SCIENCE FICTION

2018

PRELIMINARIES

Let $X_n = \{x_1, \ldots, x_n\}$ and $Y_n = \{y_1, \ldots, y_n\}$. For $P(x;y), Q(x;y) \in \mathbf{Q}[X_n; Y_n]$ we define the scalar product

$$\boxed{\langle P\,,\,Q\rangle \;=\; P(\partial_x;\partial_y)Q(x;y)\Big|_{x,y=0}}$$

For $\sigma \in S_n$, and $P(x;y) \in \mathbf{Q}[X_n;Y_n]$ we define the diagonal action

$$\sigma P(x;y) \;=\; P(x_{\sigma_1}, x_{\sigma_2}, \ldots, x_{\sigma_n}; y_{\sigma_1}, y_{\sigma_2}, \ldots, y_{\sigma_n})$$

Notice that since we have

$$\langle \sigma P\,,\,Q\rangle \;=\; \langle P\,,\,\sigma^{-1}Q\rangle$$

The orthogonal complement of an invariant subspace is also invariant.

An alternant under the diagonal action

$$\begin{array}{|c|c|c}\hline 10 & 11 & \multicolumn{1}{c}{} \\ \hline 00 & 01 & \multicolumn{1}{c|}{02} \\ \hline \end{array} \quad \Longrightarrow \quad \Delta_{3,2}(x;y) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ y_1 & y_2 & y_3 & y_4 & y_5 \\ y_1^2 & y_2^2 & y_3^2 & y_4^2 & y_5^2 \\ x_1 & x_2 & x_3 & x_4 & x_5 \\ x_1y_1 & x_2y_2 & x_3y_3 & x_4y_4 & x_5y_5 \end{pmatrix}$$

The Frobenius map $\boxed{\mathcal{F}\,\chi^\lambda \;=\; s_\lambda[X]}$

The Frobenius Characteristic of an S_n invariant module $\mathcal{M} \;=\; \bigoplus_{r,s} \mathcal{H}_{r,s}(\mathcal{M})$

$$\boxed{\mathcal{F}\mathcal{M} \;=\; \sum_{r,s} t^r q^s \mathcal{F}ch\big(\mathcal{H}_{r,s}(\mathcal{M})\big)}$$

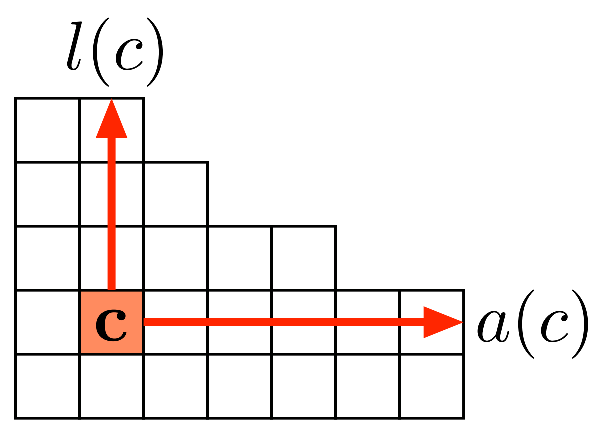
NOTATION

$$n(\mu) \;=\; \sum_{i=1}^{l(\mu)} (i-1)\mu_i \qquad\qquad T_\mu = t^{n(\mu)} q^{n(\mu')}$$

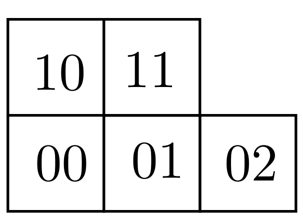
$$\mu \;=\; \{(a_1,b_1),(a_2,b_2),\ldots,(a_n,b_n)\} \qquad (a_i,b_i) \text{ the } y,x \text{ coordinates}$$

$$B_\mu(q,t) \;=\; \sum_{i=1}^{n} t^{a_i} q^{b_i} \qquad\qquad \Pi_\mu(q,t) \;=\; \prod_{i=2}^{n} (1 - t^{a_i} q^{b_i})$$

$$w_\mu(q,t) = \prod_{c\in\mu} \left(t^{l(\mu)} - q^{a(\mu)+1}\right)\left(q^{a(\mu)} - t^{l(\mu)+1}\right)$$



example



$$B_\mu(q,t) \;=\; 1 + q + q^2 + t + tq$$

$$\Pi_\mu(q,t) \;=\; (1-q)(1-q^2)(1-t)(1-qt)$$

BASIC FACTS

For a partition $\mu \vdash n$ with cells $(a_1, b_1), (a_2, b_2), \cdots, (a_n, b_n)$ set

$$\boxed{\Delta_\mu(x,y) = \det \left\| x_i^{a_j} y_i^{b_j} \right\|_{i,j=1}^{n}}$$

and define

$$\boxed{\mathcal{M}_\mu = \mathcal{L}[\partial_x^p \partial_y^q \Delta_\mu]}$$

$$\dim \mathcal{M}_\mu \leq n! \text{ (easy).}$$

Theorem (Conjectured in 1990 proved in 2000)

$$\dim \mathcal{M}_\mu = n! \text{ (hard!).}$$

$\mathcal{M}_\mu$ affords a bi-graded version of the regular representation of S_n.

The Frobenius characteristic of $\mathcal{M}_\mu$ is the modified Macdonald $\widetilde{H}_\mu(X;q,t)$

If $p(x,y) \in \mathcal{M}_\mu$ set $\boxed{\mathbf{flip}_\mu p(x,y) = p(\partial_x, \partial_y)\Delta_\mu(x,y)}$

Theorem

The map $\mathbf{flip}_\mu p(x,y) = p(\partial_x, \partial_y)\Delta_\mu(x,y)$ is non singular for $p(x,y) \in \mathcal{M}_\mu$

Proof

Say we have $p(x,y) \in \mathcal{M}_\mu$ such that $p(\partial_x, \partial_y)\Delta_\mu(x,y) = 0$

If $p(x,y) \in \mathcal{M}_\mu$ then $p(x,y) = A(\partial_x, \partial_y)\Delta_\mu(x,y)$ thus

$$p(\partial_x, \partial_y)p(x,y) = A(\partial_x, \partial_y)p(\partial_x, \partial_y)\Delta_\mu(x,y) = 0$$

Q.E.D.

BASIC FACTS

The definition of the map $\mathbf{flip}_\mu p(x,y)$ can be extended to any S_n invariant submodule by acting with $\mathbf{flip}_\mu$ on every element of a basis

Define for any symmetric $F(X;q,t)$

$$\boxed{\downarrow F(X;q,t) = \omega F(X;1/q,1/t)}$$

Theorem
If $\mathcal{M}$ has Frobenius Characteristic $\Phi_{\mathcal{M}}$, (in symbols $\mathcal{F}\,\mathcal{M} = \Phi_{\mathcal{M}}$) then

$$\boxed{\mathcal{F}\,\mathbf{flip}_\mu \mathcal{M} = T_\mu \downarrow \Phi_{\mathcal{M}}}$$

Corollary

$$\boxed{T_\mu \downarrow \widetilde{H}_\mu(X;q,t) = \widetilde{H}_\mu(X;q,t)}$$

SCIENCE FICTION

Let $\mu \vdash n+1$ be a partition with at least m removable corners.

Suppose that $\mathcal{A} = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ is an m-subset of the predecessors of μ

Conjecture $\mathbf{I_m}$

The polynomial

$$\boxed{\Phi_{\mathcal{A}}(X;q,t) = \sum_{i=1}^{m} \widetilde{H}_{\alpha_i}(X;q,t) \prod_{j=1;j\neq i}^{m} \frac{1}{1 - T_{\alpha_i}/T_{\alpha_j}}}$$

is the Frobenius characteristic of the space

$$\boxed{\mathcal{M}_{\alpha_1} \cap \mathcal{M}_{\alpha_2} \cap \cdots \cap \mathcal{M}_{\alpha_m}}$$

Notice that for $m = 2$ this reduces to

Conjecture $\mathbf{I_2}$

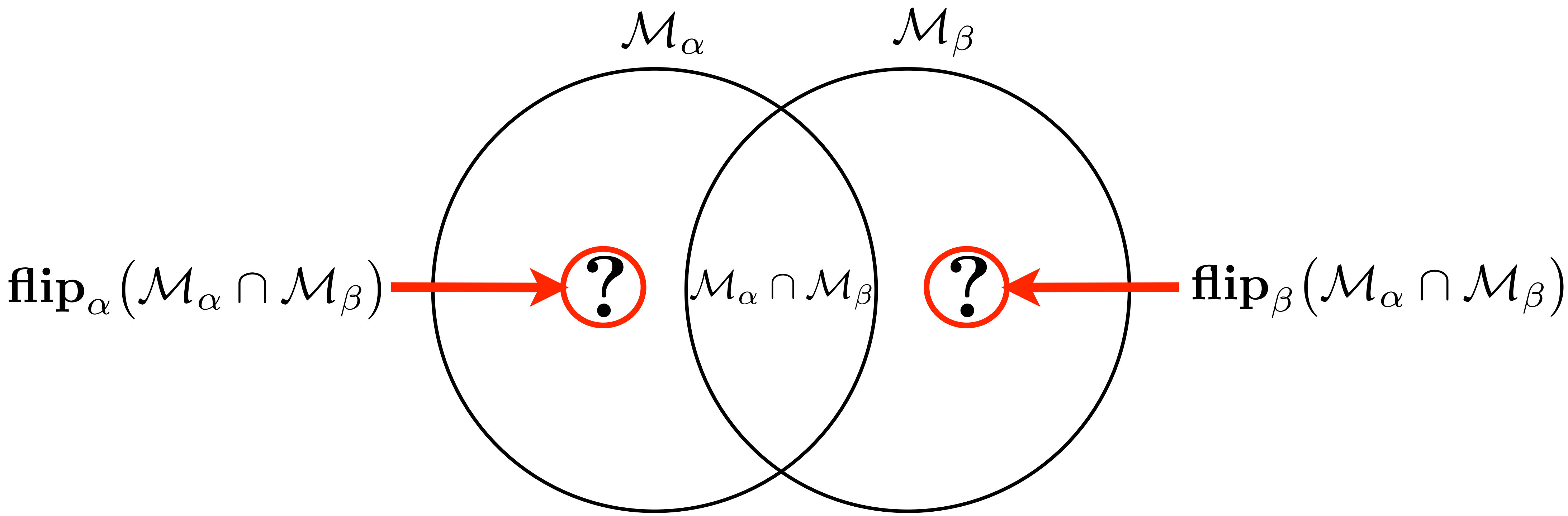
If α and β are any two predecessors of a partition μ, the polynomial

$$\boxed{\phi_{\alpha,\beta}(x;q,t) = \frac{T_\beta \widetilde{H}_\alpha - T_\alpha \widetilde{H}_\beta}{T_\beta - T_\alpha}}$$

is the Frobenius characteristic of the space

$$\boxed{\mathcal{M}_\alpha \cap \mathcal{M}_\beta}$$

OPEN PROBLEMS



$$\mathcal{M}_\alpha = (\mathcal{M}_\alpha \cap \mathcal{M}_\beta) \oplus \mathbf{flip}_\alpha(\mathcal{M}_\alpha \cap \mathcal{M}_\beta)$$

Why divided differences?

$$\mathcal{F}(\mathcal{M}_\alpha \cap \mathcal{M}_\beta) = \widetilde{H}_\alpha \frac{1}{1 - T_\alpha/T_\beta} + \widetilde{H}_\beta \frac{1}{1 - T_\beta/T_\alpha} = \frac{T_\beta \widetilde{H}_\alpha - T_\alpha \widetilde{H}_\beta}{T_\beta - T_\alpha}$$

For example if $\mathcal{A} = \{\alpha, \beta, \gamma\}$ then

$$\mathcal{F}(\mathcal{M}_\alpha \cap \mathcal{M}_\beta \cap \mathcal{M}_\gamma) = \frac{T_\gamma \mathcal{F}(\mathcal{M}_\alpha \cap \mathcal{M}_\beta) - T_\alpha \mathcal{F}(\mathcal{M}_\beta \cap \mathcal{M}_\gamma)}{T_\gamma - T_\alpha} = \Phi_\mathcal{A}$$

More generally if $\mathcal{A} = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ then

$$\Phi_\mathcal{A} = \frac{T_{\alpha_m} \mathcal{F}(\mathcal{M}_{\alpha_1} \cap \mathcal{M}_{\alpha_2} \cap \cdots \cap \mathcal{M}_{\alpha_{m-1}}) - T_{\alpha_1} \mathcal{F}(\mathcal{M}_{\alpha_2} \cap \mathcal{M}_{\alpha_2} \cap \cdots \cap \mathcal{M}_{\alpha_m})}{T_{\alpha_m} - T_{\alpha_1}}$$

DIAGONAL HARMONICS AND NABLA

Let $X_n = \{x_1, \ldots, x_n\}$ and $Y_n = \{y_1, \ldots, y_n\}$ and set

$$\boxed{\mathbf{DH}_n \;=\; \left\{P(x,y) \in \mathbf{Q}[X_n, Y_n] \;:\; \textstyle\sum_{i=1}^{n} \partial_{x_i}^{r} \partial_{y_i}^{s} P(x,y) = 0 \;\; \forall \;\; 1 \le r+s \le n\right\}}$$

In 1994 Mark Haiman (after exposure to Procesi) was able to predict that

$$\boxed{\mathcal{F}\,\mathbf{DH}_n \;=\; \sum_{\mu \vdash n} \frac{\boxed{T_\mu}\, \widetilde{H}_\mu[X;q,t]\,(1-t)(1-q)B_\mu(q,t)\Pi_\mu(q,t)}{w_\mu(q,t)}}$$

In the "*q,t-Catalan and Lagrange inversion*" paper this formula appeared together with the expansion

$$\boxed{e_n \;=\; \sum_{\mu \vdash n} \frac{\widetilde{H}_\mu[X;q,t]\,(1-t)(1-q)B_\mu(q,t)\Pi_\mu(q,t)}{w_\mu(q,t)}}$$

Francois noticed that we could write $\mathcal{F}\,\mathbf{DH}_n = \nabla \mathbf{e_n}$ by the definition

$$\boxed{\nabla \widetilde{\mathbf{H}}_\mu[\mathbf{X};\mathbf{q},\mathbf{t}] \;=\; \mathbf{T}_\mu \widetilde{\mathbf{H}}_\mu[\mathbf{X};\mathbf{q},\mathbf{t}]}$$

Thereafter we applied ∇ to everything in sight with astonishing findings!!!

SCIENCE FICTION WAS ENRICHED BEYOND BELIEF!!!

SCIENCE FICTION AND NABLA

Theorem

For $\mathcal{A} = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$

$$\boxed{\widetilde{H}_{\alpha_s}(X;q,t) \;=\; \prod_{r=1;r\neq s}^{m} \Big(1 - \frac{\nabla}{T_{\alpha_r}}\Big) \Phi_{\mathcal{A}}(X;q,t)} \qquad \text{(for all } 1 \leq s \leq m)$$

Proof

$$\prod_{r=1;r\neq s}^{m} \Big(1 - \frac{\nabla}{T_{\alpha_r}}\Big) \Phi_{\mathcal{A}}(X;q,t) \;=$$

$$= \sum_{i=1}^{m} \prod_{r=1;r\neq s}^{m} \Big(1 - \frac{\nabla}{T_{\alpha_r}}\Big) \widetilde{H}_{\alpha_i}(X;q,t) \prod_{j=1;j\neq i}^{m} \frac{1}{1 - T_{\alpha_i}/T_{\alpha_j}}$$

$$= \sum_{i=1}^{m} \prod_{r=1;r\neq s}^{m} \Big(1 - \frac{T_{\alpha_i}}{T_{\alpha_r}}\Big) \widetilde{H}_{\alpha_i}(X;q,t) \prod_{j=1;j\neq i}^{m} \frac{1}{1 - T_{\alpha_i}/T_{\alpha_j}} \;=\; \widetilde{H}_{\alpha_s}(X;q,t) \quad \textbf{Q.E.D.}$$

Corollary

$$\boxed{\widetilde{H}_{\alpha_s}(X;q,t) \;=\; \sum_{k=0}^{m-1} (-1)^k \nabla^k \Phi_{\mathcal{A}}(X;q,t) e_k\big[\tfrac{1}{T_1} + \cdots + \tfrac{1}{T_m} - \tfrac{1}{T_s}\big]}$$

and

$$\boxed{\mathcal{L}\{\widetilde{H}_{\alpha_1}, \ldots, \widetilde{H}_{\alpha_m}\} \;=\; \mathcal{L}\{\Phi_{\mathcal{A}}, \nabla\Phi_{\mathcal{A}}, \ldots, \nabla^{m-1}\Phi_{\mathcal{A}}\}}$$

MIRACLES OF SCIENCE FICTION

Let Ξ_m be the collection of words $\epsilon_1 \cdots \epsilon_m$ in $0, 1$. Define for $\epsilon_1 \cdots \epsilon_m \in \Xi_m$

$$\boxed{\Phi_{\mathcal{A}}^{\epsilon_1 \epsilon_2 \cdots \epsilon_m} = \frac{\phi_{\mathcal{A}}^{(\epsilon_1+\epsilon_2+\cdots+\epsilon_m)}}{\prod_{\epsilon_j=0}^{m} T_j}} \quad \text{with} \quad \boxed{\phi_{\mathcal{A}}^{(k)} = (-\nabla)^{m-k}\Phi_{\mathcal{A}}.}$$

Corollary

For each $1 \le s \le m$ we have the expansion

$$\boxed{\widetilde{H}_{\alpha_s} = \sum_{\epsilon_1 \epsilon_2 \cdots \epsilon_m \in \Xi_m} \Phi^{\epsilon_1 \epsilon_2 \cdots \epsilon_m} \chi(\epsilon_s = 1)}$$

Proof

We derived that

$$\widetilde{H}_{\alpha_s}(X; q, t) = \sum_{k=0}^{m-1} (-1)^k \nabla^k \Phi_{\mathcal{A}}(X; q, t) e_k\left[\tfrac{1}{T_1} + \cdots + \tfrac{1}{T_m} - \tfrac{1}{T_s}\right]$$

This can be rewritten as

$$\widetilde{H}_{\alpha_s}(X; q, t) = \sum_{k=1}^{m} \sum_{|S|=m-k} \frac{(-\nabla)^{m-k}\Phi_{\mathcal{A}}}{\prod_{j \in S} T_j} \chi(S \subseteq \{1, 2, \ldots, m\} \setminus \{s\})$$

Convert subsets of $\{1, 2 \ldots, m\}$ into words in Ξ_m

Q.E.D.

MIRACLES OF SCIENCE FICTION

Theorem

For all $\epsilon_1\epsilon_2\cdots\epsilon_m \in \Xi_m$ *and for each* $1 \le i \le m$ *we have*

$$\boxed{T_{\alpha_i} \downarrow \Phi_{\mathcal{A}}^{\epsilon_1\cdots\epsilon_{i-1}\mathbf{1}\epsilon_{i+1}\cdots\epsilon_m} = \Phi_{\mathcal{A}}^{\bar\epsilon_1\cdots\bar\epsilon_{i-1}\mathbf{1}\bar\epsilon_{i+1}\cdots\bar\epsilon_m}}$$

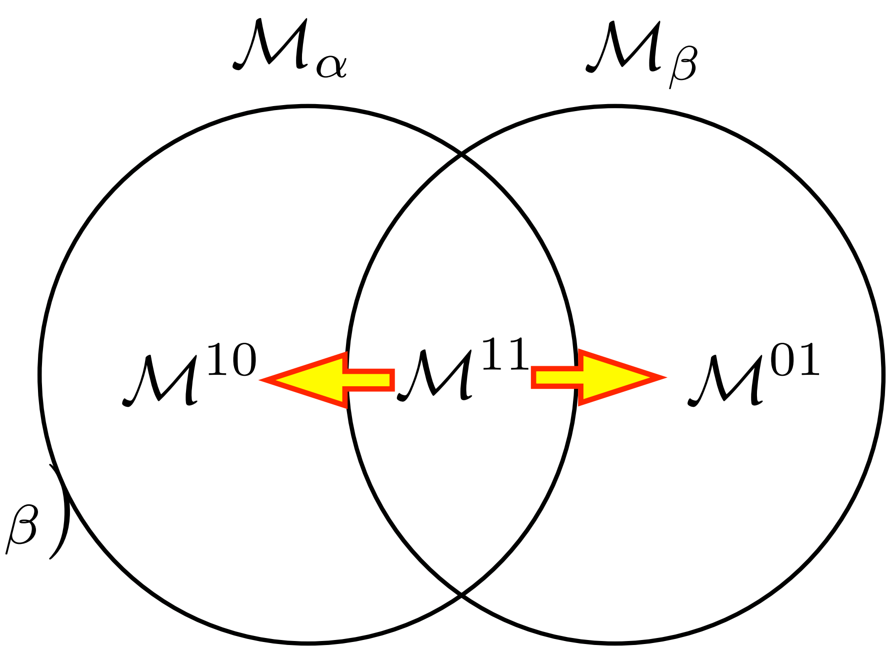
where for convenience we let $\bar\epsilon_j = 1-\epsilon_j$

For $\mathbf{m=2}$

$$\mathcal{F}ch\mathcal{M}^{10} = \mathcal{F}ch\,\mathbf{flip}_\alpha(\mathcal{M}_\alpha \cap \mathcal{M}_\beta)$$

$$\Phi_{\alpha,\beta} = \mathcal{F}ch(\mathcal{M}^{11})$$

$$\mathcal{F}ch\mathcal{M}^{01} = \mathcal{F}ch\,\mathbf{flip}_\beta(\mathcal{M}_\alpha \cap \mathcal{M}_\beta)$$



$$\mathcal{M}_\alpha = \mathcal{M}^{11} \oplus \mathcal{M}^{10} \qquad \mathcal{M}_\beta = \mathcal{M}^{11} \oplus \mathcal{M}^{01}$$

Flipping does not change dimension and Schur positivity

(By SF and the n! Theorem!)

If $\alpha \vdash n$ then $\dim \mathcal{M}^{\mathbf{10}} = \dim \mathcal{M}^{\mathbf{11}} = \dim \mathcal{M}^{\mathbf{01}} = \frac{\mathbf{n!}}{\mathbf{2}}$

$$\boxed{\text{How do we split a left regular representation?}}$$

Flipping Frobenius Characteristics

Define

$$\boxed{\mathbf{flip}_\mu \Phi(X;q,t) \;=\; T_\mu \downarrow \Phi(X;q,t)} \;=\; \big(T_\mu\, \omega\, \Phi(X;1/q,1/t)\big)$$

Theorem

$$\boxed{\mathbf{flip}_\alpha \frac{T_\beta \widetilde{H}_\alpha - T_\alpha \widetilde{H}_\beta}{T_\beta - T_\alpha} \;=\; T_\alpha \frac{\widetilde{H}_\alpha - \widetilde{H}_\beta}{T_\alpha - T_\beta}}$$

Proof

$$\mathbf{flip}_\alpha \frac{T_\beta \widetilde{H}_\alpha - T_\alpha \widetilde{H}_\beta}{T_\beta - T_\alpha} \;=\; T_\alpha \frac{T_\beta^{-1}T_\alpha^{-1}\widetilde{H}_\alpha - T_\alpha^{-1}T_\beta^{-1}\widetilde{H}_\beta}{T_\beta^{-1} - T_\alpha^{-1}} \;=\; T_\alpha \frac{\widetilde{H}_\alpha - \widetilde{H}_\beta}{T_\alpha - T_\beta} \qquad \textbf{Q.E.D.}$$

Notice: $$\widetilde{H}_\alpha \;=\; \frac{T_\beta \widetilde{H}_\alpha - T_\alpha \widetilde{H}_\beta}{T_\beta - T_\alpha} + T_\alpha \frac{T_\beta \widetilde{H}_\alpha - T_\alpha \widetilde{H}_\beta}{T_\beta - T_\alpha}$$

Notice: Flipping a Frobenius Characteristic preserves positivity

Recall that each $\widetilde{H}_\alpha[X;q,t]$ is the Frobenius of a left regular representation. Thus

$$\boxed{\widetilde{H}_\alpha[X;1,1] = e_1^n}$$

This given, what are your guesses as to the evaluation of the following limits?

$$\lim_{t,q\to 1} \frac{T_\beta \widetilde{H}_\alpha - T_\alpha \widetilde{H}_\beta}{T_\beta - T_\alpha} \;=\; h_2\, e_1^{n-2} \qquad\qquad \lim_{t,q\to 1} T_\alpha \frac{T_\beta \widetilde{H}_\alpha - T_\alpha \widetilde{H}_\beta}{T_\beta - T_\alpha} \;=\; e_2\, e_1^{n-2}$$

In other words, what is the most natural way to split e_1^n in exactly $1/2$?

SCIENCE FICTION 2018

Recall that if $\mu \vdash n+1$ and $\mathcal{A} = \{\alpha_1, \alpha_2, \ldots \alpha_m\}$ is any subset of the immediate predecessors of μ then the symmetric functions

$$\boxed{(-\nabla)^k \Phi_{\mathcal{A}} = \sum_{i=1}^{m} (-T_{\alpha_i})^k \widetilde{H}_{\alpha_i} \prod_{j=1 j\neq i}^{m} \frac{1}{1 - T_{\alpha_i}/T_{\alpha_j}}} \qquad (\text{for } 0 \leq k \leq m-1)$$

are all Schur positive. Set $\Phi_{\mathcal{A}}^{m-k} = (-\nabla)^k \Phi_{\mathcal{A}}$

Guoce Xin proved that

$$\boxed{\Phi_{\mathcal{A}}^{m-k}\Big|_{t=q=1} = e_1^{n-2m+2}\, \psi_{m,k}}$$

Where

$$\psi_{m,0} = \frac{1}{m} e_{m-1}[m\,X]\Big|_{h_i \to h_{i+1}\, h_1^{i-1}} \qquad \psi_{m,k}, = h_1^2\, \psi_{m-1,k-1} - \psi_{m-1,k}$$

This proves and extends my conjectures that

For $m=2$ $\quad \Phi_{\mathcal{A}}^2\Big|_{t=q=1} = e_1^{n-2}\, h_2$ *and* $\quad \Phi_{\mathcal{A}}^1\Big|_{t=q=1} = e_1^{n-2} e_2$

For $m=3$ and $n=5$

$$\Phi_{\mathcal{A}}^3\Big|_{t=q=1} = s_5 + 2s_{41} + 3s_{32} + s_{311} + 2s_{221}$$

$$\Phi_{\mathcal{A}}^2\Big|_{t=q=1} = s_{41} + 2s_{311} + s_{2111}$$

$$\Phi_{\mathcal{A}}^1\Big|_{t=q=1} = s_{11111} + 3s_{221} + 2s_{2111} + s_{311} + 2s_{32}$$

THANK YOU

THE CASE OF THREE PREDECESSORS

$$\mathcal{M}^{111} = \mathcal{M}_\alpha \cap \mathcal{M}_\beta \cap \mathcal{M}_\gamma$$

$$\mathcal{M}^{110} = (\mathcal{M}_\alpha \cap \mathcal{M}_\beta) \bigcap \mathcal{M}_\gamma^\perp$$

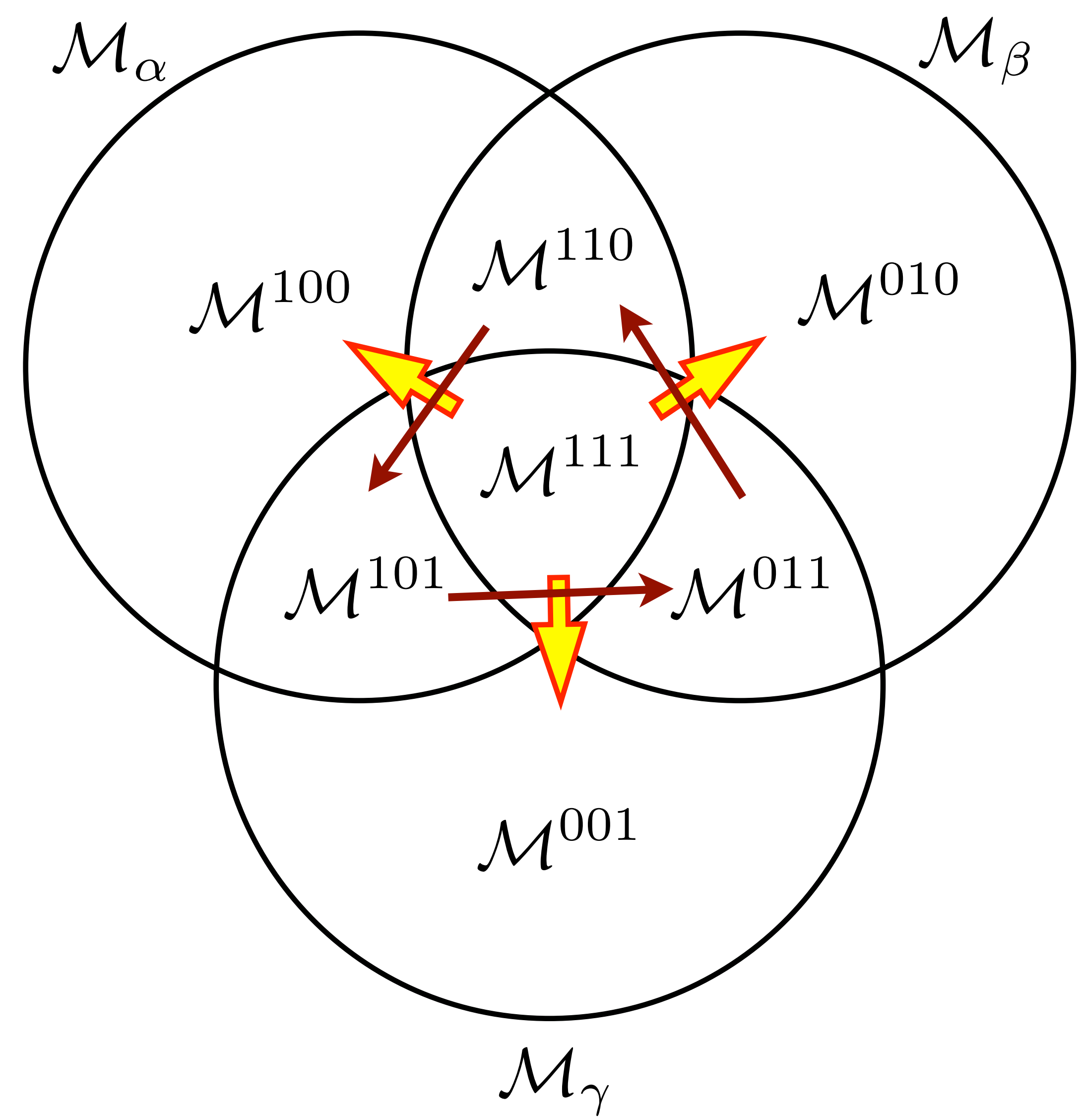
$$\mathcal{M}^{101} = (\mathcal{M}_\alpha \cap \mathcal{M}_\gamma) \bigcap \mathcal{M}_\beta^\perp$$

$$\mathcal{M}^{011} = (\mathcal{M}_\beta \cap \mathcal{M}_\gamma) \bigcap \mathcal{M}_\alpha^\perp$$

$$\mathcal{M}^{001} = (\mathcal{M}^{101} \cup \mathcal{M}^{111} \cup \mathcal{M}^{011})^\perp \bigcap \mathcal{M}_\gamma$$

$$\mathcal{M}^{010} = (\mathcal{M}^{110} \cup \mathcal{M}^{111} \cup \mathcal{M}^{011})^\perp \bigcap \mathcal{M}_\gamma$$

$$\mathcal{M}^{100} = (\mathcal{M}^{101} \cup \mathcal{M}^{111} \cup \mathcal{M}^{110})^\perp \bigcap \mathcal{M}_\gamma$$



$$\mathcal{F}\,\mathcal{M}_\alpha \cap \mathcal{M}_\beta - \mathcal{F}\,\mathcal{M}_\alpha \cap \mathcal{M}_\beta \cap \mathcal{M}_\gamma = \Phi_{\mathcal{A}}^{110}$$

$$\mathcal{F}\mathcal{M}^{\epsilon_1\epsilon_2\epsilon_3} = \Phi^{\epsilon_1\epsilon_2\epsilon_3}$$

$$\mathbf{flip}_\alpha \Phi^{111} = \Phi^{100}$$

$$\mathbf{flip}_\beta \Phi^{111} = \Phi^{010}$$

$$\mathbf{flip}_\gamma \Phi^{111} = \Phi^{001}$$

Theorem

For all $\epsilon_1\epsilon_2\cdots\epsilon_m \in \Xi_m$ *and for each* $1 \le i \le m$ *we have*

$$T_{\alpha_i} \downarrow \Phi_{\mathcal{A}}^{\epsilon_1\cdots\epsilon_{i-1}1\epsilon_{i+1}\cdots\epsilon_m} = \Phi_{\mathcal{A}}^{\bar{\epsilon}_1\cdots\bar{\epsilon}_{i-1}1\bar{\epsilon}_{i+1}\cdots\bar{\epsilon}_m}$$

where for convenience we let $\bar{\epsilon}_j = 1 - \epsilon_j$

$$\dim \mathcal{M}^{111} = \dim \mathcal{M}^{100} = n!/3$$

$$\dim \mathcal{M}^{111} = \dim \mathcal{M}^{010} = n!/3$$

$$\dim \mathcal{M}^{111} = \dim \mathcal{M}^{001} = n!/3$$

For example

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$$\mathcal{F}\mathcal{M} = \sum_{r,s} t^r q^s \mathcal{F}ch\big(\mathcal{H}_{r,s}(\mathcal{M})\big)$$

$$\widetilde{H}_\mu[X;q,t] = \sum_\lambda s_\lambda[X]\widetilde{K}_{\lambda,\mu}(q,t)$$

the marginal, modified Hall-Littlewood case $q=0

This branch of Algebraic Combinatorics resulted by my introduction

In the early 90's I discovered a recursive method for proving g

the marginal, modified Hall-Littlewood case $\widetilde{H}_\mu[X;0,t]$

THE $\bf n!\over 2$ CONJECTURE

the marginal (Hall-Littlewood) case $q = 0$

$$\widetilde{H}_\mu[X;q,t] = \sum_\lambda s_\lambda[X]\widetilde{K}_{\lambda,\mu}(q,t)$$

In 1990, working jointly with Haiman, we where led to the discovery

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must be the desired bi-graded module

the coordinates of the cells of μ and the Conjecture that

$$\Phi_{\mathcal{A}}(X;q,t) = \sum_{i=1}^{m} \widetilde{H}_{\alpha_i}(X;q,t) \prod_{j=1;j\neq i}^{m} \frac{1}{1 - T_{\alpha_i}/T_{\alpha_j}}$$

the coordinates of the cells of μ and the Conjecture that the desired module

$$\prod_{r=1;r\neq s}^{m}\left(1-\frac{\nabla}{T_{\alpha_r}}\right)\Phi_{\mathcal{A}}(X;q,t) \ =$$

$$\Phi_{\mathcal{A}}(X;q,t)=\sum_{i=1}^{m}\widetilde{H}_{\alpha_i}(X;q,t)\prod_{j=1;j\neq i}^{m}\frac{1}{1-T_{\alpha_i}/T_{\alpha_j}}$$

]

$$\prod_{r=1;r\neq s}^{m}\left(1-\frac{\nabla}{T_{\alpha_r}}\right)\Phi_{\mathcal{A}}(X;q,t) \ =$$

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