Homework #4, Due: 2/15
Math 181 (Discrete Structures), Spring 2023

Problem 1 is worth 5 points (2.5 pts each part), and Problem 2 is worth 5 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Recall that an integer x is called a *multiple* of an integer y if there is some integer z such that $x = z \times y$. Let $P(x, y)$ be the propositional formula "x is a multiple of y," where the domain of discourse is all pairs of integers. Write the meaning in English of the following propositions, and determine (with explanation) if they are true or false.

   (a) $\exists x \, \forall y \, P(x, y)$

   (b) $\exists y \, \forall x \, P(x, y)$.

2. Give a direct proof of the following theorem about sets:
   "For all sets X, Y, and Z, if $X \subseteq Y$ then $X \cup Z \subseteq Y \cup Z$."