Midterm #1 Study Guide
Math 181 (Discrete Structures), Spring 2023

1. Sets [§1.1]
   - (a) sets of numbers (integers $\mathbb{Z}$ and real numbers $\mathbb{R}$), set-builder notation, subsets ($A \subseteq B$)
   - (b) operations of union ($A \cup B$), intersection ($A \cap B$), difference ($A \setminus B$), complement (A^c)
   - (c) representing sets via Venn diagrams
   - (d) ordered pairs (x, y) and the (Cartesian) product $X \times Y$ of two sets X and Y
2. Logical propositions [§1.2, 1.3]
   - (a) operations of "or" ($p \vee q$), "and" ($p \wedge q$), "not" ($\neg p$)
   - (b) truth tables for compound propositions
   - (c) conditional a.k.a. implication a.k.a. "if... then..." ($p \to q$)
   - (d) biconditionals ($p \leftrightarrow q$) and logical equivalence ($\equiv$)
   - (e) converse $q \to p$ and contrapositive $\neg q \to \neg p$ of an implication $p \to q$
     (contrapositive is logically equivalent to original implication; converse is not!)
3. Logical arguments [§1.4]
   - (a) converting an argument from words to symbolic form and vice-versa
   - (b) proving validity using truth tables
   - (c) proving validity using the rules of inference and logical equivalences
   - (d) common forms of invalid arguments a.k.a. fallacies
4. Quantifiers [§1.5, 1.6]
   - (a) propositional formulas ($P(x)$) and domains of discourse (D)
   - (b) universal ($\forall x\ P(x)$) and existential ($\exists x\ P(x)$) quantifiers
   - (c) DeMorgan's Laws: $\neg(\forall x\ P(x)) \equiv \exists x\ \neg P(x)$ and $\neg(\exists x\ P(x)) \equiv \forall x\ \neg P(x)$
   - (d) nested quantifiers and order of quantifiers ($\forall x \exists y\ P(x, y) \not\equiv \exists y \forall x\ P(x, y)$)
5. Proofs [§2.1]
   - (a) two basic mathematical systems: the theory of integers; the theory of sets
   - (b) direct proofs for theorems of form "$\forall x_1, \ldots, x_n$ if $P(x_1, \ldots, x_n)$ then $Q(x_1, \ldots, x_n)$"
   - (c) counterexamples to universally quantified statements