THE UNIVERSITY OF CHICAGO

How to Weigh a Black Hole: Physics 386 Final Project

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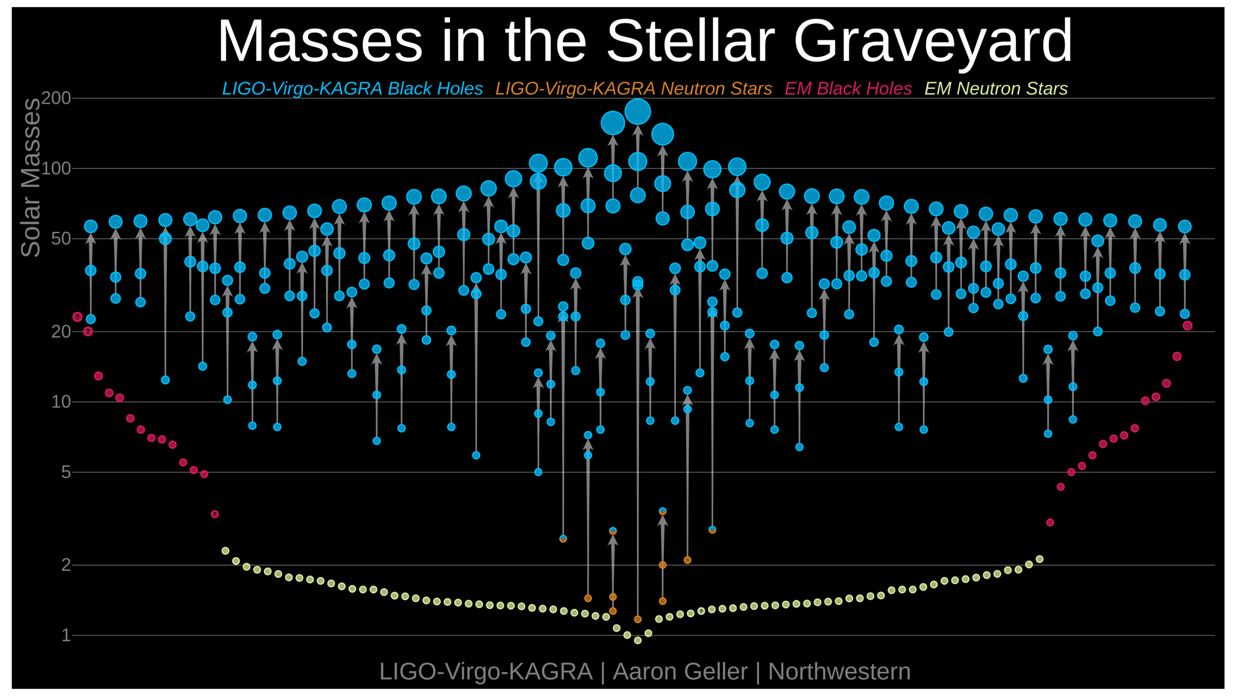
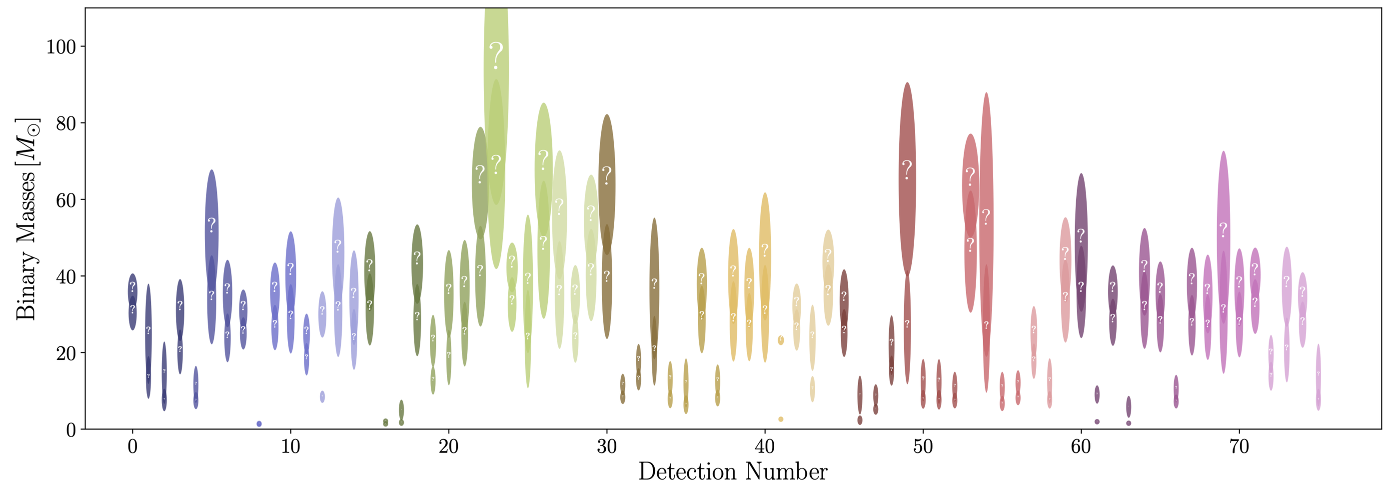
What Happens When Black Holes Collide

Black holes are the remnants of dying stars that collapse under their own weight.

On September 14, 2015, the LIGO-Virgo-KAGRA (LVK) Collaboration made its first confirmed detection of two merging black holes–GW150914. The merger was so energetic that the mass of three Suns (3 $M_\odot$) were converted directly into vibrations of space-time itself in the form of gravitational waves. As of 2023, there have been 90 such gravitational-wave detections [1].



Shown above is a schematic (no horizontal axis) of compact binary mergers detected by the LVK Collaboration. While masses are often represented as dots with single values, a more accurate representation would depict the uncertainties in our detections, as shown below for the binary progenitors shown below [2].



Overview

Project Goal

As a data analysis practicum (not for publication), the aim of this project is to simulate the statistical analysis techniques used in the LVK's first gravitational-wave detection, GW150914.

How We Estimate Parameters

Although black holes do have specific masses, in practice our measurement of the masses of two merging black holes requires the measurement of random processes requiring estimation rather than direct measurement of unknown physical parameters.

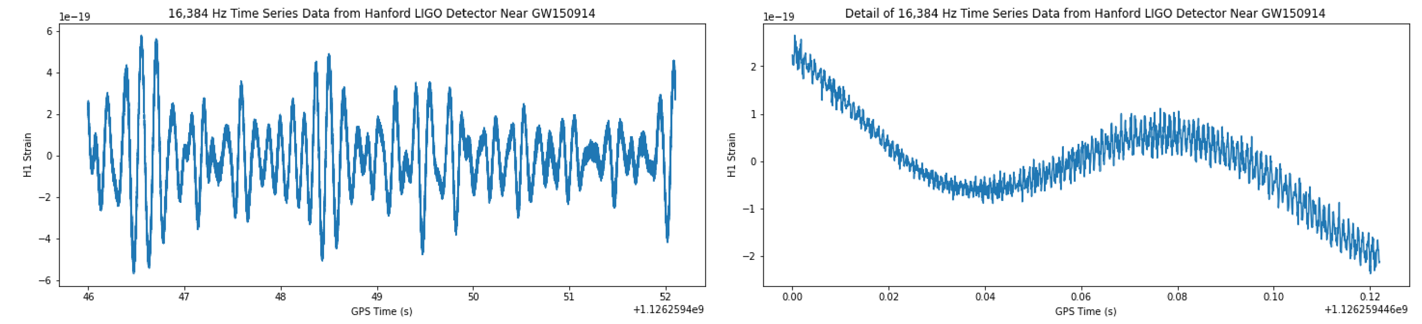
Astrophysical parameter estimation of the masses of merging black holes requires:

1. **Measuring the noise** to better distinguish it from the signal
2. **Creating models** representing a noiseless signal
3. **Calculating the likelihood** that a given signal is more than just noise, and using that likelihood to estimate the unknown parameters and confidence intervals

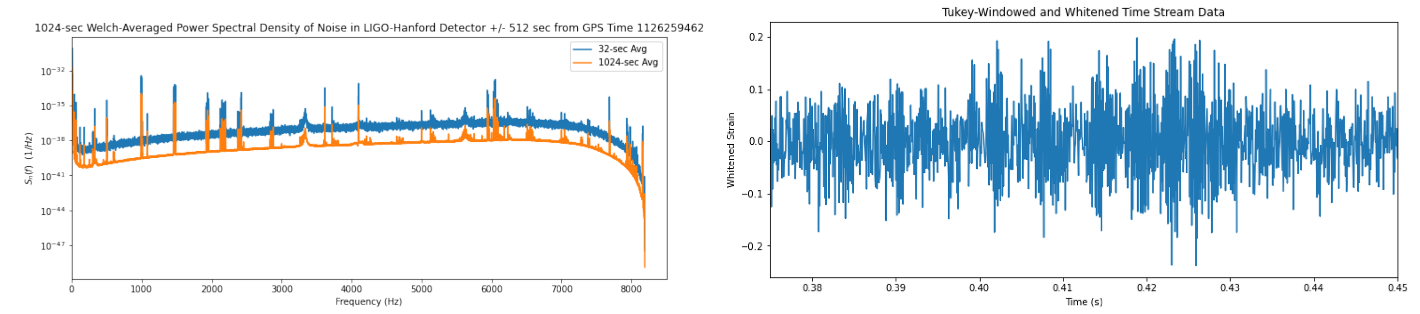
1. Finding a Signal in the Noise

The first task of analysis is to detect a signal within the noise [3]. Shown here is calibrated time stream data from the LIGO detector in Hanford, Washington. The observed variation in the data is due solely to the time-dependent noise which varies with frequency (left).

The physically meaningful signal we want to detect is visible in a close-up view of the time stream data (right).



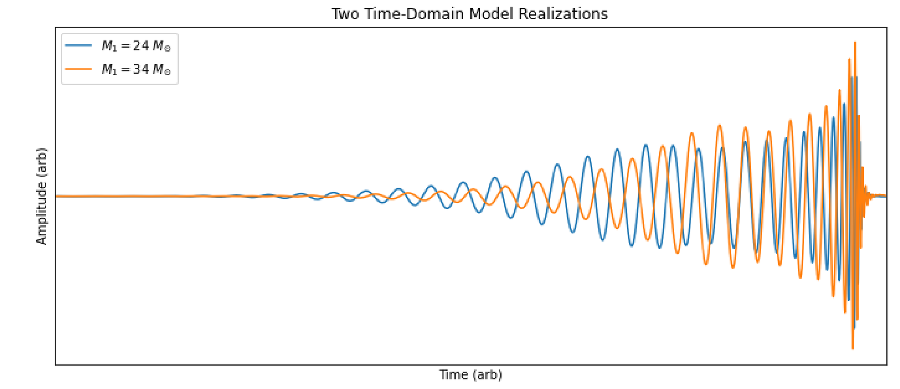
To extract the signal and characterize the features of the noise, we Fourier transform the (Tukey-windowed) time-domain data into the frequency domain. We characterize the noise by Welch averaging over two separate segments of data centered on GW150914 (32 s and 1024 s) in 4-second intervals, each interval overlapping the previous interval (by 3 s and 2 s, respectively). This averaging produces a stable representation of the detector noise (below left).

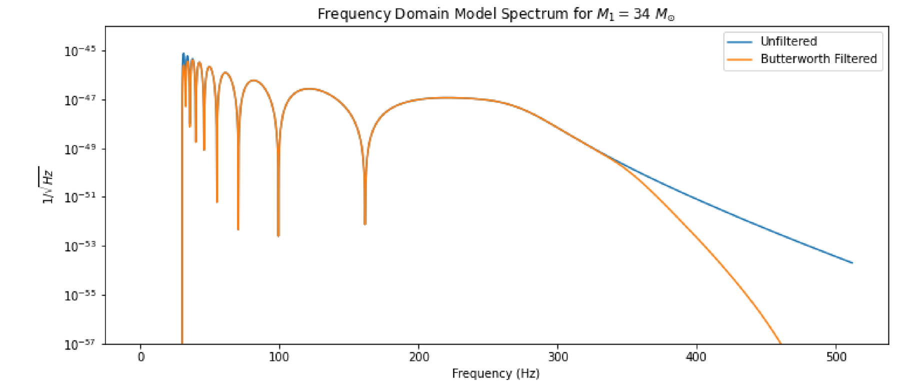


Having characterized the noise in the form of a power spectral density (PSD) we can "whiten" the (windowed) frequency-domain signal and inverse Fourier transform back to the time domain where a signal is observed (above right).

2. Creating a Model for Comparison

Using numerical relativity simulations we can generate time-domain waveforms and frequency-domain spectra. One hundred such model realizations were generated for comparison with our observed data. For each realization, the primary mass (of the larger black hole, M_1) was varied between 24 and 44 $M_\odot$ by increments of 0.2 $M_\odot$. The other parameters were set to LVK's published values [4].





3. Estimating the Primary Mass, M_1

To obtain an estimate of a given parameter θ, which in this case is the primary mass M_1, we treat the parameter as a variable and calculate the posterior probability $p(\theta|\mathsf{d},\mathsf{M},\mathsf{I})$ of obtaining a particular value of the parameter θ given our data **d**, our model **M**, and detector **I** [5]:

$$p(\theta|\mathsf{d},\mathsf{M},\mathsf{I}) = \frac{p(\theta|\mathsf{M},\mathsf{I})\ p(\mathsf{d}|\theta,\mathsf{M},\mathsf{I})}{p(\mathsf{d}|\mathsf{M},\mathsf{I})}$$

where the likelihood is given by

$$p(\mathsf{d}|\theta) = \exp\left(-\frac{1}{2}\Sigma_I\left[(\mathsf{d}_I - \mathsf{h}_I(\theta)|\mathsf{d}_I - \mathsf{h}_I(\theta)) + \int \ln(\mathsf{S}_n^I(f))df\right]\right)$$

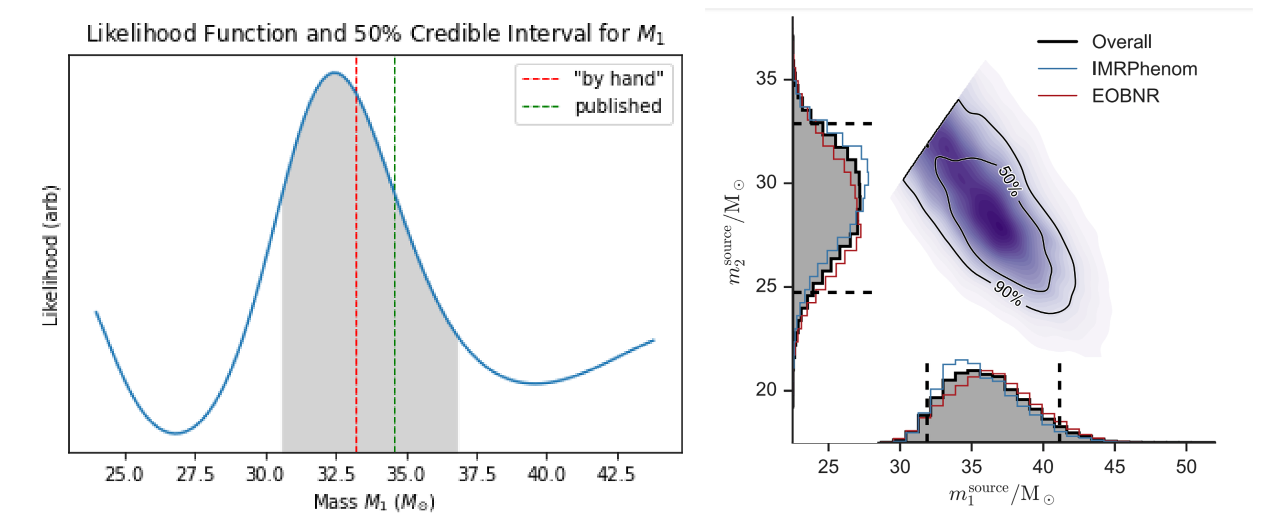
with the noise-weighted inner product defined as

$$(\,(\mathsf{d}_I - \mathsf{h}_I(\theta))\,|\,(\mathsf{d}_I - \mathsf{h}_I(\theta))\,) = 4\int_0^{f_{max}} \frac{|(\mathsf{d}_I - \mathsf{h}_I(\theta))|^2}{\mathsf{S}_n(f)}df$$

Given our 100 realizations of our model, we calculate for each one the likelihood that the data we observe could be produced by the mass value in our model in the LIGO Hanford detector. With those 100 likelihood values, we produce a likelihood function (below) that is used to determine the maximum likelihood value and Bayesian credible intervals.

Results and Future Work

Our measurement of the primary mass of GW150914, $M_1 = 33.2^{+3.7}_{-2.7}$, is in good agreement with LVK's published value of $M_1 = 34.6^{+4.4}_{-2.6}$. I'm eager to explore Markov Chain Monte Carlo techniques used for estimating parameters in multiple dimensions, as shown in this published corner plot [6].



Acknowledgments

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References

[1] A. Geller (2022) [2] T. Callister (2023) [3] P. Saulson (1994) [4] https://gwosc.org/eventapi/html/GWTC-1-confident/GW150914/v3 [5] LVC B. P. Abbott+ Class. Quantum Grav. (2020) [6] LVC B. P. Abbott+ PhysRevLett (2016)

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