Midterm #1 Study Guide
Math 157 (Calculus II), Spring 2023

1. Geometric applications of integrals [§6.1, 6.2, 6.3, 8.1, 8.2]
   (a) Area between curves [§6.1]: area between $y = f(x)$ and $y = g(x)$ is $\int_a^b |f(x) - g(x)|\, dx$.
   (b) Volume of general solid [§6.2]: if $A(x)$ = area of cross-section, then volume is $\int_a^b A(x)\, dx$.
   (c) Volume of solid of revolution [§6.2, 6.3]: "disks/washers" & "cylindrical shells" methods. For region below curve $y = f(x)$ from $x = a$ to $x = b$:
      i. rotated around x-axis, "disks method" gives volume $= \int_a^b \pi\, f(x)^2\, dx$;
      ii. rotated around y-axis, "shells method" gives volume $= \int_a^b 2\pi\, f(x)\, x\, dx$.
   (d) Arc lengths of curves [§8.1]: length of $y = f(x)$ from $x = a$ to $x = b$ is $\int_a^b \sqrt{1 + (f'(x))^2}\, dx$.
   (e) Area of surface of revolution [§8.2]:
      i. for $y = f(x)$ from $x = a$ to $x = b$ rotated about x-axis, area is $\int_a^b 2\pi f(x)\, \sqrt{1 + (f'(x))^2}\, dx$;
      ii. for $x = g(y)$ from $y = c$ to $y = d$ rotated about x-axis, area is $\int_c^d 2\pi y\, \sqrt{1 + (g'(y))^2}\, dy$.
2. Other applications of integrals [§6.4, 6.5, 8.3]
   (a) Work [§6.4]: if $F(x)$ = force as function of distance, then work done is $W = \int_a^b F(x)\, dx$.
   (b) Average of function [§6.5]: the average of $f(x)$ from $x = a$ to $x = b$ is $\frac{1}{b-a} \int_a^b f(x)\, dx$.
   (c) Center of mass [§8.3]: centroid of region below curve $y = f(x)$ located at $(\overline{x}, \overline{y})$ where $\overline{x} = \frac{1}{A} \int_a^b x f(x)\, dx$, $\quad \overline{y} = \frac{1}{A} \int_a^b \frac{1}{2}(f(x))^2\, dx$, $\quad$ and $A = \int_a^b f(x)\, dx$ is area of the region.
3. Techniques for computing integrals [§7.1, 7.2, 7.3, 7.4, 7.5]
   (a) Integration by parts [§7.1]: $\int u\, dv = uv - \int v\, du$; choose u using "LIATE" rule
   (b) Trigonometric integrals [§7.2]: for $\int \sin^n(x) \cos^m(x)\, dx$, use the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$ to isolate single factor of $\cos(x)\, dx$ or $\sin(x)\, dx$, then do a u-sub.
   (c) Trigonometric substitution [§7.3]:
      i. for $a^2 - x^2 \Rightarrow$ sub $x = a\sin(\theta)$, $dx = a\cos(\theta)\, d\theta$, and use $1 - \sin^2(\theta) = \cos^2(\theta)$;
      ii. for $a^2 + x^2 \Rightarrow$ sub $x = a\tan(\theta)$, $dx = a\sec^2(\theta)\, d\theta$, and use $1 + \tan^2(\theta) = \sec^2(\theta)$.
   (d) Integrating rational functions by partial fractions [§7.4]: find roots of denominator $Q(x)$ and solve system of equations to write $P(x)/Q(x) = A/(x-a) + B/(x-b) + ... + Z/(x-z)$ and use $\int A/(x-a)\, dx = A\ln(x-a)$; for repeated roots do $A_1/(x-a) + A_2/(x-a)^2 + \cdots$.
4. Other concepts related to integration [§7.7, 7.8]
   (a) Approximating definite integrals [§7.7]: two good approximations of $\int_a^b f(x)\, dx$ are
      i. midpoint approximation $M_n = \sum_{i=1}^n f(\overline{x}_i)\Delta x$ where $\overline{x}_i = \frac{x_{i-1} + x_i}{2}$;
      ii. trapezoid approximation $T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n))$.
   (b) Improper integrals [§7.8]: $\int_a^\infty f(x)\, dx = \lim_{t\to\infty} \int_a^t f(x)\, dx$, et cetera.