Midterm #2, 4/27
Math 157 (Calculus II), Spring 2023

Each problem is worth 10 points, for a total of 50 points. You have 50 minutes to do the exam. Remember to *show your work* on all problems!

1. Consider the parametrized curve given by $x = t^2 - 1$ and $y = t^3 + t$ for $-\infty < t < \infty$. Compute the slope of the tangent to this curve at the point $(x, y) = (0, 2)$.

2. Consider the polar curve given by $r = \sqrt{\theta(\pi - 2\theta)}$ for $0 \leq \theta \leq \frac{\pi}{2}$. Compute the area inside of this curve.

3. Consider the series $s = \displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. Let $s_n = \frac{1}{1^2} + \frac{1}{3^2} + \cdots + \frac{1}{(2n-1)^2}$ be its nth partial sum.

   (a) Compute the third partial sum s_3 as an estimate for the true value s of the series. (Do not worry about simplifying your answer.)

   (b) Let $R_3 = s - s_3$ be the corresponding remainder, i.e., the error of your estimate from part (a). Give an upper bound for R_3. (**Hint**: use an improper integral as the bound.)

4. For each of the following series, decide (with explanation) whether it converges or diverges.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{4n+1}$ (**Hint**: check the limit of the terms.)

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{2n-1}$ (**Hint:** compare to a series you know.)

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$ (**Hint:** it is an alternating series.)

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+1}{3^n-1}$ (**Hint:** look at the ratio of successive terms.)

5. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a twice-differentiable function satisfying $f(0) = 1$, $f'(0) = 2$, and $f''(0) = 3$.

   (a) Write the degree two Taylor polynomial $T_2(x)$, centered at $x = 0$, for $f(x)$.

   (b) Use your answer in part (a) to estimate the value of $f(\frac{1}{2})$. (You do not need to give any bounds on the error of your estimate.)