Draft version July 18, 2013
Preprint typeset using LaTeX style emulateapj v. 12/16/11

FRACTAL AND MULTIFRACTAL ANALYSIS AS TOOLS TO CHARACTERIZE SUPERNOVAE AND MOLECULAR CLOUDS: SUBTITLE

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Draft version July 18, 2013

ABSTRACT

Keywords: supernovae, fractals, multifractals

1. INTRODUCTION

Many physical phenomena cannot be characterized by the idealizations of Euclidean geometry alone; they exhibit “roughness”. That is, they have a detailed structure at any arbitrarily small size scale (Falconer 2003). The development of *fractal geometry* allows a mathematical treatment of the “roughness” inherent in the non-idealized phenomena of the real world. Multifractal analysis permits us to examine how those fractal characteristics themselves change with scale; in fact, they may even be fractal themselves. In this paper, we detail the application of fractal geometry to flame fronts in white dwarfs and then analyze the multifractal characteristics of star-forming molecular clouds.

1.1. *Fractal analysis of type Ia supernova flame fronts*

The accepted model for a type Ia supernova is a white dwarf that accumulates mass from a binary companion until it reaches a mass so large that the compression of the heavier elements in the core causes them to ignite and fuse, releasing more energy in the process. This deflagration results in a flame front that spreads rapidly throughout the star and causes the visible effects of supernova.

The surface of the expanding flame front can be dramatically affected by multiple types of turbulence. Landau & Lifshitz (1959) showed that the flame surface is subject to Landau-Darrieus instability in which the front is convoluted due to thermal expansion across the flame front. It is also widely established (see Kull (1991)) that Rayleigh-Taylor instability, in which a denser fluid pushes through a less-dense one under the influence of a gravitational field, can contort the deflagration front in a supernova.

These wrinkles on the surface of the flame front increase the front’s effective surface area, and because the flame speed is proportional to the flame’s surface area, the convolution of the surface can influence the rate at which the flame consumes the star. Thus, characterizing the shape of the flame front plays a crucial part in characterizing the supernova explosion process as a whole.

These wrinkles on the surface of a turbulent front can be viewed within the framework of fractal geometry, as was proposed by Mandelbrot (1975) and further developed by Timmes, F. X. (1994). Furthermore, Timmes, F. X. (1994) showed that the final effective speed of the flame front is a function of its density.

In §2.1, a method is implemented to perform fractal analysis of a supernova flame front. The results are discussed in §3.1, and the implications of the calculated fractal dimension are discussed in §4.1.

1.2. *Mutlfractal analysis of dense molecular clouds*

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2. METHODS

2.1. *Fractal analysis methods*

2.2. *Multifractal analysis methods*

3. RESULTS

3.1. *Fractal analysis methods*

3.2. *Multifractal analysis methods*

4. DISCUSSION

4.1. *Fractal analysis methods*

4.2. *Multifractal analysis methods*

Timmes eqn: The final effective speed v_{eff} of the flame front on a macroscopic scale is given by

$$v_{eff} = v_{lam} \left(\frac{l_{min}}{l_{max}} \right)^{2-D} \tag{1}$$

where v_{lam} is the laminar speed of the front through the material (that is, the speed of the flame in the absence of turbulence), l_{min} is the minimum length scale at which the turbulence takes place, l_{max} is the maximum length scale at which the turbulence takes place, and D is the fractal dimension of the flame front.

5. ACKNOWLEDGMENTS

The work of SB has been supported by Tomasz Plewa and Tim Handy at Florida State University under the Young Scholars Program.

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