

Quiz #9, Due: 11/2
Math 181 (Discrete Structures), Fall 2022

Problem 1 is worth 4 points (2 points each part), Problem 2 is worth 2 points, and Problem 3 is worth 4 points (1 point each part), for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Let $X = \{a, b\}$ and recall that X^* denotes the set of all strings over X . Define a function $f: X^* \rightarrow X^*$ by letting $f(\alpha)$ be the result of simultaneously swapping every a for a b and every b for an a in the string $\alpha \in X^*$. For instance $f(aab) = bba$.
 - (a) Write what $f(a)$, $f(bb)$, $f(baba)$, and $f(\lambda)$ are. (Recall $\lambda \in X^*$ denotes the null string.)
 - (b) Is f one-to-one? Is f onto? Explain.
2. Let R be the relation on $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$. Draw the digraph representation of R . Also draw the digraph representation of R^{-1} (the inverse relation).
3. Let R be the relation on the integers \mathbb{Z} where $(x, y) \in R$ if and only if $x - y$ is even.
 - (a) Is R reflexive? Explain.
 - (b) Is R symmetric? Explain.
 - (c) Is R anti-symmetric? Explain.
 - (d) Is R transitive? Explain.