Midterm #1, 9/26
Math 156 (Calculus I), Fall 2023

Each problem is worth 10 points, for a total of 50 points. You have 50 minutes to do the exam. Remember to *show your work* on all problems!

1. Let $f(x) = \sin(\frac{x}{2}) + 1$.
   (a) Graph $f(x)$. Be sure to include some value labels on your x- and y-axes.
   (b) Let $g(x)$ be the function whose graph is obtained from the graph of $f(x)$ by translating to the left by π and stretching vertically by a factor of 3. Write a formula for $g(x)$.

2. Let $g(x) = e^{3x} - 2$.
   (a) Describe all the horizontal and/or vertical asymptotes of the graph $y = g(x)$ of this function. Explain your answer by saying what these asymptotes mean in terms of limits.
   (b) Let $f(x) = \ln(x + 2)$. Write the formula for the composition $(f \circ g)(x)$. Make sure your formula is written in the most simplified form possible.

3. Let $f(x) = \dfrac{x^2 - x}{x^2 - 1}$. Compute the following limits, or if they do not exist explain why:
   (a) $\lim\limits_{x \to 1} f(x)$
   (b) $\lim\limits_{x \to 0} f(x)$
   (c) $\lim\limits_{x \to -1} f(x)$

4. Compute the following limits, or if they do not exist explain why:
   (a) $\lim\limits_{x \to 0} e^{\cos(x)}$
   (b) $\lim\limits_{x \to \infty} \dfrac{2x^2 - x + 4}{x^2 + 10x - 7}$
   (c) $\lim\limits_{x \to \infty} \dfrac{x^2 + 3x - 10}{5x + 9}$

5. What is the slope of the line tangent to the curve $y = x^2 - 1$ at the point $(x, y) = (0, -1)$? Explain your answer, for instance sketching a graph or by discussing a limit.