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$$X = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 4 & 2 \\ 1 & 5 & 3 \end{pmatrix}, \hat{\beta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix}, y = \begin{pmatrix} 5 \\ 6 \\ 9 \\ 10 \\ 13 \end{pmatrix}$$

$$y_i = \sum_{i=1}^{5} X_{(i)}\hat{\beta}$$

Where $X_{(i)}$ denotes the i-th row of X.

$$e_i = y_i - X_{(i)}\hat{\beta}$$

$$RSS = \|y - X\hat{\beta}\|_2^2 = \sum_{i=1}^{5}(y_i - X_{(i)}\hat{\beta})^2 = \sum_{i=1}^{5} y_i^2 + (X_{(i)}\hat{\beta})^2 - 2X_{(i)}\hat{\beta}y_i$$

To find the optimal parameters we compute $\nabla_{\hat{\beta}}$ and set it to 0.

$$\nabla_{\hat{\beta}} = \begin{pmatrix} \frac{\partial RSS}{\partial \hat{\beta}_0} \\ \frac{\partial RSS}{\partial \hat{\beta}_1} \\ \frac{\partial RSS}{\partial \hat{\beta}_2} \end{pmatrix}$$

$$\partial_{\hat{\beta}_0} RSS = 2\sum_{i=1}^{5} x_{i,1}(X_{(i)}\hat{\beta} - y_i)$$

$$\partial_{\hat{\beta}_1} RSS = 2\sum_{i=1}^{5} x_{i,2}(X_{(i)}\hat{\beta} - y_i)$$

$$\partial_{\hat{\beta}_2} RSS = 2\sum_{i=1}^{5} x_{i,3}(X_{(i)}\hat{\beta} - y_i)$$

$\nabla_{\hat{\beta}} = 0$ leads to the system:

$$\begin{pmatrix} 5 & 15 & 11 \\ 15 & 55 & 36 \\ 11 & 36 & 27 \end{pmatrix} \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{pmatrix} 43 \\ 149 \\ 102 \end{pmatrix}$$

With solution $\begin{pmatrix} 156/95 \\ 169/95 \\ 14/19 \end{pmatrix}$

To compute R^2 we use the formula:

$$R^2 = 1 - \frac{\sum_i^5 e_i^2}{\sum_i^5 (y_i - \overline{y})^2}$$

$$R^2 = 1 - \frac{\|X\hat{\beta} - y\|_2^2}{\|X\hat{\beta} - mean(y)\|_2^2} \approx 0.9359$$