# Analysis of Stegosystems

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## Introduction

## Security Definitions

### Primitives

Let $P_X$ be a probability mass function with support $\chi$, where $X$ is a discrete random variable taking the values in $\chi$. The *entropy* of $X$ is

$$H(X) = E(-\lg P_X),$$

where $E(\cdot)$ is the expected value (weighted average) function; that is,

$$H(X) = -\sum_{x \in \chi} P_X(x) \lg P_X(x). \tag{1}$$

Intuitively, the entropy of $X$ is a measure of the number of bits of uncertainty in $X$. For example, suppose $\chi$ is the set of all $n$-bit strings, and $P_X(x) = 1/2^n$ for any $x \in \chi$; that is, every $n$-bit string is equally likely to be pulled from $P_X$. This would represent a distribution of maximum uncertainty, and it is straightforward to show that Eqn. (1) evaluates to $n$ in this case. In fact, $H(X) = \lg|\chi|$ is an upper bound for $H$, where $|\chi|$ denotes the cardinality of $\chi$.

The *minimum entropy* of a distribution $P_X$ is defined as

$$H_\infty(X) = \min_{x \in \chi}\{-\lg P_X(x)\} \tag{2}$$

This can be understood as a measure of uncertainty for the "most probable" element in $\chi$ according to $P_X$. For example, if there is some element $x_0$ with $P_X(x_0) = 1$, then $H_\infty(X) = 0$ (there is no uncertainty in $X$). Suppose

the most probable element $x_0$ has probability $P_X(x_0) = 1/2$. Intuitively, the uncertainty is unity; that is, we can guess that the next value of $X$ will be $x_0$ to within a single coin flip. Indeed, evaluating Eqn. (2) for such a distribution shows that $H_\infty(X) = 1$.

Given two probability mass functions $P_X$ and $P_Y$, both with support $\chi$, the *relative entropy*, also called the Kullback-Leibler divergence, from $P_X$ to $P_Y$ is defined to be

$$D(P_X\|P_Y) = \sum_{x\in\chi} P_X(x)\lg\frac{P_X(x)}{P_Y(x)}. \tag{3}$$

Intuitively, the relative entropy is a measure of the difference between $P_X$ and $P_Y$, although it is important to note that it is not symmetric; that is, $D(P_X\|P_Y) \neq D(P_Y\|P_X)$. However, $D(P_X\|P_Y) = 0$ if and only if $P_X = P_Y$, and increases indefinitely as $P_Y$ diverges from $P_X$.

A *channel* as defined by TODO: cite is a distribution on timestamped bit sequences; i.e., a channel $\mathcal{C}$ is a distribution with support $\{(\{0,1\},t_1),(\{0,1\},t_2),\ldots\}$, where each $t_i \leq t_{i+1}$. The intent is to model communication, where not just the content but also the timing of the communication may be relevant.

## Security Definitions

Hopper paper:

Hopper, et. al TODO: cite defines the security of a stegosystem in terms of a game. Give the warden $W$ access to $M(h)$, which returns draws from $\mathcal{C}_h^b$, and an oracle $\mathcal{O}$. The oracle $\mathcal{O}$ is either $\mathsf{SE}_k$ or a function $O(\cdot,\cdot)$, where $O(m,h)$ simply returns draws from $\mathcal{C}_h^{|\mathsf{SE}_k(m,h)|}$. The warden also has access to randomness $r$. The warden's advantage against the steganographic secrecy under chosen hiddentext attack for channel $\mathcal{C}$ of stegosystem S is defined by Hopper et. al to be

$$\mathbf{Adv}_{\mathsf{S},\mathcal{C}}^{\text{ss-cha-}\mathcal{C}}(W) = \left| \Pr_{k,r,M,\mathsf{SE}}\left[ W_r^{M,\mathsf{SE}_k(\cdot,\cdot)}\text{accepts} \right] - \Pr_{r,M,O}\left[ W_r^{M,O(\cdot,\cdot)}\text{accepts} \right] \right|.$$

A stegosystem S is $(t,q,\ell,\varepsilon)$-*steganographically secret under chosen hiddenttext attack* for channel $\mathcal{C}$ (SS-CHA-$\mathcal{C}$) if, for any warden $W$ making at most $q$ queries totaling at most $\ell$ bits of hiddentext, and running in time at most $t$,

$$\mathbf{Adv}_{\mathsf{S},\mathcal{C}}^{\text{ss-cha-}\mathcal{C}}(W) \leq \varepsilon;$$

that is, the stegosystem S is insecure if an efficient warden can (with high probability) distinguish between the output of $\mathsf{SE}_k(m,h)$ and draws from $\mathcal{C}_h^{|\mathsf{SE}_k(m,h)|}$, even when given access to $\mathcal{C}_h^b$ through $M$.

A stegosystem S is $(t, q, \ell, \varepsilon)$-*universally steganographically secret under chosen hiddenttext attack* for channel $\mathcal{C}$ (USS-CHA-$\mathcal{C}$) if it is $(t, q, \ell, \varepsilon)$-SS-CHA-$\mathcal{C}$ for any channel $\mathcal{C}$ that satisfies $H_\infty(\mathcal{C}_h^b) > 1 \forall h$ drawn from $\mathcal{C}$.

Cachin paper:

# Bad Stego

# Good Stego

# Conclusion

# References