CALCULUS

INTEGRALS

COMMON CALCULUS 1 INTEGRALS

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int x^{-1}\,dx = \int \frac{1}{x}dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1} x + C$$

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$

FUNDAMENTAL THEOREM OF CALCULUS, PART

Assume $f(x)$ is continuous on $[a,b]$. If $F(x)$ is an antiderivative of $f(x)$ on $[a,\ ]$, then

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

FUNDAMENTAL THEOREM OF CALCULUS, PART II

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

$$\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))g'(x) \quad \text{(chain rule version)}$$

BASIC INTEGRATION PROPERTIES

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx \quad (a \le b \le c)$$

$$\int_a^b k\,dx = k(b-a)$$

MORE INTEGRATION PROPERTIES

$$\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|dx$$

If $f(x) \ge 0$ for $a \le x \le b$, then $\int_a^b f(x)dx \ge 0$

If $f(x) \ge g(x)$ for $a \le x \le b$, then

$$\int_a^b f(x)dx \ge \int_a^b g(x)dx$$

If $m \le f(x) \le M$ for $a \le x \le b$, then

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

COMMON CALCULUS 2 INTEGRALS

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$$

$$\int \tan x\,dx = \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int \tan^{-1} x\,dx = x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int f(kx)dx = \frac{1}{k}F(kx) + C$$

where $F(x)$ is any antiderivative of $f(x)$ and k is any nonzero constant. For example,

$$\int e^{kx}dx = \frac{1}{k}e^{kx} + C \quad \text{and} \quad \int \sin(kx)\,dx = -\frac{1}{k}\cos(kx) + C$$

INTEGRATION BY SUBSTITUTION

$$\int f(g(x))g'(x)dx = \int f(u)\,du$$

or

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where $u = g(x)$ and $du = g'(x)dx$

INTEGRATION BY PARTS

$$\int u\,dv = uv - \int v\,du$$

or

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

Remember the acronym **ILATE** when choosing u.
Inverse Trig, **L**ogarithmic, **A**lgebraic, **T**rigonometric, **E**xponential

ARC LENGTH FORMULA

The arc length differentiable function $y = f(x)$ over the interval $[a,b]$ is given by

$$\int_a^b \sqrt{1+[f'(x)]^2}dx = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

VOLUMES OF SOLIDS OF REVOLUTION

DISK METHOD: $\int_a^b \pi(\text{Radius})^2 dx = \int_a^b \pi\big(R(x)\big)^2 dx$

WASHER METHOD: $\int_a^b \pi\left(\begin{pmatrix}\text{Outer}\\\text{Radius}\end{pmatrix}^2 - \begin{pmatrix}\text{Inner}\\\text{Radius}\end{pmatrix}^2\right)dx = \int_a^b \pi\left(\big(R(x)\big)^2 - \big(r(x)\big)^2\right)dx$

SHELL METHOD: $\int_a^b 2\pi\begin{pmatrix}\text{Shell}\\\text{Radius}\end{pmatrix}\begin{pmatrix}\text{Shell}\\\text{Height}\end{pmatrix}dx$

TRIGONOMETRIC SUBSTITUTION

EXPRESSION	SUBSTITUTION	EVALUATION
$\sqrt{a^2-x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\,d\theta$	$\sqrt{a^2-a^2\sin^2\theta}$ $= a\cos\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\,d\theta$	$\sqrt{a^2+a^2\tan^2\theta}$ $= a\sec\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$ $dx = a\sec\theta\tan\theta\,d\theta$	$\sqrt{a^2\sec^2\theta-a^2}$ $= a\tan\theta$

Polar Coordinate and Parametric Calculus

Derivative

$$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos(\theta) - f(\theta)\sin\theta}$$

- For a parametric curve $(x(t), y(t))$: $d\ell = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
- For the graph of a function $y(x)$: $d\ell = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
- For a polar graph $r(\theta)$: $d\ell = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$

Series

The ratio test

If:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$$

Then the series is absolutely convergent

The root test

If:

$$\lim_{n\to\infty}\sqrt[n]{|a_n|} = L < 1$$

Then the series is absolutely convergent

Common Series

Function	Series	Interval of convergence
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$(-1,1)$
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$(-\infty,\infty)$
$\sin x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$	$(-\infty,\infty)$
$\cos x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$	$(-\infty,\infty)$
$ln\,(1+x)$	$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	$(-1,1]$
$\tan^{-1} x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$	$[-1,1]$
$(1+x)^m$	$\sum_{n=1}^{\infty} \binom{m}{n} x^k = 1 + mx + \frac{m(m-1)x^2}{2!} + \cdots$	$(-1,1)$