Dynamical Algebraic Combinatorics notes [outline]

Sam Hopkins

Chapter 0. Introduction

- §0.1 Philosophy of DAC
- §0.2 The ur examples: rotation of subsets and multisets
- §0.3 Overview of rest of notes

Chapter 1. Tableaux

- §1.1 Young diagrams, semistandard Young tableaux, Gelfand-Tsetlin patterns
- §1.2 Bender-Knuth involutions, promotion, evacuation
- §1.3 Jeu de taquin, promotion and evacuation again
- §1.4 Proof (using evacuation & jdt) that promotion of rectangular SSYTs has order n
- §1.5 Standard tableaux
- §1.6 Models for promotion of 2- and 3-rowed SYT promotion

Chapter 2. Posets

- §2.1 Basics about posets, linear extensions, promotion and evacuation
- §2.2 The rectangle redux; embedding (linear ext's of) the two triangles into the rectangle
- §2.3 Order ideals, rowmotion, toggles
- §2.4 P-partitions, piecewise linear toggles and rowmotion; the Stanley–Thomas word
- §2.5 Conjugacy of rect. promotion & rowmotion (via GT pat's); more triangle embeddings
- §2.6 The order polytope and PL maps

Chapter 3. Coxeter groups and root systems

- §3.1 Basics about Coxeter groups and root systems
- §3.2 Root posets
- §3.3 Weak order and the Edelman–Greene bijection (Type A)
- §3.4 Absolute order and the Armstrong–Stump–Thomas bijection (Type A)
- §3.5 Minuscule posets
- §3.6 Parabolic quotients and rowmotion (à la Rush–Shi)

Chapter 4. Cyclic sieving

- §4.1 Definition of cyclic sieving and ur examples via exterior/symmetric power
- §4.2 The Grassmannian and its coordinate ring, standard monomials
- §4.3 Involutions on the Grassmannian and "q=-1" phenomenon for plane partitions
- §4.4 Canonical bases & cyclic sieving for SSYT promotion (discussion only)
- §4.5 Invariant tensors, Schur–Weyl duality
- §4.6 Cyclic sieving for SYT promotion (discussion only)

Chapter 5. Invariance and homomesy

- §5.1 Invariants, cyclic descents for SYTs, the “OY invariant” for Type A root poset
- §5.2 Homomesy, basic homomesies from Stanley–Thomas word bijection
- §5.3 Symmetry of Narayana numbers, the Lalanne–Kreweras involution, rowvacuation
- §5.4 The toggleability statistics technique
- §5.5 PL homomesies

Chapter ∞. Conclusion and further topics

- §∞.1 The RSK Algorithm
- §∞.2 Birational dynamics
- §∞.3 Rowmotion beyond distributive lattices
- §∞.4 Conclusion