Quiz #12, Due: 11/30
Math 181 (Discrete Structures), Fall 2022

Problem 1 is worth 6 points (2 pts each part), and Problem 2 is worth 4 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

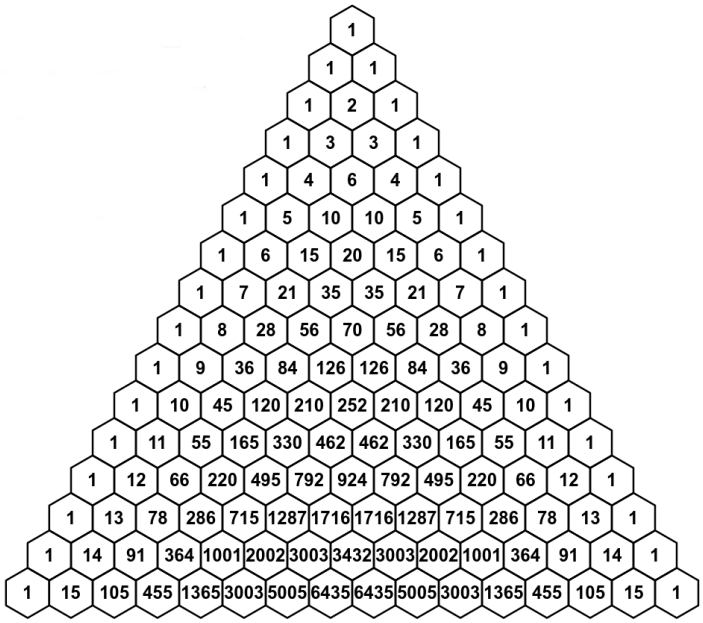
1. Recall that in class we proved the Binomial Theorem: $(x+y)^n = \sum_{k=0}^{n} C(n,k)x^k y^{n-k}$. Use the binomial theorem to prove the following identities for the binomial coefficients $C(n,k)$:

   (a) $\sum_{k=0}^{n} 2^k \cdot C(n,k) = 3^n$

   (b) $\sum_{k=0}^{n} (-1)^{n-k} \cdot 2^k \cdot C(n,k) = 1$

   (c) $\sum_{k=0}^{n} k \cdot C(n,k) = n \cdot 2^{n-1}$
   Hint: take the *derivative* of the binomial theorem, with respect to the variable x.

2. We saw how the $C(n,k)$ form Pascal's Triangle. Here are the first 16 rows of Pascal's Triangle:



   Fill in all of the odd values in this triangle, and leave the even values unfilled. Describe the resulting pattern that you see.