# (APPENDIX) Appendices {-}

# Appendix A: Updated operating model components

\beginAppA

## Updated ageing error matrix

The Sablefish age-structured assessment model relies on catch-at-age

data to estimate the true age-composition of the population; however,

observed catch-at-age data are based on otolith readings that are

imperfectly known. Failure to account for errors in otolith readings

may lead to smoothing estimates of age-classes, making it more

difficult to detect strong recruitment years or stock-recruit relationships

[@hanselman2012statistical]. Ageing errors may also bias estimates of

growth parameters, maturity schedules, and natural mortality that

can lead to overfishing or inaccurate yield projections

[@lai1987effects; @tyler1989implications]

To account for ageing-error, the Sablefish age-structured operating

model uses an ageing error matrix. In this MSE cycle, we simplified the

formulation of the ageing-error matrix from the previously

used double-geometric model to a discretized normal distribution. The

two major differences between these two formulations are (i) that the

error structure is constrained

to be symmetric for the normal formulation, while the double geometric

model allows for some skew in the error distribution; and (ii) the

normal assumes the assigned true age is the mode of the normal

density, forcing ageing errors to be on average unbiased.

We developed our ageing error matrix using otoliths that had been

read by two different readers at the DFO Pacific Biological Station

ageing lab. These data account for

approximately 15\% of the total otolith readings for BC Sablefish,

which are read first by the primary reader and then by a secondary

reader as a quality control. In the majority of cases both readers

agreed (62\%) and in cases where the two readings differ (38\%), both

readers conferred to resolve the discrepancy and agree on the final

age assigned (Pers. Comm, J. Groot, DFO). In most cases the final

age reading was that assigned by the secondary or primary reader (36\%),

but in a few cases a new age was assigned (2\%).

We applied statistical models for estimating the probability of observing

an age class (a) given the true age (b) based on methods described in

@richards1992statistical and @heifetz1999age. The model assumes a

normal ageing-error distribution where the estimated standard deviation

of the observed age for a true age b is based on three parameters

$\Phi = \{ \sigma\_1, \sigma\_A, \alpha \}$ in the form:

\begin{equation}

\sigma(b) = \left\{

\begin{array}{ll}

\sigma\_1 + (\sigma\_A - \sigma\_1) \frac{1 - e^{-\alpha(b - 1)} }{1 - e^{-\alpha(A - 1)}}, & \alpha \neq 0; \\

\sigma\_1 + (\sigma\_A - \sigma\_1) \frac{b-1}{A-1}, & \alpha = 0.\\

\end{array} \right.

\end{equation}

Parameters $\sigma\_1$ and $\sigma\_A$ are the standard deviations for

$b=1$ and $b=A$, representing the minimum and maximum ages, respectively.

The $\alpha$ parameter determines the non-linearity of the function, such

that~$\sigma(b)$ becomes linear as $\alpha \rightarrow 0$. The age-error matrix

is defined as:

\begin{align}

q(a ~|~ b, \Phi) &= \frac{x\_{ab}(\Phi)}{\sum\_{a = 1}^A x\_{ab}(\Phi) }; \\

x\_{ab} &= \frac{1}{\sqrt{2\pi}\sigma(b)} e^{-\frac12 \left[ \frac{a-b}{\sigma(b)} \right]^2}.

\end{align}

Given that the true age of the fish is unknown, it is not possible

to accurately determine bias in the age readings and whether certain

age classes are more likely to be under or over-estimated. We tested

2 different approaches for the assumed “true age”, using 1) the

mean of the two reader ages rounded to the nearest integer

[@heifetz1999age], and 2) the final age assigned. For both

approaches we set $A=90$, based on the maximum assigned age by the

readers.

The likelihood $\mathcal{L}$ of observed ages $A$ given true ages B is then defined

as:

\begin{equation}

\mathcal{L}(A|B) = \prod\_{i = 1}^I \prod\_{j = 1}^J q(a\_{ij} ~|~ b\_i \Phi),

\end{equation}

where $b\_i$ is the assumed ‘true age’ of fish $i$, and $a\_{ij}$ is

the age assigned by reader $j$ to the individual fish $i$. Maximum

likelihood parameter estimates, predicted standard deviation at age, and age-error

matrices are provided below (Table A1, Fig. A1 & A2)

## Trawl Age-Length Key and updated selectivity curve

The Sablefish age-structured operating model uses observations

of catch at age from commercial fisheries to estimate natural

mortality and gear selectivity functions. Trawl selectivity has

been identified a key determinant in reducing uncertainty in

estimates of sub-legal Sablefish catch and releases

[@cox2019evaluating], as up until now the trawl selectivity

model was heavily dependent on priors for a normal selectivity

curve estimated from tagged fish recovered (within one year

from release) in the commercial trawl fishery. To improve estimates

of legal and sub-legal fishing mortality from the

trawl sector, we leveraged catch-at-age and catch-at-length data

from BC trawl fisheries to develop a sex-specific age-length key,

which was in turn used to increase the catch-at-age sample size.

To develop our age-length key, we used all available catch-at-age

data collected from observed trips in the commercial trawl fishery.

We then used this to populate an empirical age-length frequency

matrix, binning fish into 3cm length bins and 1 year age classes.

We defined this matrix as

\begin{equation}

F = \left[ n\_{l,a} \right],

\end{equation}

where $n\_{l,a}$ is the number of fish observed in length bin $l$

and age class $a$. The matrix $A$ was converted to a probability

of age-at-length $l$ matrix $P$ by normalising the columns of $A$

\begin{equation}

P\_{l,a} = F\_{l,a} / \sum\_{a'} F\_{l,a'}.

\end{equation}

We then generated expected age composition data by applying the

matrix $P$ to length compositions $C\_l$ derived from the

commercial trawl catch-at-length data.

\begin{align}

C\_a &= P^T \cdot C\_l,

\end{align}

where $P$ is transposed so that the length dimension matches

the vector $C\_l$. We restricted $C\_l$ to catch-at-length data

from years where at least 5 trips were sampled. We defined keys

$P\_m$ and $P\_f$ for male and female fish, respectively, and

generated sex-specific age observations (Figures A3 and A4).

Length observations from unsexed fish were treated as

male specimens, as the operating model optimisation would

not converge when they were treated as females.

Inferred catch-at-age compositions had a noticable effect

on the selectivity-at-length curves for the trawl fleet (Figure A5).

The fully selected size class moved from about 42 cm to 48 cm, and the

shape of the Gamma selection curve dome was narrower, deselecting

to about 60% by the 55cm size limit, as opposed to about 80%

for the normal model in 2016.

Table A1. Ageing error model parameters for both true age cases tested.

Figure A1. Estimated standard deviation of observed ages for the two age assignment cases considered.

Figure A2. Probability of observed ages given the true age indicated in top right corner of each panel for the two age assignment cases considered.

Figure A3. Inferred male catch-at-age compositions generated by the trawl age-length key from length observations of male and unsexed fish.

Figure A4. Inferred female catch-at-age compositions generated by the trawl age-length key from length observations of female fish.

Figure A5. Trawl selectivity-at-length curves from the 2016 operating model (dashed grey line) and 2019 operating model (solid black line), and the legal size limit (vertical red dashed line). The length axis starts at the modeled length at age-1 of 32cm.