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EXECUTIVE SUMMARY OF THE THESIS

Guidance and Feedback Control Solutions for Impulsive and Continuous Rendezvous Proximity Operations

LAUREA MAGISTRALE IN SPACE ENGINEERING - INGEGNERIA SPAZIALE

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Academic year: 2024-2025

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1. Introduction

Rendezvous is a critical operation in space missions involving the approach of two or multiple spacecraft. It is widely used in satellite servicing, space station resupply, debris removal, and formation flying. Rendezvous and Docking (RVD) [4] is the process by which a chaser spacecraft approaches, aligns with, and physically connects to a target spacecraft in orbit. It involves a series of controlled orbital manoeuvres, guidance, and attitude adjustments to ensure precise positioning and safe attachment, either autonomously or with human supervision. The most critical and precision-driven phase in RVD is the final approach, where the chaser spacecraft aligns with the docking axis and closes in on the target within a defined capture range. This phase requires high-accuracy control of position, velocity, and attitude to ensure safe and successful docking, while minimizing collision risk and adhering to strict safety constraints. Formation flying is the coordinated operation of two or more spacecraft maintaining precise relative positions and orientations while orbiting, enabling cooperative tasks. In proximity operations, a guidance reference profile defines the desired trajectory, velocities, and attitudes the chaser spacecraft should follow to safely approach the target. However, due to disturbances, modeling errors, and actuator limitations, the actual motion can deviate from this ideal path. A feedback controller is therefore essential to continuously monitor the chaser's state and apply corrective actions to minimize errors and keep the spacecraft on track. Together, the guidance profile provides the planned path, and the feedback controller ensures accurate, stable, and responsive execution during the mission.

2. Contents and Contributions

This thesis makes several key contributions to the field of autonomous spacecraft proximity operations by focusing on the final approach phase of RVD missions, and extending the analysis to formation flying using eccentricity/inclination separation. At its core, the work develops a comprehensive modeling framework that captures both translational and rotational dynamics of the chaser–target system. It combines Cartesian and Relative Orbital Elements (ROE) [2, 5] representations, with a significant extension of ROE-based dynamics to accommodate eccentric orbits, an advancement that broadens the applicability of linearized control in more complex mission scenarios. A central contribution lies in the design of guidance profiles tailored for different propulsion architectures, including both impulsive and continuous thrust strategies. These are integrated into a feedback loop where Linear Quadratic Regulator (LQR) controllers [1, 7] ensure precise trajectory tracking while handling the challenges posed by discrete-thrust systems. The controllers are designed for both Cartesian and ROE domains, demonstrating flexibility and robustness across modeling frameworks. To support this analysis, a simulator was developed, emulating the full closed-loop operation of an RVD mission. The simulator includes realistic orbital dynamics, attitude control, sensor models, and actuation logic, providing a tool for validating guidance and control strategies under operational constraints. The inclusion of rotational dynamics with attitude control, assuming a cooperative target, further enhances the realism and relevance of the simulations. Together, these contributions advance the theoretical and practical tools available for autonomous GNC (Guidance, Navigation, and Control) design, offering a solid foundation for future research and real-world implementation in high-precision spacecraft proximity operations.

3. Methodology

3.1. Relative Motion Dynamics

Accurate modeling of spacecraft relative motion is critical for the success of rendezvous operations. The simulations incorporate both translational and rotational dynamics, emphasizing the assumptions and methodological choices that address the unique challenges of this study. For translational dynamics, two primary approaches are explored: the Cartesian formulation and the orbital elements-based representation. The analysis uses multiple reference frames to accurately describe spacecraft motion. The inertial frame serves as a fixed, non-rotating baseline for orbital calculations. The Local-Vertical Local-Horizontal (LVLH) frame, centred on the target spacecraft, provides a convenient reference for relative motion by aligning its z-axis with the orbital radius vector. Additionally, body-fixed frames attached to each spacecraft are used to model rotational dynamics and attitude control.

Cartesian-based Translational Dynamics

The mathematical modeling of the relative translational motion between two spacecraft, namely the chaser and the target, is accurately derived within a Cartesian coordinate framework. The dynamics are first formulated in an inertial frame for each spacecraft, and then transformed to describe the chaser's position and velocity relative to the target, ultimately expressed in the target's LVLH frame.

Several assumptions are made to simplify the analysis. The spacecraft are treated as point masses, and perturbative effects such as atmospheric drag and Earth's oblateness are neglected. The target spacecraft is assumed to follow a natural Keplerian orbit, and the relative distance between the chaser and target is small compared to the orbital radius.

Two principal dynamical models are introduced. The Tschauner-Hempel (TH) [8] equations describe the relative motion in elliptical orbits by linearizing the gravitational acceleration through a first-order Taylor expansion. These equations account for Coriolis and centrifugal forces in the rotating LVLH frame, as well as control inputs acting on the chaser spacecraft. The Hill-Clohessy-Wiltshire (HCW) equations represent a simplified special case of the time-varying linear dynamics of the Tschauner-Hempel equations applicable to circular orbits, yielding time-invariant linear dynamics that are extensively used in spacecraft rendezvous and proximity operations.

The TH and the HCW equations admit a solution consisting of a homogeneous part, described by the state transition matrix, which models the natural relative motion in the absence of control inputs, and a particular solution that incorporates the effects of external control forces. The latter has a different formulation based on the nature of the control input (impulsive or continuous).

ROE-based Translational Dynamics

A comprehensive framework for modeling spacecraft relative motion using ROE [2, 5] is presented in this study. Unlike traditional

Cartesian coordinates, Keplerian ROEs describe the relative translational dynamics in terms of differences between the orbital elements of a chaser spacecraft and a target spacecraft, offering a clearer physical interpretation of their relative positions and velocities. To overcome the singularities associated with classical Keplerian orbital elements, a quasi-nonsingular (qns) ROE formulation ($\delta\boldsymbol{\alpha}_{qns} = [\delta a/a, \delta\lambda, \delta e_x, \delta e_y, \delta i_x, \delta i_y]^T$) is employed, which is especially effective for orbits that are nearly circular and not close to the equator:

$$\delta\boldsymbol{\alpha}_{qns} = \begin{bmatrix} (a_c - a_t)/a_t \\ (M_c + \omega_c) - (M_t + \omega_t) + (\Omega_c - \Omega_t)\cos i_t \\ e_c \cos\omega_c - e_t \cos\omega_t \\ e_c \sin\omega_c - e_t \sin\omega_t \\ i_c - i_t \\ (\Omega_c - \Omega_t)\sin i_t \end{bmatrix} \quad (1)$$

Here, the subscrips t and c refer to the target and chaser vehicles. The time evolution of these ROEs is derived under the same assumptions stated for the Cartesian-based translational dynamics. The chaser spacecraft dynamics are expressed through the Gauss planetary equations, enabling the incorporation of control inputs. Furthermore, a linearized model of the relative motion about the target orbit is derived, which simplifies analysis and control design for rendezvous and formation flying missions. Transformation matrices between Keplerian elements and the qns-ROEs are also provided to facilitate practical implementation, along with the transformation matrix from Cartesian coordinates to the QNS-ROE state, in order to extend the HCW-ROE framework to the eccentric case.

The solutions of the ROE equations are given by a state transition matrix that maps the natural motion and a particular solution depending on the nature of the control input (impulsive or continuous).

Overall, this ROE-based approach enhances the understanding and management of spacecraft relative motion, making it a valuable tool for mission planning, guidance, and control in multi-spacecraft operations.

3.2. Rotational dynamics

The rotational dynamics and attitude control of the chaser spacecraft during the final approach in a rendezvous mission are derived. A simplified relative attitude dynamics model is developed under the following assumptions: fixed target attitude relative to its LVLH frame, neglect of external disturbances, and alignment of the body frame with principal inertia axes. Using Euler angles and direction cosine matrices, the relative angular velocity and kinematics are derived. Linearized Euler's equations provide a state-space representation of the chaser's rotational motion relative to the target, facilitating control design. The model captures both linear time-varying behavior for eccentric orbits and linear time-invariant dynamics for circular orbits, depending on the target's angular velocity profile.

3.3. Guidance & Control

Guidance

This study compares two guidance strategies for the final approach phase in rendezvous and docking missions: an impulsive maneuver profile and a continuous thrust profile. The impulsive strategy models the spacecraft trajectory as a series of discrete velocity changes (impulses), using known orbital dynamics solutions to compute the required velocity increments to achieve the desired final state. A multi-impulse approach based on glideslope [6] guidance is employed, combining accelerating, coasting, and decelerating phases to ensure smooth velocity and position transitions.

Conversely, the continuous guidance profile applies smoothly varying accelerations along a trapezoidal velocity profile, maintaining a straight-line path and compensating for gravitational effects. Both approaches generate a reference trajectory that is tracked by an onboard feedback controller to ensure precise rendezvous.

The comparison focuses on assessing how well impulsive maneuvers integrate with continuous feedback control and their effects on mission constraints and propellant efficiency.

Feedback control

The state error derived from guidance and navigation data is used to compute control actions that align the satellite with its reference trajectory.

The **Rendezvous Control Problem** is modeled as a multi-input multi-output (MIMO) state-space system with six states and three control inputs [4]. Two main formulations are presented: the Cartesian-based dynamics is represented by a linear time-varying (LTV) system for eccentric orbits and by a linear time-invariant (LTI) system for circular orbits. The ROE-based dynamics is also described by an LTV system due to input matrix time dependence.

The **Linear Optimal Control Problem** [1, 7] utilizes a quadratic cost function balancing state regulation and control effort through weighting matrices. The resulting optimal control law is derived by solving the Differential Riccati Equation (DRE) or Algebraic Riccati Equation (ARE) for infinite horizon problems, producing a state feedback controller with constant gain.

The **Rendezvous Reference Tracking** is implemented by minimizing the deviation between the actual and reference states. The LQR control law is applied to the state error, incorporating a feedforward term from guidance. The control structure acts as a proportional-derivative (PD) controller, sufficient for rendezvous dynamics due to their integrator-like behavior, negating the need for integral action.

4. Rendezvous Simulator

The rendezvous simulator is structured around two main components:

The **Real World** models the physical environment and spacecraft dynamics. It simulates the absolute and relative translational and rotational states of both chaser and target satellites, integrating perturbations such as J_2 gravitational effects and aerodynamic drag. The dynamics are governed by nonlinear orbital mechanics and rigid-body rotational equations, providing realistic motion and disturbance effects. The relative states of the chaser with respect to the target are computed from the absolute states to enable accurate modeling of rendezvous dynamics.

The **On-board Software** processes simulated sensor data corrupted by noise and implements mission guidance and feedback control. Translational guidance uses pre-computed reference trajectories, while navigation is approximated by adding Gaussian noise to real states. Control commands are generated via feedback loops for both translation and attitude, and adapted for hardware constraints using pulse-width modulation to manage fixed-thrust thrusters and minimum impulse bit limitations.

Together, these components enable comprehensive simulation of spacecraft rendezvous maneuvers, bridging realistic physics with control system operation.

5. Simulations & Results

This study presents simulations and case studies evaluating RVD and formation flying guidance and control algorithms. The analysis focuses on the final approach phase, using both Cartesian and ROE dynamics for control. The International Space Station (ISS) is used as the target vehicle and Soyuz as the chaser, with realistic mass, geometry, and orbital data. Key scenarios include control gain tuning, impulsive and continuous guidance strategies, and formation transitions. The translational motion is controlled via modeled thrusters, while rotational dynamics assume ideal control application. The target's docking axis is misaligned with the LVLH frame to assess 3D guidance performance. Results highlight the trade-offs between tracking precision and actuator constraints, forming a foundation for more complex mission simulations.

5.1. RVD using Impulsive Guidance

This test evaluates an RVD manoeuvre using impulsive guidance in an eccentric orbit, comparing Cartesian and ROE-based control strategies.

The guidance system relies on a sequence of impulsive hops to approximate a straight-line trajectory, enabled by high node density. The overall manoeuvre spans 29.25 minutes and involves 41 thrust pulses. The resulting position and velocity profiles exhibit a parabolic–straight–parabolic and trapezoidal pattern, respectively, with small oscillations due to impulse discretization.

Control gains for both the Cartesian-based and ROE-based approaches are derived through the backward integration of the differential Riccati equation. These gains converge to a periodic steady-state solution, as illustrated in Figure 1 and Figure 2. The control gain matrix consists of 18 elements, structured as a 3×6 matrix. For illustrative purposes, only the (3,1) component is shown here, as it effectively captures the periodic behavior associated with the steady-state convergence from the backward integration. This gain component is plotted as a function of the number of orbits (N_{orb}) used in the integration process.

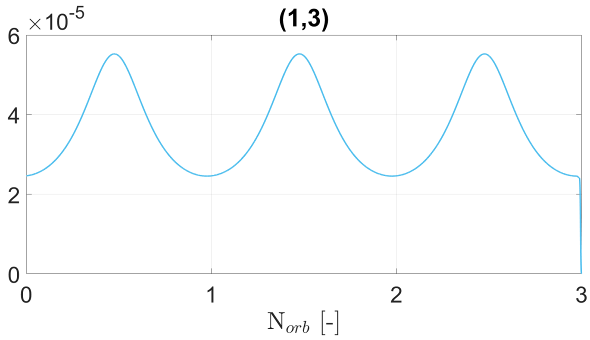

Figure 1: Cartesian control gain: DRE integration

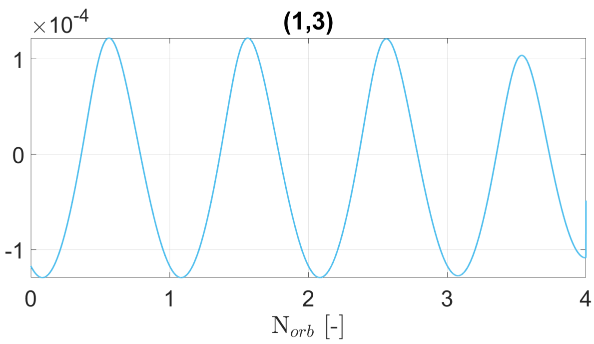

Figure 2: ROE control gain: DRE integration

Both controllers demonstrate centimeter-level tracking accuracy throughout the manoeuvre. Errors induced by discrete thrusting are minimal and corrected rapidly. The time evolution of the controlled Cartesian state is shown in Figure 3, Figure 4, and Figure 5.

Thrust commands are modulated using PWM logic and incorporate a minimum impulse bit (MIB) to ensure physical feasibility. These control inputs correspond well with the reference $\Delta\mathbf{V}$ impulses. Total propellant consumption is measured at 1.878 kg for the Cartesian controller and 1.936 kg for the ROE-based approach, with thrust vectors primarily directed in the $+x$ and $+z$ directions, aligning with the docking axis.

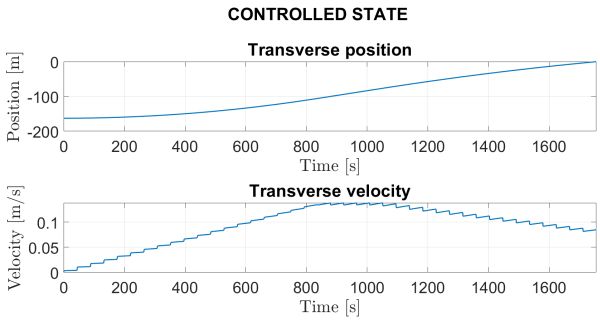

Figure 3: Time evolution of the Cartesian state (Part 1)

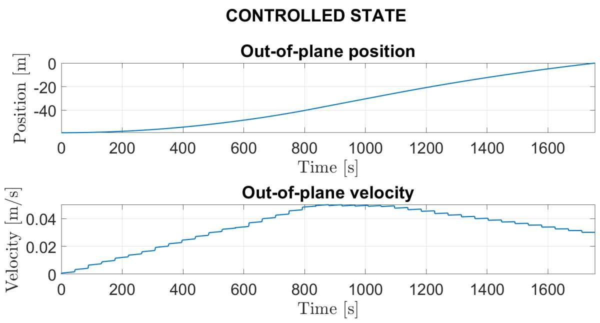

Figure 4: Time evolution of the Cartesian state (Part 2)

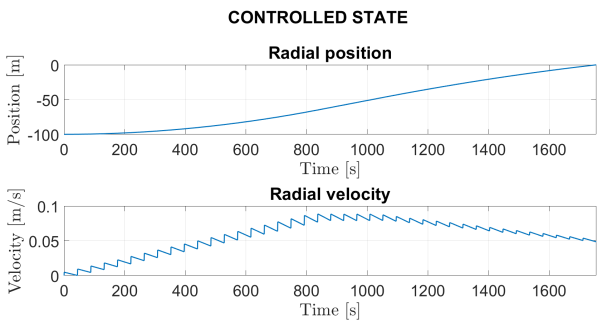

Figure 5: Time evolution of the Cartesian state (Part 3)

5.2. RVD using Continuous Guidance

This test evaluated the performance of the continuous guidance strategy for the final approach and docking phase. Unlike the impulsive guidance approach, the continuous method generates a smooth, straight line trajectory through a continuous acceleration profile. This profile follows an accelerating, coasting, decelerating pattern and leads to improved compatibility with thruster dynamics.

The maneuver duration was reduced to 370.32 s, demonstrating increased efficiency. Both the Cartesian-based and ROE-based controllers demonstrated effective tracking performance under continuous guidance. The time evolution of the controlled Cartesian state is shown in Figure 6, Figure 7, and Figure 8. The total propellant consumption was comparable between the two methods, with values of 5.752 kg and 5.761 kg, respectively. These results reflect the increased thruster activity necessitated by the continuous control approach, which favors smooth thrust commands over discrete impulses. The commanded thrust matched the reference acceleration closely in both cases.

While overall propellant consumption increased compared to the impulsive case, the smoother control commands resulted in improved system responsiveness and better exploitation of thruster capabilities. This confirms the effectiveness of continuous guidance for high-precision, short-duration rendezvous operations.

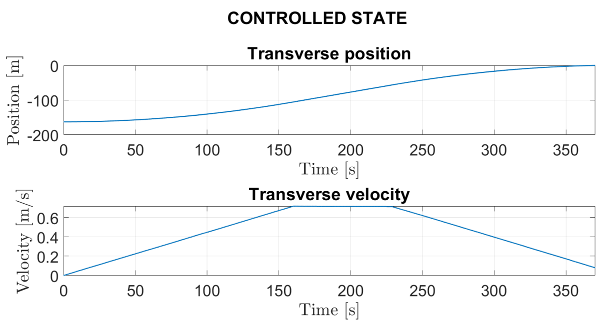

Figure 6: Time evolution of the Cartesian state (Part 1)

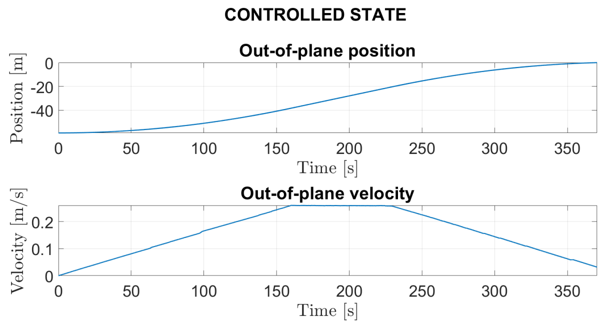

Figure 7: Time evolution of the Cartesian state (Part 2)

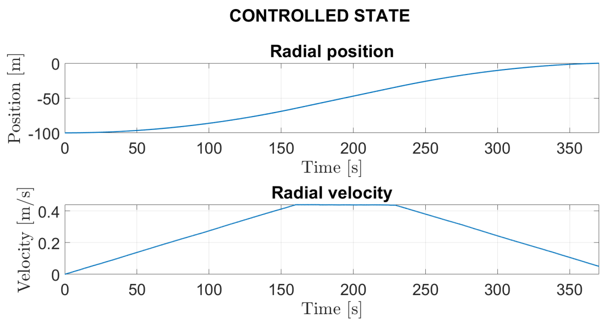

Figure 8: Time evolution of the Cartesian state (Part 3)

5.3. Formation Flying using Eccentricity/Inclination Separation Vector

This scenario addresses the challenge of maintaining a safe and controllable relative motion in the context of formation flying and proximity operations. To enhance long-term stability and avoid the geometric limitations of Cartesian coordinates, an approach based on relative orbital elements, particularly the eccentricity/inclination (e/i) vector separation [3], is adopted. This method is favored for its geometric insight and its ability to inherently prevent collisions, as it ensures that the spacecraft are not aligned in all three dimensions simultaneously.

The mission objective is to transfer from an initial state located along the target's orbit to a final e/i-separated configuration. The relative motion begins with an along-track offset and aims to achieve radial and cross-track separations in the final state. No predefined guidance trajectory is employed; instead, the controller is directly provided with the target

ROE state and is solely responsible for computing control actions based on the instantaneous tracking error.

The target is assumed to follow an eccentric orbit, and its angular velocity varies with the true anomaly. The chaser's motion is influenced by these orbital dynamics. The weight matrices used for tuning the controller gains follow the configuration adopted in the continuous guidance test case.

This setup highlights the capability of ROE-based feedback control to autonomously manage orbital reconfiguration tasks without the support of a reference trajectory, while ensuring safe separation throughout the maneuver. Over the course of four orbital periods, the controller successfully drives the chaser from its initial condition to the final configuration by regulating the quasi-nonsingular ROEs. The time history of the orbital elements (see Figure 9, Figure 10 and Figure 11) demonstrates convergence, while the control error remains bounded and progressively diminishes.

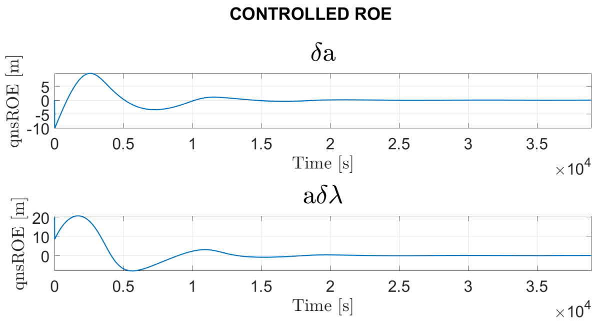

Figure 9: Time evolution of the ROE (Part 1)

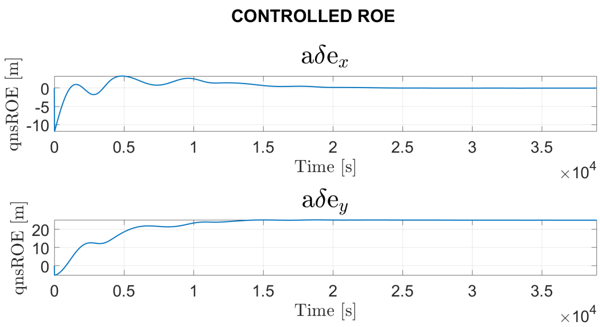

Figure 10: Time evolution of the ROE (Part 2)

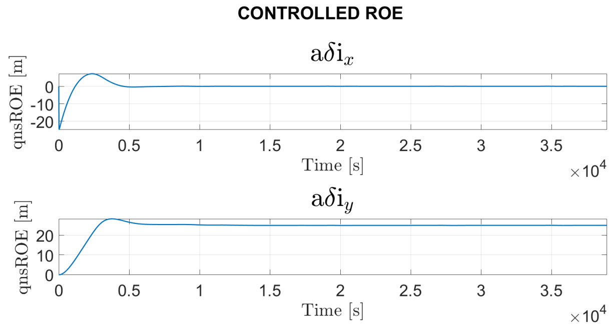

Figure 11: Time evolution of the ROE (Part 3)

6. Conclusions

This thesis investigates modeling, guidance, and control strategies for the final approach phase of Rendezvous and Docking missions, with extensions to Formation Flying. It introduces a dual-framework for translational dynamics (Cartesian and ROE), uncouples translational and rotational motion, and extends models to eccentric orbits. Both impulsive and continuous guidance strategies were developed, supported by LQR-based optimal control. A custom simulator was built to test integrated dynamics, control, and thruster models. The work also addresses attitude control for precise docking and demonstrates autonomous formation reconfiguration using feedback control.

The case studies confirm the effectiveness of the proposed guidance and control algorithms across various initial conditions and orbital regimes. Both impulsive and continuous strategies achieved accurate docking, with impulsive maneuvers offering greater fuel efficiency and continuous guidance enabling faster, more flexible operations. The choice between them depends on mission-specific trade-offs between time and fuel. The Cartesian and ROE-based control frameworks both performed well. The ROE approach showed strong tracking and is well-suited for formation flying and long-term planning, while the Cartesian model offered intuitive control during close-proximity operations. Notably, the ROE-based controller achieved comparable precision in the final approach phase. Attitude control successfully aligned the chaser with the target, ensuring correct docking orientation. A simplified pulse-modulation scheme demonstrated compatibility with discrete propulsion systems. Additionally, ROE-based methods enabled safe and effective formation reconfigurations, supporting e/i separation with collision avoidance.

7. Future Work

This study lays the groundwork for future research in several areas, including the incorporation of environmental disturbances (*e.g*, J_2 effects, drag), the implementation of robust control methods like $\mathcal{H}_\infty$, and the integration of realistic navigation filters. Further directions include the development of hybrid guidance strategies combining impulsive and continuous control, as well as extending the model to account for coupled translational and rotational dynamics for more general and realistic mission scenarios.

8. Acknowledgements

I would like to express my sincere gratitude to my academic advisor, Prof. Gabriella Vittoria Maria Gaias, and to my company supervisors, Stefano Torresan and Thomas Peters, for their guidance and support throughout this thesis. I am also grateful to the AOCS & GNC department at OHB SE System AG for the opportunity to work in such a dynamic and inspiring environment.

A deep thank you goes to my family and friends for their unwavering moral support throughout my academic journey; your presence has been a cornerstone of my achievements.

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