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a)

Given:

$$l(\beta) = \sum_{i=1}^{n} y_i \log\left(\frac{e^{f(x_i;\beta)}}{1+e^{f(x_i;\beta)}}\right) + (1-y_i)log\Big(1-\frac{e^{f(x_i;\beta)}}{1+e^{f(x_i;\beta)}}\Big)$$

We compute $\nabla_\beta l(\beta)$ as:

$$\nabla_\beta l(\beta) = \nabla_\beta \sum_{i=1}^{n} y_i\Big[f(x_i;\beta) - log(1+e^{f(x_i;\beta)})\Big] + (1-y_i)(-log(1+e^{f(x_i;\beta)})) =$$

$$= \nabla_\beta \sum_{i=1}^{n} y_i f(x_i;\beta) - y_i log(1+e^{f(x_i;\beta)}) + y_i log(1+e^{f(x_i;\beta)}) - log(1+e^{f(x_i;\beta)}) =$$

$$= \nabla_\beta \sum_{i=1}^{n} y_i f(x_i;\beta) - log(1+e^{f(x_i;\beta)}) =$$

$$= \sum_{i=1}^{n} y_i \nabla_\beta f(x_i;\beta) - \frac{e^{f(x_i;\beta)}\nabla_\beta f(x_i;\beta)}{1+e^{f(x_i;\beta)}}$$

if $f(x_i;\beta) = \beta_0 + x_{i1}\beta_1 + x_{i2}\beta_2$ then:

$$\nabla_\beta f(x_i;\beta) = \begin{pmatrix} 1 \\ x_{i1} \\ x_{i2} \end{pmatrix}$$

and

$$\nabla_\beta l(\beta) = \begin{pmatrix} \partial_{\beta_0} l(\beta) \\ \partial_{\beta_1} l(\beta) \\ \partial_{\beta_2} l(\beta) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} y_i - \frac{e^{f(x_i;\beta)}}{1+e^{f(x_i;\beta)}} \\ \sum_{i=1}^{n} y_i x_{i1} - \frac{e^{f(x_i;\beta)}x_{i1}}{1+e^{f(x_i;\beta)}} \\ \sum_{i=1}^{n} y_i x_{i2} - \frac{e^{f(x_i;\beta)}x_{i2}}{1+e^{f(x_i;\beta)}} \end{pmatrix}$$

b)

c)