Métodos Numéricos - Problem set 05

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June, 2018

In this problem set we will numerically solve a simple savings problem in a economy with idiosyncratic shocks.

Suppose there is a continuum of goat farmers that are subject to endowment shocks. A farmer's endowment is e^z, where z follows the following stochastic process:

$$z' = \rho z + \epsilon,$$

where $\epsilon \sim N(0, \sigma^2)$. The farmers instantaneous utility function is given by

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}$$

and he discounts the future with the factor $\beta \in (0, 1)$. Each farmer has access to a storage technology such that, if he sets aside q goats today, he will have 1 goat tomorrow. His budget constraint can then be writte as:

$$c + qa' = e^z + a$$

Let $\beta = q = 0.96$ and $\gamma = 1.0001$ for now.

1.a)

Let $\rho = 0.9$ and $\sigma = 0.01$. Using the Tauchen method to discretize the stochastic process in a Markov chain with 9 states, with 3 standard deviations for each side, we have the following grid for e^z and transition matrix

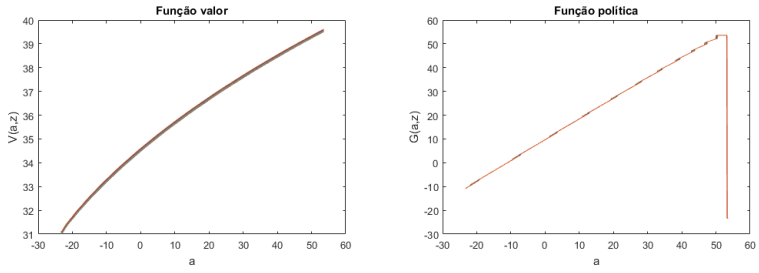
-0.6170 -0.4628 -0.3085 -0.1543 0 0.1543 0.3085 0.4628 0.6170

$$
\mathrm{P} = \begin{bmatrix}
0.5683 & 0.4025 & 0.0290 & 0.0002 & 0.0000 & 0.0000 & 0 & 0 & 0 \\
0.0843 & 0.5503 & 0.3459 & 0.0194 & 0.0001 & 0.0000 & 0.0000 & 0 & 0 \\
0.0017 & 0.1125 & 0.5829 & 0.2902 & 0.0126 & 0.0000 & 0.0000 & 0.0000 & 0 \\
0.0000 & 0.0029 & 0.1480 & 0.6034 & 0.2376 & 0.0080 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0049 & 0.1899 & 0.6104 & 0.1899 & 0.0049 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0080 & 0.2376 & 0.6034 & 0.1480 & 0.0029 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0126 & 0.2902 & 0.5829 & 0.1125 & 0.0017 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0194 & 0.3459 & 0.5503 & 0.0843 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0002 & 0.0290 & 0.4025 & 0.5683
\end{bmatrix}
$$

1.b)

Now we discretize the asset space using a grid starting from the natural debt limit under the worst endowment state up to two times the savings under the best state. This gives us a grid in $[-23.3373, 53.5624]$, with 1.000 points.

Solving the individual goat farmer problem for each state variable, using vectorized brute force on Matlab, we get the following value and policy functions:



1.c)

Next we find the stationary distribution $\pi(z, a)$ and use it to compute the aggregate savings in the economy.

First

e^z	-0.6170	-0.4628	-0.3085	-0.1543	0.0000	0.1543	0.3085	0.4628	0.6170
$P(z)$	0.0073	0.0352	0.1089	0.2143	0.2685	0.2143	0.1089	0.0352	0.0073