

Algebra Basics for High School Students

December 11, 2024

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1 Introduction to Algebra

Algebra is a branch of mathematics that uses symbols and letters to represent numbers and quantities in formulas and equations. It provides a systematic way of solving problems and understanding patterns.

2 Key Concepts in Algebra

2.1 Variables and Constants

- **Variable:** A symbol, often a letter, that represents a number whose value is unknown or can change. Example: x, y, z . - **Constant:** A fixed value. Example: $5, -3, \frac{1}{2}$.

2.2 Expressions and Equations

- **Expression:** A combination of variables, constants, and operations. Example: $3x + 2$. - **Equation:** A mathematical statement that shows two expressions are equal. Example: $3x + 2 = 11$.

2.3 Basic Operations

- Addition: $a + b$ - Subtraction: $a - b$ - Multiplication: $a \cdot b$ or ab - Division: $\frac{a}{b}$

3 Solving Linear Equations

A linear equation is an equation that forms a straight line when graphed. It has the general form:

$$ax + b = c \tag{1}$$

3.1 Steps to Solve

1. Simplify both sides of the equation if needed. 2. Isolate the variable on one side of the equation. 3. Solve for the variable.

3.2 Example 1

Solve $2x + 3 = 11$:

Step 1: Subtract 3 from both sides to isolate the term with x :

$$2x + 3 - 3 = 11 - 3$$

$$2x = 8$$

Step 2: Divide both sides by 2 to solve for x :

$$x = \frac{8}{2}$$

$$x = 4$$

3.3 Example 2

Solve $5x - 7 = 18$:

Step 1: Add 7 to both sides to isolate the term with x :

$$5x - 7 + 7 = 18 + 7$$

$$5x = 25$$

Step 2: Divide both sides by 5 to solve for x :

$$x = \frac{25}{5}$$

$$x = 5$$

3.4 Example 3

Solve $\frac{3x}{4} = 6$:

Step 1: Multiply both sides by 4 to eliminate the fraction:

$$4 \cdot \frac{3x}{4} = 6 \cdot 4$$

$$3x = 24$$

Step 2: Divide both sides by 3 to solve for x :

$$x = \frac{24}{3}$$

$$x = 8$$

4 Working with Inequalities

Inequalities express a relationship where one value is not equal to another. Symbols used include:

- $>$ (greater than)
- $<$ (less than)
- \geq (greater than or equal to)
- \leq (less than or equal to)

4.1 Example 1

Solve $3x - 7 > 5$:

Step 1: Add 7 to both sides to isolate the term with x :

$$3x - 7 + 7 > 5 + 7$$

$$3x > 12$$

Step 2: Divide both sides by 3 to solve for x :

$$x > \frac{12}{3}$$

$$x > 4$$

4.2 Example 2

Solve $4x + 2 \leq 14$:

Step 1: Subtract 2 from both sides to isolate the term with x :

$$4x + 2 - 2 \leq 14 - 2$$

$$4x \leq 12$$

Step 2: Divide both sides by 4 to solve for x :

$$x \leq \frac{12}{4}$$

$$x \leq 3$$

5 Graphing Linear Equations

The graph of a linear equation is a straight line. To graph: 1. Rewrite the equation in slope-intercept form: $y = mx + b$. 2. Identify the slope (m) and the y -intercept (b). 3. Plot the y -intercept on the graph. 4. Use the slope to find another point on the line. 5. Draw the line through the points.

5.1 Example 1

Graph $y = 2x + 1$:

Step 1: Identify the y -intercept (b): $b = 1$

Step 2: Plot the point $(0, 1)$ on the graph.

Step 3: Use the slope ($m = 2$) to find another point:

From $(0, 1)$, move up 2 units and right 1 unit to $(1, 3)$.

Step 4: Draw a straight line through the points $(0, 1)$ and $(1, 3)$.

5.2 Example 2

Graph $y = -\frac{1}{2}x + 3$:

Step 1: Identify the y -intercept (b): $b = 3$

Step 2: Plot the point $(0, 3)$ on the graph.

Step 3: Use the slope ($m = -\frac{1}{2}$) to find another point:

From $(0, 3)$, move down 1 unit and right 2 units to $(2, 2)$.

Step 4: Draw a straight line through the points $(0, 3)$ and $(2, 2)$.

6 Factoring Algebraic Expressions

Factoring is the process of breaking down an expression into a product of simpler terms.

6.1 Example 1: Common Factor

Factor $6x + 9$:

Step 1: Identify the greatest common factor (GCF) of 6 and 9:

$$GCF = 3$$

Step 2: Factor out the GCF:

$$6x + 9 = 3(2x + 3)$$

6.2 Example 2: Quadratic Expression

Factor $x^2 + 5x + 6$:

Step 1: Find two numbers that multiply to 6 and add to 5:

The numbers are 2 and 3.

Step 2: Write the factored form:

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$