Quiz #10, 11/12
Math 157 (Calculus II), Fall 2025

Problem 1 is worth 6 points, and Problem 2 is worth 4 points, for a total of 10 points. Remember to *show your work* on all problems!

1. Consider the series $s = \sum_{n=1}^{\infty} \frac{1}{n^3}$, and let $s_n = \frac{1}{1^3} + \frac{1}{2^3} + \cdots + \frac{1}{n^3}$ denote its nth partial sum.

   (a) Compute s_2, the second partial sum, as an estimate for the true value s of the series.

   (b) Let $R_2 = s - s_2$ denote the error of your estimate. Compute upper and lower bounds on this error. **Hint**: recall that $\int_{n+1}^{\infty} f(x)\,dx \leq R_n \leq \int_n^{\infty} f(x)\,dx$ for the appropriate $f(x)$.

2. For each of the following series, decide if it converges or diverges. Explain your answer.

   (a) $\sum_{n=1}^{\infty} \frac{2n^2 - n + 2}{3n^2 + 2n + 1}$ (**Hint:** look at the limit of the terms.)

   (b) $\sum_{n=1}^{\infty} \frac{1}{3^n + 1}$ (**Hint:** compare to a series you know.)

   (c) $\sum_{n=1}^{\infty} \frac{3}{3n - 1}$ (**Hint:** compare to a series you know.)

   (d) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2}$ (**Hint:** compare to a series you know.)