

Final Exam Study Guide

Math 181 (Discrete Structures), Fall 2022

1. Sets [§1.1]

- (a) sets of numbers (integers \mathbb{Z} and real numbers \mathbb{R}), set-builder notation, subsets ($A \subseteq B$)
- (b) operations of union ($A \cup B$), intersection ($A \cap B$), difference ($A \setminus B$), complement (A^c)
- (c) representing sets via Venn diagrams

2. Logical propositions [§1.2, 1.3]

- (a) operations of “or” ($p \vee q$), “and” ($p \wedge q$), “not” ($\neg p$)
- (b) truth tables for compound propositions
- (c) conditional a.k.a. implication a.k.a. “if... then...” ($p \rightarrow q$)
- (d) biconditionals ($p \leftrightarrow q$) and logical equivalence (\equiv)
- (e) converse $q \rightarrow p$ and contrapositive $\neg q \rightarrow \neg p$ of an implication $p \rightarrow q$
(contrapositive is logically equivalent to original implication; converse is not!)

3. Logical arguments [§1.4]

- (a) converting an argument from words to symbolic form and vice-versa
- (b) proving validity using truth tables
- (c) proving validity using the rules of inference and logical equivalences
- (d) common forms of invalid arguments a.k.a. fallacies

4. Quantifiers [§1.5, 1.6]

- (a) propositional formulas ($P(x)$) and domains of discourse (D)
- (b) universal ($\forall x P(x)$) and existential ($\exists x P(x)$) quantifiers
- (c) DeMorgan’s Laws: $\neg(\forall x P(x)) \equiv \exists x \neg P(x)$ and $\neg(\exists x P(x)) \equiv \forall x \neg P(x)$
- (d) nested quantifiers and order of quantifiers ($\forall x \exists y P(x, y) \not\equiv \exists y \forall x P(x, y)$)

5. Proofs [§2.1]

- (a) two basic mathematical systems: the theory of integers; the theory of sets
- (b) direct proofs for theorems of form “ $\forall x_1, \dots, x_n$ if $P(x_1, \dots, x_n)$ then $Q(x_1, \dots, x_n)$ ”
- (c) counterexamples to universally quantified statements

6. Indirect proofs [§2.2]

- (a) proof of by contrapositive: to prove $p \rightarrow q$, prove $\neg q \rightarrow \neg p$ instead
- (b) proof by contradiction: assume negation of statement, and deduce contradiction ($r \wedge \neg r$)

7. Mathematical induction [§2.4, 2.5]
 - (a) basic structure of inductive proofs: base case $P(1)$, and induction step $P(n) \rightarrow P(n+1)$
 - (b) proving $\forall(n \in \mathbb{Z}_{>0}) P(n)$ by induction, especially when $P(n)$ is an algebraic formula
 - (c) finding patterns to guess formulas involving n which can then be proved by induction
 - (d) the strong form of mathematical induction: can use $P(k)$ for all $k < n$ to prove $P(n)$
8. Functions [§3.1]
 - (a) ways to view a function $f: X \rightarrow Y$: rule to convert input $x \in X$ to output $y = f(x) \in Y$; set of ordered pairs (x, y) ; arrow diagram from X to Y
 - (b) one-to-one, onto, and bijective functions
 - (c) composition of functions, and inverse functions
 - (d) modular arithmetic functions $f(x) = x \bmod n$
9. Sequences and strings [§3.2]
 - (a) finite and infinite sequences: ordered list of elements of some set
 - (b) set of strings X^* on some finite alphabet X , and the null string $\lambda \in X^*$
 - (c) subsequences (not necessarily consecutive) versus substrings (consecutive)
10. Relations [§3.4, 3.5]
 - (a) digraph representation of a relation R on a set X
 - (b) properties that R can have: reflexive, symmetric, anti-symmetric, transitive
 - (c) partial order (reflexive, anti-symmetric, transitive): way to “compare” things in X
 - (d) equivalence relation (reflexive, symmetric, transitive): way to say certain things in X are “the same”; corresponds to a partition of X into equivalence classes
11. Basic counting principles [§6.1]
 - (a) multiplication principle: total # of possibilities = product of # of choices at each step
 - (b) addition principle: size of union of *disjoint* sets is sum of sizes of the sets
 - (c) principle of inclusion and exclusion: $\#(X \cup Y) = \#X + \#Y - \#(X \cap Y)$
12. Permutations and combinations [§6.2, 6.3]
 - (a) number of permutations (=orderings) of n element set is $n! = n \times (n-1) \times \cdots \times 1$, and number of k -permutations (=orderings of k element subsets) is $P(n, k) = n!/(n-k)!$
 - (b) for rearrangements of word with repeated letters like MISSISSIPPI use $n!/(k_1!k_2! \cdots k_m!)$
 - (c) number of k -combinations (= k element subsets) of n element set is the binomial coefficient, a.k.a. “ n choose k ” number, $C(n, k) = n!/(k! \cdot (n-k)!)$
 - (d) for selections of k things from n things with repeats allowed use $C(k+n-1, k)$
13. Binomial coefficients [§6.7]
 - (a) Binomial Theorem: $(x+y)^n = \sum_{k=0}^n C(n, k)x^k y^{n-k}$
 - (b) Pascal’s Triangle of $C(n, k)$, defined by recurrence $C(n+1, k) = C(n, k) + C(n, k-1)$
14. Pigeonhole principle [§6.8]
 - (a) if you place n pigeons in $n-1$ holes, at least one hole has at least two pigeons