Nonlinear models

I don't think we're in Kansas anymore

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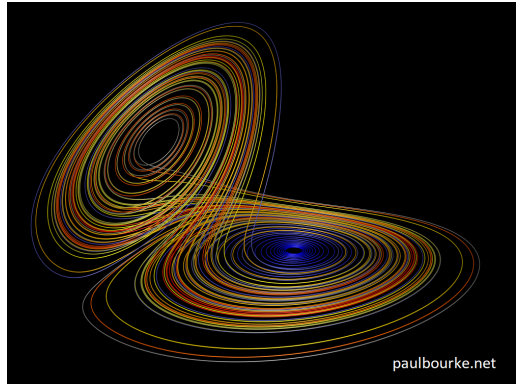
Outline

- What are nonlinear models?
- Mechanistic models
  - Some common models
  - Strategies for fitting
- Empirical models
  - Some common models
  - GAMs



The Lorenz System: a classical 3D nonlinear system

What are nonlinear models?

- **Linear models** take the form:

$$\hat{y} = X\beta = b_0 1 + b_1 x_1 ... + b_i x_i$$

$$y \sim Normal(\hat{y}, \sigma)$$

Part 1: Mechanistic models

Governing equations

Dynamics of some systems can be described by a set of equations, either in *discrete* or *continuous* time

- Exponential growth: *Discrete time*

$$n_t = n_{t-1} r$$

- Exponential growth: *Continuous time*

$$\frac{dn}{dt} = nr$$

- Predator prey cycles: *Discrete time*

$$\begin{aligned} \text{prey}_t &= \text{prey}_{t-1}(r_1 - a_1 \text{pred}_{t-1}) \\ \text{pred}_t &= \text{pred}_{t-1}(a_2 \text{prey}_{t-1} - d) \end{aligned}$$

- Predator prey cycles: *Continuous time*

$$\begin{aligned} \frac{d\text{prey}}{dt} &= r - a_1 \text{pred} \\ \frac{d\text{pred}}{dt} &= a_2 \text{prey} - d \end{aligned}$$

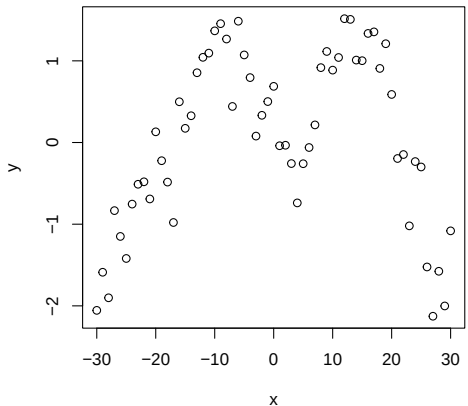
Some other common dynamic models

- Logistic growth
- Michaelis-Menten

Part 2: Empirical models

Empirical smoothing

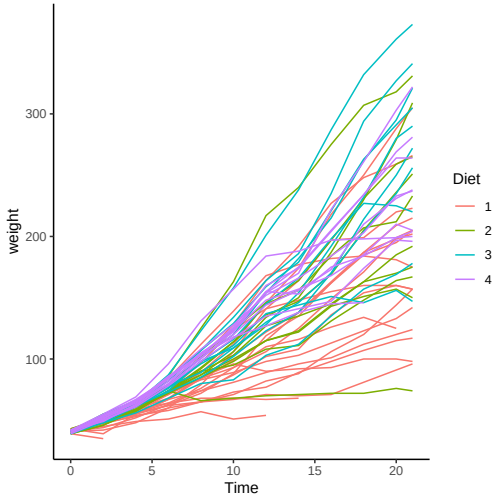
- Sometimes we don't know the specific rules that govern your system, but we want to know the *general shape*
  - e.g. population changes across time or space, temperature across seasons
- We want something that can give us *general predictions* across the range of your data without actually dealing with the underlying process
- Solution: "empirical" smoothing



"Guess the family"

Third challenge

- Try fitting a GAM to the first individual Chick in the `ChickWeight` dataset (included with R)
- Check whether the number of basis functions is appropriate. Does your answer change if you use more or less?
- If you're feeling adventurous, try fitting multiple individual chicks using a *random intercept*, and see if the `Diet` parameter changes chick growth!



To do: final nonlinear modeling challeng

- For those of you who have nonlinear or "GAMmy" data... time to update your models. If not