

Quiz #6, Due: 10/12
Math 181 (Discrete Structures), Fall 2022

Problem 1 is worth 5 points and Problem 2 is worth 5 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Prove the following: “For any two real numbers x and y , if $xy \geq 2$ then $x \geq \sqrt{2}$ or $y \geq \sqrt{2}$.”

Hint: Use a proof by contradiction or a proof by contrapositive.

2. Prove the following: “There exist irrational numbers a and b such that a^b is rational.”

Do this by employing the following **proof strategy**. Show that either

- $a = \sqrt{2}$ and $b = \sqrt{2}$ satisfy the statement;
- or, $a = \sqrt{2}^{\sqrt{2}}$ and $b = \sqrt{2}$ do.

Your proof does not have to show *which* of these two possibilities hold. You may use the fact we proved in class that $\sqrt{2}$ is irrational. You may also use the law of exponents $(x^y)^z = x^{yz}$.