Homework #10, Due: 4/5
Math 181 (Discrete Structures), Spring 2023

Problem 1 is worth 4 points (2 pts each part), and Problem 2 is worth 6 points (1.5 pts each part), for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Let $X = \{1, 2, 3\}$. Define a relation R on $X \times X = \{(a, b) : a, b \in X\}$ where $(a, b) \, R \, (c, d)$ if and only if $a + b = c + d$. For example, we have that $(2, 2) \, R \, (1, 3)$ since $2 + 2 = 4 = 1 + 3$.

   (a) Explain why R is an equivalence relation.

   (b) List one element of each equivalence class of R.

2. Let $X = \{a, b, c\}$ and let $S \subseteq X^*$ be the set of strings over the alphabet X of length four. For example, three of the elements in S are: *abab*, *acbb*, and *ccbb*.

   (a) What is $\#S$ (the number of elements in S)?

   (b) Let $S_1 \subseteq S$ be the subset of strings in S that begin with "*a*." What is $\#S_1$?

   (c) Let $S_2 \subseteq S$ be the subset of strings in S that end with "*bb*." What is $\#S_2$?

   (d) Let $S_3 \subseteq S$ be the subset of strings in S that begin with "*a*" or end with "*bb*" (or both). What is $\#S_3$? [**Hint**: use the Principle of Inclusion and Exclusion!]