

Midterm #3, 4/25  
Math 157 (Calculus II), Spring 2024

Each problem is worth 10 points, for a total of 50 points. You have 50 minutes to do the exam. Remember to *show your work* on all problems!

1. For each of the following sequence limits, state the value of the limit or state that it diverges. Explain your answer.

(a)  $\lim_{n \rightarrow \infty} \frac{5n^2 - 2n + 6}{3n^2 + 2n + 1}$

(b)  $\lim_{n \rightarrow \infty} \cos\left(\frac{2\pi}{n}\right)$

(c)  $\lim_{n \rightarrow \infty} (-1)^n \cdot \frac{n}{n+1}$

(d)  $\lim_{n \rightarrow \infty} \sqrt{n}$

2. For each of the following series, decide (with explanation) whether it converges or diverges.

(a)  $\sum_{n=1}^{\infty} \frac{n}{5n-1}$  (**Hint:** check the limit of the terms.)

(b)  $\sum_{n=1}^{\infty} \frac{3}{5n-1}$  (**Hint:** compare to a series you know.)

(c)  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n+1}$  (**Hint:** it is an alternating series.)

(d)  $\sum_{n=1}^{\infty} \frac{2^n + n}{4^n - n}$  (**Hint:** look at the ratio of successive terms.)

3. Consider the series  $s = \sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$ . Let  $s_n = \frac{1}{3^2} + \frac{1}{5^2} + \cdots + \frac{1}{(2n+1)^2}$  be its  $n$ th partial sum.

- (a) Compute the second partial sum  $s_2$  as an estimate for the true value  $s$  of the series. (Do not worry about simplifying your answer.)  
(b) Let  $R_2 = s - s_2$  be the corresponding remainder, i.e., the error of your estimate from part (a). Give an upper bound for  $R_2$ . (**Hint:** use an improper integral as the bound.)

4. Consider the function  $f(x) = e^{-2x}$ .

- (a) Express this function as a power series centered at zero:  $f(x) = \sum_{n=0}^{\infty} c_n x^n$ .  
(b) Determine the radius of convergence  $R$  of the power series you found in part (a).

5. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a twice-differentiable function satisfying  $f(0) = 3$ ,  $f'(0) = 2$ , and  $f''(0) = 1$ .

- (a) Write the degree two Taylor polynomial  $T_2(x)$ , centered at  $x = 0$ , for  $f(x)$ .  
(b) Use your answer in part (a) to estimate the value of  $f(2)$ . (You do not need to give any bounds on the error of your estimate.)