

# Final Exam Study Guide

## Math 156 (Calculus I), Fall 2022

1. Basics (domain/range, what graph looks like, etc.) for standard functions [§1.1, 1.2, 1.4, 1.5]
  - (a) algebraic functions: power functions (like  $x^3$ ), root functions (like  $\sqrt{x}$ ), polynomials (like  $x^2 - 3x + 1$ ), rational functions (like  $(x^2 - 1)/(x + 5)$ )
  - (b) trigonometric functions (like  $\sin(x)$  and  $\cos(x)$ )
  - (c) exponential functions (like  $e^x$ ) and logarithmic functions (like  $\ln(x)$ )
  - (d) piecewise functions (like absolute value  $|x|$ )
2. Algebraic operations on functions as geometric operations on graphs [§1.3]
  - (a) translation (up/down & left/right), stretching (horiz. & vert.), reflection (over axes)
  - (b) symmetry under these operations, especially even and odd functions
3. How to make new functions from old functions  $f(x), g(x)$  [§1.3]
  - (a) sum ( $f + g$ ), difference ( $f - g$ ), scaling ( $cf$ ), product ( $fg$ ), quotient ( $f/g$ )
  - (b) composition of functions:  $(f \circ g)(x) = f(g(x))$
4. Inverse functions  $f = g^{-1}$  [§1.5]
  - (a) especially exponential and logarithmic functions
  - (b) graph of inverse function is reflection across line  $y = x$
5. Intuitive definition of limit and basic reasons why a limit might not exist [§2.2]
  - (a) intuitive definition of one-sided limits
  - (b) one-sided limits must agree for usual (two-sided) limit to exist
6. How to compute limits using the limit laws [§2.3, 2.5]
  - (a) sum ( $f + g$ ), difference ( $f - g$ ), scaling ( $cf$ ), product ( $fg$ ), quotient ( $f/g$ ) limit laws
  - (b) how to deal with “0/0” by cancelling factors
  - (c) continuous functions (pushing limit thru, and direct substitution a.k.a. “plugging in”)
7. Limits at infinity and limits equal to infinity [§2.2, 2.6]
  - (a) limits at  $\pm\infty$  = horizontal asymptotes
  - (b)  $\pm\infty$ -valued limits = vertical asymptotes
8. The definition(s) of derivative [§2.1, 2.7, 2.8]
  - (a) derivative as slope of the tangent to a curve at a point
  - (b) derivative as a limit  $f'(a) = \lim_{x \rightarrow a} (f(x) - f(a))/(x - a)$

9. Derivatives of basic functions [§3.1, 3.3, 3.6]
  - (a) power functions:  $d/dx(x^n) = nx^{n-1}$
  - (b) exponential and logarithmic functions:  $d/dx(e^x) = e^x$  and  $d/dx(\ln(x)) = 1/x$
  - (c) trigonometric functions:  $d/dx(\sin(x)) = \cos(x)$  and  $d/dx(\cos(x)) = -\sin(x)$
10. Rules for derivatives of combinations of functions [§3.1, 3.2, 3.4]
  - (a) derivative is linear:  $d/dx(a \cdot f(x) + b \cdot g(x)) = a \cdot f'(x) + b \cdot g'(x)$  for  $a, b \in \mathbb{R}$
  - (b) product rule:  $d/dx(f(x) \cdot g(x)) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$
  - (c) chain rule:  $d/dx(f(g(x))) = f'(g(x)) \cdot g'(x)$
  - (d) quotient rule:  $d/dx(f(x)/g(x)) = (g(x) \cdot f'(x) - f(x) \cdot g'(x))/(g(x))^2$   
*[don't have to separately memorize quotient rule, it follows from other rules]*
11. Implicit differentiation [§3.5]
  - (a) for  $y$  defined implicitly via equation  $p(x, y) = 0$ , find  $dy/dx$  by taking  $d/dx$  of both sides
  - (b) use this to find the slope of the tangent at any point on the curve
12. Related rates [§3.9]
  - (a) if two quantities  $f(t), g(t)$  are related, then their rates of change  $df/dt, dg/dt$  are related: like with implicit differentiation, just differentiate the relation between  $f(t)$  and  $g(t)$
13. Extreme values [§4.1, 4.3]
  - (a) local versus absolute (global) minimum and maximum values, Extreme Value Theorem
  - (b) the Closed Interval Method: extreme values of continuous  $f$  on closed interval must occur at endpoints or at critical points (values  $x$  where  $f'(x) = 0$  or is not defined)
  - (c) 1st and 2nd Derivative Tests for deciding if critical points are min.'s or max.'s
14. What derivatives tell us about shape of graph [§4.2, 4.3, 4.5]
  - (a)  $f'(x) > 0$  means  $f$  is increasing,  $f'(x) < 0$  means  $f$  is decreasing
  - (b)  $f''(x) > 0$  means  $f$  is concave up (smile),  $f''(x) < 0$  means  $f$  is concave down (frown)
15. L'Hôpital's rule [§4.4]
  - (a) for indeterminate form limits (meaning " $\pm\infty$ " or " $\frac{0}{0}$ "),  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$
16. Anti-derivatives, a.k.a. indefinite integrals [§4.9, 5.4, 5.5]
  - (a) basic anti-derivatives/indefinite integrals:  $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ ,  $\int e^x dx = e^x + C$ ,  $\int \frac{1}{x} dx = \ln(x) + C$ ,  $\int \sin(x) dx = -\cos(x) + C$ ,  $\int \cos(x) dx = \sin(x) + C$
  - (b) integral is linear:  $\int a \cdot f(x) + b \cdot g(x) dx = a \int f(x) dx + b \int g(x) dx$  for  $a, b \in \mathbb{R}$
  - (c) the  $u$ -substitution technique: can treat the " $dx$ " in an integral as a differential
17. Definite integrals [§5.1, 5.2, 5.3]
  - (a) definite integral  $\int_a^b f(x) dx$  as area under the curve  $y = f(x)$  from  $x = a$  to  $x = b$ , or more precisely as limit of "Riemann" (rectangle) sums  $\lim_{n \rightarrow \infty} \sum_{i=0}^n f(x_i^*) \Delta x$
  - (b) Fundamental Theorem of Calculus:  $\int_a^b f(x) dx = F(b) - F(a) = \int f(x) dx \Big|_a^b$