# ASSIGNMENT 5

Financial Engineering

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## 1 Point 0

We computed the daily $VaR$ and $ES$ with a 3y estimation via Gaussian parametric approach, considering an equally weighted portfolio with notional €10 MIO and significance level $\alpha = 0.95$. First, we calculated the logreturns, their mean and their variance, therefore the mean and the variance of the losses of the portfolio as:

$$\underline{\mu_{portfolio}} = -\underline{w_t} \cdot \underline{\mu}$$

$$\sigma^2_{portfolio} = \underline{w_t} \cdot \underline{\underline{\Sigma}} \cdot \underline{w_t}$$

where:

- $\underline{w_t}$ is the vector of the weights of the portfolio
- $\mu$ is the mean of the returns
- $\underline{\underline{\Sigma}}$ is the covariance matrix of the returns

So, we determined the VaR and the ES as:

$$VaR_{\alpha,portfolio} = \delta\mu + \sqrt{\delta}\sigma Var^{std}_{\alpha}$$

$$ES_{\alpha,portfolio} = \delta\mu + \sqrt{\delta}\sigma ES^{std}_{\alpha}$$

where:

- $VaR^{std}_{\alpha} = N^{-1}(\alpha)$
- $ES^{std}_{\alpha} = \frac{\Phi(N^{-1}(\alpha))}{1-\alpha}$ where $N$ and $\Phi$ are respectively the cdf and the pdf of the standard normal distribution

We obtained:

| $ValueatRisk(VaR)$ | $ExpectedShortfall(ES)$ |
|---|---|
| 147027.4829 | 185698.0435 |

# 2 Point 1

## 2.1 Section A

We computed daily VaR and ES with a 3y estimation for the portfolio 1 (with significance level $\alpha = 0.99$) in two different ways.

**HISTORICAL SIMULATION** We determined the weights of the portfolio considering the number of stocks $n_{i,t}$ and their today's value $S_i(t)$ (20th of March 2019) and the value of the portfolio $V(t)$:

$$w_{i,t} = \frac{n_{i,t} \cdot S_i(t)}{V(t)}$$

We computed logreturns and the losses of the portfolio, then we sorted them. We found the index of the largest integer that doesn't exceed $n \cdot (1 - \alpha)$, so we determined the VaR of the portfolio as the loss corresponding to that index and the ES of the portfolio as the mean of the losses:

$$Var_\alpha = L^{(n(1-\alpha),n)}$$

$$ES_\alpha = mean(L^{i,n}, i = [n(1-\alpha), ....., 1])$$

We obtained:

| $ValueatRisk(VaR)$ | $ExpectedShortfall(ES)$ | $PlausibilityCheck$ |
|---|---|---|
| 130182.33240 | 187986.81971 | 131141.39227 |

**STATISTICAL BOOTSTRAP** We proceeded as before but instead of the totality of the realized returns we considered just the ones corresponding to the indexes that we generated randomly and with replacement (with 200 simulation). We obtained:

| $ValueatRisk(VaR)$ | $ExpectedShortfall(ES)$ | $PlausibilityCheck$ |
|---|---|---|
| 123080.41371 | 254617.14503 | 128869.99214 |

## 2.2 Section B

We computed the daily VaR and ES with a 3y estimation for the portfolio 2 with the Weighted Historical Simulation approach (with $\lambda = 0.97$ and significance level $\alpha = 0.99$). We computed the logreturns and the losses of the portfolio then we determined the weights of the losses as:

$$w = c\lambda^t$$

where c is given by

$$c = \frac{1-\lambda}{1-\lambda^n}$$

We sorted the losses and we found the largest value $i^*$ such that:

$$\sum_{i=1}^{i^*} w_i \leq 1 - \alpha$$

so we determined the VaR and the ES of the portfolio as

$$VaR_\alpha = L^{i^*,n}$$

$$ES_\alpha = \frac{\sum_{i=1}^{i^*} w_i L^{(i,n)}}{\sum_{i=1}^{i^*} w_i}$$

We obtained:

| $ValueatRisk(VaR)$ | $ExpectedShortfall(ES)$ | $PlausibilityCheck$ |
|---|---|---|
| 140137.80478 | 180280.70420 | 170822.85606 |

## 2.3 Section C

Considering an equally weighted portfolio of the first 20 companies in the provided csv we computed the 10 days VaR and ES with 3 years estimation using the dataset provided via a Gaussian paramatric PCA approach using the first n components where n = 1,...,6. We proceed as follows:

- computed $\Sigma$ the covariance matrix of the returns
- decompose $\Sigma$ as $\Sigma = \Gamma\Lambda\Gamma'$ where $\Gamma$ is the matrix containing ordered orthonormal eigenvectors of $\Sigma$ and $\Lambda$ is the diagonal matrix containing its ordered eigenvalues
- considered $\underline{\hat{\mu}} = \Gamma'\underline{\mu}$ and $\underline{\hat{w}} = \Gamma'\underline{w}$
- computed $\sigma^2_{red} = \sum_{i=1}^{k} \hat{w_i^2}\lambda_i$ and $\mu_{red} = \sum_{i=1}^{k} \hat{w}_i\hat{\mu}_i$ where k is the number of principal components considered

Then we computed the request VaR and ES using the following formulas:

$$VaR_\alpha = Notional \cdot (-\Delta\mu^{red} + \sqrt{\Delta}\sigma^{red} VaR_\alpha^{std})$$

$$ES_\alpha = Notional \cdot (-\Delta\mu^{red} + \sqrt{\Delta}\sigma^{red} ES_\alpha^{std})$$

where $VaR_\alpha^{std} = N^{-1}(\alpha))$ and $ES_\alpha^{std} = \frac{1}{1-\alpha} \cdot \Phi(N^{-1}(\alpha))$.
We obtained the following results:

| | $VaR$ | $ES$ |
|---|---|---|
| $k = 1$ | 630818.85486 | 758352.59216 |
| $k = 2$ | 629485.54991 | 764377.31748 |
| $k = 3$ | 632626.73221 | 762179.21583 |
| $k = 4$ | 631529.87660 | 764484.66921 |
| $k = 5$ | 631430.24950 | 764834.73413 |
| $k = 6$ | 631737.82774 | 764577.40224 |
| Plausibility Check | 719911.55479 | |

Plotting the value of the VaR and ES with respect to different number of components we obtained:

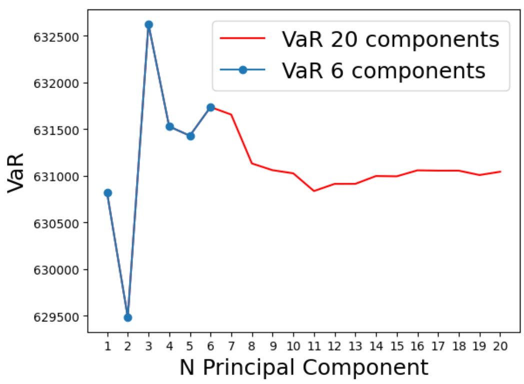


Figure 1: VaR using PCA

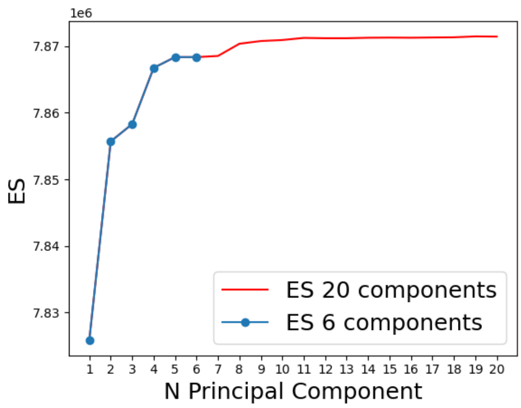


Figure 2: ES using PCA

We can notice that the biggest oscillation for the VaR value is of the order of 0.5% and the biggest error with respect to the VaR computed with all the data is 0.25%. So the results are acceptable even considering a smaller number of principal components.

### 2.4 Section D

We tested the values we obtained for the VaR with the plausibility check. For the plausibility check we proceeded as follows: after deriving the logreturns, we computed the lower and the upper percentiles

$$l_i = VaR_{X_i}(1-\alpha)$$

$$u_i = VaR_{X_i}(\alpha)$$

We then calculated the signed VaR as:

$$sVaR = sens_i \cdot \frac{|l_i| + |u_i|}{2}$$

where as sensitivities we used the weights of the stocks. We obtained the estimation of the VaR as:

$$VaR = (-\delta\mu + \sqrt{\delta}\sqrt{sVaR \cdot C \cdot sVaR})V$$

where C is the correlation matrix and V is the portfolio value.

# 3 Point 2

We computed the VaR via Full Monte-Carlo, Delta and Delta-Gamma Normal approaches for a portfolio composed by Vonovia stocks for 25.870.000€(i.e. 1000000 stocks) and the same number of put options. Using BS for the put price, we obtained:

| $PortfolioValue$ € |
|---|
| $2.618816e + 07$ |

We simulated the stock prices at $t + \Delta$ as:

$$S_{t+\Delta} = S_t e^{X_t + \Delta}$$

and we calculated the put prices at t and t+$\Delta$ using BS. Then we computed the losses for both the stock and the put as:

$$L_p(X_t; \Delta) = -n_p(P(t + \Delta) - P(t))$$

$$L_S(X_t; \Delta) = -n_S(S_{t+\Delta} - S_t)$$

We then derived the losses of the portfolio via :

$$L_{portfolio} = L_p + L_S$$

and we sorted them, so we found the largest integer s.t. doesn't exceed $n \cdot (1 - \alpha)$ and finally we determined the VaR of the portfolio as:

$$Var_\alpha = L^{(n(1-\alpha),n)}$$

obtaining:

| $VaR_{Portfolio}$ (€) |
|---|
| 1171723.9665230056. |

For the Delta normal method, we considered the first order expansion of the loss of the portfolio:

$$L(X_t) = -\sum_{i=1}^{d} sens_i(t) X_{t,i}$$

where the sensitivity is $sens_i(t) = \delta_i S_i(t)$

So, we computed the BS Delta of the put (the Delta of the stock is equal to 1). We then proceeded via HS and found that:

| $VaR_{DeltaNormal}$ (€) |
|---|
| 3281411.5805413723 |

We also implemented the Delta-Gamma method, where we considered a second order expansion:

$$L(X_{t,i}) = -[(n_p \delta_p + n_s) S_t X_{t,i} + n_p \gamma_p S_t^2 X_{t,i}^2]$$

where $\gamma = \gamma_{put}$

After computing the BS Gamma of the put (the Gamma of the stock is equal to 0), we followed the scheme of the previous point and we obtained:

| $VaR_{DeltaGammaNormal}$ (€) |
|---|
| 1211409.2899795123 |

# 4 Point 3

To price each payment we utilised the following closed formula:

$$Price_i = \frac{B(t_0, t_i)}{B(t_0, t_{i-1})} \cdot (\frac{B(t_0, t_{i-1})}{B(t_0, t_i)} N(d_1) - N(d_2))$$

$$d_1 = \frac{ln(\frac{B(t_0,t_{i-1})}{B(t_0,t_i)}) + \frac{\sigma^2}{2}(t_i - t_{i-1})}{\sigma \sqrt{t_i - t_{i-1}}}$$

$$d_2 = d_1 - \sigma \sqrt{t_i - t_{i-1}}$$

$$Price_{OPTION} = S_0 \sum_i Price_i$$

The formula was obtained following the Black and Scholes model.

We obtained the following results:

| | $Price$ (€) |
|---|---|
| Cliquet with analytical formula | 22566654.38051 |
| Cliquet with analytical formula considering default | 22228314.82092 |

We figured that ISP would price the option without regarding their possibility to default, but as buyer we need to take in to account the fact that ISP could fail and as a consequence not pay the option.