

# Midterm #1 Topics Study Guide

## Math 181 (Discrete Structures), Fall 2022

### 1. Sets [§1.1]

- (a) sets of numbers (integers  $\mathbb{Z}$  and real numbers  $\mathbb{R}$ ), set-builder notation, subsets ( $A \subseteq B$ )
- (b) operations of union ( $A \cup B$ ), intersection ( $A \cap B$ ), difference ( $A \setminus B$ ), complement ( $A^c$ )
- (c) representing sets via Venn diagrams

### 2. Logical propositions [§1.2, 1.3]

- (a) operations of “or” ( $p \vee q$ ), “and” ( $p \wedge q$ ), “not” ( $\neg p$ )
- (b) truth tables for compound propositions
- (c) conditional a.k.a. implication a.k.a. “if... then...” ( $p \rightarrow q$ )
- (d) biconditionals ( $p \leftrightarrow q$ ) and logical equivalence ( $\equiv$ )
- (e) converse  $q \rightarrow p$  and contrapositive  $\neg q \rightarrow \neg p$  of an implication  $p \rightarrow q$   
(contrapositive is logically equivalent to original implication; converse is not!)

### 3. Logical arguments [§1.4]

- (a) converting an argument from words to symbolic form and vice-versa
- (b) proving validity using truth tables
- (c) proving validity using the rules of inference and logical equivalences
- (d) common forms of invalid arguments a.k.a. fallacies

### 4. Quantifiers [§1.5, 1.6]

- (a) propositional formulas ( $P(x)$ ) and domains of discourse ( $D$ )
- (b) universal ( $\forall x P(x)$ ) and existential ( $\exists x P(x)$ ) quantifiers
- (c) DeMorgan’s Laws:  $\neg(\forall x P(x)) \equiv \exists x \neg P(x)$  and  $\neg(\exists x P(x)) \equiv \forall x \neg P(x)$
- (d) nested quantifiers and order of quantifiers ( $\forall x \exists y P(x, y) \not\equiv \exists y \forall x P(x, y)$ )

### 5. Proofs [§2.1]

- (a) two basic mathematical systems: the theory of integers; the theory of sets
- (b) direct proofs for theorems of form “ $\forall x_1, \dots, x_n$  if  $P(x_1, \dots, x_n)$  then  $Q(x_1, \dots, x_n)$ ”
- (c) counterexamples to universally quantified statements