Midterm #3 Study Guide
Math 156 (Calculus I), Fall 2023

1. Area under the curve [§5.1, 5.2]

   (a) can approximate area under the curve $y = f(x)$ from $x = a$ to $x = b$ by the rectangle (Riemann) sum $A_n = \sum_{i=1}^{n} f(x_i^*)\,\Delta x$, where $\Delta x = \frac{b-a}{n}$, $x_i = a + i \cdot \Delta x$ for $i = 0, 1, \ldots, n$, and any choice of sample points $x_i^* \in [x_{i-1}, x_i]$

   (b) usual choices: $x_i^* = x_{i-1}$ (left endpoints $A_n = L_n$); $x_i^* = x_i$ (right endpoints $A_n = R_n$); or $x^* = \frac{x_i + x_{i-1}}{2}$ (midpoints of intervals)

   (c) if $f(x)$ is continuous, all give same limit $A = \lim_{n\to\infty} A_n$, the true area under the curve

2. Definite integrals [§5.2, 5.3]

   (a) definite integral $\int_a^b f(x)\,dx$ is the area "under" the curve $y = f(x)$ from $x = a$ to $x = b$ as defined above: $A = \lim_{n\to\infty} A_n$; this counts area below the x-axis negatively

   (b) Fundamental Theorem of Calculus: $\int_a^b f(x)\ dx = F(b) - F(a) = \int f(x)\ dx\Big]_a^b$, where $F(x) = \int f(x)\,dx$ is an anti-derivative of $f(x)$

   (c) another way to think of FTC: integral of rate of change is net change (e.g., integral of velocity is displacement)

3. Anti-derivatives, a.k.a. indefinite integrals [§4.9, 5.4, 5.5]

   (a) basic anti-derivatives/indefinite integrals: $\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C$, $\int e^x\ dx = e^x + C$, $\int \frac{1}{x}\ dx = \ln(x) + C$, $\int \sin(x)\ dx = -\cos(x) + C$, $\int \cos(x)\ dx = \sin(x) + C$

   (b) integral is linear: $\int a \cdot f(x) + b \cdot g(x)\ dx = a \int f(x)\ dx + b \int g(x)\ dx$ for $a, b \in \mathbb{R}$

   (c) the u-substitution technique: can treat the "dx" in an integral as a differential, so if we let $u = g(x)$ then we can substitute $du = g'(x)\,dx$ in an integral