# Application of Voting Rules on the Electoral College

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**Samuel Marks**
Rensselaer Polytechnic Institute
markss@rpi.edu

**Tejus Surendran**
Rensselaer Polytechnic Institute
surent@rpi.edu

**Abstract**

The primary focus of this study is to explore how the application of different rank-based voting rules and the introduction of a multi-stage voting system would impact the result of the U.S. presidential election. Historical data from the 2016 and 2020 elections are utilized to generate a distribution of voter preferences, or a preference profile, for each state. Voting rules introduced include Plurality, Plurality with Runoff, Borda, Copeland, 2-Approval, Veto, and Single Transferable Vote (STV). Two alternative voting systems are then considered and simulated:

1. A rank-based voting rule is implemented at the state level. The winner of the vote receives all of the electoral votes of that state. At the national level, the winner is the candidate with the majority of electoral votes.
2. A rank-based voting rule is implemented at the state level. The resulting scores of the vote are used to generate preferences for the electors of that state. Then, a second voting rule is applied to the resulting profile. The winner of this second round is the winner at the national level.

## Introduction

The electoral college has long been a point of contention within American politics. There are those who believe that because the electoral college is outlined within the constitution, it is definitive of our identity as a nation and therefore must be preserved. However, many have called into question the validity and fairness of this voting system. Critics are quick to point out that five times throughout history, candidates who have received more votes than their opponent have lost the election, two instances occurring within the last 21 years. In 2000, Al Gore ran against George W. Bush and lost after winning the popular vote. Similarly Hillary Clinton lost against Donald Trump during the 2016 election. One reason for these occurrences is the concept of winner-take-all, where the candidate that wins the plurality vote at the state level takes all of the electoral votes in that state no matter the margin of victory. In some cases, studies have indicated that due to these practices, nearly 80% of the ballots cast in the U.S. presidential election do not influence the result.

Over the past 200 years, there have been over 700 proposals to Congress to reform or eliminate the electoral college. However, none have been widely successful. The closest attempt thus far is the The National Popular Interstate Compact, that says the winner of the aggregated plurality vote at the national level will receive the electoral votes of that state. However, only 15 states have agreed.

The topic of the electoral college as well as the general structure of the U.S. presidential election offers many opportunities to explore the application of different voting rules and voting systems. Because the current system relies on state boundaries instead of allowing all voter's ballots to aggregate at the national level, we can consider the implementation of a multi-stage voting rule. That is, the application of a voting rule at the state level and the application of either the same rule or a different rule at the national level when aggregating results from each state.

While there has been extensive research conducted on topics surrounding voting rules, specific applications of voting rules have not been explored in terms of the U.S. presidential election and the electoral college. In this study, we explore how redefining the way voting occurs in the U.S. presidential election impacts the result of the election.

## Data Collection

In order to simulate the impacts of different voting rules on the U.S. presidential election, we recognized the need to collect historical data from past elections. Thankfully, data from prior elections is readily available and provides insights broken down by state including the number of voters that voted for each candidate, the total number of votes cast, and the number electoral votes assigned to that state. Because our goal is to implement rank-based voting rules, this data alone is not sufficient to implement the rules and leaves the need to systematically generate preferences for each voter/agent and similarly a preference profile for each state.

To view the raw input data used for each election, please see the accompanying *2020_election_data.txt* and *2016_election_data.txt* files.

## Alternatives

The current voting system allows, in many states, voters to make use of write-in ballots, meaning voters can choose to vote for a candidate who is not already listed on the election ballot. Due to the marginal percentage that these write-in ballots account for in each state and for simplicity of this study, we have decided only to consider the four most popular candidates from each election. The alternatives for each election, 2016 and 2020, are as follows:

2016: Trump (T), Clinton (C), Johnson (J), Stein (S)

2020: Biden (B), Trump (T), Jorgensen (J), Hawkins (H)

## Profile Generation

In order to implement various rank-based voting rules, we first needed to generate preference profiles for each state. We began by listing all possible preferences an agent could have based on the given alternatives. For example, all possible preferences for the 2020 election are listed below.

| | | |
|---|---|---|
| B > T > J > H | T > J > B > H | J > H > B > T |
| B > T > H > J | T > J > H > B | J > H > T > B |
| B > J > T > H | T > H > B > J | H > B > T > J |
| B > J > H > T | T > H > J > B | H > B > J > T |
| B > H > T > J | J > B > T > H | H > T > B > J |
| B > H > J > T | J > B > H > T | H > T > J > B |
| T > B > J > H | J > T > B > H | H > J > B > T |
| T > B > H > J | J > T > H > B | H > J > T > B |

Next, we utilized alternative parameterization to generate a probability for each preference.

Alternate Parameterization:

$$\pi_\theta(a_1 > a_2 > \dots > a_m) = \frac{\lambda_1}{\lambda_1+\dots+\lambda_m} \times \frac{\lambda_2}{\lambda_2+\dots+\lambda_m} \times \dots \times \frac{\lambda_{m-1}}{\lambda_{m-1}+\lambda_m}$$

$\theta = ( \lambda_1, \lambda_2, \dots , \lambda_m )$ $\quad \lambda_i =$ "quality" of alternative i

Here, the lambda value, or the "quality", for each alternative *i* is assigned the proportion of voters that voted for that alternative within a particular state (given by the historical data). Generated for each preference is the probability of an agent having that preference. The resulting probability distribution is used to generate the number of voters in the state with that preference, or the preference profile.

## Voting Rules

The voting rules that we decided to implement along with a brief description of each is listed below.

*Plurality*: The winner is the candidate with the most first place votes (1-approval).

*Plurality with Runoff*: The election has two rounds. In the first round, all alternatives except the two with the highest plurality scores drop out. In the second round, the alternative preferred by more voters wins.

*Borda*: The winner is the candidate with the most points where points are assigned to candidates based on their ranking (in this case, 3 points for first, 2 points for second, 1 point for third, and 0 points for fourth).

*Copeland*: The winner is the alternative with the highest Copeland score, or the greatest number of pairwise wins.

*2-Approval*: The winner is the candidate with the most first or second place votes.

*Veto*: The winner is the candidate with the least last-place preferences.

*Single Transferable Vote (STV)*: The election has *n* - 1 rounds where *n* is the number of alternatives. In each round the candidate with the least plurality points drops out. The remaining alternative after the *n* - 1 round wins.

## Implementation

We decided to explore two possible voting systems. Both involve generating preference profiles for each state and implementing each rule for those profiles.

In the first system (System I), we determined the winner under each rule at the state level. The winner would then receive all electoral votes from that state. At the national level, the winner is the candidate with the majority of electoral votes.

The second system (System II) involves using the result of the voting rule at the state level to generate preferences for the electors of that state. Then, a second voting rule is applied to this new profile. The overall winner is the candidate to win this second stage. Here, we are considering a multi-stage voting rule. However, due to time restrictions, we were only able to implement two variations of this system, including Plurality in the first stage with Borda in the second stage and Borda in the first stage and Veto in the second stage.

## Results

The results from the 2020 election simulations (System I) are below. It is important to note that Biden won the 2020 elections under the existing voting system.

| | |
|---|---|
| Plurality: Biden | Plurality with Runoff: Biden |
| Borda: Biden | Copeland: Biden |
| 2-Approval: Biden | Veto: Biden |
| STV: Biden | |

To see a detailed report of the simulation results, please refer to the accompanying *2020_election_sim_results.txt* file.

The results from the 2016 election simulations (System I) are as follows. It is important to note that

Trump won the 2016 elections under the existing voting system.

| | |
|---|---|
| Plurality: Trump | Plurality with Runoff: Trump |
| Borda: Trump | Copeland: Trump |
| 2-Approval: Trump | Veto: Trump |
| STV: Trump | |

To see a detailed report on the simulation results, please refer to the accompanying *2016_election_sim_results.txt* file.

Due to time limitations, we were unable to perform as many System II simulations as desired. However, we were able to conduct two simulations, utilizing the 2020 election data. The first implemented Plurality in the first stage with Borda in the second stage. The second implemented Borda in the first stage and Veto in the second stage. The results of these System II simulations are as follows.

Plurality/Borda: Biden

Borda/Veto: Tie between Biden and Trump

## Conclusion

After obtaining the results from the various voting simulations, we gained some insight into how the result of the election could be impacted by the presence of a different voting system.

In the System I simulations, we were initially surprised to see that every election result produced a 306 to 232 electoral vote split between the top two candidates for both the 2020 and 2016 data, perfectly aligning with the true outcomes of each election, as both elections resulted in a 306-232 electoral vote split (taking faithless electors into consideration). While we assumed this was a likely possibility for the 2020 election simulations given the greater performance differential between the two leading candidates, we did not anticipate to see the same consistency in the results for the 2016 simulated elections. Due to the closeness of the popular vote between candidates Trump (T) and Clinton (C), we expected to see varied results in the System I simulations. However, Trump won in all simulationed elections.

We assume that these results are due to two facts: (1) the performance differentials between each candidate in the data were vast enough apart to produce profiles that resulted in the same winner no matter the voting rule implemented and (2) despite the margin of victory at the state level, the winning candidate in System I receives the entire amount of electoral votes for that state in the same manner as in the existing voting system, producing less-than-interesting results. We realize that in order to more accurately account for the margin of victory at the state level, System II (or a similar voting system) would need to be implemented.

Consequently, the major aspect of this study that we would have liked to explore in more depth is the implementation of more multi-stage simulated (System II) elections, as it has the potential to return more interesting results.

Overall, we have accomplished what we set out to achieve, as we were able to simulate U.S. presidential elections with the implementation of a variety of voting rules and systems and determine their outcome based on preference profiles generated from real, historical data.

For full details, please visit https://bit.ly/3xGUBDf.

## Future Work

Due to the limited time that we were able to explore and apply the concepts at hand, there are many areas of this study that could continue to be developed and expanded upon. This includes the implementation of more voting rules and systems. We would like to explore the implementation of additional rank-based voting rules as well as implementing more variations of multi-stage voting rules (System II). Furthermore, we would like to utilize additional parameters to more accurately generate preference profiles for each state (i.e., take into account more information to parameterize the candidates). Lastly, we would like to explore the possible/necessary winner problem as it relates to the electoral college. That is, determining whether a given candidate has the possibility of winning or is necessarily the winner given partial, or incomplete, preference profiles.

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