Midterm #2, 11/15
Math 156 (Calculus I), Fall 2022

Each problem is worth 10 points, for a total of 50 points. Remember to *show your work* on all problems!

1. Compute the derivative $f'(x)$ for the following functions $f(x)$:

   (a) $f(x) = \sqrt{x^3}$

   (b) $f(x) = xe^x$

   (c) $f(x) = \cos(2x)$

   (d) $f(x) = \ln(x^2)$

2. Consider the curve in the x, y-plane defined by the implicit equation $x^2 + xy + y^2 = 1$. Compute the slope of the tangent to this curve at the point $(x, y) = (1, 0)$.

3. Let $f \colon \mathbb{R} \to \mathbb{R}$ be given by $f(x) = xe^x$. Find all the critical points of f. For each critical point: is it a local minimum, a local maximum, or neither?

4. As in the previous problem, let $f(x) = xe^x$. Compute the second derivative $f''(x)$. Then find the intervals where the graph of $f(x)$ is concave up, and where it is concave down.

5. Use L'Hôpital's rule to compute the following limits:

   (a) $\displaystyle \lim_{x \to 0} \frac{\sin(x)}{e^x - 1}$

   (b) $\displaystyle \lim_{x \to 0^+} \frac{\ln(x)}{x^{-1}}$