

Marco Reina 7066486  
Samuele Serri 7069839

1. **a)**

if  $\lambda \rightarrow 0$  then the solution of (2.1) and (2.2) will be the same as minimizing RSS.

if  $\lambda \rightarrow \infty$  then the coefficients  $\beta_j$  for  $j = 1, \dots, p$  will be close to zero.

**b)**

Lasso regularization has the property of setting some coefficients exactly to zero with a  $\lambda$  large enough, therefore if our goal is to do variable selection we should use  $L_1$  regularization.

2. **a)**

$$f(Y|X, \beta) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|x_i|}{\sigma_2}\right)$$

**b)**

$$p(\beta) = \prod_{j=1}^p \frac{1}{2b} \exp\left(-\frac{|\beta_j|}{b}\right)$$

The posterior distribution takes the form:

$$p(\beta|X, Y) \propto f(Y|X, \beta)p(\beta|X) = f(Y|X, \beta)p(\beta) \quad [1]$$

$$\begin{aligned}
p(\beta|X, Y) &= \prod_{i=1}^n \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|x_i|}{\sigma^2}\right) \prod_{j=1}^p \frac{1}{2b} \exp\left(-\frac{|\beta_j|}{b}\right) = \\
&= \prod_{i=1}^n \prod_{j=1}^p \frac{1}{\sqrt{2\pi\sigma^2}2b} \exp\left(\frac{-|x_i|}{\sigma^2} + \frac{-|\beta_j|}{b}\right)
\end{aligned}$$

**c)**

[1] The formula for the posterior distribution was taken from the book *James, G., Witten, D., Hastie, T., & Tibshirani, R. (2013). An introduction to statistical learning* p. 251