Dynamical Algebraic Combinatorics notes [outline]

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Chapter 0. Introduction

- §0.1 Philosophy of DAC
- §0.2 The ur examples: rotation of subsets and multisets
- §0.3 Overview of rest of notes

Chapter 1. Tableaux

- §1.1 Young diagrams, semistandard Young tableaux, Gelfand-Tsetlin patterns
- §1.2 Bender-Knuth involutions, promotion, evacuation
- §1.3 Jeu de taquin; promotion and evacuation again
- §1.4 Knuth equivalence
- §1.5 Proof (using evacuation & jdt) that promotion of rectangular SSYTs has order n
- §1.6 Standard tableaux
- §1.7 Models for promotion of 2- and 3-rowed SYT promotion

Chapter 2. Posets

- §2.1 Basics about posets
- §2.2 Linear extensions, promotion and evacuation
- §2.3 The rectangle redux; embedding (linear ext's of) the two triangles into the rectangle
- §2.4 Order ideals, rowmotion, toggles
- §2.5 P-partitions, piecewise linear toggles and rowmotion
- §2.6 Conjugacy of rect. promotion & rowmotion (via GT pat's)
- §2.7 The Stanley–Thomas word and more triangle embeddings
- §2.8 The order polytope and PL maps

Chapter 3. Coxeter groups and root systems

- §3.1 Basics about Coxeter groups and root systems
- §3.2 Root posets
- §3.3 Reduced words and the Edelman–Greene bijection (in Type A)
- §3.4 Absolute order and the Armstrong–Stump–Thomas bijection (in Type A)
- §3.5 Minuscule posets
- §3.6 Parabolic quotients and rowmotion (à la Rush–Shi)

Chapter 4. Cyclic sieving

- §4.1 Definition of cyclic sieving and ur examples via exterior/symmetric power
- §4.2 The Grassmannian and its coordinate ring, standard monomials
- §4.3 Involutions on the Grassmannian and "q=-1" phenomenon for plane partitions

- §4.4 Canonical bases & cyclic sieving for SSYT promotion [discussion only]
- §4.5 Invariant tensors, Schur–Weyl duality
- §4.6 Cyclic sieving for SYT promotion [discussion only]

Chapter 5. Invariance and homomesy

- §5.1 Invariants; basic examples of invariants from content rotation
- §5.2 Cyclic descents for SYTs
- §5.3 The “OY invariant” for Type A root poset
- §5.4 Homomesy; basic rowmotion homomesies from Stanley–Thomas word bijection
- §5.5 Symmetry of Narayana numbers & the Lalanne–Kreweras involution, rowvacuation
- §5.6 The toggleability statistics technique

Chapter ∞. Further topics and conclusion

- §∞.1 The RSK Algorithm
- §∞.2 Birational dynamics
- §∞.3 Rowmotion beyond distributive lattices
- §∞.4 Conclusion