

# **Dynamical Algebraic Combinatorics notes [outline]**

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## **Chapter 0. Introduction**

- §0.1 Philosophy of DAC
- §0.2 The ur examples: rotation of subsets and multisets
- §0.3 Overview of rest of notes

## **Chapter 1. Tableaux**

- §1.1 Young diagrams, semistandard Young tableaux, Gelfand-Tsetlin patterns
- §1.2 Bender-Knuth involutions, promotion, evacuation
- §1.3 Jeu de taquin; promotion and evacuation again
- §1.4 Proof (using evacuation & jdt) that promotion of rectangular SSYTs has order  $n$
- §1.5 Standard tableaux
- §1.6 Models for promotion of 2- and 3-rowed SYT promotion

## **Chapter 2. Posets**

- §2.1 Basics about posets, linear extensions, promotion and evacuation
- §2.2 The rectangle redux; embedding (linear ext's of) the two triangles into the rectangle
- §2.3 Order ideals, rowmotion, toggles; the Stanley–Thomas word
- §2.4 P-partitions, piecewise linear toggles and rowmotion
- §2.5 Conjugacy of rect. promotion & rowmotion (via GT pat's); more triangle embeddings
- §2.6 The order polytope and PL maps

## **Chapter 3. Coxeter groups and root systems**

- §3.1 Basics about Coxeter groups and root systems
- §3.2 Root posets
- §3.3 Weak order and the Edelman–Greene bijection (Type A)
- §3.4 Absolute order and the Armstrong–Stump–Thomas bijection (Type A)
- §3.5 Minuscule posets
- §3.6 Parabolic quotients and rowmotion (à la Rush–Shi)

## **Chapter 4. Cyclic sieving**

- §4.1 Definition of cyclic sieving and ur examples via exterior/symmetric power
- §4.2 The Grassmannian and its coordinate ring, standard monomials
- §4.3 Involutions on the Grassmannian and “ $q=-1$ ” phenomenon for plane partitions
- §4.4 Canonical bases & cyclic sieving for SSYT promotion (discussion only)
- §4.5 Invariant tensors, Schur–Weyl duality
- §4.6 Cyclic sieving for SYT promotion (discussion only)

## **Chapter 5. Invariance and homomesy**

- §5.1 Invariants; cyclic descents for SYTs, the “OY invariant” for Type A root poset
- §5.2 Homomesy; basic homomesies from Stanley–Thomas word bijection
- §5.3 Symmetry of Narayana numbers & the Lanne–Kreweras involution, rowvacuation
- §5.4 The toggleability statistics technique
- §5.5 PL homomesies

## **Chapter $\infty$ . Conclusion and further topics**

- § $\infty$ .1 The RSK Algorithm
- § $\infty$ .2 Birational dynamics
- § $\infty$ .3 Rowmotion beyond distributive lattices
- § $\infty$ .4 Conclusion