

Dynamical Algebraic Combinatorics notes

[outline as of 03/05/2022]

Sam Hopkins

Chapter 0. Introduction

- §0.1 Philosophy of DAC
- §0.2 The basic examples: rotation of subsets and multisets
- §0.3 Overview of rest of notes

Chapter 1. Tableaux

- §1.1 Young diagrams, semistandard Young tableaux, Gelfand-Tsetlin patterns
- §1.2 Bender-Knuth involutions, promotion and evacuation
- §1.3 Reading words and Knuth equivalence
- §1.4 Jeu de taquin; promotion and evacuation again
- §1.5 Proof (using evacuation & jdt) that promotion of rectangular SSYTs has order n
- §1.6 Standard tableaux
- §1.7 Models for promotion of 2- and 3-rowed SYT promotion

Chapter 2. Posets

- §2.1 Basics about posets
- §2.2 Linear extensions, promotion and evacuation
- §2.3 Embedding linear ext's of the two "triangles" into the rectangle
- §2.4 Order ideals, rowmotion and toggles
- §2.5 The Stanley–Thomas word
- §2.6 P-partitions, piecewise linear toggles and rowmotion
- §2.7 Conjugacy of rect. promotion & rowmotion (via GT pat's)
- §2.8 Embedding P-partitions of the triangles into the rectangle
- §2.9 The order polytope and PL maps

Chapter 3. Coxeter groups and root systems

- §3.1 Basics about Coxeter groups and root systems
- §3.2 The symmetric group (Type A) in detail
- §3.3 Root posets
- §3.4 Reduced words and the Edelman–Greene bijection (in Type A)
- §3.5 Absolute order and the Armstrong–Stump–Thomas bijection (in Type A)
- §3.6 Parabolic quotients
- §3.7 Minuscule posets
- §3.8 Toggling as reflection for minuscule posets (à la Rush–Shi)

Chapter 4. Cyclic sieving

- §4.1 Definition and basics of cyclic sieving
- §4.2 The basic examples via exterior/symmetric power
- §4.3 General and special linear group representations
- §4.4 The Grassmannian and its coordinate ring, standard monomials
- §4.5 Involutions on the Grassmannian and “ $q=-1$ ” phenomenon for plane partitions
- §4.6 Canonical bases & cyclic sieving for SSYT promotion [discussion only]
- §4.7 Invariant tensors, Schur–Weyl duality
- §4.8 Cyclic sieving for SYT promotion [discussion only]

Chapter 5. Invariance and homomesy

- §5.1 Invariance and homomesy; explanation of how they are “dual”
- §5.2 Promotion invariants (cyclic descents)
- §5.3 Rowmotion invariants (the “OY invariant”)
- §5.4 Promotion homomesies (reciprocal sums)
- §5.5 Rowmotion homomesies (order ideal & antichain cardinality, toggleability statistics)
- §5.6 Symmetry of Narayana numbers & the Lanne–Kreweras involution, rowvacuation

Appendix A. The RSK correspondence

- §A.1 Row insertion description of RSK
- §A.2 Toggle description of RSK
- §A.3 Greene’s theorem + generalization for “global” description of RSK
- §A.4 Symmetries of RSK: transposition and 180 degree rotation
- §A.5 The “sandwich theorem” for rectangular tableaux

Appendix B. Birational dynamics

- §B.1 Birational toggling and birational rowmotion
- §B.2 Nonintersecting lattice paths formula
- §B.3 Birational homomesies
- §B.4 Noncommutative dynamics

Appendix C. Beyond distributive lattices

- §C.1 Rowmotion as acting on a distributive lattice
- §C.2 Semidistributive lattices
- §C.3 Trim lattices