Index Number: ______________________ Programme: ______________________

UNIVERSITY OF ENERGY AND NATURAL RESOURCES, SUNYANI, GHANA

SCHOOL OF ENGINEERING

DEPARTMENT OF COMPUTER AND ELECTRICAL ENGINEERING

LEVEL 200 END OF SECOND SEMESTER EXAMINATION, 2016/2017

Bachelor of Science (Electrical and Electronics Engineering)

Bachelor of Science (Computer Engineering)

ELNG 208: Electromagnetic Field Theory

May, 2017 **Time: 2 Hrs**

Instructions: ***The paper consist of two SECTIONS (A and B). Answer all questions in SECTION A and THREE (3) in SECTION B.***

SECTION A – Answer all questions in SECTION A. Shade correctly the letter corresponding to the correct option from A to D on the shading sheet provided. Any correct answer carries 1 mark.

1. Let $\boldsymbol{F} = 2\boldsymbol{a}_x - 6\boldsymbol{a}_y + 10\boldsymbol{a}_z$ and $\boldsymbol{T} = \boldsymbol{a}_x + T_y\boldsymbol{a}_y + 5\boldsymbol{a}_z$. If F and T have the same unit vector, find T_y

   A. 6 B. 0
   C. −3 D. −6

2. Point charges $Q_1 = 1$ nC and $Q_2 = 2$ nC are at a distance apart. Which of the following statement(s) is/are incorrect?

   I. The force on Q1 is repulsive
   II. The force on Q2 is the same in magnitude as that on Q1
   III. As the distance between them decreases, the force on Q1 increases linearly
   IV. The force on Q2 is along the line joining them
   V. A point charge Q3 = −3 nC located at the midpoint between Q1 and Q2 experiences no net force

   A. I, II, IV and V only B. I, III and IV only
   C. III and V only D. I and IV only

3. Given $\mathbf{A} = 3\mathbf{a}_r + 2\mathbf{a}_\theta - 6\mathbf{a}_\phi$ and $\mathbf{B} = 4\mathbf{a}_r + 3\mathbf{a}_\phi$, determine $\mathbf{A} \bullet \mathbf{B}$.

   A. 34 B. −6
   C. 6 D. 14

4. Inside a hollow conducting sphere ______________.
   A. electric field is zeros
   B. electric field is non zero constant
   C. electric field changes with magnitude of the charge
   D. electric field changes with distance from the centre

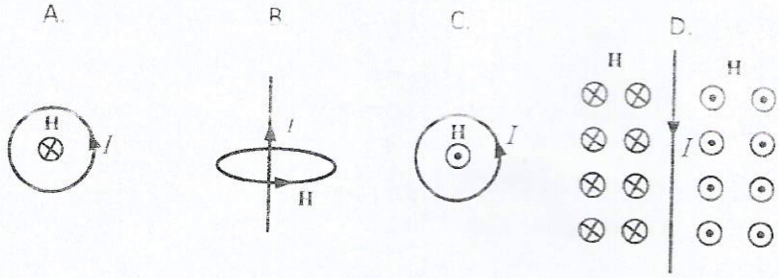
5. Given $\mathbf{B} = -9.3\mathbf{a}_x - 6\mathbf{a}_y + 2\mathbf{a}_z$, Find the projection of A along $\mathbf{a}_y$.

   A. −12 B. −6
   C. 18 D. 2

6. The energy stored in the magnetic field at the solenoid 30 cm long and 3 cm diameter wound with 100 turns of wire carrying a current at 10 A is ______________.

   A. 0.015 J B. 0.5 J
   C. 0.15 J D. 1.15 J

7. Pick the odd one out in the figure below with the representation of I and **H**. 15.



8. The z-axis carries filamentary current of 10π A along $\hat{\mathbf{a}}_z$. Which of these is incorrect?

A. $\mathbf{H} = -\hat{a}_x\ A/m$ at (0, 5, 0) B. $\mathbf{H} = \hat{a}_\phi$ A/m $(5, \pi/4, 0)$ 16.

C. $\mathbf{H} = 0.8\hat{a}_x - 0.6\hat{a}_y$ at $(-3, 4, 0)$ D. $\mathbf{H} = -\hat{a}_\phi$ at $(5, 3\pi/2, 0)$

9. Which of the following statements is not characteristic of static magnetic field? 17.
A. It is solenoidal.
B. Magnetic flux lines are always closed.
C. The total number of flux lines entering a given region is equal to the total number of flux lines leaving the region.
D. It is conservative.

10. If $\mathbf{R} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$, the position vector of point (x, y, z) and $R = |\mathbf{R}|$, which of the following is incorrect? 18.

A. $\nabla \bullet \mathbf{R} = 1$ B. $\nabla R = \mathbf{R}/R$

C. $\nabla \times \mathbf{R} = 0$ D. $\nabla^2 (\mathbf{R} \bullet \mathbf{R}) = 6$

11. The dot product of a vector with itself gives ______________.
A. the square of the vector
B. the square of the magnitude
C. the angle between the vector and ground
D. reverse the unit vector of the multiplicand 19.

12. The cross product of two vectors A and B yields ________________.
A. a scalar quantity given by the product of the magnitude of the two vectors and a cos of the angle between them.
B. a scalar quantity given by the product of the two vectors and a cos of the angle between them. 20
C. a vector that is perpendicular to both A and B, and therefore normal to the plane containing them.
D. none of the above

13. The energy stored in electrostatic filed or electromagnetic field is called ______________.

A. electromagnetic energy B. kinetic energy

C. potential energy D. magnetron energy

14. The flux of a field passing through an arbitrary area is maximum when ______________. 2
A. the field is orthogonal to the area vector
B. the field is normal to the area vector
C. the field is in the direction of the area vector
D. none of the above

15. Which of these is true about Lorentz force equation?
    I. A region is said to be characterized by an electric field if a particle of charge q moving with a velocity v experiences a force F_e, independent of v
    II. A region is said to be characterized by an electric field if a particle of charge q moving with a velocity v experiences a force F_e dependent on v
    III. A region is said to be characterized by a magnetic field, if a charged particle experiences a force F_m which depends on v

    A. Both I and III B. Both II and III
    C. Both I and II D. I, II and III

16. If the distance between two point charges is increased by a factor of 3, the new force on either charge will be ______________.
    A. remain the same B. decreased by a factor 3
    C. decreased by a factor of 9 D. increased by a factor 3

17. Two particles are held in equilibrium by the gravitational and electrostatic forces between them. Particle A has mass m_A, and charge q_A. Particle B has mass m_B and charge q_A. The distance between the charges is d. Which of the following changes will cause the charges to accelerate towards one another?
    A. m_A is doubled and m_B is halved B. m_A is doubled and m_B is doubled
    C. q_A is doubled and q_B is halved D. q_A is halved and m_A is halved

18. Charges A, B, C and D are charged particles forming a square with 1.0 m sides as shown in the Figure 1. A and D have charges of +2 C; B and C have charge −4 C. If q is a particle with a charge of +1 C sitting directly in the middle of the square, what is the net force on q due to the other particles?
    A. 1.4 N
    B. 5.6 N
    C. 4 N
    D. 0 N

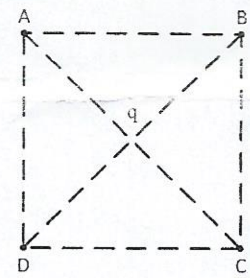


Figure 1: Question 17

19. A particle with charge of +2 C and mass of 1 g is exposed to an electric field with strength 5 N/C. How far will the particle move in 10 secs?
    A. 5×10^2 m B. 1×10^3 m
    A. 2×10^5 m D. 5×10^5 m

20. Why is it that the magnitude of magnetomotive force required for air gap is much greater than that required for iron part of a magnetic circuit?
    A. Because air is a gas.
    B. Because air has the highest relative permeability.
    C. Because air is a conductor of magnetic flux.
    D. Because air has the lowest relative permeability.

21. The permeability of a material having a flux density of 5 Wb/m^2 is 10^{-5} H/m. What is the value of magnetizing force?
    A. 5×10^{-5} N/Wb B. 500×10^3 N/Wb
    C. $4\pi \times 10^{-7}$ N/Wb D. $4\pi \times 10^7$ N/Wb

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Section B – *Answer ALL questions in PART I and ONE in PART II*

Part I – Answer ALL questions

Question 22 [15 Marks]

a) Determine the total charge, Q

i. On line $0 < x < 5$ m if $\rho_L = 12x^3$, C/m.

ii. On the cylinder $\rho = 3,\ 0 < z < 4$ m if $\rho_S = \rho z^2$ nC/m^2.

iii. Within the sphere $r = 6$ m if $\rho_V = \dfrac{10}{r\sin\theta}$ C/m^3.

b) Two concentric spherical electrodes are biased at $V_{ac} = 100$ V. The outer sphere is grounded. The radii of the two spheres are $a = 1$ m and $c = 4$ m, respectively. The volume between the two spheres consists of two concentric layers. The first layer is defined by $a \le r \le b$ where $b = 2$ m. The relative permittivity first layer is $\varepsilon_{r1} = 10$. That of the second layer is defined by $b \le r \le c$ and $\varepsilon_{r2} = 1$.

i. Find the electric field as a function of the distance r from the centre in both regions: $\vec{E}_1(r)$ and $\vec{E}_2(r)$. (Hint: $\varepsilon = \varepsilon_o \varepsilon_r$)

ii. Consider the range $a \le r \le c$, calculate the maximum value, $E_{\max}$, if the distance, $r = r_{\max}$ is the distance where $\vec{E}$ field is maximum.

iii. Find the free-charge surface density at the two electrodes, ρ_{sa} and ρ_{sc}. Take V = 100 V as stated above.

iv. Determine the capacitance C of the structure and the total stored electrostatic energy W_e.

Question 23 [15 Marks]

a) A 650 mm length of coaxial cable with an inner radius of 1.5 cm and an outer radius of 5 cm. The space between the conductors is assumed to be filled with air. When connected to a charge source, the total charge on the inner conductor is 35 nC. Determine:

i. the charge density on each conductor.

ii. the ***E*** and ***D*** fields.

b) i. Given the magnetic field $\vec{H} = \rho^2 \hat{a}_\phi,\ A/m$, determine the total current I passing through the circular surface $0 \le \rho \le 1, 0 \le \phi \le 2\pi, z = 0$ in the $\hat{a}_z$ direction.

ii. Explain how electric energy is produced from a time varying magnetic field.

iii. State Gauss' law of electrostatics.

iv. State three conditions defining a Gaussian surface.

c) Uniform line charge of density $\rho_l = 5$ nC/m lies on a circular ring of radius $R = 3$ m in the $z = 0$ plane. The circle is centred at the origin.

i. Find the expression for the potential V at the point $P(0, 0, z)$.

ii. What is the value of V if $z = 4$ m and the medium is vacuum?

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Part II – Answer only ONE question in this part.

Question 24 [10 Marks]

a) Coordinate conversion:

i. Give the rectangular coordinate of the point $C(\rho = 4.4, \phi = -115^\circ, z = 2)$.

ii. Give the cylindrical coordinates of point $D(x = -3.1, y = 2.6, z = -3)$

iii. Specify the distance from C to D

b) Given the magnetic vector potential $A = -\frac{\rho^2}{4} a_z$ Wb/m, calculate the total magnetic flux crossing the surface $\phi = \frac{\pi}{2}, 1 \le \rho \le 2$ m, $0 \le z \le 5$ m.

Question 25 [10 Marks]

a) Find the flux crossing the portion of the plane $\phi = \pi/4$ defined by $0.01 < r < 0.05$ m and $0 < z < 2$ m (see Figure 2). A current filament of 2 .50 A along the z-axis is in the $\hat{a}_z$ direction.

b) If $\vec{J} = \frac{1}{R^3}(2\cos\theta \hat{a}_R + \sin\theta \hat{a}_\theta)$ A/m², calculate the current passing through:

i. A hemispherical shell of radius 25 cm.

ii. A sphere shell of radius 10 cm.

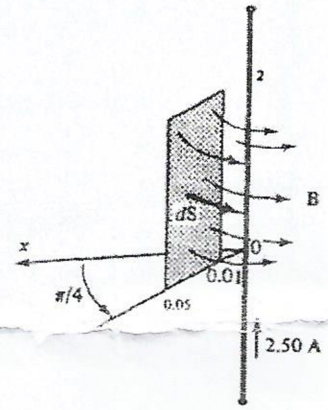

Figure 2: Question 25a

$-1.85a_x - 3.987a_y + 2a_z$

$P = 4.04$

$\phi = \tan^{-1}\left(\frac{2.6}{-3.1}\right)$

$= -39.98$

$z = -3$

$CD = OD - OC$

$$\begin{pmatrix} 4.04 \\ -39.98 \\ -3 \end{pmatrix} - \begin{pmatrix} -1.85 \\ -3.987 \\ 2 \end{pmatrix} = \begin{pmatrix} 5.89 \\ -36 \\ -1 \end{pmatrix} \Rightarrow 5.89a_x - 36a_y - 9z$$

2

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he $z = 0$

$\nabla \cdot \nabla \Phi = \nabla^2 \Phi$
$\nabla \cdot \nabla \times \mathbf{A} = 0$
$\nabla \times \nabla \Phi = 0$
$\nabla \times \nabla \times \mathbf{A} = \nabla \nabla \cdot \mathbf{A} - \nabla^2 \mathbf{A}$

DIFFERENTIAL ELEMENTS

Cartesian coordinates

$d\mathbf{l} = dx\mathbf{a}_x + dy\mathbf{a}_y + dz\mathbf{a}_z$

$d\mathbf{s} = dydz\mathbf{a}_x + dxdz\mathbf{a}_y + dxdy\mathbf{a}_z$; $dv = dxdydz$

Cylindrical coordinates

$d\mathbf{l} = d\rho\mathbf{a}_\rho + \rho d\phi\mathbf{a}_\phi + dz\mathbf{a}_z$

$d\mathbf{s} = \rho d\phi dz\mathbf{a}_\rho + d\rho dz\mathbf{a}_\phi + \rho d\rho d\phi\mathbf{a}_z$; $dv = \rho d\rho d\phi dz$

Spherical coordinates

$d\mathbf{l} = dr\mathbf{a}_r + rd\theta\mathbf{a}_\theta + r\sin\theta d\phi\mathbf{a}_\phi$

$d\mathbf{s} = r^2\sin\theta d\theta d\phi\mathbf{a}_r + r\sin\theta drd\phi\mathbf{a}_\theta + rdrd\theta\mathbf{a}_\phi$; $dv = r^2\sin\theta drd\theta d\phi$

COORDINATE TRANSFORMATIONS

Rectangular ↔ Cylindrical

$$\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \\ z &= z \end{aligned} \qquad \begin{aligned} \rho &= (x^2+y^2)^{1/2} \\ \phi &= \arctan(y/x) \\ z &= z \end{aligned}$$

Rectangular ↔ Spherical

$$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned} \qquad \begin{aligned} r &= (x^2+y^2+z^2)^{1/2} \\ \theta &= \arccos[z/(x^2+y^2+z^2)^{1/2}] \\ \phi &= \arctan(y/x) \end{aligned}$$

Cylindrical ↔ Spherical

$$\begin{aligned} \rho &= r\sin\theta \\ \phi &= \phi \\ z &= r\cos\theta \end{aligned} \qquad \begin{aligned} r &= (\rho^2+z^2)^{1/2} \\ \phi &= \phi \\ \theta &= \arccos[z/(\rho^2+z^2)^{1/2}] \end{aligned}$$

VECTOR TRANSFORMATIONS

Rectangular Components ↔ Cylindrical Components

$$\begin{aligned} A_x &= A_\rho\cos\phi - A_\phi\sin\phi \\ A_y &= A_\rho\sin\phi + A_\phi\cos\phi \\ A_z &= A_z \end{aligned} \qquad \begin{aligned} A_\rho &= A_x\cos\phi + A_y\sin\phi \\ A_\phi &= -A_x\sin\phi + A_y\cos\phi \\ A_z &= A_z \end{aligned}$$

Rectangular Components ↔ Spherical Components

$$\begin{aligned} A_x &= A_r\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi \\ A_y &= A_r\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi \\ A_z &= A_r\cos\theta - A_\theta\sin\theta \end{aligned}$$

$$\begin{aligned} A_r &= A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta \\ A_\theta &= A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta \\ A_\phi &= -A_x\sin\phi + A_y\cos\phi \end{aligned}$$

Note: θ and ϕ are the position angles of the observation point.

Cylindrical Components ↔ Spherical Components

$$\begin{aligned} A_\rho &= A_r\sin\theta + A_\theta\cos\theta \\ A_\phi &= A_\phi \\ A_z &= A_r\cos\theta - A_\theta\sin\theta \end{aligned} \qquad \begin{aligned} A_r &= A_\rho\sin\theta + A_z\cos\theta \\ A_\theta &= A_\rho\cos\theta - A_z\sin\theta \\ A_\phi &= A_\phi \end{aligned}$$

Note: θ is the position angle of the point at which the vector exists.

SOME CONSTANTS

$\varepsilon_0 \approx 8.854187\times10^{-12}$ F/m

$\mu_0 = 4\pi\times10^{-7}$ H/m; $g \approx 9.8$ m/s^2 (Earth's acceleration)

$q_e \approx -1.6022\times10^{-19}$ C; $m_e \approx 9.1094\times10^{-31}$ kg (electron charge/mass)

ELECTROMAGNETIC EQUATIONS

Maxwell's equations (differential form)

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t};\ \nabla\times\mathbf{H} = \frac{\partial\mathbf{D}}{\partial t} + \sigma\mathbf{E} + \mathbf{J}^i;\ \nabla\cdot\mathbf{D} = \rho;\ \nabla\cdot\mathbf{B} = 0$$

Coaxial line

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)},\ \text{F/m};\ L' = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) + \frac{\mu_0}{8\pi},\ \text{H/m}$$

END OF FORMULA TABLE

CONTINUED

FORMULA SHEET ELNG208

TRIGONOMETRIC IDENTITIES

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$\sin A + \sin B = 2\sin[(A+B)/2]\cos[(A-B)/2]$

$\sin A - \sin B = 2\cos[(A+B)/2]\sin[(A-B)/2]$

$\cos A + \cos B = 2\cos[(A+B)/2]\cos[(A-B)/2]$

$\cos A - \cos B = -2\sin[(A+B)/2]\sin[(A-B)/2]$

$\sin^2 A = (1-\cos 2A)/2$

$\cos^2 A = (1+\cos 2A)/2$

$\sin 2A = 2\sin A \cos A = 2\tan A/(1+\tan^2 A)$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\int x\sin(ax)dx = [\sin(ax) - ax\cos(ax)]/a^2$

$\int x\cos(ax)dx = [\cos(ax) + ax\sin(ax)]/a^2$

$\int \sinh(ax)dx = a^{-1}\cosh(ax)$, $\quad \int \cosh(ax)dx = a^{-1}\sinh(ax)$

$\int \text{arcsinh}(x/a)dx = x\cdot\text{arcsinh}(x/a) - \sqrt{x^2+a^2}$

$$\int \text{arccosh}(x/a)dx = \begin{cases} x\cdot\text{arccosh}(x/a) - \sqrt{x^2-a^2}, & \text{if arccosh}(x/a) > 0 \\ x\cdot\text{arccosh}(x/a) + \sqrt{x^2-a^2}, & \text{if arccosh}(x/a) < 0 \end{cases}$$

$$\int \frac{dx}{(ax^2+b)\sqrt{fx^2+g}} = \frac{1}{\sqrt{b}\sqrt{ag-bf}}\arctan\left(\frac{x\sqrt{ag-bf}}{\sqrt{b}\sqrt{fx^2+g}}\right), (ag > bf)$$

VECTOR IDENTITIES

$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})$

$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

$\nabla(\Phi+\Psi) = \nabla\Phi + \nabla\Psi$

$\nabla\cdot(\mathbf{A}+\mathbf{B}) = \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B}$

$\nabla\times(\mathbf{A}+\mathbf{B}) = \nabla\times\mathbf{A} + \nabla\times\mathbf{B}$

$\nabla(\Phi\Psi) = \Phi\nabla\Psi + \Psi\nabla\Phi$

$$\nabla\left(\frac{\Phi}{\Psi}\right) = \frac{\Psi\nabla\Phi - \Phi\nabla\Psi}{\Psi^2}$$

$\nabla\Phi^n = n\Phi^{n-1}\nabla\Phi$

$\nabla\cdot(\Phi\mathbf{A}) = \mathbf{A}\cdot\nabla\Phi + \Phi\nabla\cdot\mathbf{A}$

$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\nabla\times\mathbf{A} - \mathbf{A}\cdot\nabla\times\mathbf{B}$

$\nabla\times(\Phi\mathbf{A}) = \nabla\Phi\times\mathbf{A} + \Phi\nabla\times\mathbf{A}$

$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\nabla\cdot\mathbf{B} - \mathbf{B}\nabla\cdot\mathbf{A} + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$ †

$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

†Note: $\mathbf{A}\cdot\nabla = A_x\frac{\partial}{\partial x} + A_y\frac{\partial}{\partial y} + A_z\frac{\partial}{\partial z}$

BASIC INTEGRALS OF ELEMENTARY FUNCTIONS

$\int x^n dx = \frac{x^{n+1}}{n+1}$, $n \neq -1$

$\int e^x dx = e^x$, $\int a^x dx = \frac{a^x}{\ln a}$

$\int \sin x dx = -\cos x$,

$\int \cos x dx = \sin x$

$\int \frac{dx}{\sin^2 x} = -\cot x$

$\int \tan x dx = -\ln\cos x$

$\int \frac{dx}{x} = \ln x$

$\int \frac{dx}{\cos^2 x} = -\tan x$

$\int \cot x dx = \ln\sin x$

$\int \frac{dx}{\cos x} = \ln\left[\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right]$

$\int \frac{dx}{\sin x} = \ln\left(\tan\frac{x}{2}\right)$

$$\int_0^{2\pi} \cos mx\cdot\cos nx\ dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \neq 0 \end{cases}$$

$$\int_0^{2\pi} \sin mx\cdot\cos nx\ dx = 0$$

$$\int_0^{\pi} \sin mx\cdot\sin nx\ dx = \begin{cases} 0, & m \neq n \\ \pi/2, & m = n \neq 0 \end{cases}$$

$$\int_0^{\pi} \cos mx\cdot\cos nx\ dx = \begin{cases} 0, & m \neq n \\ \pi/2, & m = n \neq 0 \end{cases}$$

$$\int_0^{\pi} \sin mx\cdot\cos nx\ dx = \begin{cases} 0, & m+n = \text{even number} \\ \frac{2m}{m^2-n^2}, & m+n = \text{odd number} \end{cases}$$

$$\int_0^{\pi} \frac{(a-b\cos x)}{(a^2+b^2-2ab\cos x)}dx = \begin{cases} \pi/a, & a > b > 0 \\ 0, & b > a > 0 \end{cases}$$

$$\int_0^{2\pi} \sin mx\cdot\sin nx\ dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \neq 0 \end{cases}$$