

# Midterm #2 Study Guide

## Math 157 (Calculus II), Spring 2025

### 1. More geometric applications of integrals [§8.1, 8.2]

- (a) Arc lengths of curves [§8.1]: length of  $y = f(x)$  from  $x = a$  to  $x = b$  is  $\int_a^b \sqrt{1 + (f'(x))^2} dx$ .
- (b) Area of surface of revolution [§8.2]:
  - i. for  $y = f(x)$  from  $x = a$  to  $x = b$  rotated about  $x$ -axis, area is  $\int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} dx$ ;
  - ii. for  $y = f(x)$  from  $x = a$  to  $x = b$  rotated about  $y$ -axis, area is  $\int_a^b 2\pi x \sqrt{1 + (f'(x))^2} dx$ .

### 2. Parametrized curves [§10.1, 10.2]

- (a) Curve of form  $x = f(t)$  and  $y = g(t)$  for some auxiliary variable  $t$  (“time”) [§10.1]
- (b) Slope of tangent [§10.2] to curve given by chain rule:  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$
- (c) Arc length [§10.2] is  $\int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt = \int_a^b \sqrt{g'(t)^2 + f'(t)^2} dt$

### 3. Polar coordinates and polar curves [§10.3, 10.4]

- (a) Cartesian vs. polar [§10.3]:  $(x, y) = (r \cos \theta, r \sin \theta)$  and  $(r, \theta) = (\sqrt{x^2 + y^2}, \arctan(\frac{y}{x}))$
- (b) Area inside [§10.4] polar curve  $r = f(\theta)$  for  $\alpha \leq \theta \leq \beta$  is  $\int_\alpha^\beta \frac{1}{2} r^2 d\theta = \int_\alpha^\beta \frac{1}{2} f(\theta)^2 d\theta$
- (c) Slope of tangent [§10.4] to polar curve  $r = f(\theta)$  given by chain and product rules:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}(r \sin \theta)}{\frac{d}{d\theta}(r \cos \theta)} = \frac{f(\theta) \cos \theta + f'(\theta) \sin \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}$$