

# Midterm #1 Study Guide

## Math 157 (Calculus II), Spring 2024

### 1. Geometric applications of integrals [§6.1, 6.2, 6.3]

- (a) Area between curves [§6.1]: area between  $y = f(x)$  and  $y = g(x)$  is  $\int_a^b |f(x) - g(x)| dx$ .
- (b) Volume of general solid [§6.2]: if  $A(x)$  = area of cross-section, then volume is  $\int_a^b A(x) dx$ .
- (c) Volume of solid of revolution [§6.2, 6.3]: “disks/washers” & “cylindrical shells” methods.  
For region below curve  $y = f(x)$  from  $x = a$  to  $x = b$ :
  - i. rotated around  $x$ -axis, “disks method” gives volume  $= \int_a^b \pi f(x)^2 dx$ ;
  - ii. rotated around  $y$ -axis, “shells method” gives volume  $= \int_a^b 2\pi f(x) x dx$ .

### 2. Other applications of integrals [§6.4, 6.5]

- (a) Work [§6.4]: if  $F(x)$  = force as function of distance, then work done is  $W = \int_a^b F(x) dx$ .
- (b) Average of function [§6.5]: the average of  $f(x)$  from  $x = a$  to  $x = b$  is  $\frac{1}{b-a} \int_a^b f(x) dx$ .

### 3. Techniques for computing integrals [§7.1, 7.2, 7.3, 7.4, 7.5]

- (a) Integration by parts [§7.1]:  $\int u dv = uv - \int v du$ ; choose  $u$  using “LIATE” rule
- (b) Trigonometric integrals [§7.2]: for  $\int \sin^n(x) \cos^m(x) dx$ , use the Pythagorean identity  $\sin^2(x) + \cos^2(x) = 1$  to isolate single factor of  $\cos(x) dx$  or  $\sin(x) dx$ , then do a  $u$ -sub.
- (c) Trigonometric substitution [§7.3]:
  - i. for  $a^2 - x^2 \Rightarrow$  sub  $x = a \sin(\theta)$ ,  $dx = a \cos(\theta) d\theta$ , and use  $1 - \sin^2(\theta) = \cos^2(\theta)$ ;
  - ii. for  $a^2 + x^2 \Rightarrow$  sub  $x = a \tan(\theta)$ ,  $dx = a \sec^2(\theta) d\theta$ , and use  $1 + \tan^2(\theta) = \sec^2(\theta)$ .
- (d) Integrating rational functions by partial fractions [§7.4]: find roots of denominator  $Q(x)$  and solve system of equations to write  $P(x)/Q(x) = A/(x-a) + B/(x-b) + \dots + Z/(x-z)$  and use  $\int A/(x-a) dx = A \ln(x-a)$ ; for repeated roots do  $A_1/(x-a) + A_2/(x-a)^2 + \dots$ .

### 4. Other concepts related to integration [§7.7, 7.8]

- (a) Approximating definite integrals [§7.7]: two good approximations of  $\int_a^b f(x) dx$  are
  - i. midpoint approximation  $M_n = \sum_{i=1}^n f(\bar{x}_i) \Delta x$  where  $\bar{x}_i = \frac{x_{i-1} + x_i}{2}$ ;
  - ii. trapezoid approximation  $T_n = \frac{\Delta x}{2} (f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n))$ .
- (b) Improper integrals [§7.8]:  $\int_a^\infty f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx$ , et cetera.