Revision of `TOMS-2021-0028`

Dear reviewers,

We are deeply grateful for the comments, questions, and suggested references you have all provided. In accordance with your feedback we have made significant changes to the manuscript to improve clarity, expand on references to related literature, and incorporate comparisons with other spline software. We hope you find these changes have resulted in an overall better product.

Each numbered item and italicized text block below is a reviewer comment or summary of multiple similar comments, our responses immediately follow.

(1) *The authors should include references to more related works and justify the differences between this work and the many prior works on C^2 interpolation. [given 23 references]*

> *M. Abbas, A. A. Majid, J. M. Ali, Monotonicity-preserving C^2 rational cubic spline for monotone data, Applied Mathematics and Computation, 219(2012), 2885-2895.*
>
> *R. K. Beatson, Monotone and Convex approximation by splines: Error estimates and a curve fitting algorithm, SIAM J. Numer. Anal., 19(1982), 1278–1285*
>
> *P. Costantini, On monotone and convex spline interpolation, Math. Comp., 46(1986), 203–214.*
>
> *I. Cravero, C. Manni, shape-preserving interpolants with high smoothness, Journal of computational and applied mathematics, 157(2003), 383–405.*
>
> *R. Delbourgo and J. A. Gregory, C^2 rational quadratic spline interpolation to monotonic data, IMA J. Numer. Anal., 3(1983), 141–152.*
>
> *R. Delbourgo, Accurate C^2 rational interpolations in tension, SIAM J. Numer. Anal., 30(1993), 595–607.*
>
> *R. A. DeVore, Monotone approximation by splines, SIAM J. Math. Anal., 8 (1977), 891-905.*
>
> *R. A. DeVore, Monotone approximation by polynomials, SIAM J. Math. Anal., 8 (1977), 906-921.*
>
> *R. L. Dougherty, A. Edelman, and J. M. Hyman, Nonnegativity-, monotonicity- convexity-preserving cubic and quintic Hermite interpolation, Math. Comp., 52(1989), 471–494.*
>
> *J. A. Gregory and R. Delbourgo, Piecewise rational quadratic interpolation to monotonic data, IMA J. Numer. Anal., 2(1982), 123–130.*
>
> *J. C. Fiorot and J. Tabka, Shape-preserving C^2 cubic polynomial interpolating splines, Math. Comp., 57(1991), 291–298.*
>
> *X. Han, Shape-preserving piecewise rational interpolation with higher order continuity, Applied Mathematics and Computation 337 (2018) 1-13.*
>
> *H. T. Huynh, Accurate monotone cubic interpolation, SIAM J. Numer. Anal., 30(1993), 57–100.*
>
> *B. I. Kvasov, Monotone and convex interpolation by weighted quadratic splines, Adv. Comput. Math., 40(2014), 91-116.*
>
> *C. Manni and P. Sablonnière, Monotone interpolation of order 3 by C^2 cubic splines IMA J. Numer.Anal., 17(1997), 305–320.*
>
> *C. Manni, On shape preserving C^2 Hermite interpolation, BIT Numerical Mathematics, 41(2001),*

127–148.

S. Pruess, Shape preserving C^2 cubic spline interpolation, IMA J. Numer. Anal., 13(1993), 493–507.

Y. Zhu, X. Han, C^2 rational quartic interpolation spline with local shape preserving property, Appl. Math. Lett., 46(2015), 57-63.

Y. Zhu, X. Han, Shape preserving C^2 rational quartic interpolation spline with two parameters, International Journal of Computer Mathematics, 92(2015), 2160-2177.

Response

(2) The authors should include references to and comparisons with the following software packages.

Costantini, P. An algorithm for computing shape-preserving interpolating splines of arbitrary degree, Journal of Computational and Applied Mathematics, 1988, 22(1), pp. 89-136.

Response

() The monotonicity of $Q(x)$ is not proved.

Respopnse

() The approximation order of the algorithm is not shown.

Response

() Why use piecewise degree 5? In principle, piecewise degree 3 is enough for C^2. The additional degrees of freedom for making the interpolant monotone could be obtained by adding knots between the breakpoints. It would be great if you could motivate your choice in more detail.

In general it is not possible to form a C^2 approximation of arbitrary data without the incorporation of additional knots. The restriction of a fixed set of knots is somewhat arbitrary, but that is the application targeted by this line of research.

() Some statements depend on two variables being "approximately equal", but what does that mean? Equal up to some small threshold?

Response

() In Algorithm 1 (and also in the text), you use the term "curvature" to refer to the second derivative. I would avoid this, since "curvature" is a term reserved for curves, but you are dealing with functions. Similarly, the term "minimum curvature derivative" is inappropriate.

Response

() In Algorithm 2, it might be better to say that f is a "degree 5" polynomial, instead of "order 6".

Response

() Your binary search results in a particular set of derivatives, for which the local quintic piece is guaranteed to be monotone, but it is clearly not a unique choice. Alternatively, one could try to find, among all possible sets of derivatives and corresponding monotone quintic interpolants, the best, according to some criterion, e.g. least L2 norm. This would turn the problem into a constrained optimization problem, which is probably harder to solve, but it might, at the same time, give better results and overcome the limitation shown in Fig. 1. More motivating details would be appreciated.

Response

() *It is quite probable that a user would want to convert the piecewise Hermite representation of the interpolant to quintic B-spline form. But the authors say in Sections 3 and 4 that they solve a banded linear system of length n (n the number of data points) to make this conversion. However, this is not necessary. At each data point, only three of the quintic B-splines are non-zero. Therefore, the three associated B-spline coefficients can be computed from the value, first, and second derivatives of the interpolant at that single point. Thus one can just solve a small 3*3 linear system to get one group of 3 B-spline coefficients. Well, it requires a bit of algebraic work to write out this 3*3 linear system, but it can be done.*

Response

() *I'm confused by the O(1) and O(n) discussion in Section 4. If there are n data points then surely the main algorithm (estimating first and second derivatives at each data point) must require at least an O(n) algorithm, not O(1).*

Response

() *The output are values – why not the spline coefficients? If the output are values one would hope for a visualization package.*

Response

() *I was not familiar with the term/notion "quadratic facet model". The term facet made me believe that the paper was on bivariate interpolation. Nothing in the abstract or title prevents this misconception.*

To the best of our knowledge, rejection based on a single review has not been a journal policy and we respectfully request additional reviews. We believe this algorithm and the accompanying code has great potential for TOMS, noting that the quadratic and cubic works preceding this have remained state-of-the-art for forty years. Thank you for both your time and consideration.

Sincerely,

Thomas Lux, on behalf of all authors.