Midterm #2 Study Guide
Math 181 (Discrete Structures), Fall 2022

1. Indirect proofs [§2.2]
   (a) proof of by contrapositive: to prove $p \to q$, prove $\neg q \to \neg p$ instead
   (b) proof by contradiction: assume negation of statement, and deduce contradiction $(r \wedge \neg r)$
2. Mathematical induction [§2.4, 2.5]
   (a) basic structure of inductive proofs: base case $P(1)$, and induction step $P(n) \to P(n+1)$
   (b) proving $\forall (n \in \mathbb{Z}_{>0})\ P(n)$ by induction, especially when $P(n)$ is an algebraic formula
   (c) finding patterns to guess formulas involving n which can then be proved by induction
   (d) the strong form of mathematical induction: can use $P(k)$ for all $k < n$ to prove $P(n)$
3. Functions [§3.1]
   (a) ways to view a function $f\colon X \to Y$: rule to convert input $x \in X$ to output $y = f(x) \in Y$; set of ordered pairs (x, y); arrow diagram from X to Y
   (b) one-to-one, onto, and bijective functions
   (c) composition of functions, and inverse functions
   (d) modular arithmetic functions $f(x) = x \mod n$
4. Sequences and strings [§3.2]
   (a) finite and infinite sequences: ordered list of elements of some set
   (b) set of strings X^* on some finite alphabet X, and the null string $\lambda \in X^*$
   (c) subsequences (not necessarily consecutive) versus substrings (consecutive)
5. Relations [§3.4, 3.5]
   (a) digraph representation of a relation R on a set X
   (b) properties that R can have: reflexive, symmetric, anti-symmetric, transitive
   (c) partial order (reflexive, anti-symmetric, transitive): way to "compare" things in X
   (d) equivalence relation (reflexive, symmetric, transitive): way to say certain things in X are "the same"; corresponds to a partition of X into equivalence classes
6. Basic counting principles [§6.1]
   (a) multiplication principle: total # of possibilities = product of # of choices at each step
   (b) addition principle: size of union of *disjoint* sets is sum of sizes of the sets
   (c) principle of inclusion and exclusion: $\#(X \cup Y) = \#X + \#Y - \#(X \cap Y)$