There are quicker ways to deal with spatial and temporal effects, including spatial partial differential equations (`R-INLA` and `TMB`, see here here) or Gaussian predictive process (`plgp`) models
- We're going to use an approach you've already used: additive models!
- GAMs are a common way to deal with this problem, as they move the ST variation from the covariance matrix into a random effect, which speeds up estimation



Covariance vs Random Effects

Estimate covariance explicitly

$$
\begin{aligned}
\hat{y} &= X\beta \\
y &\sim Normal(\hat{y}, \Sigma) \\
\Sigma &= \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,j} \\ \sigma_{2,1} & \sigma_{2,2} & \cdots & \sigma_{2,j} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{i,1} & \sigma_{i,2} & \cdots & \sigma_{i,j} \end{pmatrix} \\
\sigma_{i,j} &= \alpha^2 \exp(-\frac{1}{2\rho^2} D_{i,j}^2)
\end{aligned}
$$

Approximate covariance with basis functions

$$
\begin{aligned}
\hat{y} &= X\beta + Zu \\
y &\sim Normal(\hat{y}, \sigma) \\
u &\sim Normal(0, \lambda S)
\end{aligned}
$$

GAM approach

Third challenge

- For my PhD work, I studied bees in canola fields. One of the species I studied was *Megachile rotundata*, the alfalfa leafcutting bee. I counted bees visiting flowers at distances away from their shelters (“hives”), and some of the data is found here (`seedFieldDat.csv`).
- Using a GAM, model how visitation changes with distance from shelter (`sDist`), edge of field (`eDist`), while controlling for observation time (`time`), day of year (`StartTime`), and different Fields. You can either do a model for each `Year` separately, or have `Year` as a term in the model

Two-column slide