Homework #5, Due: 2/15
Math 181 (Discrete Structures), Spring 2023

Problem 1 is worth 5 points (2.5 pts each part), and Problem 2 is worth 5 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Recall that an integer x is called a *multiple* of an integer y if there is some integer z such that $x = z \times y$. Let $P(x, y)$ be the propositional formula "x is a multiple of y," where the domain of discourse is the set $\mathbb{Z} \times \mathbb{Z}$ of pairs of integers. Write the meaning in English of the following propositions, and determine (with explanation) if they are true or false.

   (a) $\exists x \, \forall y \, P(x, y)$

   (b) $\exists y \, \forall x \, P(x, y)$.

2. Give a direct proof of the following theorem about sets:
   "For all sets X, Y, and Z, if $X \subseteq Y$ then $X \cup Z \subseteq Y \cup Z$."