

Homework #10, Due: 4/5  
Math 181 (Discrete Structures), Spring 2023

Problem 1 is worth 4 points (2 pts each part), and Problem 2 is worth 6 points (1.5 pts each part), for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Let  $X = \{1, 2, 3\}$ . Define a relation  $R$  on  $X \times X = \{(a, b) : a, b \in X\}$  where  $(a, b) R (c, d)$  if and only if  $a + b = c + d$ . For example, we have that  $(2, 2) R (1, 3)$  since  $2 + 2 = 4 = 1 + 3$ .
  - (a) Explain why  $R$  is an equivalence relation.
  - (b) List one element of each equivalence class of  $R$ .
2. Let  $X = \{a, b, c\}$  and let  $S \subseteq X^*$  be the set of strings over the alphabet  $X$  of length four. For example, three of the elements in  $S$  are:  $abab$ ,  $acbb$ , and  $cbb$ .
  - (a) What is  $\#S$  (the number of elements in  $S$ )?
  - (b) Let  $S_1 \subseteq S$  be the subset of strings in  $S$  that begin with “a.” What is  $\#S_1$ ?
  - (c) Let  $S_2 \subseteq S$  be the subset of strings in  $S$  that end with “bb.” What is  $\#S_2$ ?
  - (d) Let  $S_3 \subseteq S$  be the subset of strings in  $S$  that begin with “a” or end with “bb” (or both). What is  $\#S_3$ ? **Hint:** use the Principle of Inclusion and Exclusion!