Visualizing uncertainty for improved decision making

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Abstract. Decision making often depends on the analysis and evaluation of large amounts of data for which information visualization proved to be a valuable approach. The recognition of uncertainty in the data is crucial and it should therefore be appropriately represented. However, suitable methods for its visualization are in many cases not available.

This paper provides a general view on uncertainty with respect to a widely accepted information model. Out of this view it explains why the recognition of uncertainty plays an important role for decisions, and it depicts available but especially missing visualization concepts today, e.g. for the display of uncertain relationships. To fill that gap new ideas are derived for an improved visual decision support.

Introduction

Rational decisions are usually based on the analysis and evaluation of problem specific data. Today there is an increasing amount of such data available due to its extensive generation e.g. by surveys or simulation studies and an advanced information infrastructure. At the same time many decisions e.g. in business or politics are embedded in a growing system of dependencies and lead to far-reaching consequences so that not only the possibility but also the need for substantiated data analysis rises. This means many decisions have to be made within increasingly complex and wide information spaces.

To discover relevant facts and patterns therein which support or oppose an argument a wealth of methods has been developed, e.g. in the fields knowledge discovery in data bases (KDD) or data mining. A different but also well proven approach is visualization. Here raw data is mapped to a graphical representation that effectively exploits the very capable human visual data processing capacity. This way visualization is able to support the confirmation of assumed facts and also to discover yet unknown phenomena within the raw data.

Despite the vast amount of different visualization techniques available an important point has often been neglected: uncertainty. The data used to create a representation and later on to base a decision on is often not 100% sure. Missing values, information originating from a source of low credibility, deviations of measured values, lack of precision or strong variability in an information base might – if not carefully considered – lead to sub optimal or even disastrous decisions. Therefore the user of a visualization system has to be made aware of relevant uncertainty in the data.

How valuable the integration of uncertainty into a display can be the following example demonstrates: Here the graphical reconstruction of a medieval building includes uncertainty about the architecture. The foundations are known for sure but the further from the ground the less certain is the equivalence of the assumed and true shape. To represent this fact uncertainty is encoded into transparency (see figure 1):

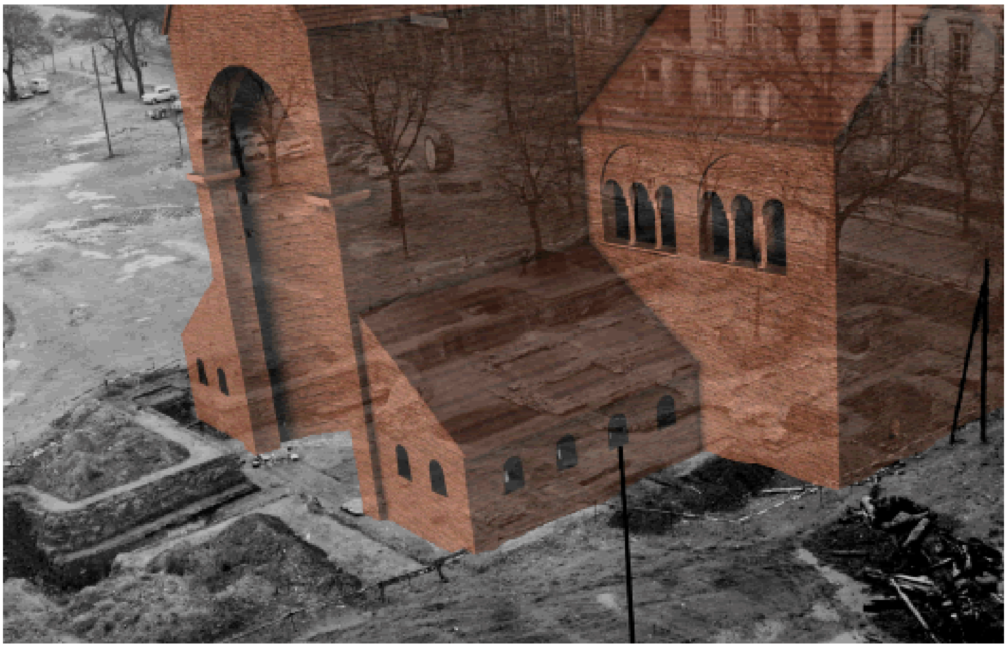

Figure 1. Reconstruction of the medieval Kaiserpfalz: Uncertainty about the true architecture is encoded into transparency. (from [11])

Unfortunately such visual integration of uncertainty is hardly considered for complex information structures also including abstract data and hierarchical or other relationships. But many decision situations depend on such information and would definitely profit from new techniques that depict the trustworthiness of the underlying data.

Consider a manager who wants to improve the predictions of the development of a special stock portfolio. For this he might consider different possible influences like oil price, exchange rates, commodity indices, political stability, merger rumors or consumer acceptance. If he analyses the correlation of a certain share price and consumer acceptance it plays e.g. an important role where the data comes from. Previous share prices are typically precisely recorded but an indication of consumer acceptance might be given by an actual number of purchases or by some results from a user survey. This results in a different reliability of the data that the manager should be made aware of not to draw a wrong conclusion.

The aim of this paper after first introducing basic information visualization concepts and second after suitably narrowing down the term uncertainty for the tackled field information visualization, drawing the connection between uncertainty and information and providing necessary measures usable for the visual mapping is therefore third to present some of the few known approaches to finally come to the derivation of new ideas for the visualization of uncertainty within complex information spaces as the main goal. An awareness shall be created for the demand for new results within this young research topic, for its faced problems and expected prospects for visually supported decision making.

1. Basic Information Visualization Concepts

A research area that deals with the effective representation of the complex information structures required by many decision situations is the field of information visualization. It provides powerful methods to display and visually explore large amounts of attribute values and/or their relationships.

The different methods rely on a basic information model which shall be introduced now.

1.1 The basic information model

Information visualization is based on an information model that can shortly be characterized as follows (according to e.g. [17] and [18]):

The paper follows Bertin [4] to distinguish the *information content* and the *structure* in which the information is organized. Furthermore, it follows Wendt [30] who introduces the concept of *information objects* as a necessary abstraction of *data*, which represent information in an information processing system.

Thus datasets are encapsulated in information objects IO. A discrete countable set of information objects is denoted as IM = {IO_1, ..., IO_n}. Every IO has an associated set of attributes, where A is defined as a single attribute and AM as the corresponding attribute set.

As an example see table 1 that provides hypothetical data to our portfolio manager:

Table 1. Exemplary information objects and attributes.

IO_i \ A_i	Time	Oil price	Exchange rate	Political stability
IO_1	01.07.00	27,3	1,45	high
IO_2	01.08.00	28,0	1,32	high
IO_3	01.09.00	27,9	1,30	high

The table shows the values of four attributes (columns) for three information objects (rows).

The information structure IS is defined as the set of all structure elements SE existing in the data which represent by a pair (IO_i, IO_j) a relationship between two information objects IO_i and IO_j. In the example above there might be a relation between consecutive information objects so that their difference in time is always one month. Then we would get |IM|-1 structure elements (IO_i, IO_{i+1}).

In a different context the information objects might be people where a relation indicates if a person is brother, sister of child of another person.

To distinguish different kinds of relationships like “brother of” or “greater than” the term structure set STM can be coined as a subset of IS.

A SE may have structure attributes SA attached to it, e.g. “reason for the relation” or “duration of the relation”. All existing SA define the structure attribute set SAM. The information space IR of an application can now be described as IR = {IM, AM, IS, SAM}.

Different visualization techniques usually focus on different elements of this basic information model and e.g. display attribute value distributions. The model supports most tasks in information visualization. For special cases, e.g. the inclusion of finer granular relations than between information objects, it can be extended easily.

1.2 General information visualization methods

Although information visualization is still a fairly young research area it already brought up a wealth of different concepts. The main focus was laid on (see e.g. [26], [18]):

- The development of space-saving display techniques:

  It is a challenge to display large amounts of data on the limited screen space. An exemplary approach to this problem are focus&context techniques which display regions of interest in more detail while de-emphasizing other regions. (see figure 2)
- The visualization of information objects and attributes:

  A goal could be to analyze individual objects or attribute values to discover unknown relations or trends. Known techniques are e.g. Parallel Coordinates (see e.g. [10]) or the use of special arrangement algorithms. Parallel Coordinates map attributes of a dataset to axes arranged in parallel which are scaled according to attribute domains. The dataset is then represented as a polyline connecting the corresponding points on the axes (see figure 2). A category of techniques that arrange information objects relies on the metaphor of via springs connected objects that attract or repel each other. The similarity of objects is represented by equal force of attraction and repulsion of the springs respectively and enables the calculation of the object layout (see figure 2).

- The visualization of the information structure:

  The aim here is to display large tree or graph structures in the data. An accepted technique is the Tree-map [13] that realizes an axes parallel nesting by mapping the root of a tree to a box, children to smaller boxes within that contain their children as boxes, and so on … (see figure 2)

- The development of intuitive interaction techniques:

  Due to different possible objectives and the limited screen space different interaction techniques are usually offered to navigate through, select and manipulate data and its representation.

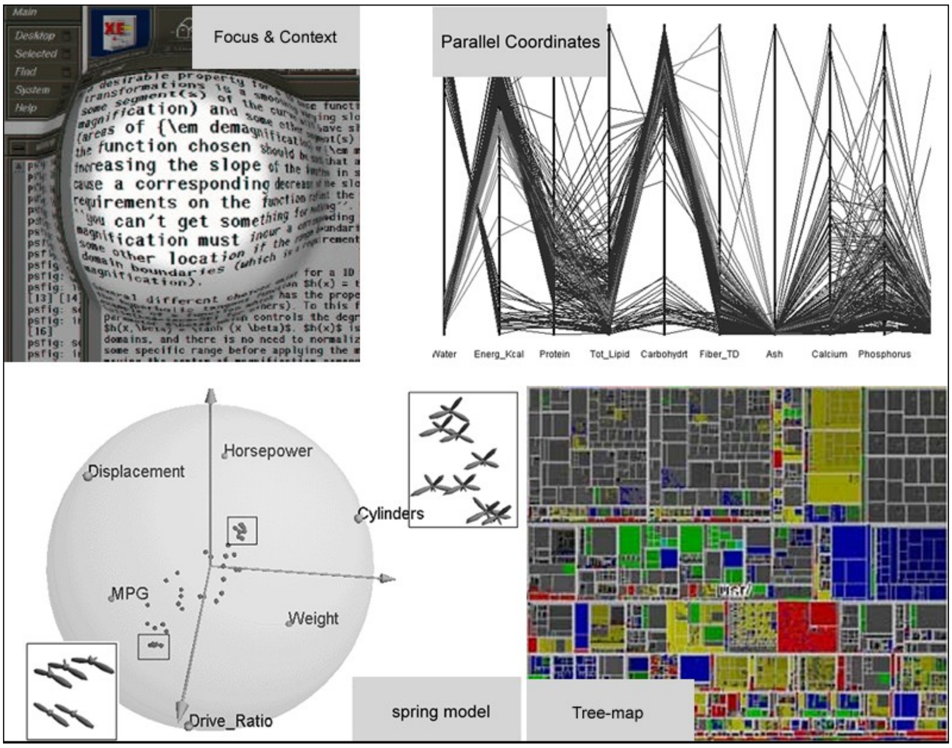

Figure 2. Information visualization examples.
Top left: a focus & context-display. (from [14]) Top right: Parallel Coordinates for the representation of the substances in different cereals where each product is represented by a polyline that intersects the vertical substance axes. (from [26]) Bottom left: visualization of car data with a spring model. The position of an object depends on the values of the positioned attributes. (from [26]) Bottom right: Tree-map giving an overview of a file system of several thousand files. Each rectangle size is determined by the file's size; color represents file type. (according to [7])

These concepts proved to be quite effective within their problem domains – but they usually don't consider uncertainty. To discuss how it can be integrated into a visualization the term uncertainty has to be clarified.

2. The Term Uncertainty

2.1 Sources

Reasons for uncertainty – some yet undefined distrust in the data – are manifold. There could be errors in measuring devices or simulation runs, statistical variations, unavoidable or for performance reasons intended losses in the transformation or even in the presentation of the data. In principle uncertainty can be introduced in any step from the investigation / acquisition of the data over filtering and mapping operations to the final picture [19].

2.2 Definition

In the literature there is no consensus about the perception of uncertainty (often also denoted as "data quality problems") or about a universal way to represent it [22]. One of the few closed definitions of uncertainty can be found in [9] where it is stated as: „degree to which the lack of knowledge about the amount of error is responsible for hesitancy in accepting results and observations without caution".

In general uncertainty is understood as a composition of different concepts (see e.g. [2, 15, 22, 29]) such as:

- error – outlier or deviation from a true value,
- imprecision – resolution of a value compared to the needed resolution (e.g. values are highly accurately given for countries but are needed for states),
- subjectivity – degree of subjective influence in the data,
- non-specificity – lack of distinctions for objects (e.g. an attribute value is known to be one of several alternatives but not which one).

This short list is just a hint on the different facets of uncertainty. There is a wide variety of further concepts available that could be considered for a representation.

Having defined relevant components of uncertainty for an application it is possible to include them into the information base.

2.3 Linking Uncertainty to the Information Model

In the literature the linking of uncertainty to information is usually constrained to information objects and their attribute values, e.g.: how accurate is a position, which measured values are missing, how probable does a value seem, …?

One of the few exceptions are [8] and [27]. In [8] uncertainty is also attached to dimensions (attributes) where it gives an impression on the average uncertainty of the values of all information objects belonging to one kind of attribute. This way the user can e.g. discover that the variability of the oil price over time is generally higher than of the exchange rates. [27] also considers "random and fuzzy phenomena" in spatial and temporal relationships. But an examination of the occurrence of uncertainty in an information space in general has so far not been provided.

In principle different kinds of uncertainty can be assigned as metadata to all of the mentioned elements of the information model. So to stay application independent this paper suggests to examine uncertainty with respect to information objects, attributes, attribute values, structure elements, structure attributes, … . The following examples serve to demonstrate this to be reasonable for individual model elements:

- Information object: dataset represents a persons opinion (subjectivity),
- Structure element: wrongly classified as sister (error),

- Structure attribute: time of discovery of a relation is known by month but is needed by day (imprecision),
- Attribute set: aggregated uncertainty to provide an overview (e.g. overall reliability).

Despite many more assignable examples of practical meaning attempts of such a complete consideration of the linkage of uncertainty to information were to our knowledge not known before and therefore deserve further investigation. But this is beyond the scope of this paper and shall be left open for now. Nevertheless the point to make here is the obvious complexity that accompanies an extensive integration of different uncertainty concepts and an assignment to individual element of the information model. It leads to a multiplied storage effort but also to a dramatically more complex transformation and graphical presentation of the data.

To capture the concept of uncertainty for practical usage in a visualization system suitable measures have to be provided that will be discussed next.

2.4 Measures of Uncertainty

Most often uncertainty is quantitatively described by scalar values like probability, error percentage, distance (e.g. from the true value) or standard deviation. For example [29] shows how to intuitively express concepts like error, precision, completeness, consistency or subjectivity with standard deviation measures. [15] points out that other formal concepts than probability like fuzzy, plausibility, belief, possibility or necessity measures are more suitable for certain applications and should therefore be regarded. Other possibilities include assignments to defined uncertainty classes, ranges of values or sets of possible values (e.g. to express non-specificity). Also qualitative descriptions like “values are second-hand” or “estimated by three experts” might be used.

These measures can be determined by comparisons with former results, specifications or experience [28], comparisons to known or postulated distributions, assumed relations or expected value ranges [3], distances from interpolated to measured values [6] or results from Monte-Carlo-Simulations [16]. If this is not possible distances between approximating and exact methods or between different interpolation or integration methods [24], differences according to distinct parameterizations of methods [23] or tests of reconstruction techniques for missing values against randomly deleted, known values (cross validation) [3] can be used. Otherwise also subjective estimations of uncertainty might be reasonable for certain scenarios.

These measures may be automatically or interactively attached as metadata to the elements of the information model what forms the basis for a visualization that creates awareness of the underlying uncertainty and thus improves the quality of decision making.

3. Visualization of Uncertainty

3.1 A preparatory step: Reducing the complexity

Lets assume our portfolio manager has access to a history of exchange rates. Therein some values for certain days are missing because they were accidentally deleted or there was no trading these days. Furthermore certain other values could not be precisely gathered and had to be interpolated or guessed. If such uncertainty is also part of other recorded information like oil price or consumer acceptance an extensive visual analysis becomes very difficult that truly accounts for the data quality not to mislead the decisions of the manager.

Generally it can be stated that if there are many different kinds of uncertainty like errors, imprecision or subjectivity recorded for different elements of the information model the resulting data structure will be quite complex.

This complexity usually prevents a complete presentation of the data and the underlying uncertainty on the limited screen. Therefore a subset of the different kinds of uncertainty should be chosen that contains the decision relevant parts, e.g. by interactive selection or by definition of thresholds or critical values. Another possibility to reduce the complexity is to support an aggregation of uncertainty values. The easiest way is to incorporate averages. More sophisticated methods to calculate with or reason about quantitatively expressed uncertainty can be found e.g. in [15]. Such methods are also needed to integrate uncertainty introduced within the data transformation or visualization process [29].

More difficult is the handling of qualitative measures. For an aggregation or reasonable display these measures first have to be assigned to ordered classes or if possible to a quantitative equivalent.

An interesting reverse approach is the usage of uncertainty to guide the data selection process as mentioned e.g. in [25] or with the term "data quality filter" in [21] and [19]. According to this approach distrusted information above a certain threshold can just be omitted so that screen space is saved and the graphical complexity is reduced.

The complexity reduction should result in a relevant and manageable description of uncertainty prepared for visualization.

3.2 Known Techniques

The few available techniques for the display of uncertainty were mainly developed for certain specialized domains. These are in particular cartography, flow and volume visualization. The emerged techniques can be distinguished as follows:

- Utilization of free graphical variables:
  Uncertainty is mapped on yet unused graphical attributes (see e.g. [2], [22]) like color, size, position, angle or on focus, clarity, fuzziness, transparency and edge crispness. Figure 3 (left) shows an example where color is used to encode uncertainty in the future development of a rural environment.
- Integration of additional graphical objects:
  Typical examples are the inclusion of difference images or uncertainty glyphs [24] (see figure 3 right), labels, isosurfaces with an introduced thickness indicating spatial uncertainty [12] or an overlaid grid with varying thickness, sharpness or transparency of the grid lines to indicate local uncertainty [6].
- Use of animation:
  To represent uncertainty in a dynamic way parameters like speed, duration, blinking, motion blur, range or extend of motion, specific order (what moves when) or oscillation qualify for a mapping [24].
- Interactive representation:
  An impression on uncertainty could be provided to the user by interaction. A known example is the "clickable map" where uncertainty can be discovered by mouse interaction [31].
- Addressing other human senses:
  Uncertainty can also be incorporated by the use of acoustic, e.g. pitch, volume or rhythm [20] or haptic senses, e.g. touch [25] or vibration [5].

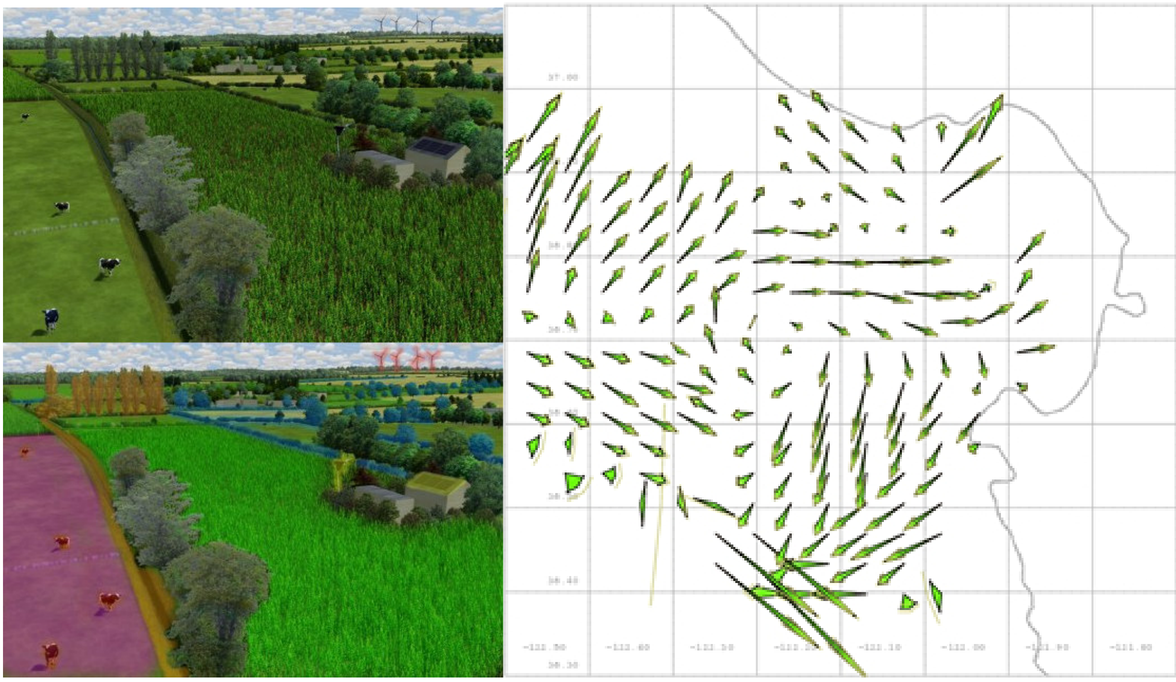

Figure 3. Uncertainty visualization examples.
Left: Usage of color to indicate different degrees of uncertainty in a future environmental setting (from [1]). Right: Uncertainty vector glyphs (arrows) over Monterey Bay indicate magnitude und directional uncertainty in a flow vector field (from [24]).

The mentioned techniques are able to communicate underlying distrust in the data to the decider. As expected, primarily supported are the main application fields cartography, flow and volume visualization. But even within those fields it is stated that new visualization metaphors and techniques are needed for an integrated display of uncertainty (see e.g. [29]).

3.3 Visualizing Uncertainty to Support Decisions

3.3.1 Problem Description and Need for New Approaches

As depicted by the portfolio manager example decisions can depend on abstract data that have no spatial dimension like cartographic, volume or flow data. The truthful representation of therein included uncertainty strongly influences the decision quality. If the manager e.g. wants to predict the development of a certain stock value he has to know if a merger rumor is based on a signed contract or a boulevard press speculation. He should also be able to detect if certain values (e.g. commodity values) are corrupted or interpolated. This means he needs a visualization of abstract data that creates awareness of all kinds of relevant uncertainty.

Unfortunately most existing techniques are – as already argued – not developed for abstract data. Furthermore the main concern of them is the representation of a single uncertainty value. Sources in literature about the treatment of higher dimensional measures, e.g. error, precision and validity at the same time, are hardly to be found. One of the few, [25], suggests techniques from multivariate data visualization like Parallel Coordinates (see section 1.2) but there definitely is a demand for new approaches.

Beside this there is a lack of methods to display uncertainty in relationships. Relations in the data are of high importance for many decision situations. For example for pest control predator-prey-relationships have to be considered to avoid an unintended decease of another species. Uncertainty in relations should therefore be carefully and truly displayed.

A general challenge is the development of graphical metaphors that draw an intuitive mental connection between uncertainties and the data they relate to so that an user does not miss this connection.

To emphasize the lack and need for various suitable uncertainty visualization techniques was one of the major concerns of this paper. It motivates the goal to increase the research effort to fill the gap and to develop information systems that enable an effective modeling and interactive display of information objects and information structure in a way that also expresses the reliability of the information. Some ideas for this shall now be introduced.

3.3.2 New Ideas Motivating Further Research

It was shown that general methods to display different kinds of uncertainty for different elements of the information model are needed.

To cope with the complexity of higher dimensional uncertainty it is reasonable – and shall be given as a suggestion here – first to display an overview e.g. by representing aggregated uncertainty for the whole information space IR, the information set IM, the attribute set AM and/or attributes A. For example if dataset values are displayed in a table like table 1 one could use an extra row and column to include aggregate uncertainty measures for single attributes and information objects with minimal additional screen space consumption. These again could be aggregated for the information set and attribute set and represented at the intersection of the mentioned row and column. Beside this different uncertainty kinds could be aggregated and displayed as scalar values with the known methods. This aggregation should include the overall impression and indicate extreme values of a certain kind of uncertainty at the same time. This way the manager becomes at least aware of the existence of unreliable data. To discover which data values and if e.g. errors or subjectivity are responsible for the displayed uncertainty suitable interaction techniques are to be included – e.g. focus & context.

The other mentioned open question is the inclusion of uncertainty in the visualization of relationships. For the portfolio manager relations are e.g. the hierarchy of the portfolio consisting of stocks, commodities and derivatives in which stocks are further partitioned according to trading places and so on, or dependencies between oil price and exchange rates or between merger rumors and certain stock values.

Relationships are usually represented explicitly by edges in a tree or a graph or implicitly by e.g. graphically nesting components of a hierarchy or by positioning (where close position means a strong connection). For these visualizations the following new ideas where derived:

- For a visualization by edges the lines indicating the relations can be varied. Blurred, distinguished in color, wavy or dotted lines are able to indicate less trusted relationships. If there is non-specificity, e.g. if it is known that a company has given an order to one of two other companies but not to which one, a link might be drawn from the originator-node to a place between the closely placed contractor-nodes without directly connecting to them. Another approach would be to group the two contractor-nodes e.g. by a surrounding circle and then to connect the circle to the originator-node. So the user becomes aware of the relation but it stays unclear who the actual contractor is.
- Hierarchy visualizations that nest components can integrate uncertainty by changing the appearance and structure of the nesting. As an example consider a Tree-map (see section 1.2): Here e.g. the variation of the box line style as mentioned for edges results in the intended mapping of uncertainty. To represent overlapping relations

like the two-contractor-originator-situation instead of boxes general polygons should be allowed that overlap if necessary.

- If relations are displayed by proximity of objects a regular arrangement of objects can be used to indicate highly trusted data whereas an irregular arrangement depicts uncertainty. If such arrangement counteracts the visibility of the data structures an overlaid regular/irregular grid serves as another possibility.
- A somewhat contrary idea is to use uncertainty as a threshold (as mentioned e.g. in [25]) to display only trusted relations. Different thresholds could be used to classify relations and display them according to their class membership.

These ideas present first alternatives to fill a gap in uncertainty visualization: the display of higher order uncertainty and especially the display of distrust in relationships. They represent possible new features for improved visualization guided decision making and provide a chance for our portfolio manager to better understand his data. This forms a starting point for further research, what is already planned for the near future.

Conclusion

The aim of this paper was to show the importance of taking into account uncertainty in displays when visualization systems are used as a means to guide decisions.

For this purpose the main concepts of uncertainty were generalized, critically discussed and afterwards linked to the elements of an adapted version of the information model used in information visualization. Combined with suitable measures it was possible to show how uncertainty can be integrated in representations. On the other hand current limits were depicted, too: e.g. in the visualization of higher order uncertainty or of uncertain relationships. The need to address these problems was emphasized and first approaches were presented that help to transform all aspects of uncertainty in the data into a visualization – as truly as for decisions necessary.

To summarize it can be stated that the main contributions of this paper are a complete consideration of the linkage of uncertainty to the components of the information model, the discovering of important but missing concepts for the visualization of uncertainty and the development of first ideas for their realization and therefore an improved decision making.

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