

Midterm #2 Study Guide

Math 156 (Calculus I), Fall 2022

1. Derivatives of basic functions [§3.1, 3.3, 3.6]

- (a) power functions: $d/dx(x^n) = nx^{n-1}$
- (b) exponential and logarithmic functions: $d/dx(e^x) = e^x$ and $d/dx(\ln(x)) = 1/x$
- (c) trigonometric functions: $d/dx(\sin(x)) = \cos(x)$ and $d/dx(\cos(x)) = -\sin(x)$

2. Rules for derivatives of combinations of functions [§3.1, 3.2, 3.4]

- (a) derivative is linear: $d/dx(c \cdot f(x) + d \cdot g(x)) = c \cdot f'(x) + d \cdot g'(x)$ for $c, d \in \mathbb{R}$
- (b) product rule: $d/dx(f(x) \cdot g(x)) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$
- (c) chain rule: $d/dx(f(g(x))) = f'(g(x)) \cdot g'(x)$
- (d) quotient rule: $d/dx(f(x)/g(x)) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$
[don't have to separately memorize quotient rule, it follows from other rules]

3. Implicit differentiation [§3.5]

- (a) for y defined implicitly via equation $p(x, y) = 0$, find dy/dx by taking d/dx of both sides
- (b) use this to find the slope of the tangent at any point on the curve

4. Related rates [§3.9]

- (a) if two quantities $f(t), g(t)$ are related, then their rates of change $df/dt, dg/dt$ are related
- (b) as with implicit differentiation, just differentiate the relation between $f(t)$ and $g(t)$

5. Extreme values [§4.1, 4.3]

- (a) local versus absolute (global) minimum and maximum values
- (b) Extreme Value Theorem: continuous f on closed interval has (absolute) min. & max.
- (c) the Closed Interval Method: extreme values must occur at endpoints or at critical points (values x where $f'(x) = 0$ or is not defined)
- (d) 1st and 2nd Derivative Tests for deciding if critical points are min.'s or max.'s

6. What derivatives tell us about shape of graph [§4.2, 4.3, 4.5]

- (a) $f'(x) > 0$ means f is increasing, $f'(x) < 0$ means f is decreasing
- (b) $f''(x) > 0$ means f is concave up (smile), $f''(x) < 0$ means f is concave down (frown)

7. L'Hôpital's rule [§4.4]

- (a) for "indeterminate form" limits (meaning " $\pm\infty$ " or " $\frac{0}{0}$ "), $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$