

# Midterm #2 Study Guide

## Math 156 (Calculus I), Fall 2023

### 1. Derivatives of basic functions [§3.1, 3.3, 3.6]

- (a) power functions:  $d/dx(x^n) = nx^{n-1}$
- (b) exponential and logarithmic functions:  $d/dx(e^x) = e^x$  and  $d/dx(\ln(x)) = 1/x$
- (c) trigonometric functions:  $d/dx(\sin(x)) = \cos(x)$  and  $d/dx(\cos(x)) = -\sin(x)$

### 2. Rules for derivatives of combinations of functions [§3.1, 3.2, 3.4]

- (a) derivative is linear:  $d/dx(a \cdot f(x) + b \cdot g(x)) = a \cdot f'(x) + b \cdot g'(x)$  for  $a, b \in \mathbb{R}$
- (b) product rule:  $d/dx(f(x) \cdot g(x)) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$
- (c) chain rule:  $d/dx(f(g(x))) = f'(g(x)) \cdot g'(x)$
- (d) quotient rule:  $d/dx(f(x)/g(x)) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$   
*[don't have to separately memorize quotient rule, it follows from other rules]*

### 3. Implicit differentiation and related rates [§3.5, 3.9]

- (a) for  $y$  defined implicitly via equation  $p(x, y) = 0$ , find  $dy/dx$  by taking  $d/dx$  of both sides, and use this to find the slope of the tangent at any point on the curve
- (b) if two quantities  $f(t), g(t)$  are related, then their rates of change  $df/dt, dg/dt$  are related: like with implicit differentiation, just differentiate the relation between  $f(t)$  and  $g(t)$

### 4. Extreme values [§4.1, 4.3]

- (a) local versus absolute (global) minimum and maximum values, Extreme Value Theorem
- (b) the Closed Interval Method: extreme values of continuous  $f$  on closed interval must occur at endpoints or at critical points (values  $x$  where  $f'(x) = 0$  or is not defined)
- (c) 1st and 2nd Derivative Tests for deciding if critical points are min.'s or max.'s

### 5. What derivatives tell us about shape of graph [§4.2, 4.3, 4.5]

- (a)  $f'(x) > 0$  means  $f$  is increasing,  $f'(x) < 0$  means  $f$  is decreasing
- (b)  $f''(x) > 0$  means  $f$  is concave up (smile),  $f''(x) < 0$  means  $f$  is concave down (frown)

### 6. L'Hôpital's rule [§4.4]

- (a) for indeterminate form limits (meaning " $\pm\infty$ " or " $\frac{0}{0}$ "),  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$